# CHAPTER III

## MATRIX METHODS IN SHORT CIRCUIT STUDIES

### 3.1. Network Equation.

In a network, if loads are represented by their equivalent admittances, and generators by their equivalent current sources and equivalent internal shunt admittances, Kirchhoff's law may be applied to form the relation of busbar voltages and injected currents at those busbars. In matrix form, they are related by

$$\left[ I \right] = \left[ Y \right] \left[ V \right] \qquad (3.1)$$

where $\left[ Y \right]$ is the nodal admittance matrix of the network with load equivalent admittances and generator internal admittances added to the appropriated busbars.

An alternative relation may be obtained by inverting $\left[ Y \right]$. If $\left[ Z \right]$ is the invert of $\left[ Y \right]$, then

$$\left[ V \right] = \left[ Z \right] \left[ I \right] \qquad (3.2)$$

For a network of n busbars, (3.2) may fully expressed as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \cdot \\ \cdot \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdot & \cdot & Z_{1n} \\ Z_{21} & Z_{22} & Z_{23} & \cdot & \cdot & Z_{2n} \\ Z_{31} & Z_{32} & Z_{33} & \cdot & \cdot & Z_{3n} \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ Z_{n1} & Z_{n2} & Z_{n3} & \cdot & \cdot & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \cdot \\ \cdot \\ I_n \end{bmatrix} \qquad (3.3)$$

From (3.3)

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 + \ldots\ldots + Z_{1n} I_n$$

$V_1$ may be explained to be caused by the superposition of the injection of the current into the network when current sources are applied to their connected busbars one at a time.

The other V's are obtained in the same manner. The voltage at busbar K is

$$V_K = Z_{K1} I_1 + Z_{K2} I_2 + Z_{K3} I_3 + \ldots\ldots + Z_{Kn} I_n \qquad (3.4)$$

## 3.2. Three-Phase Short Circuit.

When there is a three-phase short circuit at busbar K, there will a current $I_f$ flowing out of the system or network. The current may be considered as that produced by a negative current source connected to the busbar . That is the current source injected a negative current into busbar K.

If $V_K$ in equation (3.4) is the voltage at busbar K before the occurrence of the short circuit. The new superimposed $V_K$ is $V_{K(f)}$ and is express by

$$V_{K(f)} = Z_{K1}I_1 + Z_{K2}I_2 + Z_{K3}I_3 + \ldots + Z_{KK}I_K + \ldots + Z_{Kn}I_n + (-I_f)Z_{KK}$$

Since it is a short circuit through zero impedance, the fault $V_{K(f)} = 0$. Hence

$$0 = Z_{K1}I_1 + Z_{K2}I_2 + Z_{K3}I_3 + \ldots + Z_{KK}I_K + \ldots Z_{Kn}I_n + (-I_f)Z_{KK} \quad (3.5)$$

Substracting (3.5) from (3.4), we get

$$V_K = I_f Z_{KK}$$

of

$$I_f = \frac{V_K}{Z_{KK}} \quad (3.6)$$

Similarly, by superposition theorem , a $(-I_f)Z_{Km}$ will be added to the prefault voltage at any busbar m to obtain a fault voltage at the busbar. Hence the fault voltage distribution is obtained from

$$V_{m(f)} = V_m - I_f Z_{Km} \qquad (3.7)$$

Once the fault voltage distribution in the short circuited system has been obtained, the fault current flowing in a system element can be determined from the characteristic of the element and the fault voltage at the busbars it is connected to.

For a branch connected to busbars m and n, the series admittance Y and the shunt susceptance b are used to determine the current from the equqtion



$$I_{mn} = (V_{m(f)} - V_{n(f)})Y + V_{m(f)}b \qquad (3.9)$$

For a shunt line of admittance Y connects to busbar m

$$I_m = V_{m(f)}Y \qquad (3.10)$$

For a untapped transformer connected to busbars m and n, if Y is the equivalent admittance of the transformer, then

$$I_{mn} = (V_{m(f)} - V_{n(f)})Y \qquad (3.11)$$

For a tapped transformer with a off-nominal turn-ratio n on side n of it.

$$I_{mn} = (n\, V_{m(f)} - V_{n(f)})(nY) \qquad (3.12)$$

and

$$I_{nm} = \left(\frac{V_{n(f)}}{n} - V_{m(f)}\right)(nY) \qquad (3.13)$$

For a generator at busbar m generating a prefault current $Y_g E_g$ and internal admittance $Y_g$,

$$I_m = Y_g E_g - Y_g V_{m(f)} \qquad (3.14)$$

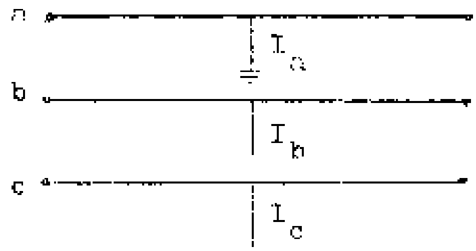
For a load at busbar m whose equivalent admittance is $Y_1$

$$I_m = Y_1 V_{m(f)} \qquad (3.15)$$

## 3.3. One-Phase Short Circuit.

If there is a short circuit of busbar m in phase a of a three-phase system. The system will then be unbalanced. The fault currents in each phase at the location of fault are the currents flowing in three hypothetical stubs with zero impedance as shown in Fig. 4. The stub in phase a is connected to ground to cause the short circuit. The other two are just opened. The current in each stub is

$$I_a = I_a \text{ and } I_b = I_c = 0 \qquad (3.16)$$




<u>Fig. 4 Connection diagram of the hypothetical stubs for one phase short circuit</u>

The sequence components of the phase currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

That is, $I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a$ (3.17)

The sequence components of the fault voltage at busbar m are determined by the superposition theorem in each sequence network; from (3.7)

$$V_{a1} = V_{m1} - I_{a1}Z_{mm1}$$

$$V_{a2} = V_{m2} - I_{a2}Z_{mm2} \qquad (3.18)$$

$$V_{a0} = V_{m0} - I_{a0}Z_{mm0}$$

Since under the prefault condition, the system is balanced,

$$V_{m1} = V_m \text{ and } V_{m2} = V_{m0} = 0 \qquad (3.19)$$

Then (3.18) becomes

$$V_{a1} = V_m - I_{a1}Z_{mm1} \qquad (3.20)$$

$$V_{a2} = -I_{a1}Z_{mm2} \qquad (3.21)$$

$$V_{a0} = -I_{a1}Z_{mm0} \qquad (3.22)$$

Then $V_a = V_{a1} + V_{a2} + V_{a0}$

$$= V_m - I_{a1}(Z_{mm1} + Z_{mm2} + Z_{mm3})$$

But $V_a$ at busbar m is zero, since the short circuit is through zero impedance. Hence

$$I_{a1} = \frac{V_m}{Z_{mm1} + Z_{mm2} + Z_{mm0}} \qquad (3.23)$$

and the fault voltage distribution at any busbar K in the positive sequence network is determined by

$$V_{K(f)} = V_K - I_{a1} Z_{mK1} \qquad (3.24)$$

in a negative sequence network, by

$$V_{K(f)} = - I_{a1} Z_{mK2} \qquad (3.25)$$

in a zero sequence network, by

$$V_{K(f)} = - I_{a1} Z_{mK0} \qquad (3.26)$$

Once the fault voltages in each sequence network are determined, the sequence components of current in any network elements can be calculated from the sequence characteristics of the element and the correspondent sequence voltages at the busbars which it is connected to the equations (3.9) to (3.15) are used for different kinds of elements.

The phase values of current in each element are obtained from

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} \qquad (3.27)$$