CHAPTER II

THE PROOF OF MAXWELL'S EQUATIONS FOR THE FIELD OF A UNIFORMLY MOVING CHARGE.

Maxwell's equations for the behaviour of the electromagnetic field in empty space are as follows

$$\nabla \cdot \vec{E} = 0 , \qquad \nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t},$$

$$\nabla \cdot \vec{B} = 0 , \qquad \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}.$$

We shall show that these equations hold for the fields produced by a charged particle moving with uniform velocity taking the formulas found in chapter I as axiomatic .

Axiom The components of the electric and magnetic fields at the point (x,y,z) produced by a particle of charge q that is situated at the origin of the xyz coordinate system and has a uniform velocity v in the ox direction are,

$$E_x = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) x , \quad E_y = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) . y, \quad E_z = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) z,$$

and $B_x = 0$, $B_y = -\frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} . z$, $B_z = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right)\frac{v}{c} . y$.

where $s^2 = x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)$.

To prove $\nabla . \vec{E} = 0$ I

Proof To prove the above equation we use the formulas

$$E_x = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) x , \quad E_y = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) y , \quad E_z = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) z.$$

By differentiating E_x, E_y, E_z with respect to x, y, z respectively, we have first

$$\frac{\partial E_x}{\partial x} = \frac{\partial}{\partial x}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) x \right\}$$

$$= q\left(1 - \frac{v^2}{c^2}\right) \frac{\partial}{\partial x}\left(x s^{-3} \right)$$

$$= q\left(1 - \frac{v^2}{c^2}\right)\left\{ - 3 x s^{-4} \frac{\partial s}{\partial x} + s^{-3} \right\}$$

$$= q\left(1 - \frac{v^2}{c^2}\right)\left[- 3x \left\{ x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2) \right\}^{-5/2} x + \left\{ x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2) \right\}^{-3/2} \right],$$

that is

$$\frac{\partial E_x}{\partial x} = \frac{-3q\, x^2\left(1 - \frac{v^2}{c^2}\right)}{\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{5/2}} + \frac{q\left(1 - \frac{v^2}{c^2}\right)}{\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{3/2}}. \quad \ldots\ldots\ldots(1.1)$$

Similarly

$$\frac{\partial E_y}{\partial y} = \frac{\partial}{\partial y}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) y \right\}$$

$$= \frac{-3qy^2\left(1 - \frac{v^2}{c^2}\right)^2}{\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{5/2}} + \frac{q\left(1 - \frac{v^2}{c^2}\right)}{\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{3/2}}, \quad \ldots(1.2)$$

and

$$\frac{\partial E_z}{\partial z} = \frac{\partial}{\partial z}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) z \right\}$$

$$= \frac{-3qz^2\left(1 - \frac{v^2}{c^2}\right)^2}{\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{5/2}} + \frac{q\left(1 - \frac{v^2}{c^2}\right)}{\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{3/2}}. \quad \ldots(1.3)$$

Since

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} ,$$

substituting for $\frac{\partial E_x}{\partial x}$, $\frac{\partial E_y}{\partial y}$, $\frac{\partial E_z}{\partial z}$ from equations (1.1),(1.2) (1.3) we obtain

$$\nabla \cdot \vec{E} = \frac{-3q\,x^2(1-\frac{v^2}{c^2}) - 3qy^2(1-\frac{v^2}{c^2})^2 - 3qz^2(1-\frac{v^2}{c^2})^2}{\left\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\right\}^{5/2}} + \frac{3\,q\,(1-\frac{v^2}{c^2})}{\left\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\right\}^{3/2}}$$

$$= \frac{-3qx^2(1-\frac{v^2}{c^2}) - 3qy^2(1-\frac{v^2}{c^2})^2 - 3qz^2(1-\frac{v^2}{c^2})^2 + 3qx^2(1-\frac{v^2}{c^2}) + 3qy^2(1-\frac{v^2}{c^2})^2}{\left\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\right\}^{5/2}} + \frac{3qz^2(1-\frac{v^2}{c^2})^2}{\left\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\right\}^{5/2}}$$

$= 0$,

which is I.

<u>To prove</u> $\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$ .... (II)

<u>Proof</u> The components of $\nabla \times \vec{B}$ are

$$(\nabla \times B)_x = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right),$$

$$(\nabla \times B)_y = -\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right),$$

and $(\nabla \times B)_z = \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)$.

Since $B_z = \frac{q}{s^3}(1-\frac{v^2}{c^2})\frac{v}{c}\,.\,y$, we obtain

$$\frac{\partial B_z}{\partial y} = \frac{\partial}{\partial y}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot y \right\} .$$

Hence

$$\frac{\partial B_z}{\partial y} = \frac{qv\left(1 - \frac{v^2}{c^2}\right)}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{3/2}} - \frac{3qvy^2\left(1 - \frac{v^2}{c^2}\right)^2}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} . \quad \ldots.(2.1)$$

Since

$$B_y = -\frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot z \ , \qquad \text{we obtain}$$

$$\frac{\partial B_y}{\partial z} = -\frac{\partial}{\partial z}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot z \right\} .$$

Hence

$$\frac{\partial B_y}{\partial z} = \frac{-qv\left(1 - \frac{v^2}{c^2}\right)}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{3/2}} + \frac{3qvz^2\left(1 - \frac{v^2}{c^2}\right)^2}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} . \quad \ldots.(2.2)$$

Subtracting (2.2) from (2.1) we obtain

$$(\bar{v} \times B)_x = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) = \frac{2qvx^2\left(1-\frac{v^2}{c^2}\right) - qv\left(1-\frac{v^2}{c^2}\right)^2(y^2+z^2)}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} . \quad \ldots.(2.3)$$

From

$$B_z = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot y \ , \qquad \text{we obtain}$$

$$\frac{\partial B_z}{\partial x} = \frac{\partial}{\partial x}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot y \right\}$$

$$= \frac{-3qvxy\left(1 - \frac{v^2}{c^2}\right)}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} , \qquad \ldots\ldots\ldots.(2.4)$$

and since

$$B_x = 0$$

we have $\frac{\partial B_x}{\partial z} = 0$ . ... (2.5)

Subtracting (2.5) from (2.4) we obtain

$$(\nabla \times B)_y = -\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right) = \frac{3qvxy\left(1-\frac{v^2}{c^2}\right)}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} . \quad \text{... (2.6)}$$

Again from

$$B_y = -\frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right)\frac{v}{c} . z \quad , \text{ we obtain}$$

$$\frac{\partial B_y}{\partial x} = -\frac{\partial}{\partial x}\left\{\frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right)\frac{v}{c} . z\right\}$$

$$= \frac{3qvxz\left(1-\frac{v^2}{c^2}\right)}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} , \quad \text{........ (2.7)}$$

and since

$$B_x = 0 ,$$

we have $\frac{\partial B_x}{\partial y} = 0$ . ... (2.8)

Subtracting (2.8) from (2.7) we obtain

$$(\nabla \times B)_z = \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = \frac{3qvxz\left(1-\frac{v^2}{c^2}\right)}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} . \quad \text{.....(2.9)}$$

Since $E_x = \frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right)x$, we obtain

$$\frac{\partial E_x}{\partial t} = \frac{\partial}{\partial t}\left\{\frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right) x\right\} .$$

Now x, y, z denote the distances in the ox, oy, and oz directions between the moving charged particle and the point in space where the derivatives of the electromagnetic field are calculated. Since the velocity of the charged particle is $v_x = v$, $v_y = 0$, $v_z = 0$ we must put $\frac{dx}{dt} = -v$, $\frac{dy}{dt} = 0$, $\frac{dz}{dt} = 0$

This gives

$$\frac{\partial E_x}{\partial t} = \frac{2qvx^2\left(1-\frac{v^2}{c^2}\right) - qv\left(1-\frac{v^2}{c^2}\right)^2(y^2+z^2)}{\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}}, \quad \text{.......(2.10)}$$

Similarly,

since $E_y = \frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right)y$, we obtain

$$\frac{\partial E_y}{\partial t} = \frac{\partial}{\partial t}\left\{\frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right)y\right\},$$

which gives

$$\frac{\partial E_y}{\partial t} = \frac{3qxvy\left(1-\frac{v^2}{c^2}\right)}{\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}}, \quad \text{.............. (2.11)}$$

and since

$E_z = \frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right)z$, we obtain

$$\frac{\partial E_z}{\partial t} = \frac{\partial}{\partial t}\left\{\frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right)z\right\},$$

which gives

$$\frac{\partial E_z}{\partial t} = \frac{3qvxz\left(1-\frac{v^2}{c^2}\right)}{\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}}. \quad \text{............... (2.12)}$$

But $\nabla\times\vec{B} = \left((\nabla\times B)_x, (\nabla\times B)_y, (\nabla\times B)_z\right)$

and $$\frac{\partial \vec{E}}{\partial t} = \left\{ \frac{\partial E_x}{\partial t} , \frac{\partial E_y}{\partial t} , \frac{\partial E_z}{\partial t} \right\}$$

so from equations (2.3),(2.6),(2,9) and (2.10),(2.11),(2.12) we obtain

$$\nabla \times \vec{B}$$
$$= \left(\frac{2qvx^2(1-\frac{v^2}{c^2})-qv(1-\frac{v^2}{c^2})^2\{z^2+y^2\}}{c\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\}^{5/2}} , \frac{3qvxy\ (1-\frac{v^2}{c^2})}{c\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\}^{5/2}} , \frac{3qvxz\ (1-\frac{v^2}{c^2})}{c\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\}^{5/2}} \right)$$

and $$\frac{\partial \vec{E}}{\partial t}$$

$$= \left(\frac{2qvx^2(1-\frac{v^2}{c^2})-qv(1-\frac{v^2}{c^2})^2\{z^2+y^2\}}{\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\}^{5/2}} , \frac{3qvxy(1-\frac{v^2}{c^2})}{\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\}^{5/2}} , \frac{3qvxz(1-\frac{v^2}{c^2})}{\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\}^{5/2}} \right)$$

Hence $$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} . \qquad \ldots\ldots\ldots\ldots \quad II$$

<u>To prove</u> $$\nabla . \vec{B} = 0 \qquad \ldots\ldots\ldots\ldots \quad III$$

<u>Proof</u> Since $B_x = 0$, we have

$$\frac{\partial B_x}{\partial x} = 0. \qquad \ldots\ldots\ldots\ldots \quad (3.1)$$

From $B_y = -\frac{q}{s^3}(1-\frac{v^2}{c^2}) \frac{v}{c} . z$, we obtain

$$\frac{\partial B_y}{\partial y} = -\frac{\partial}{\partial y}\left\{ \frac{q}{s^3} (1-\frac{v^2}{c^2}) \frac{v}{c} . z \right\}$$

which gives

$$\frac{\partial B_y}{\partial y} = \frac{3qvyz\ (1-\frac{v^2}{c^2})^2}{c\{x^2+(1-\frac{v^2}{c^2})(y^2+z^2)\}^{5/2}} . \qquad \ldots\ldots\ldots\ldots \quad (3.2)$$

Again from

$$B_z = \frac{q}{s^3}(1-\frac{v^2}{c^2}) \frac{v}{c} . y$$, we obtain

$$\frac{\partial B_z}{\partial z} = \frac{-3qvyz\left(1-\frac{v^2}{c^2}\right)^2}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} \qquad \ldots\ldots\ldots(3.3)$$

Since $\quad \nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$,

Substituting for $\frac{\partial B_x}{\partial x}$, $\frac{\partial B_y}{\partial y}$, $\frac{\partial B_z}{\partial z}$ from equations (3.1),(3.2), (3.3), we obtain

$$\nabla \cdot \vec{B} = 0 + \frac{3qvyz\left(1-\frac{v^2}{c^2}\right)^2}{c\left\{x^2\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} + \frac{-3qvyz\left(1-\frac{v^2}{c^2}\right)^2}{c\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}}$$

$$= 0 ,$$

which is III .

<u>To prove</u> $\quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \ldots\ldots\ldots\ldots\ (IV)$

<u>Proof</u> The components of $\nabla \times \vec{E}$ are

$$(\nabla \times E)_x = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) ,$$

$$(\nabla \times E)_y = -\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) ,$$

and $\quad (\nabla \times E)_z = \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$.

Since $\quad E_z = \frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right) z$, we obtain

$$\frac{\partial E_z}{\partial y} = \frac{\partial}{\partial y}\left\{\frac{q}{s^3}\left(1-\frac{v^2}{c^2}\right) z\right\} .$$

Hence $\quad \frac{\partial E_z}{\partial y} = \frac{-3qyz\left(1-\frac{v^2}{c^2}\right)^2}{\left\{x^2+\left(1-\frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} . \qquad \ldots\ldots\ldots(4.1)$

Since $E_y = \frac{q}{s^3}(1 - \frac{v^2}{c^2})\, y$, we obtain

$$\frac{\partial E_y}{\partial z} = \frac{\partial}{\partial z}\left\{ \frac{q}{s^3}(1 - \frac{v^2}{c^2})\, y \right\} .$$

Hence

$$\frac{\partial E_y}{\partial z} = \frac{-3qyz\,(1 - \frac{v^2}{c^2})^2}{\left\{ x^2 + (1 - \frac{v^2}{c^2})(y^2+z^2)\right\}^{5/2}} . \qquad \text{........(4.2)}$$

Subtracting (4.2) from (4.1) we obtain

$$(\nabla \times E)_x = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) = 0 . \qquad \text{..........(4.3)}$$

From

$$E_z = \frac{q}{s^3}(1 - \frac{v^2}{c^2})\, z \text{ , we obtain}$$

$$\frac{\partial E_z}{\partial x} = \frac{\partial}{\partial x}\left\{ \frac{q}{s^3}(1 - \frac{v^2}{c^2})\, z \right\}$$

$$= \frac{-3q\,(1 - \frac{v^2}{c^2})xz}{\left\{ x^2 + (1 - \frac{v^2}{c^2})(y^2+z^2)\right\}^{5/2}} , \qquad \text{...........(4.4)}$$

and since

$$E_x = \frac{q}{s^3}(1 - \frac{v^2}{c^2})\, x \text{ , we obtain}$$

$$\frac{\partial E_x}{\partial z} = \frac{\partial}{\partial z}\left\{ \frac{q}{s^3}(1 - \frac{v^2}{c^2})\, x \right\} .$$

Hence

$$\frac{\partial E_x}{\partial z} = \frac{-3q\,(1 - \frac{v^2}{c^2})^2\, xz}{\left\{ x^2 + (1 - \frac{v^2}{c^2})\right\}^{5/2}} . \qquad \text{...........(4.5)}$$

Subtracting (4.5) from (4.4) we obtain

$$(\nabla \times E)_y = -\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) = \frac{3qxv^2 z\,(1 - \frac{v^2}{c^2})}{c^2\left\{ x^2 + (1 - \frac{v^2}{c^2})(y^2+z^2)\right\}^{5/2}} . \qquad \text{...(4.6)}$$

Again from $E_y = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) y$, we obtain

$$\frac{\partial E_y}{\partial x} = \frac{\partial}{\partial x}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) y \right\}$$

$$= \frac{-3qxy\left(1 - \frac{v^2}{c^2}\right)}{\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{5/2}}, \quad \ldots\ldots\ldots(4.7)$$

and since $E_x = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) x$, we obtain

$$\frac{\partial E_x}{\partial y} = \frac{\partial}{\partial y}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) x \right\} .$$

Hence

$$\frac{\partial E_x}{\partial y} = \frac{-3qxy\left(1 - \frac{v^2}{c^2}\right)^2}{\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{5/2}} \quad \ldots\ldots\ldots(4.8)$$

Subtracting (4.8) from (4.7) we obtain

$$(\nabla \times E)_z = \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = \frac{-3qv^2xy\left(1 - \frac{v^2}{c^2}\right)}{c^2\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{5/2}} . \quad \ldots(4.9)$$

Since $B_x = 0$, we have

$$\frac{\partial B_x}{\partial t} = 0 . \quad \ldots\ldots\ldots\ldots(4.10)$$

Since $B_y = -\frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right)\frac{v}{c} \cdot z$, we obtain

$$\frac{\partial B_y}{\partial t} = -\frac{\partial}{\partial t}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right)\frac{v}{c} \cdot z \right\},$$

which gives

$$\frac{\partial B_y}{\partial t} = \frac{-3qv^2xz\left(1 - \frac{v^2}{c^2}\right)}{c\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{5/2}} \quad \ldots\ldots\ldots\ldots(4.11)$$

when we put $\frac{dx}{dt} = -v, \ \frac{dy}{dt} = 0, \ \frac{dz}{dt} = 0,$ as explained in the proof of II.

Since $B_z = \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot y$, we obtain

$$\frac{\partial B_z}{\partial t} = \frac{\partial}{\partial t}\left\{ \frac{q}{s^3}\left(1 - \frac{v^2}{c^2}\right) \frac{v}{c} \cdot y \right\} ,$$

which gives

$$\frac{\partial B_z}{\partial t} = \frac{3qv^2xy\left(1 - \frac{v^2}{c^2}\right)}{c\left\{x^2 + \left(1 - \frac{v^2}{c^2}\right)(y^2 + z^2)\right\}^{5/2}} . \quad \ldots\ldots\ldots\ldots(4.12)$$

But $\nabla \times \vec{E} = \left((\nabla \times E)_x, \ (\nabla \times E)_y , \ (\nabla \times E)_z \right)$

and $\frac{\partial \vec{B}}{\partial t} = \left(\frac{\partial B_x}{\partial t} , \ \frac{\partial B_y}{\partial t} , \ \frac{\partial B_z}{\partial t} \right)$

so from equations (4.3),(4.6),(4.9), (4.10),(4.11),(4.12) we obtain

$$\nabla \times \vec{E} = \left(0 , \ \frac{3qxv^2z\left(1 - \frac{v^2}{c^2}\right)}{c^2\left\{x^2+\left(1- \frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} , \ \frac{-3qv^2xy\left(1 - \frac{v^2}{c^2}\right)}{c^2\left\{x^2+\left(1- \frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} \right),$$

and

$$\frac{\partial \vec{B}}{\partial t} = \left(0 , \ \frac{-3qxv^2z\left(1 - \frac{v^2}{c^2}\right)}{c\left\{x^2+\left(1- \frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} , \ \frac{3qv^2xy\left(1 - \frac{v^2}{c^2}\right)}{c\left\{x^2+\left(1- \frac{v^2}{c^2}\right)(y^2+z^2)\right\}^{5/2}} \right).$$

Hence $\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$, ...IV