CHAPTER 4

MATERIAL BALANCE

The material balance equation has long been regarded as one of the basis tools of reservoir engineers for interpreting and predicting reservoir performance. In this chapter, the zero dimensional material balance is derived and subsequently applied, using mainly the interpretive technique of Havlena and Odeh to gain an understanding of reservoir drive mechanisms under primary recovery conditions. Finally, some of the uncertainties attached to estimation of in situ pore compressibility, a basic component in the material balance equation, are qualitatively discussed. Although the classical material balance techniques, once applied, have now largely been superseded by numerical simulators, which are essentially multidimensional, multi-phase, dynamic material balance programs, the classical approach is well worth studying since it provides a valuable insight into the behaviour of hydrocarbon reservoirs.

The general form of material balance equation is derived as a volume balance which equates the cumulative observed production, expressed as an underground withdrawal, to the expansion of the fluids in the reservoir resulting from a finite pressure drop. If the total observed surface production of oil and gas is expressed in terms of an underground withdrawal, evaluated at a lower pressure p, which means effectively taking all the surface production back down to the reservoir at this lower pressure can be expressed in the terms as below:

Underground withdrawal = Expansion of oil + originally dissolved gas

+ Expansion of gas cap

+ Reduction in Hydrocarbon Pore Volume (HCPV) due to connate water expansion and decrease in pore volume

Before evaluating the various components mentioned above it is necessary to define the following parameters:

$$N = V\phi(1-S_{wc})/B_{oi} \qquad STB \tag{4.1}$$

m = initial HC volume of gascap/initial HC volume of oil

N_p = cumulative oil production in STB

R_p = cumulative GOR   SCF/STB

4.1 Expansion of oil + originally dissolved gas

-Liquid expansion:

The N (stb) will occupy a reservoir volume of NB_{oi} (rb), at the initial pressure, while at the lower pressure p, the reservoir volume occupied by the N stb will be NB_o, where B_o is the oil formation volume factor at the lower pressure. The difference gives the liquid expansion as:

$$N(B_o-B_{oi}) \quad \text{(rb)} \tag{4.2}$$

-Liberated gas expansion:

Since the initial oil is in equilibrium with the gascap, the oil must be at saturation or bubble point pressure. Reducing the pressure below pi will result in the liberation of solution gas. The total amount of solution gas in the oil is NR_{si} scf. Therefore, the gas volume liberated during the pressure drop Δp, expressed in reservoir barrels at the lower pressure is:

$$N(R_{si}-R_s)B_g \quad \text{(rb)} \tag{4.3}$$

4.2 Expansion of gascap gas

The total volume of gascap gas is mNB_{oi} rb, which in scf may be expressed as

$$G = \frac{mNB_{oi}}{B_{gi}} \quad \text{(scf)} \tag{4.4}$$

This amount of gas at the reduced pressure p, will occupy a reservoir volume

$$mNB_{oi} \frac{B_g}{B_{gi}} \quad \text{(rb)} \tag{4.5}$$

Therefore, the expansion of the gascap is

$$mNB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) \quad \text{(rb)} \tag{4.6}$$

4.3 Change in HCPV due to connate water expansion and pore volume reduction

The total volume change due to these combined effects can be mathematically expressed as

$$d(HCPV) = -dV_w + dV_f \qquad (4.7)$$

or, as a reduction in hydrocarbon pore volume as

$$d(HCPV) = -(c_w V_w + c_f V_f)\Delta p \qquad (4.8)$$

where V_f is the total pore volume = HCPV/(1-S_{wc})

and V_w is the connate water volume = Vf x S_{wc} = (HCPV)S_{wc}/(1-S_{wc})

Since the total HCPV, including the gascap is

$$(1+m)NB_{oi} \qquad \text{(rb)} \qquad (4.9)$$

then the HCPV reduction can be expressed as

$$-d(HCPV) = (1+m)NB_{oi}\left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\Delta p \qquad (4.10)$$

This reduction in the volume can be occupied by the hydrocarbons at the lower pressure, p , must correspond to an equivalent amount of fluid production expelled from the reservoir, and hence should be added to the fluid expansion terms.

4.4 Underground withdrawal

The observed surface production during the pressure drop Δp is N_p stb of oil and $N_p R_p$ scf of gas. When these volume of oil plus dissolved gas will be $N_p B_o$ rb. All that is known about the total gas production is that, the lower pressure, $N_p R_s$ scf will be dissolved in the N_p stb of oil. The remaining produced gas, $N_p(R_p-R_s)$ scf is therefore, the total amount of liberated and gascap gas produced during the pressure drop Δp and will occupy a volume $N(R_p-R_s)B_g$ rb at the lower pressure. The total underground withdrawal term is therefore

$$N_p(B_o + (R_p - R_s)B_g) \qquad \text{(rb)} \qquad (4.11)$$

The total material balance equation becomes as follows:

$$N_p\left[B_o + (R_p - R_s)B_g\right] = NB_{oi}\left[\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + m\left(\frac{B_g}{B_{gi}} - 1\right) + (1+m)\frac{(c_w S_w + c_f)}{(1 - S_w)}\Delta p\right] + (W_e - W_p)B_w$$

(4.12)

where:

B_o	=	oil formation volume factor
B_g	=	gas formation volume factor
B_w	=	water formation volume factor
C_w	=	water compressibility
c_f	=	rock pore volume compressibility
m	=	ratio of gas cap pore volume to oil pore volume
N_p	=	cumulative oil production
N	=	initial oil in place
p	=	average reservoir pressure; subscript i = initial
R_s	=	solution gas-oil ratio
R_p	=	cumulative production gas-oil ratio
S_w	=	water saturation
W_e	=	cumulative aquifer influx
W_p	=	total water produced

4.5 The material balance expressed as a linear equation

The material balance equation of later developed further to be expressed as a linear equation. Havlena and Odeh developed two interesting papers which described the technique of interpreting the material balance as the equation of the straight line, and also illustrating the application to reservoir case histories.

The way Havlena and Odeh presented requires the definition of the following terms:

Underground withdrawal

$$F = N_p\left[B_o + (R_p - R_s)B_g\right] + W_p B_w \qquad \text{(rb)} \qquad (4.13)$$

Expansion of oil and its originally dissolved gas

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g \qquad \text{(rb/stb)} \qquad (4.14)$$

Expansion of gascap gas

$$E_g = B_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) \qquad \text{(rb/stb)} \qquad (4.15)$$

Expansion of connate water and reduction in the pore volume

$$E_{f,w} = (1+m)B_{oi}\frac{(c_w S_w + c_f)}{(1-S_w)}\Delta p \qquad \text{(rb/stb)} \qquad (4.16)$$

Using these terms the material balance equation can be written as

$$F = N(E_o + mE_g + E_{f,w}) + W_e B_w \qquad (4.17)$$

Havlena and Odeh have shown in many cases that the above equation can be interpreted as a linear function. For instance, in the case of a reservoir which has no gas cap, negligible water influx and for which the connate water and rock compressibility term may be neglected; the equation can be reduced to

$$F = NE_o$$

in which the observed production, evaluated as an underground withdrawal, should plot as a linear function of the expansion of oil plus its originally dissolved gas, the latter being calculated from a knowledge of the PVT parameters at the current reservoir pressure. This interpretation technique is useful, in that, if a simple linear relationship is expected for a reservoir and yet the actual plot turns out to be non linear, then this deviation can itself be diagnostic in determining the actual drive mechanisms in the reservoir.