References

E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen. 1999. LAPACK Users' Guide. Society for Industrial and Applied Mathematics, Philadelphia, PA, third edition. ISBN 0-89871-447-8 (paperback).

C. Bradford Barber, David P. Dobkin, and Hannu Huhdanpaa. 1996. The quickhull algorithm for convex hulls. ACM Trans. Math. Soft. 22,4:469–483. URL citeseer.ist.psu.edu/article/barber95quickhull.html .

A. Bicchi and V. Kumar. 2000. Robotic grasping and contact: A review. In IEEE Int. Conf. on Robotics and Automation.

Antonio Bicchi. 2000. Hands for dexterous manipulation and robust grasping: A difficult road toward simplicity. IEEE Trans. Robot. Autom. 16,16:652–662.

Antonio Bicchi. 1995. On the closure properties of robotic grasping. Int. J. Robot. Research 14 ,4:319–334.

Robert Bix. 1994. Topics in Geometry. Academic Press.

CGAL Editorial Board. 2006. CGAL User and Reference Manual. URL http://www.cgal.org/Manual/3.2/doc_html/cgal_manual/index.html .

Ch. Borst, M. Fischer, and G. Hirzinger. 2003. Grasping the dice by dicing the grasp. In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems.

Ch. Borst, M. Fischer, and G. Hirzinger. 2004. Grasp planning: How to choose a suitable task wrench space. In IEEE Int. Conf. on Robotics and Automation.

Stephen Boyd and Lieven Vandenberghe. 2004. Convex Optimization. Cambridge University Press.

Randy C. Brost. 1991. Analysis and planning of planar manipulation tasks. PhD thesis, Carnegie-Mellon University School of Computer Science.

Randy C. Brost and Matthew T. Mason. 1989. Graphical analysis of planar rigid-body dynamics with multiple frictional contacts. In International Symposium on Robotics Research, pp. 293–300, Tokyo, Japan. Cambridge, Mass: MIT Press.

R.C. Brost and K. Goldberg. 1996. A complete algorithm for designing planar fixtures using modular components. IEEE Trans. Robot. Autom. 12,1:31–46.

C. Burnikel, K. Mehlhorn, and S. Schirra. 1994. On degeneracy in geometric computations. In Proceedings of 5th ACM-SIAM Symposium on Discrete algorithms, pp. 16–23.

J. Butterfaß, G. Hirzinger, S. Knoch, and H. Liu. 1998. Dlr's multisensory hand part i: Hard- and software architecture. In IEEE Int. Conf. on Robotics and Automation.

J. Butterfaß, M. Grebenstein, H. Liu, and G. Hirzinger. 2001. DLR-hand II: Next generation of a dextrous robot hand. In IEEE Int. Conf. on Robotics and Automation, pp. 109–114.

Jae-Sook Cheong and A. Frank van der Stappen. 2005. Output-sensitive computation of all form-closure grasps of a semi-algebraic set. In IEEE Int. Conf. on Robotics and Automation.

Jae-Sook Cheong and A. Frank van der Stappen. 2007a. Computing all independent form-closure grasp regions of a rectilinear polyhedron. In Proc. of the IEEE Conference on Automation Science and Engineering.

Jae-Sook Cheong and A. Frank van der Stappen. 2007b. Computing all form-closure grasps of a rectilinear polyhedron with seven frictionless point fingers,. In Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS).

Jae-Sook Cheong, Herman J. Haverkort, and A. Frank van der Stappen. 2006. On computing all immobilizing grasps of a simple polygon with few contacts. Algorithmica 44:117–136.

K. L. Clarkson, K. Mehlhorn, and R. Seidel. 1993. Four results on randomized incremental constructions. Computational Geometry Theory and Application 3,4:185–212.

Jordi Cornella and Raul Suarez. 2005. Fast and flexible determination of force-closure independent regions to grasp polygonal objects. In IEEE Int. Conf. on Robotics and Automation, pp. 778–783, Barcelona, Spain.

Jordi Cornella and Raul Suarez. 2006. A new framework for planning three-finger grasps of 2d irregular objects. In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 5688–5694.

Dan Ding, Yun-Hui Liu, Michael Yu Wang, and Shuhuo Wang. 2001a. Automatic selection of fixturing surfaces and fixturing points for polyhedral workpieces. IEEE Trans. Robot. Autom. 17,6:833–841.

Dan Ding, Yun-Hui Liu, and Shuguo Wang. 2001b. Computation of 3-D form-closure grasps. IEEE Trans. Robot. Autom. 17,4:515–522.

A. Edsinger-Gonzales and J. Weber. 2004. Domo: a force sensing humanoid robot for manipulation research. In Humanoid Robots, 2004 4th IEEE/RAS International Conference on, volume 1, pp. 273–291.

Bernard Faverjon and John Ponce. 1991. On computing two-finger force-closure grasps of curved 2D objects. In IEEE Int. Conf. on Robotics and Automation, pp. 424–429, Sacramento, CA.

C. Ferrari and J.F. Canny. 1992. Planning optimal grasps. In IEEE Int. Conf. on Robotics and Automation, pp. 2290–2295, Nice, France.

Chandrasekhar Gatla, Ron Lumia, John Wood, and Greg Starr. 2004. An efficient method to compute three fingered planar object grasps using active contour models. In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 3674–3679.

E. G. Gilbert, D. W. Johnson, and S. S. Keerthi. 1988. A fast procedure for computing the distance between complex objects in three-dimentional space. IEEE Trans. Robot. Autom. 4: 193–203.

Elmer G. Gilbert and Chek-Peng Foo. 1990. Computing the distance between gemeral convex objects in three-dimensional space. IEEE Trans. Robot. Autom. 6,1:53–61.

A.J. Goldman and A.W. Tucker. 1956. Polyhedral convex cones. In H.W. Kihn and A.W. Tucker, editors, Linear Inequalities and Related Systems. Princeton University Press.

Jacob E. Goodman and Joseph O'Rourke, editors. 1997. Handbook of discrete and computational geometry. CRC Press, Inc., Boca Raton, FL, USA. ISBN 0-8493-8524-5.

Torbjorn Granlund. 2006. The GNU Multiple Precision Arithmetic Library. Free Software Foundation, Inc., 51 Franklin Street, Fifth Floor, Boston, MA 02110-1301, USA, 4.2.1 edition. URL http://www.swox.com/gmp/manual/ .

J. L. Guibas, L. Ramshaw, and J. Stolfi. 1983. The kinetic framework for computational geometry. In Proceeding of the 24th Annual Symposium of Foundations of Computer Sciene.

Li Han, Jeff C. Trinkle, and Zexiang X. Li. 2000. Grasp analysis as linear matrix inequality problems. IEEE Trans. Robot. Autom. 16,6:663–674.

K. Harada, S. Kajita, H. Saito, M. Morisawa, F. Kanehiro, K. Fujiwara, K. Kaneko, and H. Hirukawa. 2005. A humanoid robot carrying a heavy object. In IEEERA, pp. 1712–1717.

Yasuhisa Hasegawa, Junya Matsuno, and Toshio Fukuda. 2000. Regrasping behavior and object transition generated by ep. In Proceedings of the Congress on Evolutionary Computation.

K. Hsiao and T. Lozano-Perez. 2006. Imitation learning of whole-body grasps. In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 5657–5662. doi: 10.1109/IROS.2006. 282366.

S.C. Jacobsen, J.E. Wood, D.F. Knutti, and K.B. Biggers. 1984. The Utah-MIT Dextrous Hand: Work in progress. International Journal of Robotics Research 3,4:21–50.

Yan-Bin Jia. 2000. Grasping curved objects through rolling. In IEEE Int. Conf. on Robotics and Automation, pp. 377–382.

Yan-Bin Jia. 2001. Localization on curved objects using tactile information. In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 701–706.

Yan-Bin Jia. 2002a. Curvature-based computation of antipodal grasps. pp. 1571–1577.

Yan-Bin Jia. 2002b. Computation on parametric curves with applications in localization and grasping. In Proceedings of the 5th InternationalWorkshop on Algorithmic Foundations of Robotics.

Yan-Bin Jia. 2004. Computation on parametric curves with an application in grasping. Int. J. Robot. Research 23,7–8:825–855.

Yan-Bin Jia. 1995. On computing optimal planar grasps. In IEEE Int. Conf. on Robotics and Automation.

Yoshiaki Katada, Mikhail Svinin, Kazuhiro Ohkura, and Kanji Ueda. 2001. Stable grasp planning by evolutionary programming. IEEE Trans. Robot. Autom. 48,4:749–756.

J.R. Kerr and B. Roth. 1986. Analysis of multi-fingered hands. International Journal of Robotics Research 4,4.

D.G. Kirkpatrick, B. Mishra, and C.K. Yap. 1990. Quantitative Steinitz's theorems with applications to multifingered grasping. In 20^{th} ACM Symp. on Theory of Computing, pp. 341–351, Baltimore, MD.

K. Lakshminarayana. 1978. Mechanics of form closure. Technical Report 78-DET-32, ASME.

Jia-Wei Li, Hong Liu, and He-Gao Cai. 2003. On computing three-finger force-closure grasps of 2-d and 3-d objects. IEEE Trans. Robot. Autom. 19,1:155–161.

Ying Li, Jiaxin L. Fu, and Nancy S. Pollard. 2007. Data driven grasp synthesis using shape matching and task-based pruning. IEEE Transactions on Visualization and Computer Graphics 3,4:732–747.

Zexiang Li and S. Shankar Sastry. 1988. Task-oriented optimal grasping by multifingered robot hand. IEEE Trans. Robot. Autom. 4,1:32–44.

H. Liu, P. Meusel, J. Butterfaß, and G. Hirzinger. 1998. Dlr's mulitsensory articulated hand part ii: The parallel torque/position control system. In IEEE Int. Conf. on Robotics and Automation.

Yun-Hui Liu. 1999. Qualitative test and force optimization of 3-d frictional form-closure grasps using linear programming. IEEE Trans. Robot. Autom. 15,1:163–173.

Yun-Hui Liu. 1998. Computing n-finger force-closure grasps on polygonal objects. In IEEE Int. Conf. on Robotics and Automation.

Yun-Hui Liu, Miu-Ling Lam, and Dan Ding. 2004. A complete and efficient algorithm for searching 3-d form-closure grasps in the discrete domain. IEEE Trans. Robot. 20,5: 805–816.

C. S. Lovchik and M. A. Diftler. 1999. The robonault hand: A dexterous robot hand for space. In IEEE Int. Conf. on Robotics and Automation, pp. 907–912.

H. Hashimoto M. Buss and J. Moore. 1996. Dexterous hand grasping force optimization. IEEE Trans. Robot. Autom. 12,3:406–418.

X. Markenscoff and C.H. Papadimitriou. 1989. Optimum grip of a polygon. International Journal of Robotics Research 8,2:17–29.

Xanthippi Markenscoff, Luqun Ni, and Christos H. Papadimitriou. 1990. The geometry of grasping. Int. J. Robot. Research 9,1:61–74.

Matthew T. Mason. 2001. Mechanics of Robotic Manipulation. Th MIT Press.

Matthew T. Mason. 1991. Two graphical methods for planar contact problems. In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 443–448, Osaka, Japan.

J.P. Merlet. 1988. Singular configurations of parallel manipulators and grassmann geometry. In J.D. Boissonnat and J.P. Laumont, editors, Geometry and Robotics, volume 391 of Lecture Notes in Computer Science, pp. 194–212. Springer-Verlag.

B. Mirtich and J.F. Canny. 1993. Optimum force-closure grasps. Technical Report ESRC 93-11/RAMP 93-5, Robotics, Automation, and Manufacturing Program, University of California at Berkeley.

B. Mishra and N. Silver. 1989. Some discussion of static gripping and its stability. IEEE Systems, Man, and Cybernetics 19,4:783–796.

B. Mishra, J.T. Schwartz, and M. Sharir. 1987. On the existence and synthesis of multifinger positive grips. Algorithmica, Special Issue: Robotics 2,4:541–558.

Antonio Morales, Tamim Asfour, Pedram Azad, Steffen Knoop, and Rudiger Dillmann. 2006. Integrated grasp planning and visual object localization for a humanoid robot with five-fingered hands. In Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on, pp. 5663–5668. doi: 10.1109/IROS.2006.282367.

Van-Duc Nguyen. 1986. Constructing force-closure grasps. In IEEE Int. Conf. on Robotics and Automation.

Van-Duc Nguyen. 1988. Constructing force-closure grasps. Int. J. Robot. Research 7,3:3–16.

Van-Duc Nguyen. 1989. Constructing stable grasps. Int. J. Robot. Research 8,1:27–37.

Nattee Niparnan and Attawith Sudsang. 2004. Fast computation of 4-fingered force-closure grasps from surface points. In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems.

Nattee Niparnan and Attawith Sudsang. 2006a. Computing all force-closure grasps of 2d objects from contact point set. In IEEE/RSJ Int. Conf. on Intelligent Robots and Systems.

Nattee Niparnan and Attawith Sudsang. 2006b. A heuristic approach for computing frictionless force-closure grasps of 2d objects from contact point set.pdf. In Proceedings of 2006 IEEE International Conference on Robotics, Automation & Mechatronics.

J. Pertin-Troccaz. 1989. Grasping: a state of the art. In O. Khatib, J. Craig, and T. Lozano-Pérez, editors, The Robotics Review 1. MIT Press.

Nancy S. Pollard. 2004. Closure and quality equivalence for efficient synthesis of grasps from examples. Int. J. Robot. Research 23,6:595–614.

Nancy S. Pollard. 1994. Parallel algorithms for synthesis of whole hand grasps. Technical Report AI-TR 1464., MIT Artificial Intelligence Laboratory.

Nancy S. Pollard. 1996. Synthesizing grasps from generalized prototypes. In IEEE Int. Conf. on Robotics and Automation.

Nancy S. Pollard and Victor B. Zordan. 2005. Physically based grasping control from example. In ACM SIGGRAPH / Eurographics Symposium on Computer Animation, pp. 311–318.

J. Ponce and B. Faverjon. 1995. On computing three-finger force-closure grasps of polygonal objects. IEEE Trans. Robot. Autom. 11,6:868–881.

J. Ponce, S. Sullivan, J-D. Boissonnat, and J-P. Merlet. 1993. On characterizing and computing three- and four-finger force-closure grasps of polyhedral objects. In IEEE Int. Conf. on Robotics and Automation, pp. 821–827, Atlanta, Georgia.

J. Ponce, A. Sudsang, S. Sullivan, B. Faverjon, J.-D. Boissonnat, and J.-P. Merlet. 1997. On computing four-finger equilibrium and force-closure grasps of polyhedral objects. Int. J. Robot. Research 16,1:11–35.

William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery. 1992. Numerical Recipes in C. Cambridge University Press.

F. Reuleaux. 1876. The kinematics of machinery. MacMillan, NY. Reprint, Dover, NY, 1963.

F. Reuleaux. 1963. The Kinematics of Machinery. Dover, NY.

E. Rimon and A. Blake. 1996. Caging 2D bodies by one-parameter two-fingered gripping systems. In IEEE Int. Conf. on Robotics and Automation, pp. 1458–1464, Minneapolis, MN.

Elon Rimon and Joel Burdick. 1996. On force and form closure for multiple finger grasps. In IEEE Int. Conf. on Robotics and Automation, volume 2, pp. 1795–1800.

Elon Rimon and Joel Burdick. 1998a. Mobility of bodies in contact–part I: A 2nd-order mobility index for multiple-finger grasps. IEEE Trans. Robot. Autom. 14,5:696–708.

Elon Rimon and Joel Burdick. 1998b. Mobility of bodies in contact–part II: How forces are generated by curvature effects. IEEE Trans. Robot. Autom. 14,5:709–717.

J.K. Salisbury. 1982. Kinematic and force analysis of articulated hands. PhD thesis, Stanford University, Stanford, CA.

J.K. Salisbury and B. Roth. 1982. Kinematic and force analysis of articulated hands. ASME J. Mechanisms, Transmissions, and Automation in Design 105:33–41.

Ashutosh Saxena, Justin Driemeyer, Justin Kearns, and Andrew Y. Ng. 2007. Robotic grasping of novel objects. In Proceedings of the Conference on Neural Information Processing Systems (NIPS).

K. B. Shimoga. 1996. Robot grasp synthesis algorithms: a survey. Int. J. Robot. Research 15,3: 203–266.

P. Somov. 1900. Über Gebiete von Schraubengeschwindigkeiten eines starren Korpers bie verschiedener Zahl von Stutzflachen. Zeitricht für Mathematik und Physik 45:245–306.

David E. Stewart. 2000. Rigid-body dynamics with friction and impact. SIAM Review 42,1: 3–39.

J. Stolfi. 1988. Primitives for Computational Geometry. PhD thesis, Stanford University, Department of Computer Science.

Raul Suarez, Israel Vazquez, and Jose M. Ramirez. 2003. Planning four grasping points from images of planar objects. In Proceedings of the IEEE International Symposium on Assembly and Task Planning, pp. 169–174.

A. Sudsang, J. Ponce, and N. Srinivasa. 2000. Grasping and in-hand manipulation: Geometry and algorithms. Algorithmica 26,3:466–493.

Attawith Sudsang. 2002. A sufficient condition for capturing an object in the plane with disc-shaped robots. In IEEE Int. Conf. on Robotics and Automation, pp. 682–687.

Attawith Sudsang and Thanathorn Phoka. 2005. Geometric reformulation of 3-fingered force-closure condition. In IEEE Int. Conf. on Robotics and Automation.

William T. Townsend. 2000. The BarrettHand grasper–programmably flexible part handling and assembly. Industrial Robot: An International Journal 27,3:181–188.

J.C. Trinkle. 1992. On the stability and instantaneous velocity of grasped frictionless objects. IEEE Transactions on Robotics and Automation 8(5):560–572.

Chao-Ping Tung and Avinash C. Kak. 1996. Fast construction of force-closure grasps. IEEE Trans. Robot. Autom. 12,4:615–626.

Gino van den Bergen. 1999. A fast and robust gjk implementation for collision detection of convex objects. Journal of Graphics Tools 4,2:7–25.

A. Wallack and J.F. Canny. 1994. Planning for modular and hybrid fixtures. In IEEE Int. Conf. on Robotics and Automation, pp. 520–527, San Diego, CA.

Michael Yu Wang. 2000. An optimum design for 3-d fixture synthesis in a point set domain. IEEE Trans. Robot. Autom. 16,6:839–846.

D.E. Willard. 1985. New data structures for orthogonal queries. SIAM Journal of Computing 14: 232–253.

Shao-Po Wu, Lieven Vandenberghe, and Stephen Boyd. 1996. MAXDET Software for Determinant Maximization Problems User Guide. Stanford University, alpha version edition.

Yu Zheng and Wen-Han Qian. 2005. Coping with the grasping uncertainties in force-closure analysis. Int. J. Robot. Research 24,4:311–327. ISSN 0278-3649. doi: http://dx.doi.org/10.1177/0278364905049469.

Yu Zheng and Wen-Han Qian. 2006. An enhanced ray-shooting approach to force-closure problems. Journal of Manufacturing Science and Engineering 128,4:960–968.

Xiangyang Zhu and Han Ding. 2006. Computation of force-closure grasps: An iterative algorithm. IEEE Trans. Robot. 22,1:172–179.

Xiangyang Zhu and Jun Wang. 2003. Synthesis of force-closure grasps on 3-d objects based on the q distance. IEEE Trans. Robot. Autom. 19,4:669–679.

Xiangyang Zhu, Han Ding, and Jun Wang. 2003. Grasp analysis and synthesis based on a new quantitative measure. IEEE Trans. Robot. Autom. 19,6:942–953.

Xiangyang Zhu, Han Ding, and S. K. Tso. 2004. A pseudodistance function and its applications. IEEE Trans. Robot. Autom. 20,2:344–352.

APPENDICES

APPENDIX A

IMPLEMENTATION OF CASE (B)

In this chapter we provide implementation of the conditions of Case (b), which are Lemma 5.6 and Subcase (ii) and Subcase (iii) of Lemma 5.8. We further divide Lemma 5.6 into two more subcases: (i) the case where G_a is an actual contact fan, (ii) the case where G_a is formed by two boundary wrenches of different fans. Let us denote the anchor fan being considered as F_i and let F_i intersects an arbitrary candidate fan G_a. Let us define a *pairing wrench* of a boundary wrench $\boldsymbol{w}_{il}$ of a fan F_i as the other boundary wrench of F_i, e.g., $\boldsymbol{w}_{1l}$ is a pairing wrenches of $\boldsymbol{w}_{1r}$, and vice versa.

All conditions discussed in this chapter rely on one common operation: side identification of a wrench with respect to a plane. Let us consider this operation in force dual representation. An arbitrary plane P through the origin is represented by a line l, except for the $z = 0$ plane which is represented by a line at infinity. Vectors lying on the same side with respect to P are represented by points lying on the same side of the half plane divided by the line l. However, a vector might be represented by its negative. The side of a negative vector is inverted. Hence, a vector also lies on the same side when its negative lies on the *different side* of the half plane divided by the line. Given a plane P, a positive dual point and a zero torque dual point lying on one side of $\Omega(P)$ and a negative dual points lying on the other side of $\Omega(P)$ are all lying on the same side of P.

Though Case (b) cannot directly use FINDINTS as Case (a) or Case (c), FINDINTS is still useful in this case since the intermediate data from FINDINTS is used in the implementation of Case (b). In FINDINTS, there is a step that the boundary wrenches of every fan are classified into subset V_L and V_R, and subsequently into several other subset according to their signs. Let us refer to this step as a classifying step. It is the data of this classification step that is used in the implementation of Case (b).

It should be noted that FINDINTS considers a *positive* anchor fan and identify *negative* fans that intersects with the anchor fan. However, Case (b) considers a negative anchor fan and identify positive fans. By Corollary 5.3, we can transform the condition requiring a negative fan to lie inside a positive span of fans into the equivalent condition requiring a positive fan to lie inside a positive span of negative fans. For integrity with the conditions given in Section 5.3, the following discussion assume that FINDINTS considers the negative fan instead.

A.1 Implementation of Subcase (i) of Lemma 5.6

Let us denote this case as Case (b1-i). In this case, one boundary wrench of $-F_i$, says $-\boldsymbol{w}_{il}$, intersects a relative interior of the other fan, says $F_k = \text{SPAN}^+(\{\boldsymbol{w}_{kl}, \boldsymbol{w}_{kr}\})$. We first identify this case from the intermediate data of FINDINTS. Since FINDINTS is used in the calculation of Case (a), we can utilize the intermediate data from the routine without any additional cost. FINDINTS classifies the boundary wrenches into two subsets: the left subset V_L and the right subset V_R. Let us assume that we are classifying the boundary wrenches of F_k. Let p and q be the force dual representation of $\boldsymbol{w}_{kl}$ and $\boldsymbol{w}_{kr}$, respectively. When $-\boldsymbol{w}_{il}$ lies inside $\text{RI}(F_k)$, the point p, q and $\Omega(-T\boldsymbol{w}_{il})$ must lie on the same line. This can be detected easily from the alpha angle. Assume that both p and q are positive dual points, the summation of the alpha angle of p and q must equal to π and they must lie on the different side of f'_y axis (see Figure A.1a). Note that one of them can safely be a point at infinity. When either p or q is a negative dual point, both must have the same alpha angle and be members of the same set V_L or V_R. The point that is a positive dual point must also be closer to $\Omega(-T\boldsymbol{w}_{il})$ (see Figure A.1b and Figure A.1c). This closeness can be easily determined from the beta angle. The condition for p and q in this case can be represented as follows.

$$
\begin{aligned}
(\alpha_p + \alpha_q = \pi \quad &\wedge \quad p \in V_R^+ \wedge q \in V_L^+) \vee \\
(\alpha_p + \alpha_q = \pi \quad &\wedge \quad p \in V_L^+ \wedge q \in V_R^+) \vee \\
(\alpha_p + \alpha_q = \pi \quad &\wedge \quad p \in V_L^+ \wedge q \in V_R^0) \vee \\
(\alpha_p + \alpha_q = \pi \quad &\wedge \quad p \in V_R^+ \wedge q \in V_L^0) \vee \\
(\alpha_p = \alpha_q \wedge \beta_p < \beta_q \quad &\wedge \quad p \in V_R^+ \wedge q \in V_R^-) \vee \\
(\alpha_p = \alpha_q \wedge \beta_p < \beta_q \quad &\wedge \quad p \in V_L^+ \wedge q \in V_L^-)
\end{aligned}
\tag{A.1}
$$

When p and q satisfy Equation (A.1), we knows that F_k and F_i form a half space which is bounded by a line joining p and q. The solution is every wrench whose negative lies inside the half space. Negative dual points lying above the line and positive dual points and points at infinity lying below the line form force closure grasps with F_i and F_k. Identifying such points can be done easily using the alpha angle. Let V_M, the middle set, be a set of point not being the member of V_L nor V_R. Similar to V_L and V_R, V_M is divided into V_M^+, V_M^0 and V_M^+ according to the sign of the dual points. Let r be a dual representation of an arbitrary wrench. The condition of r is as follows.

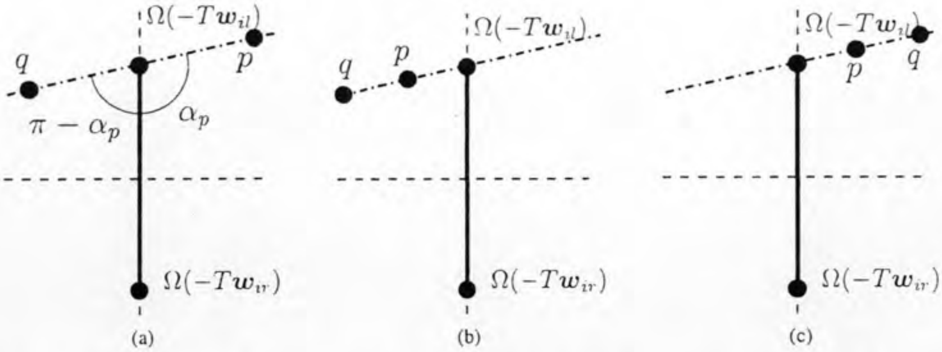

Figure A.1: The case when the boundary of $-F_i$, says $-\boldsymbol{w}_{il}$, intersects F_k. (a) Both p and q are positive dual points. They have to be on the opposite side. (b)–(c) The point p is a positive dual point while q is a negative dual points. They have to be on the same side and the distance from $\Omega(-T\boldsymbol{w}_{il})$ of q of has to be more than p. Note that this can be inferred efficiently from the beta angle of the points.

$$\begin{aligned}
(\alpha_p > \alpha_r \quad &\wedge \quad r \in V_L^+ \cup V_L^0) \vee \\
(\pi - \alpha_p > \alpha_r \quad &\wedge \quad r \in V_R^+ \cup V_R^0) \vee \\
(\alpha_p < \alpha_r \quad &\wedge \quad r \in V_L^-) \vee \\
(\pi - \alpha_p < \alpha_r \quad &\wedge \quad r \in V_R^-) \vee \\
(\alpha_r = 0 \quad &\wedge \quad r \in V_M^+ \cup V_M^0) \vee \\
(\alpha_r = \pi \quad &\wedge \quad r \in V_M^-)
\end{aligned} \tag{A.2}$$

In the actual implementation, an additional set $\mathfrak{F}_\alpha$ is created to store any fan that satisfies Equation (A.1) in the dividing step. After FINDINTS completes its normal operation, every fan in $\mathfrak{F}_\alpha$ is enumerated. For each fan, Equation (A.2) is used to identify all points forming force closure grasps.

To identify a point r that satisfies Equation (A.2), we maintain a set of ordered lists $\mathcal{U} = \{\mathcal{U}_L^+, \mathcal{U}_L^0, \mathcal{U}_L^-, \mathcal{U}_R^+, \mathcal{U}_R^0, \mathcal{U}_R^-\}$. The lists, using alpha angle as an ordering element, contain points in V_L^+, V_L^0, V_L^-, V_R^+, V_R^0, V_R^-, respectively. The construction of $\mathcal{U}$ is $O(n \lg n)$. For each fan in $\mathfrak{F}_\alpha$, a point r satisfying Equation (??) can be identified easily from $\mathcal{U}$ in $O(\lg n + K)$. Since there are at most n point in $\mathfrak{F}_\alpha$, all satisfying wrenches can be identified in $O(n \lg n + K)$ which is dominated by $O(n \lg^2 n + K)$ of FINDINTS. This does not affect the overall complexity.

The discussion so far concerns F_k that contains $-\boldsymbol{w}_{il}$. The remaining case when F_k contains $-\boldsymbol{w}_{ir}$ can be done in the same manner by using beta angle instead of alpha angle in Equation (A.1) and (A.2).

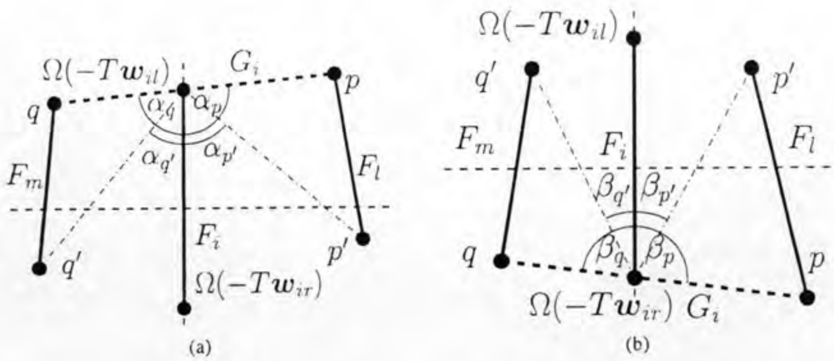

Figure A.2: The case (b1-ii) in force dual representation. The thick dashed line in the top is the candidate fan (G_a). (a) The point p, p', q, q' are positive dual points satisfying Equation (A.1) and (A.2). (b) The point p, p', q, q' are positive dual points satisfying the beta version of Equation (A.1) and (A.2)

A.2 Implementation of Subcase (ii) of Lemma 5.6

Let us denote this case as Case (b1-ii). In this case G_a is not a contact fan. Equation (A.1) and (A.2) still hold in this case. However, the boundary wrenches of G_a are from different contact fans, together with F_i, they are already counted for three fans. The algorithm needs not to identify any other wrench r. Rather, it has to report every candidate fan that satisfies the condition similar to the Case (b1-i). Assume that the boundary wrenches of G_a come from the contact fan F_l and F_m. Also let p and q be the force dual representation of the boundary wrench of G_a. The wrench that has to satisfies Equation (A.2) must be the remaining boundary wrench of F_l and F_m. Let p' and q' denoted the force dual representation of the pairing wrench associated with p and q, respectively.

The major difference between this case and Case (b1-i) is that, in Case (b1-i), we can identify F_k immediately in the dividing step and we have to identify the remaining fan in the final step. However, in Case (b1-ii), G_a cannot be identify immediately since p and q comes from different fan. Additional step is required to identify p and q. Moreover, the additional requirement of p' and q' must be satisfied also. By replacing r with p', the condition of p' can be inferred directly from Equation (A.2). Similarly, the condition of q' can be inferred from Equation (A.2) by replacing p and r by q and q', respectively. Figure A.2a illustrates example of points that satisfy the equations.

To identify wrenches satisfying these conditions in this case, additional modification is applied to FINDINTS as follows. At the classifying of FINDINTS, p and its pairing wrench p' are also tested whether they satisfy Equation (A.2). If they satisfy the condition, p is added to a set S. S is divided into $S_R^+, S_R^0, S_R^-, S_L^+, S_L^0, S_L^-$ according to signs and sides of its members. Ordered lists ${}^S\mathcal{U}_R^+, {}^S\mathcal{U}_R^0, {}^S\mathcal{U}_R^-, {}^S\mathcal{U}_L^+, {}^S\mathcal{U}_L^0, {}^S\mathcal{U}_L^-$ are constructed respectively to the partitions of S, using the alpha angle of p as a key. Intuitively, S contains every point p whose pairing wrench lies in the half space formed by the anchor fan and the point p itself and any point q that satisfies Equation (A.1), which is to be identified afterward. It should be noted that p and q satisfying Equation (A.2) definitely do not satisfy Equation (A.1) and it will not be detected by Case (b1-i).

After all wrenches are classified and the respective range trees are constructed, we enumerate every point in S and identify every point q that satisfies Equation (A.1). The point q must be the member of S. By using Equation (A.1), we could identify the requirement of the angle of q. The point satisfying the Equation can be identify directly from the ordered list associated with the respective partition of S. For each point in S, the satisfying point q can be identified in $O(\lg n + K)$. The construction of the ${}^S\mathcal{U}$ can be done in $O(n \lg n)$. This also does not affect the overall complexity.

Similar to the Case (b1-i), the condition so far assumes that $-\boldsymbol{w}_{il}$ intersects G_a. The remaining case when $-\boldsymbol{w}_{ir}$ intersects G_a can be done by replacing alpha angle by beta angle in Equation (A.1) and (A.2). However, be noted that when a point p is put into S, we immediately know that its pairing wrench p' also satisfy the beta version of Equation (A.2). Hence, the set S for the beta version is identified automatically.

A.3 Implementation of Subcase (ii) of Lemma 5.8

Let us denote this case as Case (b2-ii). This case is very similar to Case (b1-i). The difference is that a wedge is used instead of a half space to mark the area where wrenches that form force closure grasps must lies upon. A wedge is bounded by two half spaces; one of them is bounded the line joining p, q and the dual representation of $-\boldsymbol{w}_{il}$. This is exactly the same half space $\mathcal{H}_{\boldsymbol{v}}$ as in Case (b1-i). The other half space $\mathcal{H}_{\boldsymbol{u}}$ is bounded by the f'_y axis. We have to include this half space into Equation (A.2). The other difference is that p (or q) must coincide with the dual representation of $-\boldsymbol{w}_{il}$. In the wrench classifying step, when a wrench, coincides with $\Omega(-T\boldsymbol{w}_{il})$, its pairing wrench is stored in $\mathfrak{F}'_\alpha$.

For each wrench in $\mathfrak{F}'_\alpha$, we can easily identify wrenches that lie inside the wedge using their alpha angles, signs and sides. Let a point stored in $\mathfrak{F}'_\alpha$ be denoted by p' and let p be its pairing wrench. Assume that p' is a positive dual point or a point at infinity, the half space $\mathcal{H}_{\boldsymbol{u}}$ contains all wrenches lying on the same side as p', e.g., when p' is a member of left subset, $\mathcal{H}_{\boldsymbol{u}}$ includes all wrench facing left side. On the contrary, when p' is a negative dual point, the side where $\mathcal{H}_{\boldsymbol{u}}$ contains must be the opposite side. Figure A.3a and Figure A.3b illustrate examples when p' is a positive and a negative dual point, respectively. The areas and the signs of points that satisfy the

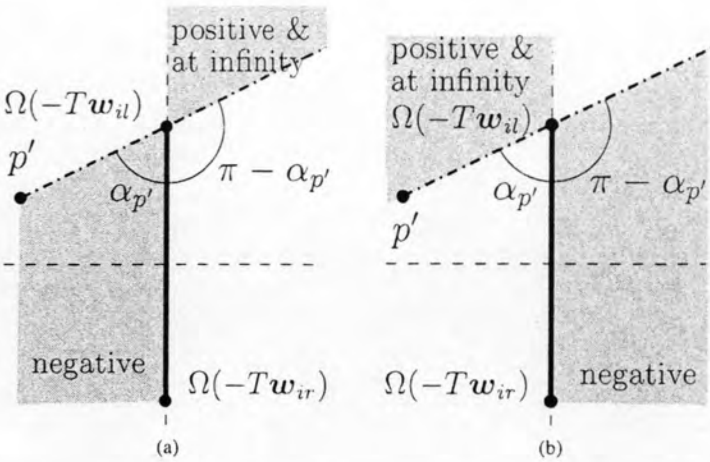

Figure A.3: Case (b2-ii) represented in the force dual representation. (a) The satisfying region when p' is a positive dual point. Every positive dual point and point at infinity in the bottom left shaded region and every negative dual point in the top right region satisfy the condition. (b) The similar satisfying region when p' is a red point.

condition are marked by the shaded region. Let r be the point to be identified. The condition for this case is as follows.

$$
\begin{aligned}
(\alpha_{p'} > \alpha_r \quad &\wedge \quad p' \in V_L^+ \cup V_L^0 \wedge r \in V_L^+ \cup V_L^0) \vee \\
(\alpha_{p'} > \alpha_r \quad &\wedge \quad p' \in V_R^+ \cup V_R^0 \wedge r \in V_R^+ \cup V_R^0) \vee \\
(\pi - \alpha_{p'} < \alpha_r \quad &\wedge \quad p' \in V_L^+ \cup V_L^0 \wedge r \in V_R^-) \vee \\
(\pi - \alpha_{p'} < \alpha_r \quad &\wedge \quad p' \in V_R^+ \cup V_R^0 \wedge r \in V_L^-) \vee \\
(\alpha_{p'} < \alpha_r \quad &\wedge \quad p' \in V_L^- \wedge r \in V_L^+ \cup V_L^0) \vee \\
(\alpha_{p'} < \alpha_r \quad &\wedge \quad p' \in V_R^- \wedge r \in V_R^+ \cup V_R^0) \vee \\
(\pi - \alpha_{p'} > \alpha_r \quad &\wedge \quad p' \in V_L^- \wedge r \in V_R^-) \vee \\
(\pi - \alpha_{p'} > \alpha_r \quad &\wedge \quad p' \in V_R^- \wedge r \in V_L^-)
\end{aligned} \tag{A.3}
$$

The identification of r can be done by searching in the ordered lists in $\mathcal{U}$ which is already generated from Case (b1-i). Similarly, overall complexity in this case is $O(n \lg n + K)$. This also does not affect the overall complexity. Similar to the other cases, the condition so far assumes that $-\boldsymbol{w}_{il}$ intersects F_k. The remaining case when $-\boldsymbol{w}_{ir}$ intersects F_k can be done by substituting alpha angle by beta angle in Equation (A.3).

A.4 Implementation of Subcase (iii) of Lemma 5.8

Finally, we consider the last case Lemma 5.8. This case can be identified by having two different fans such that the boundary wrenches of them coincides with the boundary wrenches of $-F_i$. From the classification in the Case (b2-ii), let $\mathfrak{F}'_\alpha$ and $\mathfrak{F}'_\beta$ be the set that contain pairing wrenches of wrenches that coincide with $\boldsymbol{w}_{il}$ and $\boldsymbol{w}_{ir}$, respectively. For every point p' in $\mathfrak{F}'_\alpha$, we can identify a wrench being antipodal of p' easily by searching for a point in $\mathfrak{F}'_\alpha$ that has the same alpha and beta angle lying on the same side but the signs are different. In the case of point at infinity, the points must lie on the different side and the summation of their alpha (resp. beta) angle must equal to π. Let q' denotes a point in $\mathfrak{F}'_\beta$. The condition can be listed as follows.

$$\begin{aligned}
(\alpha_{p'} = \alpha_{q'} \wedge \beta_{p'} = \beta_{q'} \quad &\wedge \quad p' \in V_L^+ \wedge q' \in V_L^-) \vee \\
(\alpha_{p'} = \alpha_{q'} \wedge \beta_{p'} = \beta_{q'} \quad &\wedge \quad p' \in V_R^+ \wedge q' \in V_R^-) \vee \\
(\alpha_{p'} + \alpha_{q'} = \pi \wedge \beta_{p'} + \beta_{q'} = \pi \quad &\wedge \quad p' \in V_L^0 \wedge q' \in V_R^0) \vee \\
(\alpha_{p'} + \alpha_{q'} = \pi \wedge \beta_{p'} + \beta_{q'} = \pi \quad &\wedge \quad p' \in V_R^0 \wedge q' \in V_L^0)
\end{aligned} \tag{A.4}$$

An additional set of ordered lists is introduced in this case. A set ${}^{\mathfrak{F}}\mathcal{U} = \{{}^{\mathfrak{F}}\mathcal{U}_R^+, {}^{\mathfrak{F}}\mathcal{U}_R^0, {}^{\mathfrak{F}}\mathcal{U}_R^-, {}^{\mathfrak{F}}\mathcal{U}_L^+, {}^{\mathfrak{F}}\mathcal{U}_L^0, {}^{\mathfrak{F}}\mathcal{U}_L^-\}$ contains points in $\mathfrak{F}'_\beta$, according to their signs and sides. For each point p' in $\mathfrak{F}'_\alpha$, the satisfying point q' can be identified from the respective ${}^{\mathfrak{F}}\mathcal{U}$ within $O(\lg n)$. The construction of ${}^{\mathfrak{F}}\mathcal{U}$ can be done in $O(n \lg n)$. Hence, the overall complexity is $O(n \lg n)$.

APPENDIX B

LIST OF PUBLICATIONS

Parts of this work are published in the following articles.

International Journals

1. Nattee Niparnan, Attawith Sudsang, Prabhas Chongstitvatana, "Positive Span of Force and Torque Components in Three Dimensional Four Finger Force Closure Grasps", Advanced Robotics. (Accepted)

International Conference Proceedings

1. Thanathorn Phoka, Nattee Niparnan and Attawith Sudsang ,"Hierarchical Simplification for 5-Fingered 3D Regrasp Planning on Triangular Mesh Objects", Proc. of the IEEE Int. Conf. on Robotics and Biomimetics (IEEE ROBIO), China, 2007
2. Nattee Niparnan, Attawith Sudsang, "Positive Span of Force and Torque Components of Four-Fingered Three-Dimensional Force-Closure Grasps", IEEE Int. Conf. on Robotics and Automation (IEEE ICRA), Roma, Italy , 2007.
3. Nattee Niparnan, Thanathorn Phoka and Attawith Sudsang, "Computing Frictionless Force-Closure Grasps of 2D Objects from Contact Point Set", IEEE Int. Conf. on Robotics and Biomimetics (IEEE ROBIO), 2006
4. Thanathorn Phoka, Nattee Niparnan and Attawith Sudsang, "Planning Optimal Force-Closure Grasps for Curved Objects", IEEE Int. Conf. on Robotics and Biomimetics (IEEE ROBIO), 2006
5. Nattee Niparnan and Attawith Sudsang, "Computing All Force-Closure Grasps of 2D Objects from Contact Point Set", IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IEEE IROS), 2006
6. Nattee Niparnan and Attawith Sudsang, "A Heuristic Approach for Computing Frictionless Force-Closure Grasps of 2D Objects from Contact Point Set", Proc. of the IEEE International Conference on Robotics, Automation and Mechatronics (IEEE RAM), 2006

7. Chalermsub Sangkhavijit, N Niparnan and Prabhas Chongstitvattana, "Computing 4-Fingered Force-Closure Grasps from Surface Points using Genetic Algorithm", Proc. of the IEEE International Conference on Robotics, Automation and Mechatronics (IEEE RAM), 2006

8. Thanathorn Phoka, Nattee Niparnan and Attawith Sudsang, Planning Optimal Force-Closure Grasps for Curved Objects by Genetic Algorithm, Proc. of the IEEE International Conference on Robotics, Automation and Mechatronics (IEEE RAM), 2006

9. Attawith Sudsang, Thanathorn Phoka, Peam Pipattanasomporn and Nattee Niparnan, "Re-grasp Planning of Four-Fingered Hand for Parallel Grasp of a Polygonal Object", IEEE Int. Conf. on Robotics and Automation (IEEE ICRA), 2005

10. Nattee Niparnan and Attawith Sudsang, "Fast Computation of 4-Fingered Force-Closure Grasps from Surface Points", IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IEEE IROS), 2004

Local Conference Proceedings

1. Mahisorn Wongphati, Nattee Niparnan and Attawith Sudsang, "An Indoor Mapping for a Mobile Robot Using an Omnidirectional Camera", 5th National Computer Science and Engineering Conference (NCSEC), Thailand, 2005

Biography

Nattee Niparnan was born in Bangkok, Thailand, on May, 1979. He received B.Eng. and M.Eng., both in computer engineering, from Chulalongkorn University, Thailand, in 2001 and 2003, respectively. His undergraduate study, from 1997 to 2001, was supported by Scholarship from Charoen Pokphand Group. In his master degree and the first few years in his doctorate, he also serves as a teaching assistant at the Department of Computer Engineering, Chulalongkorn University. His bachelor degree and his master master degree have been supervised by Dr. Prabhas Chonstitvatana. His doctorate has been under the supervision of Dr. Attawith Sudsang. In summer 2004, he was granted a visiting scholarship by Toshiba International Foundation to participate in training at Humancentric Laboratory, Research and Development Center, Toshiba Corporation, Kawasaki. Since 2005, he has received a grant from the Thailand Research Fund through the Royal Golden Jubilee Ph.D. Program under Grant No. Ph.D. 1.O.CU/48/A.1. In 2007, he additionally received a grant from the 90^{th} Anniversary of Chulalongkorn University Fund through the Ratchadapiseksomphot Fund. His field of interest includes various topics in Robotics with emphasis on grasp planning, grasp analysis and dexterous manipulation. He is also educated in the field of Evolutionary Computation.