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นางสาวนภารัตน์ ศิริพิทักษ์ชัย

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CONTROL OF THE NUMBER OF DOTS IN InAs QUANTUM DOT MOLECULES FOR QUANTUM COMPUTING

Miss Naparat Siripitakchai

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วิทยานิพนธ์ฉบับนี้ นำเสนอการปลูกโครงสร้างควอนตัมดอตโมเลกุลแบบกระจายตัวทางด้านข้างด้วยกระบวนการจัดเรียงตัวเอง โดยเครื่องปลูกผลึกแบบลำโมเลกุลด้วยเทคนิคการปลูกกลบด้วยชั้นบางและการปลูกซ้ำ การกลบด้วยชั้นบางและการปลูกซ้ำบนควอนตัมดอต (quantum dot, QD) ที่อุณหภูมิต่ำทำให้เกิดโครงสร้างนาโนโฮล (nanohole) รูปร่าง ขนาด และ ความลึกของนาโนโฮลสามารถควบคุมได้ด้วยความหนาของชั้นกลบ หลังจากนั้นการปลูกซ้ำด้วยอินเดียมอาร์เซไนด์หนา 0.6 โมโนเลเยอร์ (monolayer, ML) บนโครงสร้างนาโนโฮลที่แตกต่างทำให้เกิดโครงสร้างนาโนพล็อพเพลเลอร์ (nano-propeller) ที่มีขนาดใบแตกต่างกัน ความยาวของใบนาโนพล็อพเพลเลอร์ สามารถควบคุมด้วยความหนาของชั้นกลบและอุณหภูมิที่ใช้กลบ เมื่อปริมาณอินเดียมอาร์เซไนด์เพิ่มขึ้นเป็น 1.2 และ 1.5 โมโนเลเยอร์ ทำให้เกิดโครงสร้างควอนตัมดอตโมเลกุลที่มีรูปร่างแตกต่างกัน โดยที่ดอตตรงกลางจะเกิดขึ้นหลังจากที่อินเดียมอะตอมลงไปเติมหลุมนาโนโฮล จนกระทั่งความเครียดที่กลางหลุมนาโนโฮลลดลงจนมีค่าต่ำสุด อินเดียมอะตอมที่เหลือจะทำพันธะที่ปีกของนาโนพล็อพเพลเลอร์ การตรวจสอบความสม่ำเสมอของดอต และ ขนาดของดอตสามารถทำได้ด้วยการวัดโฟโตลูมิเนสต์เซนต์ที่อุณหภูมิ 77 เคลวิน นอกจากนี้ผลโฟโตลูมิเนสต์ยังแสดงให้เห็นถึงการเลื่อนยอดของแถบพลังงานไปที่ระดับพลังงานสูงขึ้นเมื่อพลังงานกระตุ้นมากขึ้น ซึ่งคาดว่าเป็นผลเนื่องมาจากอิทธิพลของการคลับปิงระหว่างดอตกลางและดอตข้าง มีผลทำให้แถบพลังงานของควอนตัมดอตโมเลกุลขยายไปบริเวณที่มีพลังงานสูงขึ้น

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NAPARAT SIRIPITAKCHAI: CONTROL OF THE NUMBER OF DOTS IN InAs QUANTUM DOT MOLECULES FOR QUANTUM COMPUTING, THESIS ADVISOR: PROF. SOMSAK PANYAKEOW, D.Eng., 85 pp.

Self-assembled lateral InAs quantum dot molecules (QDMs) are grown by solid-source molecular beam epitaxy (MBE) technique using the thin-capping-and-regrowth MBE process. Thin-capping of GaAs on as-grown InAs quantum dots (QDs) at low temperatures leads to nanohole templates. The shape, size and depth of nanoholes are controlled by capping thickness. Subsequent regrowth with 0.6 ML of InAs on the templates result in nano-propeller QDs with different blades' dimensions. We found that the length of the propeller blades is controlled by either the capping temperature or the thickness of capping layer. When the amount of InAs regrowth layer increases from 0.6 to 1.2 and 1.5 ML, QDMs with different shapes are created. The center dots are formed by filling up nanoholes at the beginning phase until the strain relaxation at center becomes minimized. Later on, the rest of InAs regrowth material starts to form satellite dots along the blade of nano-propeller where remaining strain fields are still distributed at the perimeter of nano-propeller blades. The dot uniformity and their dot size of all QDM samples are examined by photoluminescence (PL) measurements at 77 K. Furthermore, the excitation power dependence of PL spectra of all samples is also conducted. The PL results from some case of different QDMs also exhibit the shift of the PL peaks toward higher energies. We believe that the coupling between center dot and satellite dots lead to the extending in their band structures.

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m^*	effective mass
$\hbar$	reduced Planks constant
$r = (x, y, z)$	carrier position vector
$V(r)$	confinement potential due to band offset
$F(r)$	envelope wave function
E	carrier energy
E_{bulk}	carrier energy in case of bulk
E_{QW}	carrier energy in case of quantum well (QW)
E_{QWR}	carrier energy in case of quantum wire (QWR)
E_{QD}	carrier energy in case of quantum dot
k	amplitude of wave vector
$k_{\parallel}$	amplitude of in-plane (y-z) wave vector
$k_{\perp}$	amplitude of wave vector in x-direction
$\mathbf{k} = (k_x, k_y, k_z)$	wave vector of carriers
$E_{l,x}$	quantized energy in x-direction
$E_{m,y}$	quantized energy in y-direction
$E_{n,z}$	quantized energy in z-direction
$V(r)$	confinement energy
l	quantum number in x-direction
m	quantum number in y-direction
n	quantum number in z-direction
$D_{\mathrm{bulk}}(E)$	bulk density of state
$D_{\mathrm{QW}}(E)$	quantum well (QW) density of state
$D_{\mathrm{QWR}}(E)$	quantum wire (QWR) density of state
$D_{\mathrm{QD}}(E)$	quantum dot (QD) density of state
θ	Heavisides unit step function
N_{wi}	area density of the quantum wires
δ	delta function
N_D	volume density of QD
a	lattice constant of deposited material
a_0	lattice constant of substrate material
ε	misfit
d_1	thickness of strained epitaxial film
d_2	thickness of substrate

c_{11}, c_{12}	elastic constant of the epitaxial layer
$\varepsilon_{\parallel}$	in-plane strain
$\varepsilon_{\perp}$	strain in perpendicular to the growth direction
ε_{xx}	strain in the x-direction
ε_{yy}	strain in the y-direction
ε_{zz}	strain in the z-direction
h_c	critical thickness
$v_{PR} = c_{12}/(c_{11} + c_{12})$	Poisson ratio