CHEPTER V

Analyze the Effect of Genetic Parameters

5.1 Test Problems

In this work, we analyze the effect of genetic parameters (mutation probability and crossover probability) used in Multiple Objective Genetic Algorithm (MOGA, Fonseca and Fleming 1993) , Non-dominated Sorting Genetic Algorithm (NSGA), (Srinivas and Deb, 1994), Niched-Pareto Genetic Algorithm (NPGA) (Horn, Nafpliotis and Goldberg 1994), Elitish non-dominated Sorting Genetic Algorithm (NSGA-II) (**Deb, Agrawal, Pratap and Meyarivan, 2000**) and Strength Pareto Evolutionary Algorithm (SPEA) (Zitzler and Thiele, 2000). These algorithms can be classified into two classes. The first one is non-elitist multi-objective evolutionary algorithms. The algorithms belonged to this class are MOGA, NSGA and NPGA. The other class is elitist multi-objective evolutionary algorithms. The algorithms belonged to this class are NSGA-II and SPEA. All algorithms were implemented in Matlab (version 7). The optimal values of genetic parameter were selected based on three criteria which are the distance between obtained Pareto fronts and true Pareto front, the ratio between solution in Pareto front and outside Pareto front, and the distribution of solutions on the obtained Pareto front.

Other operators and parameters were defined as shown in the table 5.1-5.5

Table 5.1 MOGA operator and parameters is used for determination

Chromosome representation	Real-value chromosome
Selection strategy	Stochastic universal selection
Crossover type	Single-point crossover
Mutation type	Non-uniform mutation
Niching parameters	0.1
Maximum number of generation	100
Termination criterion	specified number of generations
Constraint-handing	Penalty function

Table 5.2 NSGA operator and parameters is used for determination

Chromosome representation	Real-value chromosome
Selection strategy	Roulette-wheel selection
Crossover type	Arithmetical crossover
Mutation type	Non-uniform mutation
Niching parameters	0.1
Maximum number of generation	100
Termination criterion	specified number of generations
Constraint-handing	Penalty function

Table 5.3 NPGA operator and parameters is used for determination

Chromosome representation	Real-value chromosome
Selection strategy	Binary tournament selection
Crossover type	Arithmetical crossover
Mutation type	Non-uniform mutation
Niching parameters	0.1
Domination pressure t_{dom}	10
Maximum number of generation	100
Termination criterion	specified number of generations
Constraint-handing	Penalty function

Table 5.4 NSGA-II operator and parameters is used for determination

Chromosome representation	Real-value chromosome
Selection strategy	Binary tournament selection
Crossover type	Arithmetical crossover
Mutation type	Non-uniform mutation
Maximum number of generation	100
Termination criterion	specified number of generations
Constraint-handing	Penalty function

Table 5.5 SPEA operator and parameters is used for determination

Chromosome representation	Real-value chromosome
Selection strategy	Binary tournament selection
Crossover type	Arithmetical crossover
Mutation type	Non-uniform mutation
Maximum number of generation	100
Maximum number of External pop	1000
Termination criterion	specified number of generations
Constraint-handing	Penalty function

5.1.1 Problem No 1:

$$\begin{aligned} &\text{Minimize} \quad f_1(x) = x_1^2 \\ &\text{Minimize} \quad f_2(x) = \frac{1 + x_2^2}{x_1^2} \\ &\text{Subject to} \quad \sqrt{0.1} \le x_1 \le 1, \\ &\qquad\qquad 0 \le x_2 \le \sqrt{5} \end{aligned} \tag{5.1}$$

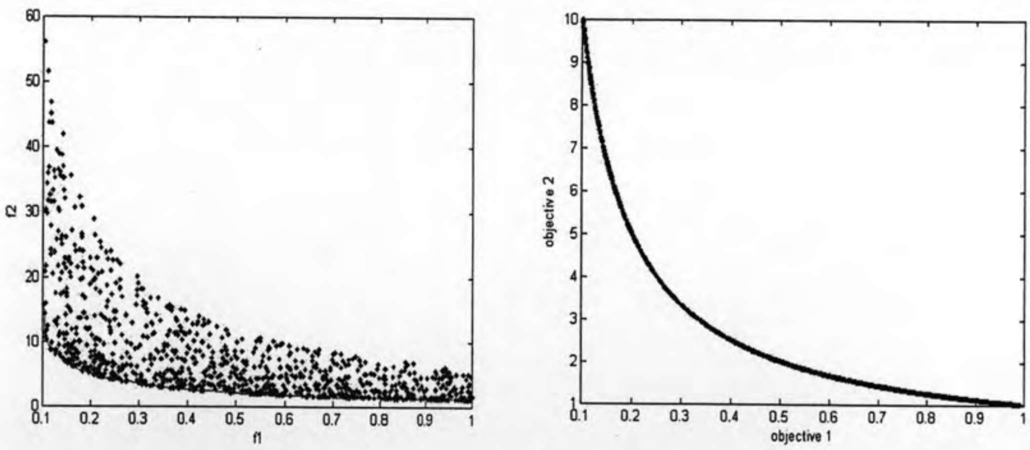

Figure 5.1: Feasible objective space of problem No.1 shown in the left hand side and the true Pareto optimal solution of problem No.1 shown in the left hand side

Problem No. 1 is a two variables problem. The feasible objective space and true Pareto optimal solution of problem No.1 are shown in Fig. 5.1. The true Pareto optimal solution is a convex-connect set.

Case Non-Elitist Multi-Objective Evolutionary Algorithms

We applied the MOGA, NSGA and NPGA on problem No.1. First, we used the algorithms without a mutation operator (mutation probability = 0). We used a fixed σ_{share}=0.1. The results show that three algorithms without the mutation operator cannot find the true Pareto optimal solution. It is shown in Fig. 5.2. When three algorithms are applied to non-uniform mutation, a different scenario emerges. The algorithms can find more solution near the true Pareto optimal solution, but when the value of mutation probability is larger than 0.2, the algorithms show inability to find a good spread of the Pareto-optimal solutions.

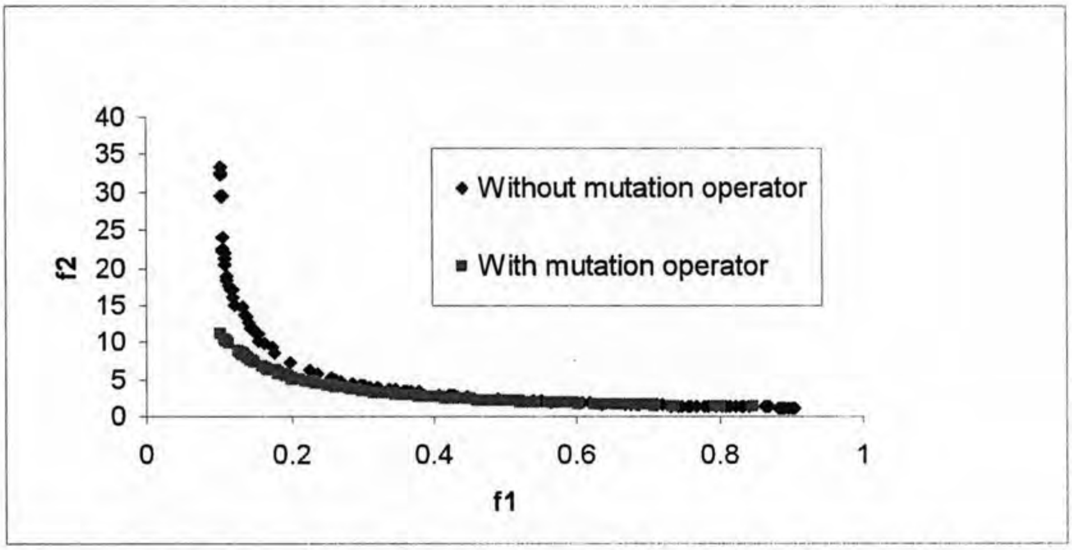

Figure 5.2: Pareto optimal solution obtained after 100-th generation with an MOGA on problem No. 1.

The influence of crossover probability was analyzed. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations but if too large crossover probability is used, the population quickly loses its diversity. Consequently, the population converges to only one portion of Pareto-optimal front. But when the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front.

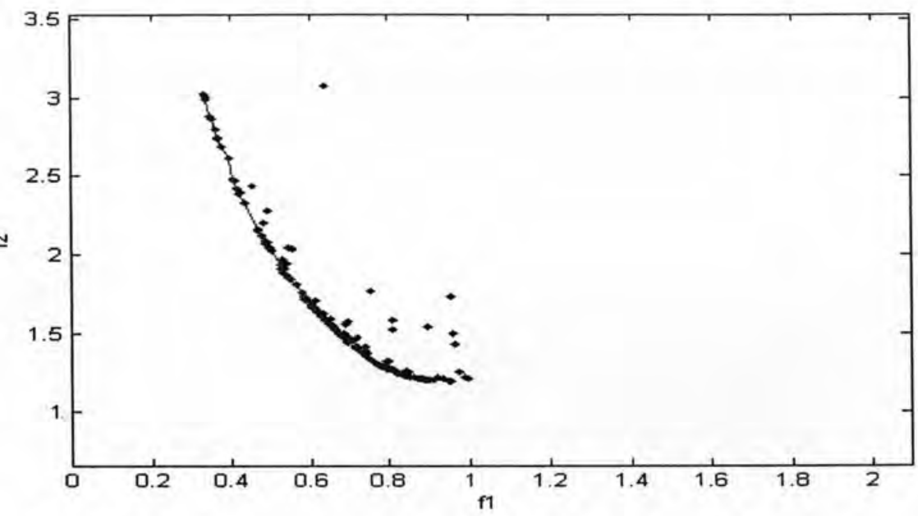

Figure 5.3: Population after 100-th generation obtained with an NPGA (mutation probability 0.22 and crossover probability 0.6) on problem No.1.

- In MOGA, the value of crossover probability between the ranges of 0.35-0.9 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.9, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.35 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an MOGA is shown in Fig. 5.4.
- In NSGA, the value of crossover probability between the ranges of 0.4-0.9 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.9, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.4 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an NSGA is shown in Fig. 5.5.
- In NPGA, the value of crossover probability between the ranges of 0.4-0.9 can produce a good Pareto-optimal solution. If the value of crossover probability was set lager than 0.9, the algorithm cannot find a good spread of Pareto-optimal front. The true Pareto-optimal front cannot be obtained, if the crossover probability was set lower than 0.4. Pareto optimal solution obtained with an NPGA is shown in Fig. 5.6.

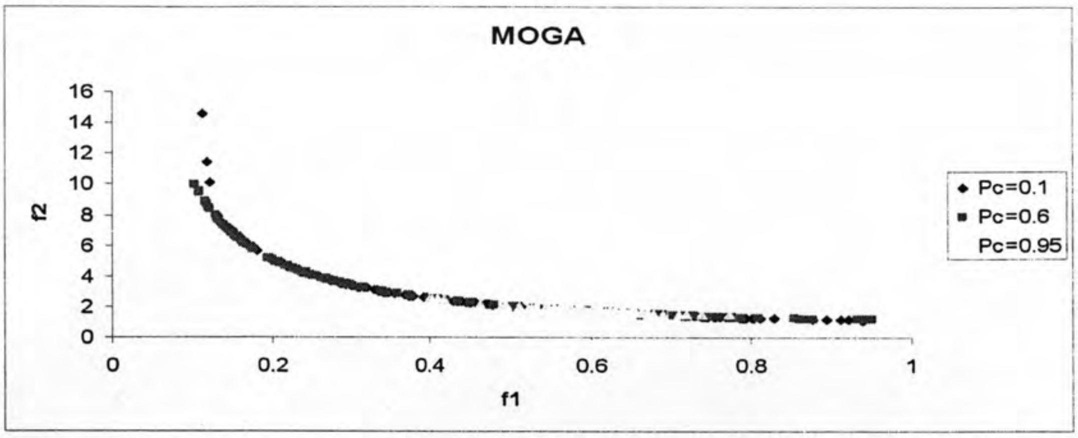

Figure 5.4: The Pareto-optimal solution for problem No.1 after 100-th generation obtained with an MOGA (mutation probability 0.05).

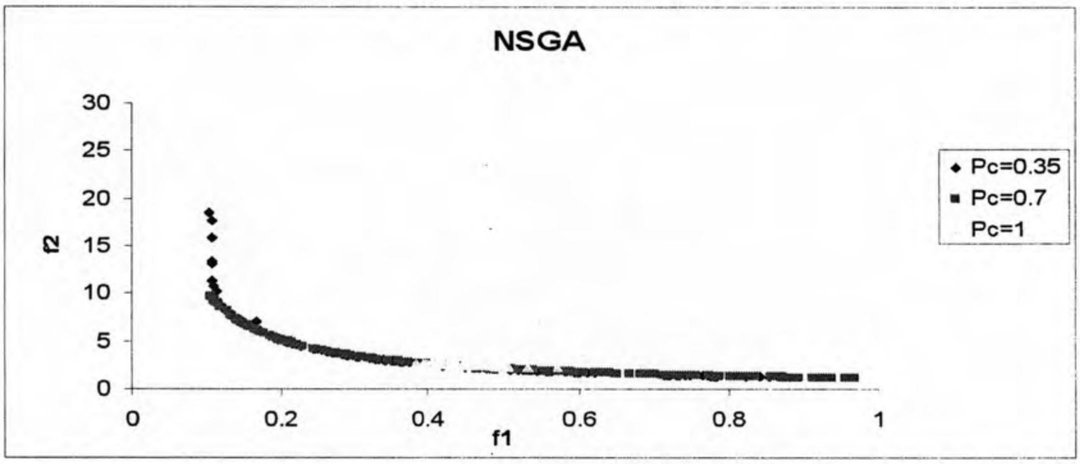

Figure 5.5: The Pareto-optimal solution for problem No. 1 after 100-th generation obtained with an NSGA (mutation probability 0.05).

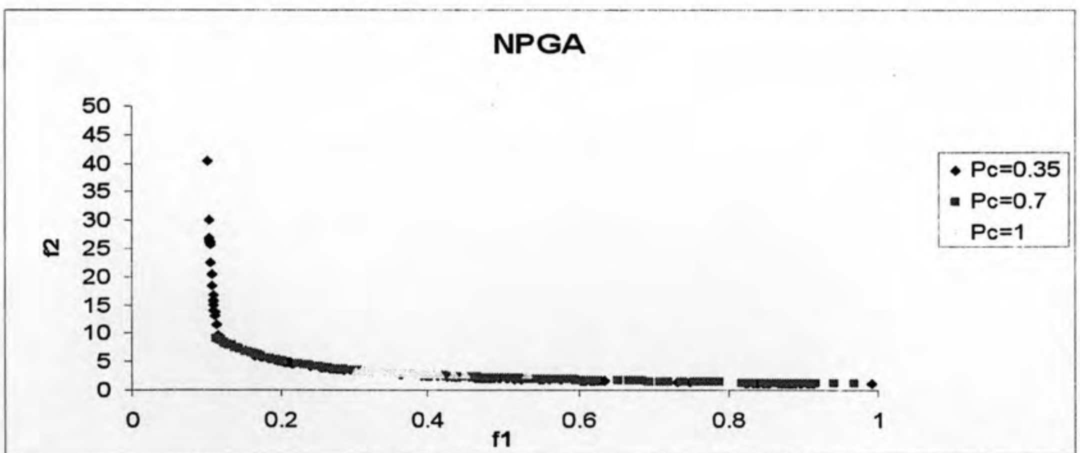

Figure 5.6: The Pareto-optimal solution for problem No.1 after 100-th generation obtained with an NPGA (mutation probability 0.05).

The distribution of solutions on the obtained Pareto optimal front was determined. In MOGA, the value of crossover probability between the ranges of 0.35-0.9 can produce a Pareto-optimal solution with a similar average niche value. In NSGA and NPGA, the value of crossover probability between the ranges of 0.4-0.9 can produce a Pareto-optimal solution with a similar average niche value. The relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability is shown in Fig. 5.7.

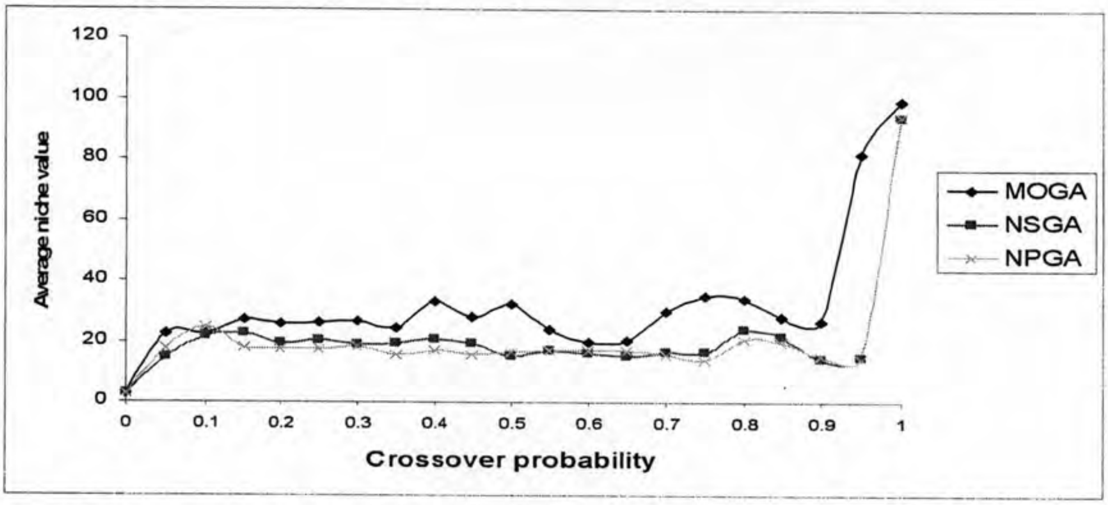

Figure 5.7: Relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability with MOGA, NSGA and NPGA on problem No.1.

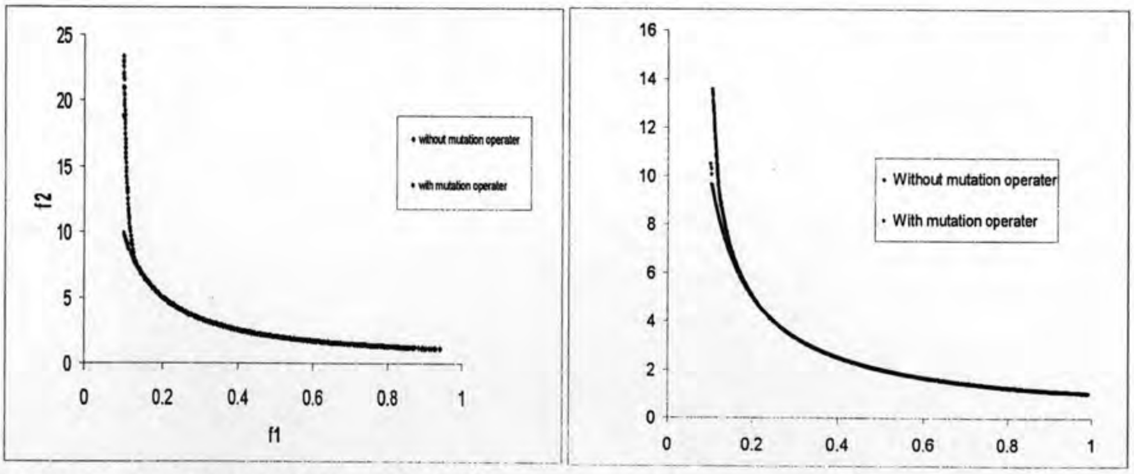

Figure 5.8: Pareto optimal solution obtained of problem No.1 with an NSGA-II (left) and an SPEA (right) for compare both algorithms with mutation operator and without mutation operator.

Case Elitist Multi-Objective Evolutionary Algorithms

Next, we applied the NSGA-II and SPEA on problem No.1. Both algorithms without the mutation operator cannot find the true Pareto-optimal solutions as shown in Fig. 5.8. When we used algorithms with mutation operator, the mutation probability value has a small effect to find a good spread of Pareto-optimal solutions as shown in Fig. 5.9.

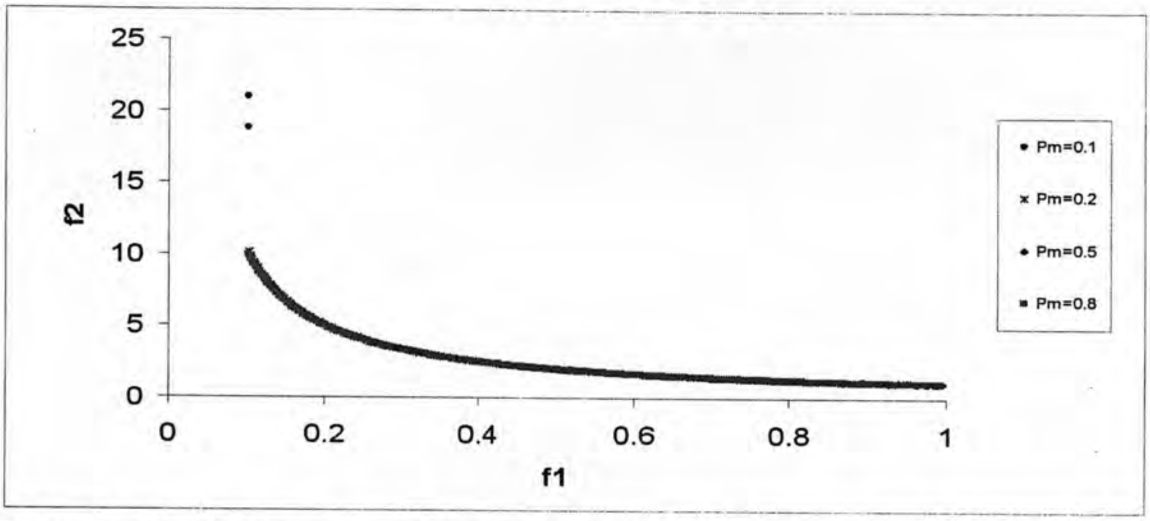

Figure 5.9: Pareto optimal solution obtained of problem No.1 after 100-th generation with an NSGA-II (effect of mutation probability).

The effect of crossover probability was analyzed. When the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front for both algorithms. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations.

-In NSGA-II, the value of crossover probability between the ranges of 0.25-1 can produce a good Pareto-optimal solution. The Pareto front obtained is close to the true Pareto optimal front. Figure 5.10 shows the obtained Pareto optimal front.

-In SPEA, the value of crossover probability between the ranges of 0.2-1 can produce a good Pareto-optimal solution. The Pareto front obtained shows in Fig. 5.11.

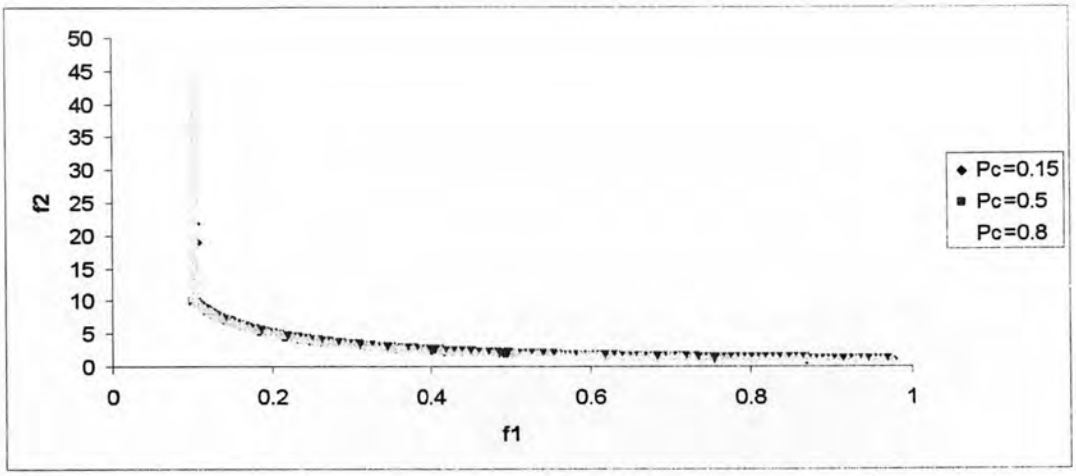

Figure 5.10: Pareto optimal solution obtained of problem No.1 after 100^{-th} generation with an NSGA-II (effect of crossover probability).

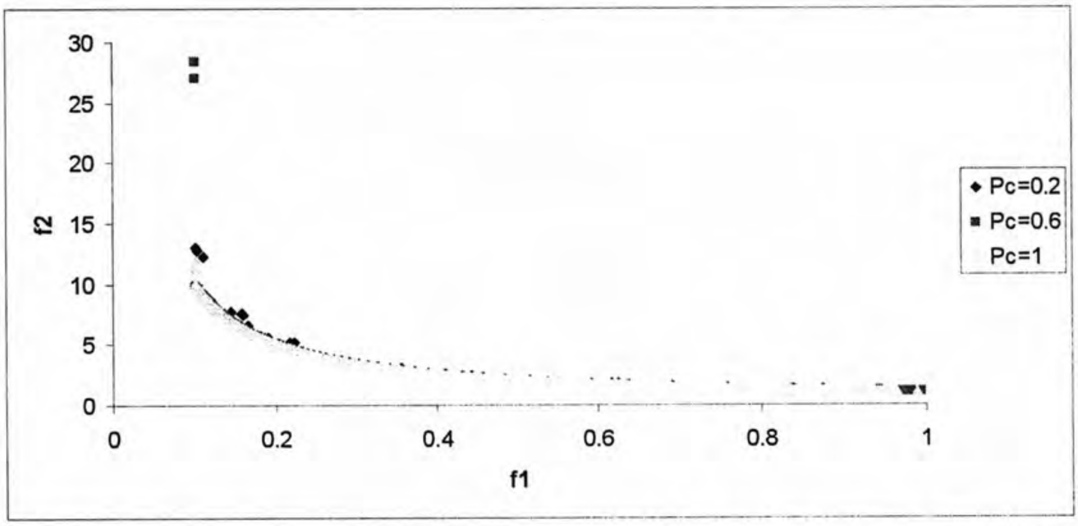

Figure 5.11: Pareto optimal solution obtained after 100^{-th} generation with an SPEA (effect of crossover probability).

Next, we determined distribution of solutions on the obtained Pareto front. In NSGA-II, the value of crossover probability between the ranges of 0.25-1 can produce the obtained Pareto optimal front with a similar average distance of solution on obtained Pareto optimal front. NSGA-II with crossover probability in this rank can create all solutions converged to true Pareto optimal solution. In SPEA, the value of crossover between the ranges of 0.45-1 can create the size of external population more than the population size.

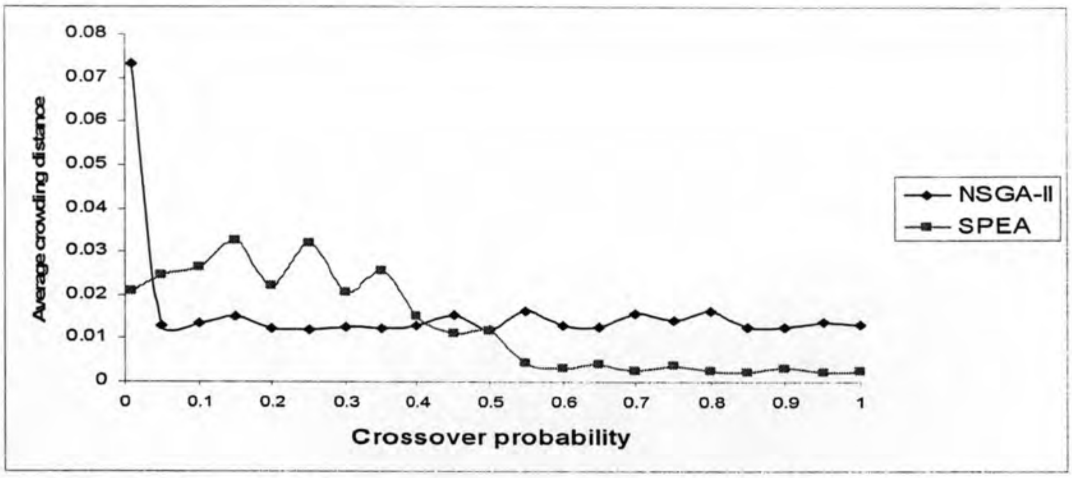

Figure 5.12: Relationship between average crowding distance and crossover probability of problem No.1.

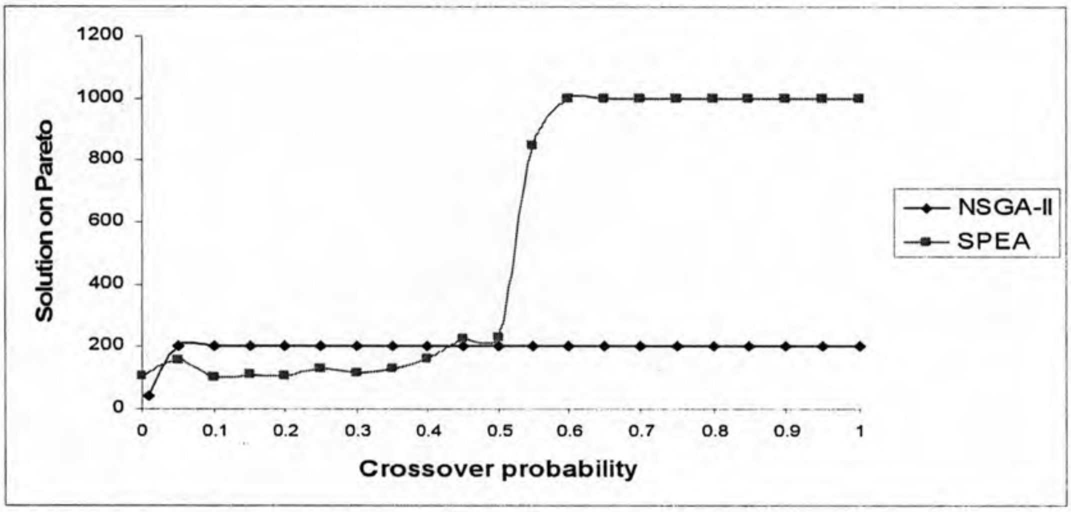

Figure 5.13: Relationship between number of solution on Pareto optimal front and crossover probability of problem No.1.

Hence, in this problem, we conclude that the optimal value of mutation probability is between 0.02 - 0.2 for Non-Elitist Multi-Objective Evolutionary Algorithms and the optimal value of mutation probability is more than 0.02 for Elitist Multi-Objective Evolutionary Algorithms. While the suitable value of crossover probability for MOGA, NPGA and NSGA is between 0.4 and 0.8, and the suitable value of crossover probability for NSGA-II is between 0.25-1 and between 0.45-1 for SPEA.

5.1.2 Problem No 2:

$$\begin{aligned}
&\text{Minimize} \quad f_1(d,l) = \rho\frac{\pi d^2}{4}l \\
&\text{Minimize} \quad f_2(d,l) = \delta = \frac{64Pl^3}{3E\pi d^4} \qquad (5.2)\\
&\text{Subject to} \quad \begin{matrix}\sigma_{\max} \le S_y, \\ \delta \le \delta_{\max}\end{matrix}
\end{aligned}$$

Where the maximum stress is calculated as follows:

$$\sigma_{\max} = \frac{32Pl}{\pi d^3}$$

The following parameter values are used:

ρ = 7800 kg/m^3, P= 1kN, E = 207 GPa,

S_y =300 MPa , $\delta_{\max}$ = 5 mm.

The feasible decision variable space in the overall space is enclosed by $10 \le d \le 50$ mm and $200 \le l \le 1000$ mm.

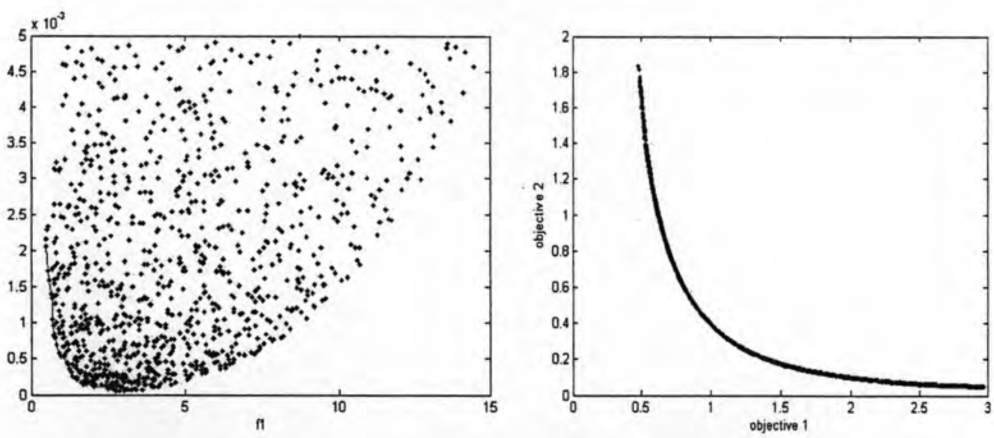

Figure 5.14: Feasible objective space of problem No.2 shown in the left hand side and the true Pareto optimal solution of problem No.2 shown in the left hand side.

Problem No.2 is a two variables problem. The characteristic of feasible objective space and true Pareto optimal solution of problem No.2 are shown in Fig. 5.14. The true Pareto optimal solution is a convex-connect set.

Case Non-Elitist Multi-Objective Evolutionary Algorithms

We applied the MOGA, NSGA and NPGA on problem No.2. First, we used the algorithms without a mutation operator. The results show that three algorithms without the mutation operator cannot find the true Pareto optimal solution. It is shown in Fig. 5.15. When three algorithms are applied to non-uniform mutation, a different scenario emerges. The algorithms can find more solution near the true Pareto optimal solution, but when the value of mutation probability is larger than 0.1, algorithms show inability to find a good spread of the Pareto-optimal solutions.

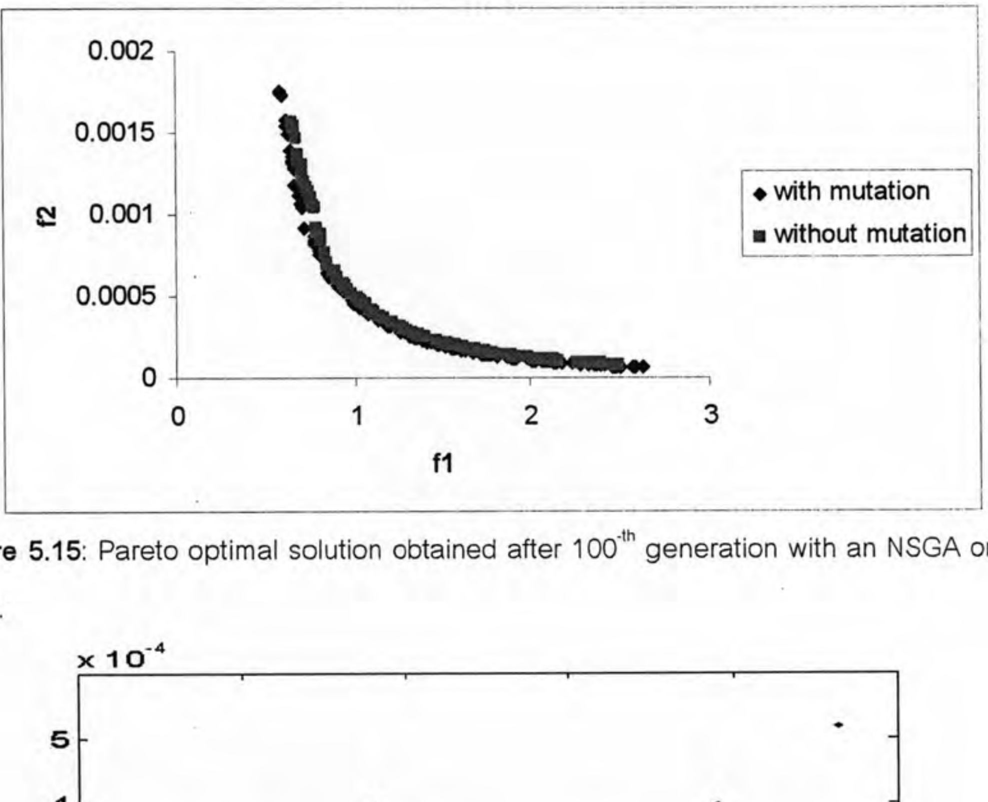

Figure 5.15: Pareto optimal solution obtained after 100^{-th} generation with an NSGA on problem No.2.

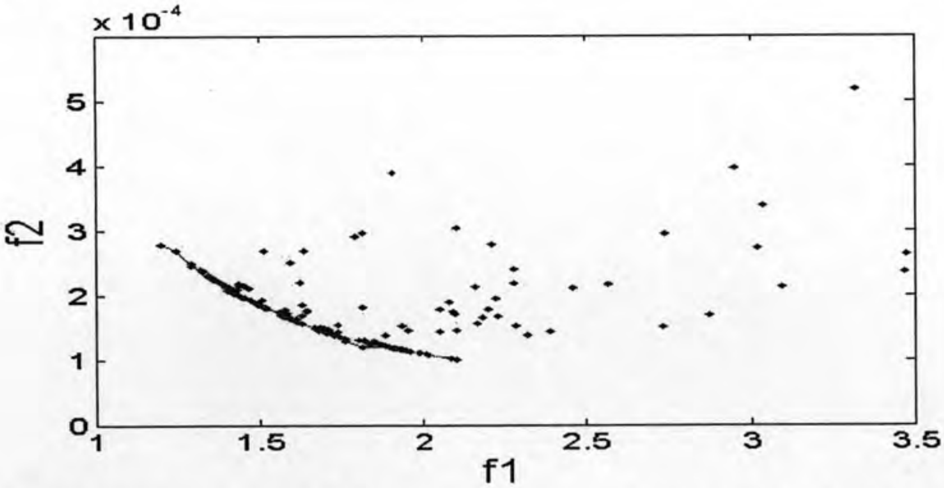

Figure 5.16: Population after 100^{-th} generation obtained with an NSGA (mutation probability 0.12 and crossover probability 0.6) on problem N0.2.

The influence of crossover probability of problem No.2 is similar with problem No.1. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations but if too large crossover probability is used, the population quickly loses its diversity. Consequently, the population converges to only one portion of Pareto-optimal front. But when the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front.

- In MOGA, the value of crossover probability between the ranges of 0.4-0.85 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.85, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.4 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an MOGA is shown in Fig. 5.17.
- In NSGA, the value of crossover probability between the ranges of 0.4-0.85 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.85, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.4 is used, the obtained Pareto-optimal solution is away from the true actual Pareto-front. Pareto optimal solution obtained with an NSGA is shown in Fig. 5.18.
- In NPGA, the value of crossover probability between the ranges of 0.4-0.8 can produce a good Pareto-optimal solution. If the value of crossover probability was set lager than 0.8, the algorithm cannot find a good spread of Pareto-optimal front. The true Pareto-optimal front cannot be obtained, if the crossover probability was set lower than 0.4. Pareto optimal solution obtained with an NPGA is shown in Fig. 5.19.

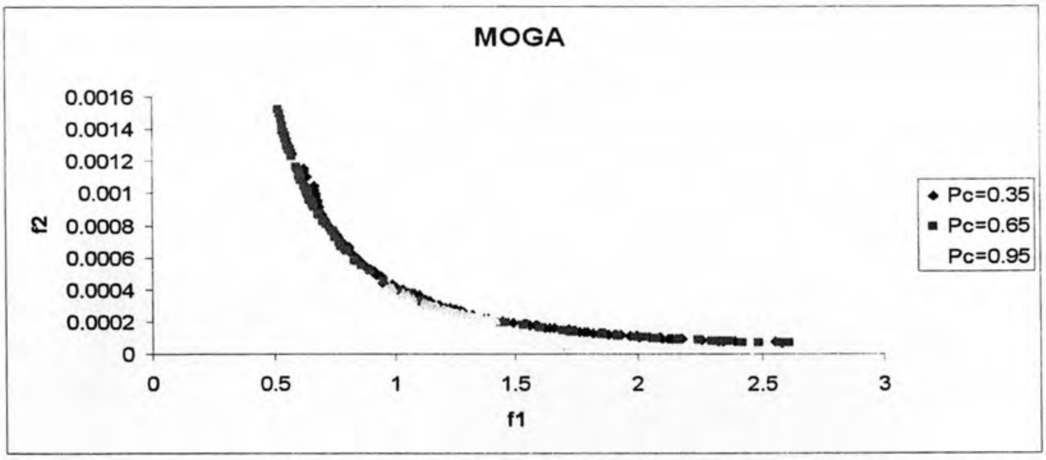

Figure 5.17: The Pareto-optimal solution for problem No.2 after 100-th generation obtained with an MOGA (mutation probability 0.05).

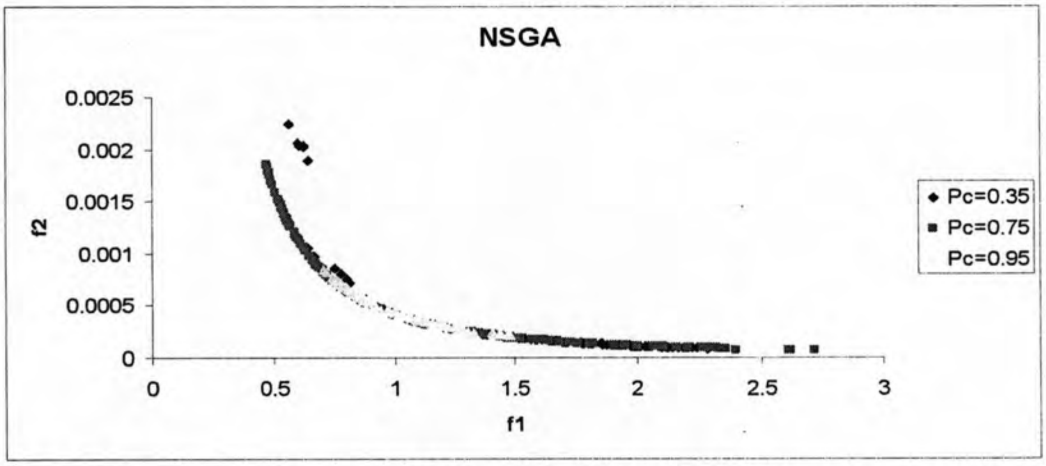

Figure 5.18: The Pareto-optimal solution for problem No.2 after 100-th generation obtained with an NSGA (mutation probability 0.05).

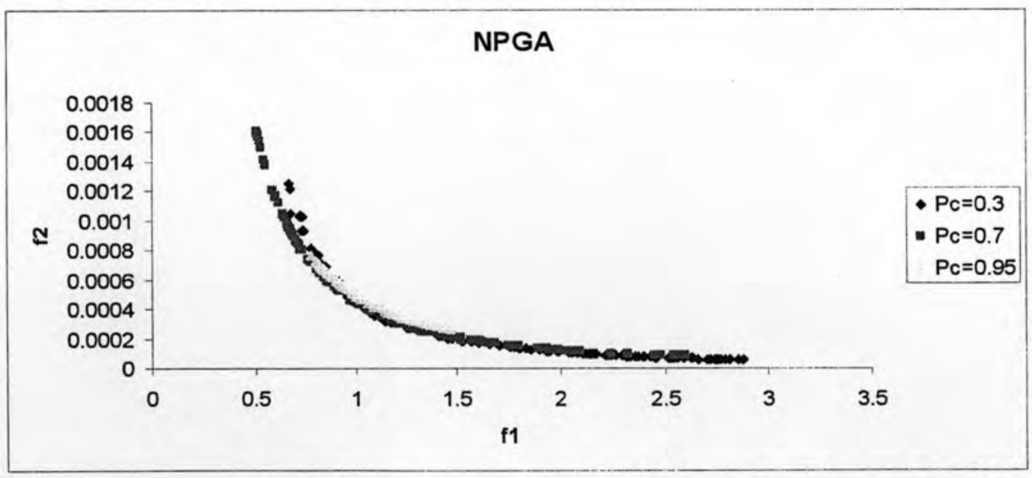

Figure 5.19: The Pareto-optimal solution for problem No.2 after 100-th generation obtained with an NPGA (mutation probability 0.05).

The distribution of solutions on the obtained Pareto optimal front was determined. In MOGA NSGA and NPGA, the value of crossover probability between the ranges of 0.4-0.8 can produce a Pareto-optimal solution with a similar average niche value. The relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability is shown in Fig. 5.20.

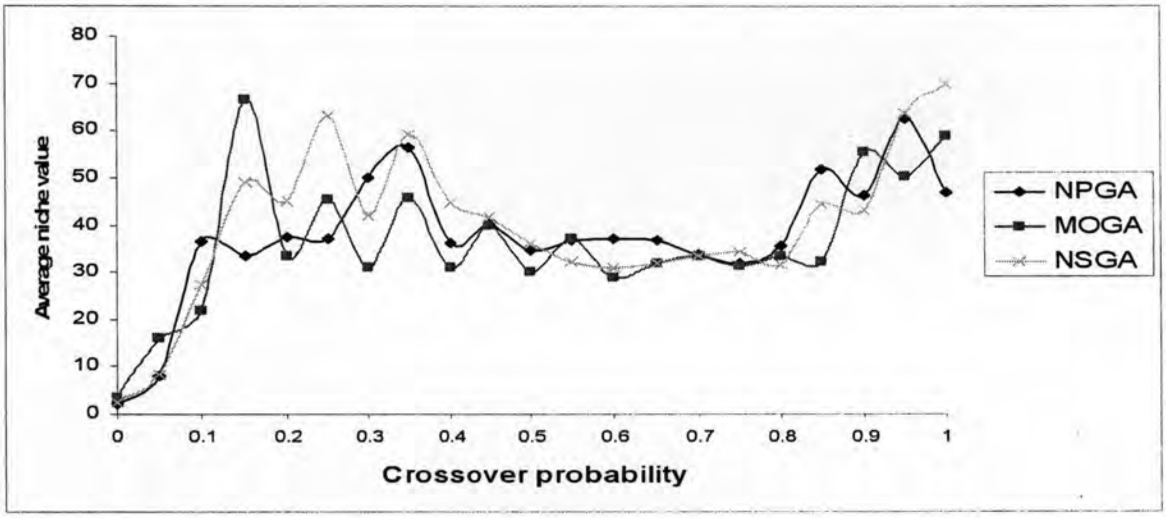

Figure 5.20: Relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability with MOGA, NSGA and NPGA on problem No.2.

Case Elitist Multi-Objective Evolutionary Algorithms

Next, we applied the NSGA-II and SPEA on problem No.2. Both algorithms without the mutation operator cannot find the true Pareto-optimal solutions as shown in Fig. 5.21. When we used algorithms with mutation operator, the mutation probability value has a small effect to find a good spread of Pareto-optimal solutions as shown in Fig. 5.22.

The effect of crossover probability was analyzed. When the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front for both algorithms. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations.

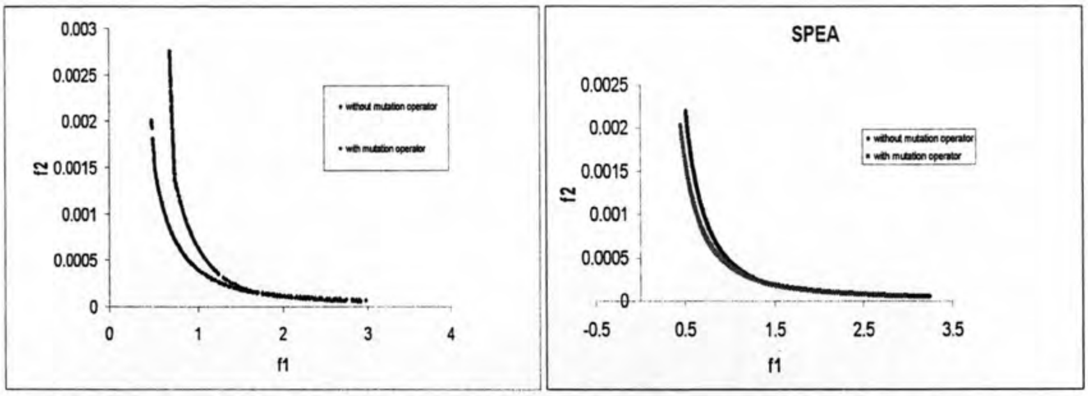

Figure 5.21: Pareto optimal solution obtain of problem No.2 with an NSGA-II (left) and an SPEA (right) for compare both algorithms with mutation operator and without mutation operator.

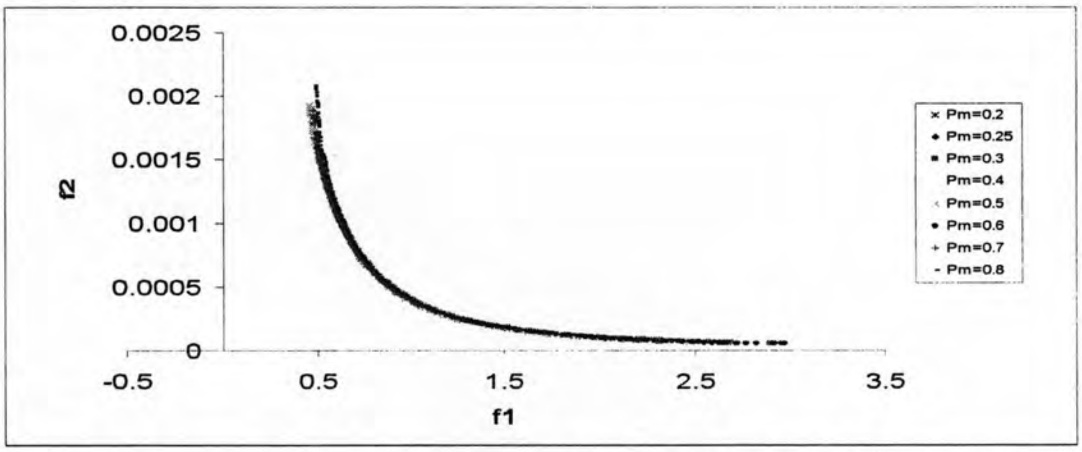

Figure 5.22: Pareto optimal solution obtained of problem No.2 after 100-th generation with an SPEA (show effect of mutation probability).

-In NSGA-II, the value of crossover probability between the ranges of 0.4-1 can produce a good Pareto-optimal solution. The Pareto front obtained is close to the true Pareto optimal front. Figure 5.23 shows the obtained Pareto optimal front.

-In SPEA, the value of crossover probability between the ranges of 0.4-1 can produce a good Pareto-optimal solution. The Pareto front obtained shows in Fig. 5.24.

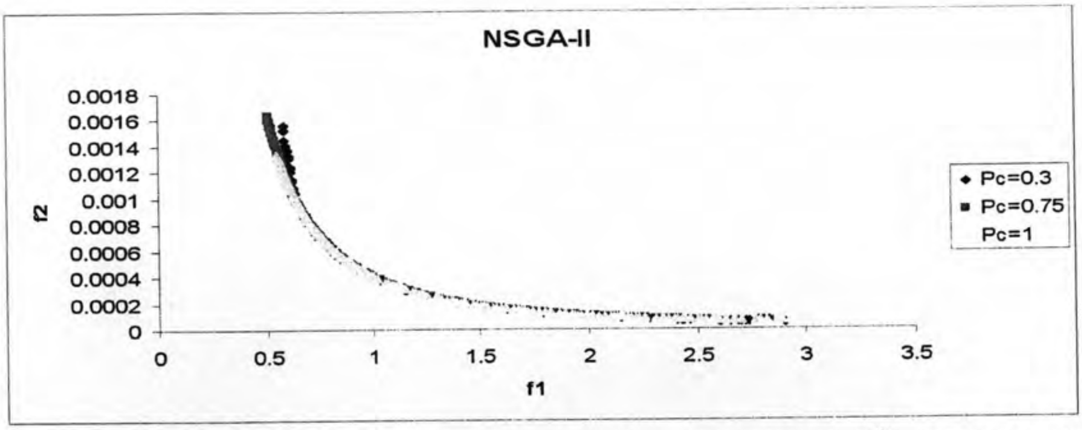

Figure 5.23: Pareto optimal solution obtained of problem No.2 after 100^{-th} generation with an NSGA-II (show effect of crossover probability).

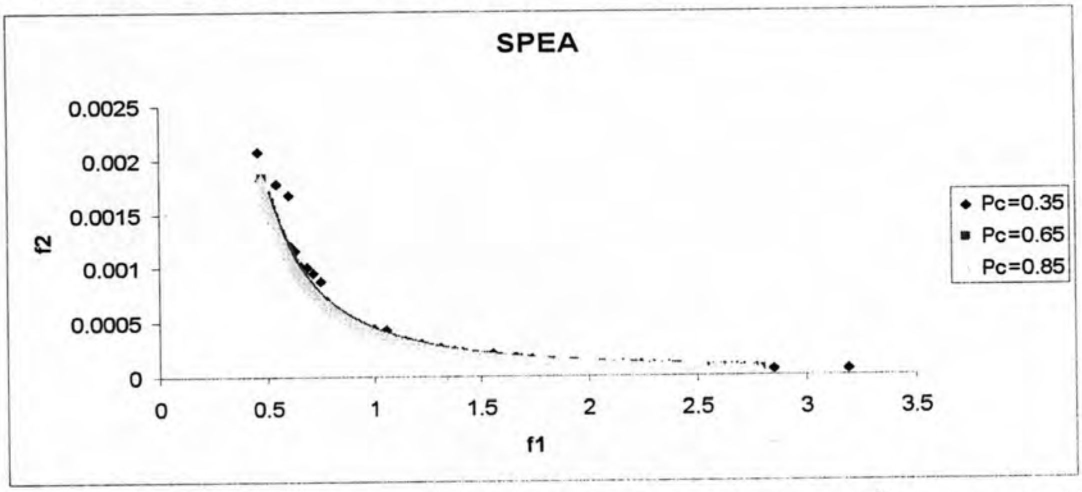

Figure 5.24: Pareto optimal solution obtained of problem No.2 after 100^{-th} generation with an SPEA (show effect of crossover probability)

Next, we determined distribution of solutions on the obtained Pareto front. In NSGA-II, the value of crossover probability between the ranges of 0.4-1 can produce the obtained Pareto optimal front with a similar average distance of solution on obtained Pareto optimal front. NSGA-II with crossover probability in this rank can create all solutions converged to true Pareto optimal solution. In SPEA, the value of crossover between the ranges of 0.55-1 can create the size of external population more than the population size.

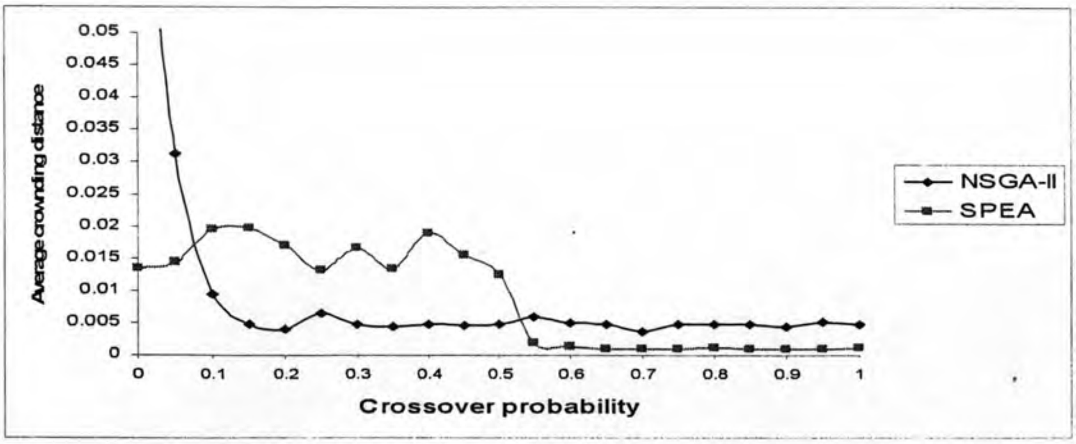

Figure 5.25: Relationship between average crowding distance and crossover probability of problem No.2.

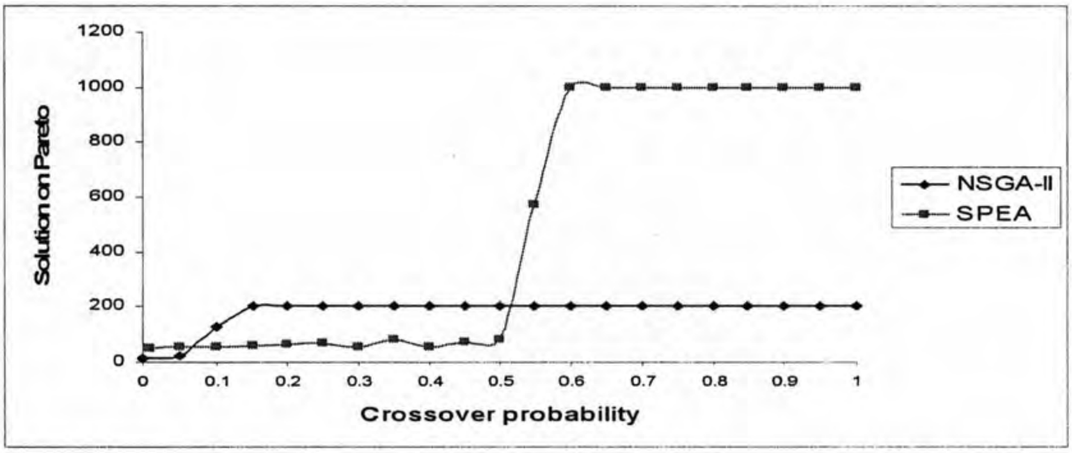

Figure 5.26: Relationship between number of solution on Pareto optimal front and crossover probability of problem No.2.

Hence, in this problem, we conclude that the optimal value of mutation probability is between 0.02 - 0.1 for Non-Elitist Multi-Objective Evolutionary Algorithms and the optimal value of mutation probability is more than 0.02 for Elitist Multi-Objective Evolutionary Algorithms. While the suitable value of crossover probability for MOGA, NPGA and NSGA is between 0.4 and 0.8, and the suitable value of crossover probability for NSGA-II is between 0.4-1 and between 0.55-1 for SPEA.

5.1.3 Problem No 3:

$$\text{Minimize} \quad f_1(x) = 1 - \exp(-\sum_{i=1}^{n}(x_i - \frac{1}{\sqrt{n}})^2$$

$$\text{Minimize} \quad f_2(x) = 1 - \exp(-\sum_{i=1}^{n}(x_i + \frac{1}{\sqrt{n}})^2 \qquad (5.3)$$

$$\text{Subject to} \quad -4 \le x_i \le 4$$

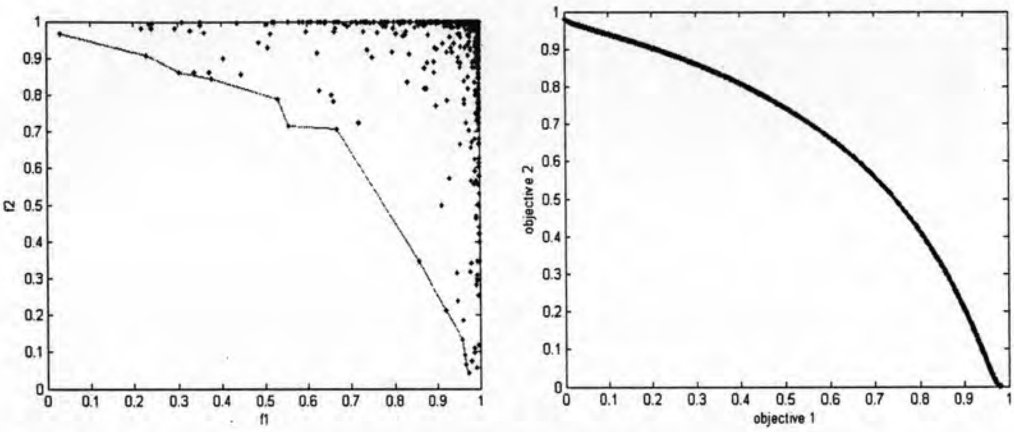

Figure 5.27: Feasible objective space of problem No.3 shown in the left hand side and the true Pareto optimal solution of problem No.3 shown in the left hand side.

The characteristic of feasible objective space and true Pareto optimal solution of problem No.3 are shown in Fig. 5.27. The true Pareto optimal solution is non-convex set.

Case Non-Elitist Multi-Objective Evolutionary Algorithms

We applied the MOGA, NSGA and NPGA on problem No.3. First, we used the algorithms without a mutation operator. The results show that three algorithms without the mutation operator can find the true Pareto optimal solution. But the obtained Pareto optimal front isn't good spread of solution on Pareto front. It is shown in Fig. 5.28. When three algorithms are applied to non-uniform mutation, a different scenario emerges. The algorithms can find the good spread of solution on Pareto optimal front, but when the value of mutation probability is larger than 0.3, algorithms show inability to find a good spread of the Pareto-optimal solutions. It is shown in Fig. 5.29.

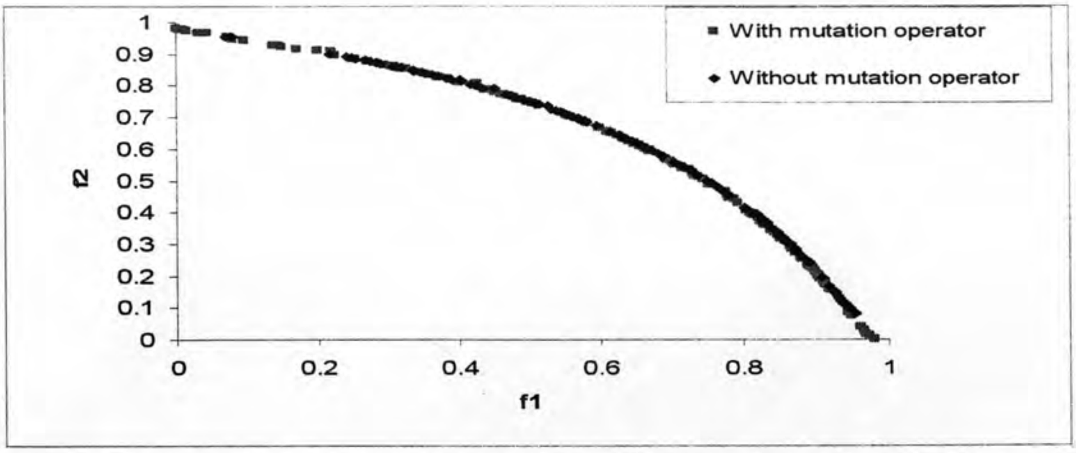

Figure 5.28: Pareto optimal solution obtained after 100$^{\text{-th}}$ generation with an NPGA on problem No.3.

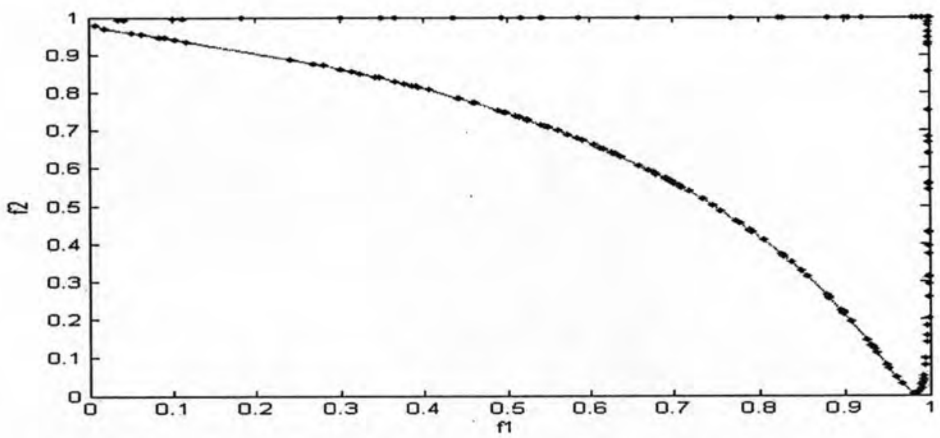

Figure 5.29: Population after 100$^{\text{-th}}$ generation obtained with an NPGA (mutation probability 0.3 and crossover probability 0.6) on problem No.3.

The influence of crossover probability was analyzed. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations but if too large crossover probability is used, the population quickly loses its diversity. Consequently, the population converges to only one portion of Pareto-optimal front. But when the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front.

- In MOGA, the value of crossover probability between the ranges of 0.15-0.95 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.95, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.15 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an MOGA is shown in Fig. 5.30.

- In NSGA, the value of crossover probability between the ranges of 0.1-0.9 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.9, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.1 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an NSGA is shown in Fig. 5.31.
- In NPGA, the value of crossover probability between the ranges of 0.1-0.95 can produce a good Pareto-optimal solution. If the value of crossover probability was set lager than 0.95, the algorithm cannot find a good spread of Pareto-optimal front. The true Pareto-optimal front cannot be obtained, if the crossover probability was set lower than 0.1. Pareto optimal solution obtained with an NPGA is shown in Fig. 5.32.

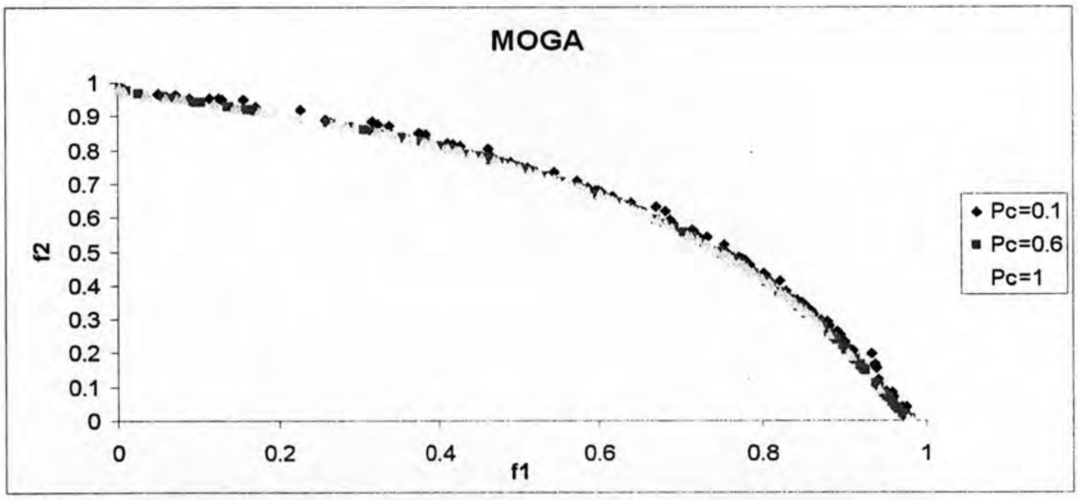

Figure 5.30: The Pareto-optimal solution for problem No.3 after 100^{-th} generation obtained with an MOGA (mutation probability 0.05).

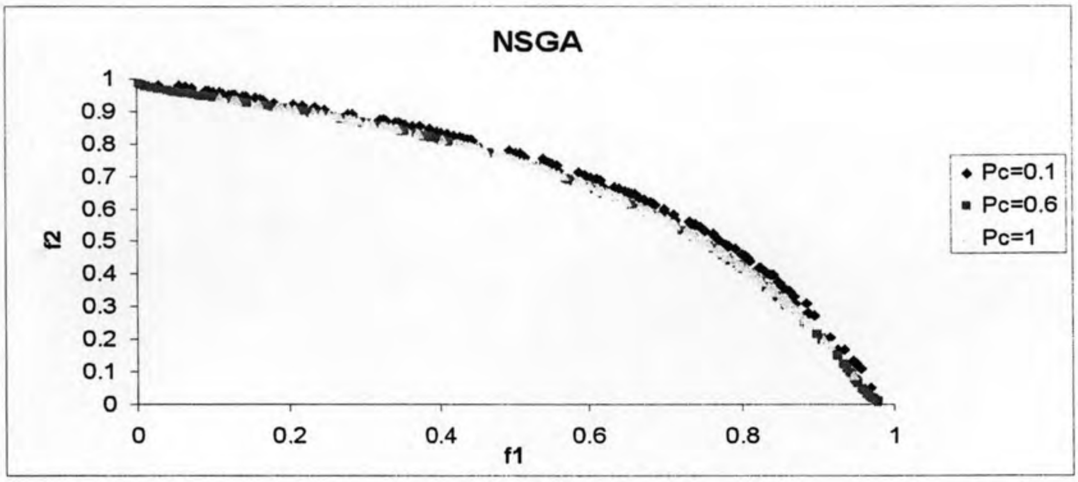

Figure 5.31: The Pareto-optimal solution for problem No.3 after 100-th generation obtained with an NSGA (mutation probability 0.05).

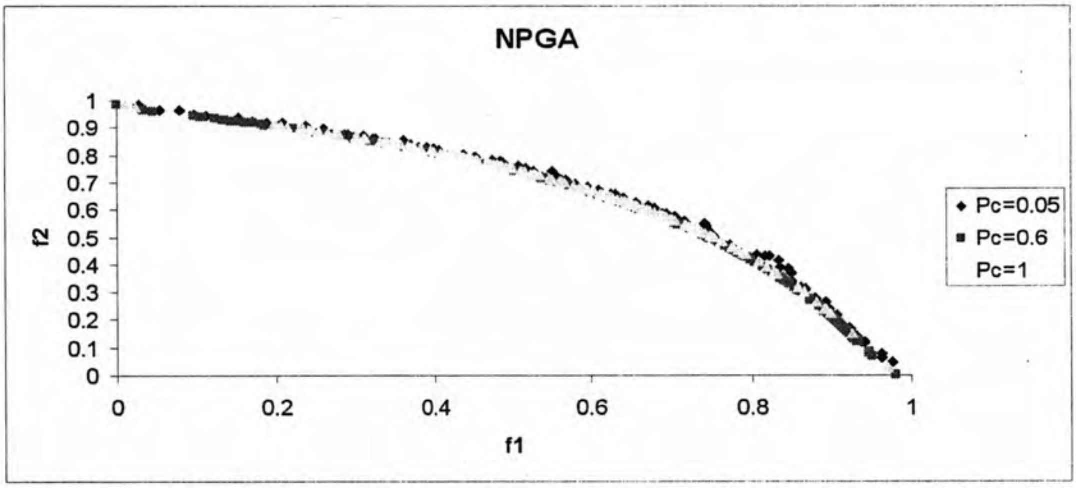

Figure 5.32: The Pareto-optimal solution for problem No.3 after 100-th generation obtained with an NPGA (mutation probability 0.05).

The distribution of solutions on the obtained Pareto optimal front was determined. In MOGA NSGA and NPGA, the value of crossover probability between the ranges of 0.4-0.9 can produce a Pareto-optimal solution with a similar average niche value. The relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability is shown in Fig. 5.33

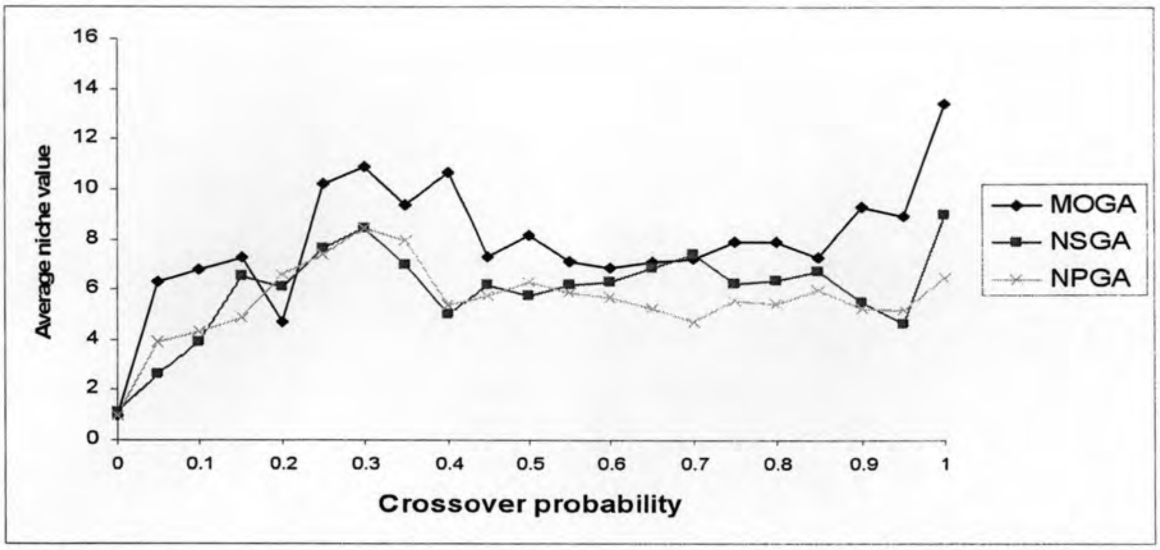

Figure 5.33: Relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability with MOGA, NSGA and NPGA on problem No.3.

Case Elitist Multi-Objective Evolutionary Algorithms

Next, we applied the NSGA-II and SPEA on problem No.3. Both algorithms without the mutation operator can find the true Pareto-optimal solutions with a good spread of Pareto-optimal solutions show in Fig. 5.34. The value of mutation probability has non effect for both algorithms on problem No.3

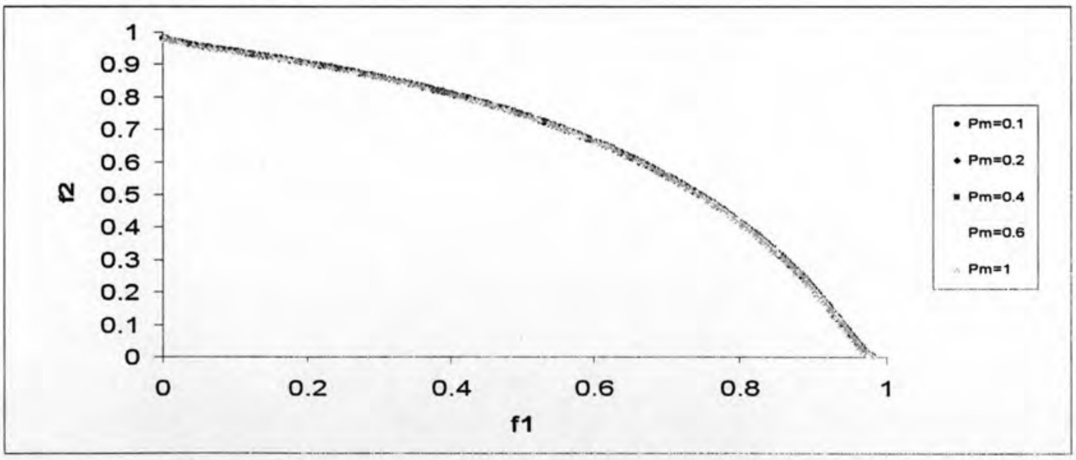

Figure 5.34: Effect of the value of mutation probability on problem No.3 after 100th generation with an NSGA-II.

The effect of crossover probability was analyzed. When the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front for both algorithms. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations.

-In NSGA-II, the value of crossover probability between the ranges of 0.1-1 can produce a good Pareto-optimal solution. The Pareto front obtained is close to the true Pareto optimal front. Figure 5.35 shows the obtained Pareto optimal front.

-In SPEA, the value of crossover probability between the ranges of 0.1-1 can produce a good Pareto-optimal solution. The Pareto front obtained shows in Fig. 5.36.

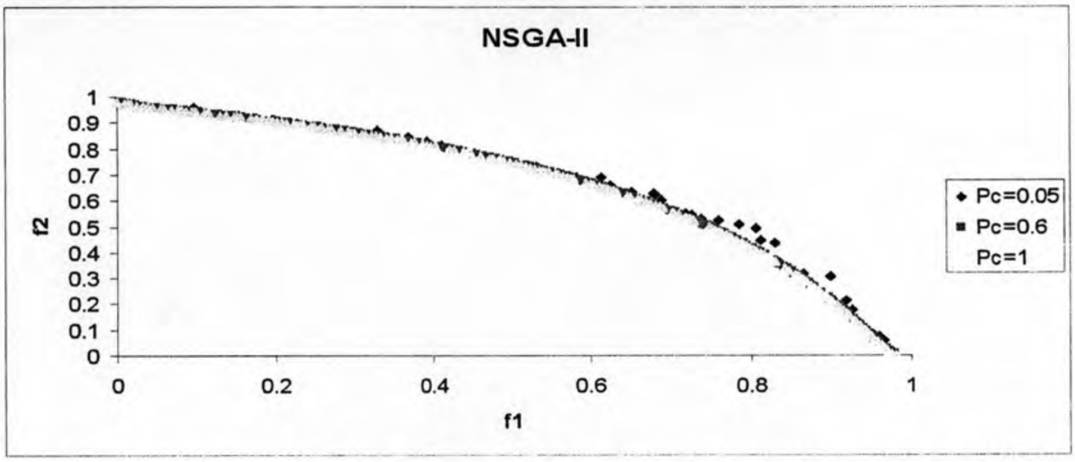

Figure 5.35: Pareto optimal solution obtained of problem No.3 after 100^{-th} generation with an NSGA-II (effect of crossover probability).

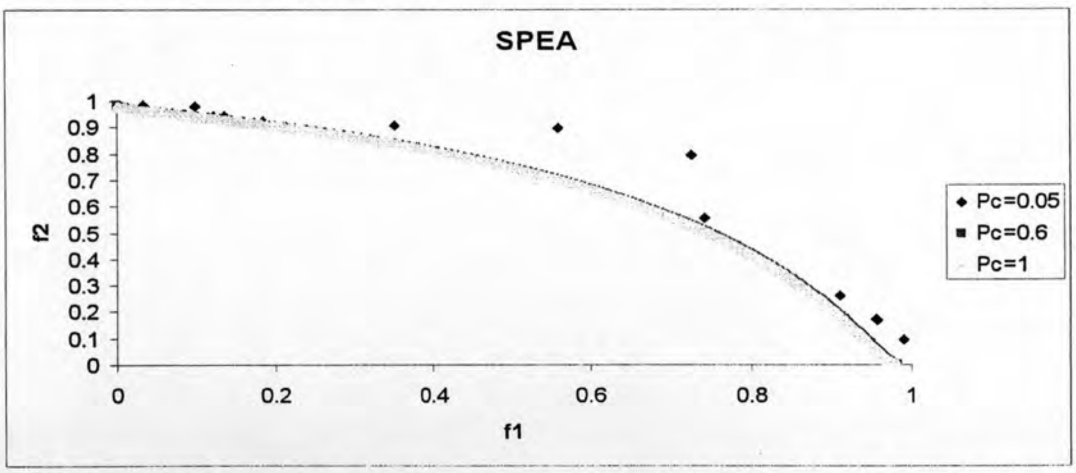

Figure 5.36: Pareto optimal solution obtained of problem No.3 after 100^{-th} generation with an SPEA (effect of crossover probability).

Next, we determined distribution of solutions on the obtained Pareto front. In NSGA-II, the value of crossover probability between the ranges of 0.1-1 can produce the obtained Pareto optimal front with a similar average distance of solution on obtained Pareto optimal front. NSGA-II with crossover probability in this rank can create all solutions converged to true Pareto optimal solution. In SPEA, the value of crossover between the ranks of 0.2-1 can create the size of external population more than the population size.

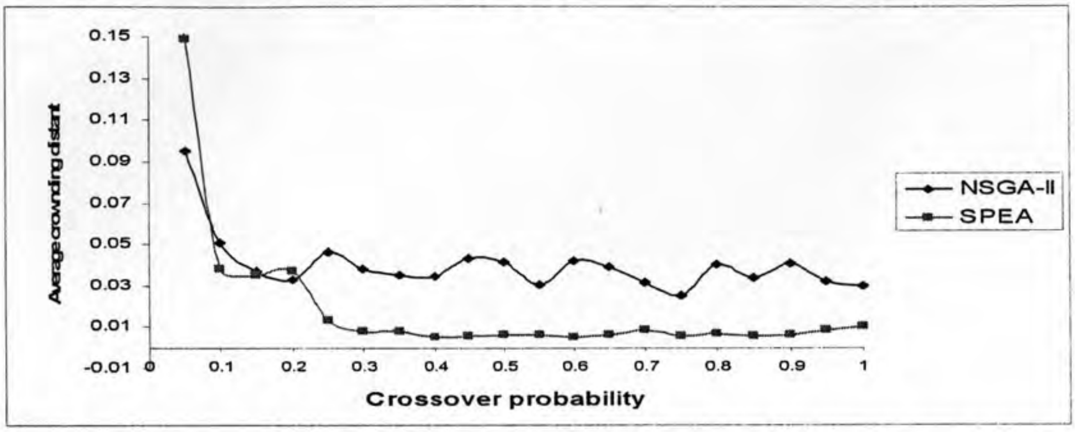

Figure 5.37: Relationship between average crowding distance and crossover probability of problem No.3.

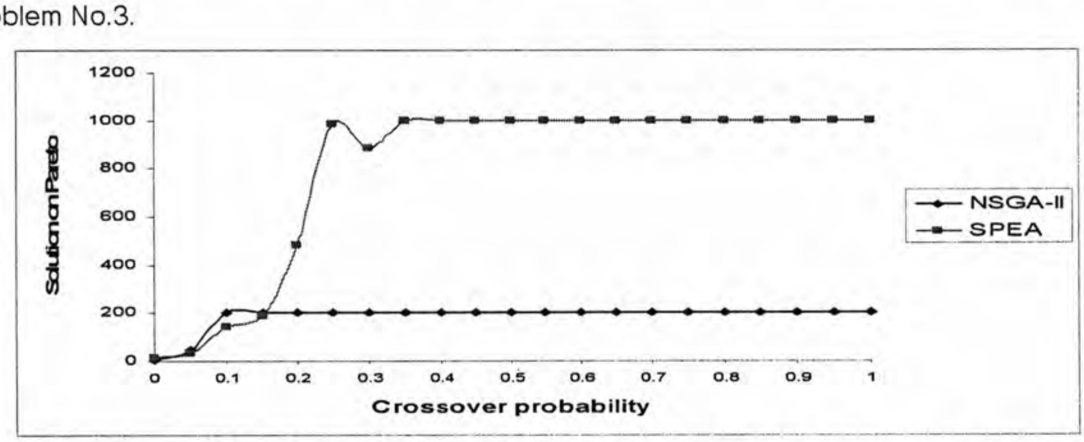

Figure 5.38: Relationship between number of solution on Pareto optimal front and crossover probability of problem No.3.

Hence, in this problem, we conclude that the optimal value of mutation probability is between 0.02 for Non-Elitist Multi-Objective Evolutionary Algorithms. The value of mutation probability has small effect for Elitist Multi-Objective Evolutionary Algorithms. While the suitable value of crossover probability for MOGA, NPGA and NSGA is between 0.4 and 0.9, and the suitable value of crossover probability for NSGA-II is between 0.1-1 and between 0.2-1 for SPEA.

5.1.4 Problem No 4:

$$
\begin{aligned}
&\text{Minimize} \quad f_1(x) = [1 + (A_1 - B_1)^2 + (A_2 - B_2)^2] \\
&\text{Minimize} \quad f_2(x) = (x+3)^2 + (y+1)^2 \qquad (5.4) \\
&\quad A_1 = 0.5\sin 1 - 2\cos 1 + \sin 2 - 1.5\cos 2 \\
&\textit{where} \quad A_2 = 1.5\sin 1 - \cos 1 - 2\sin 2 - 0.5\cos 2 \\
&\quad B_1 = 0.5\sin x - 2\cos x + \sin y - 1.5\cos y \\
&\quad B_2 = 1.5\sin x - \cos x - 2\sin y - 0.5\cos y
\end{aligned}
$$

$$
\text{Subject to} \quad \begin{aligned} -\pi \le x \le \pi \\ -\pi \le y \le \pi \end{aligned}
$$

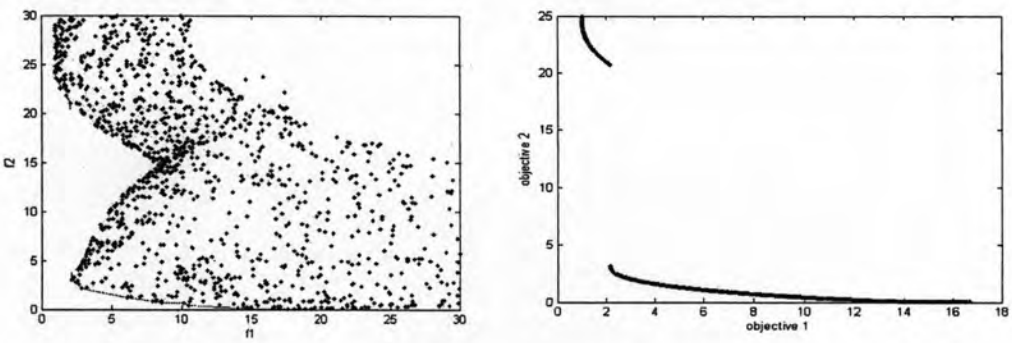

Figure 5.39: Feasible objective space of problem No.4 shown in the left hand side and the true Pareto optimal solution of problem No.4 shown in the left hand side.

The characteristic of feasible objective space and true Pareto optimal solution of problem No.4 are shown in Fig. 5.39. This problem has a local Pareto optimal front, so that it is very difficulty to find the global Pareto optimal solution.

Case Non-Elitist Multi-Objective Evolutionary Algorithms

We applied the MOGA, NSGA and NPGA on problem No.4. First, we used the algorithms without a mutation operator. The results show that three algorithms without the mutation operator cannot find the true Pareto optimal solution. It is shown in Fig. 5.40. When three algorithms are applied to non-uniform mutation, a different scenario emerges. The algorithms can find more solution near the true Pareto optimal solution, but when the value of mutation probability is larger than 0.1, algorithms show inability to find a good spread of the Pareto-optimal solutions.

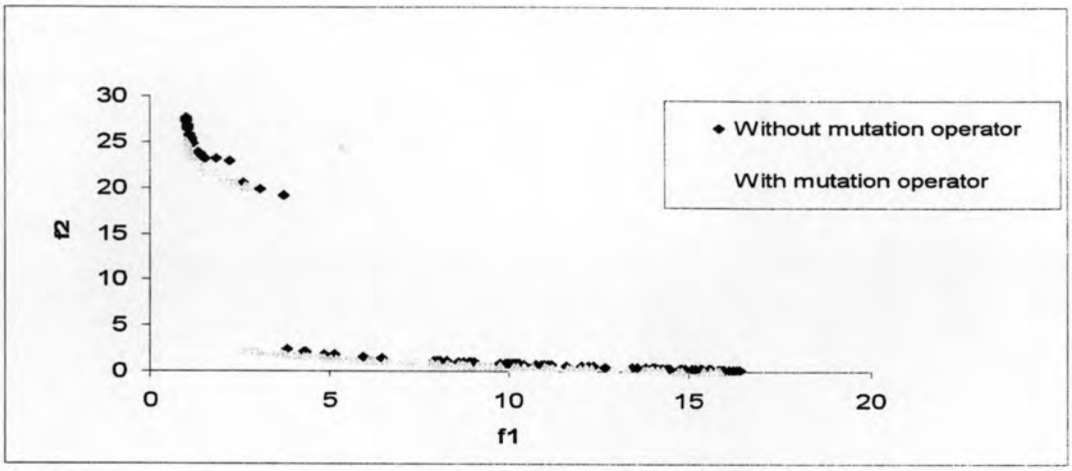

Figure 5.40: Pareto optimal solution obtained after 100^{-th} generation with an NSGA on problem No.4.

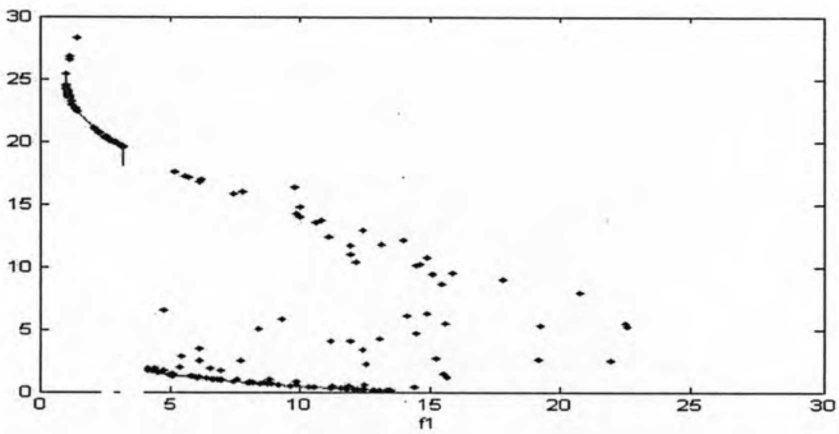

Figure 5.41: Population after 100^{-th} generation obtained with an NSGA (mutation probability 0.12 and crossover probability 0.6) on problem No.4.

The influence of crossover probability was analyzed. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations but if too large crossover probability is used, the population quickly loses its diversity. Consequently, the population converges to local Pareto-optimal front. But when the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front.

- In MOGA, the value of crossover probability between the ranges of 0.4-0.65 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.65, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.4 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an MOGA is shown in Fig. 5.42.

- In NSGA, the value of crossover probability between the ranges of 0.4-0.6 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.6, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.4 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an NSGA is shown in Fig. 5.43.
- In NPGA, the value of crossover probability between the ranges of 0.4-0.6 can produce a good Pareto-optimal solution. If the value of crossover probability was set lager than 0.8, the algorithm cannot find a good spread of Pareto-optimal front. The true Pareto-optimal front cannot be obtained, if the crossover probability was set lower than 0.4. Pareto optimal solution obtained with an NPGA is shown in Fig. 5.44.

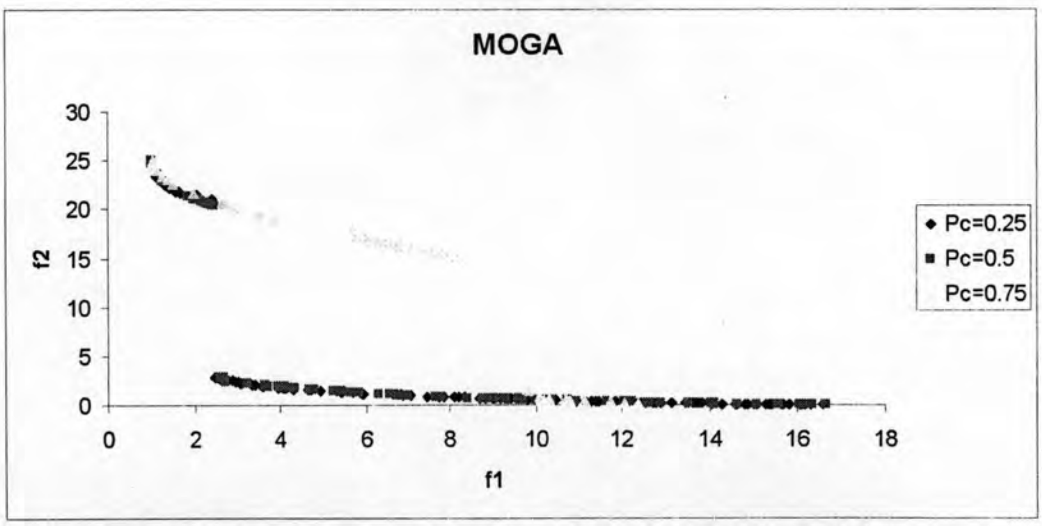

Figure 5.42: The Pareto-optimal solution for problem No.4 after 100^{-th} generation obtained with an MOGA (mutation probability 0.05).

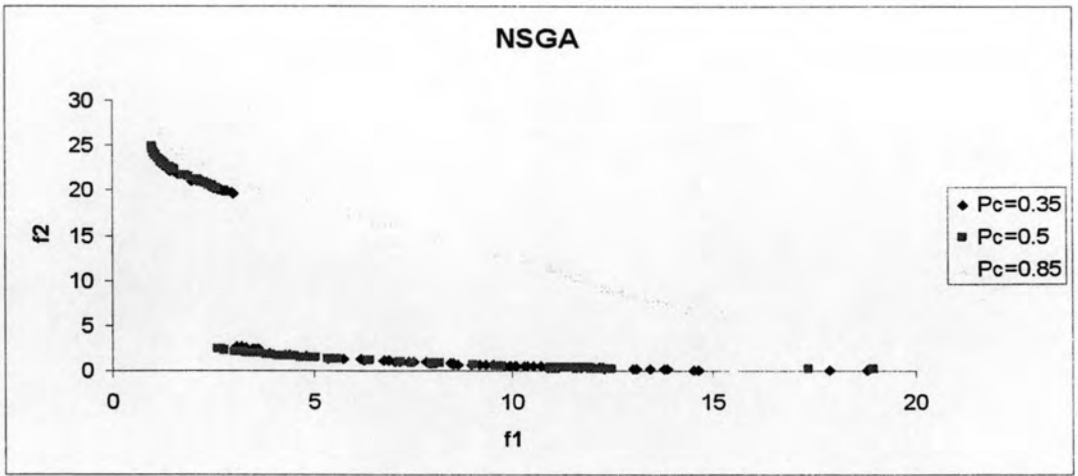

Figure 5.43: The Pareto-optimal solution for problem No.4 after 100^{-th} generation obtained with an NSGA (mutation probability 0.05).

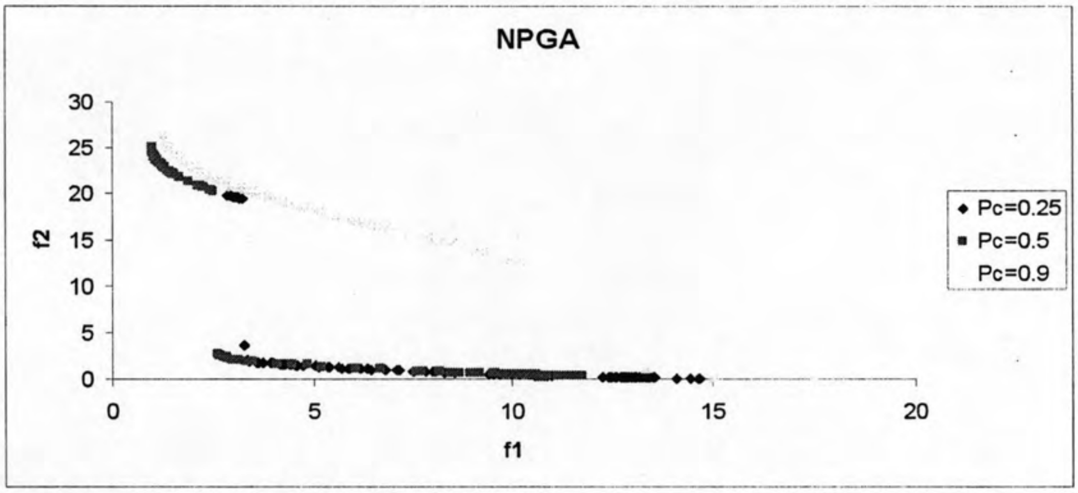

Figure 5.44: The Pareto-optimal solution for problem No.4 after 100^{-th} generation obtained with an NPGA (mutation probability 0.05).

The distribution of solutions on the obtained Pareto optimal front was determined. In MOGA NSGA and NPGA, the value of crossover probability between the ranges of 0.4-0.6 can produce a Pareto-optimal solution with a similar average niche value. The relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability is shown in Fig. 5.45.

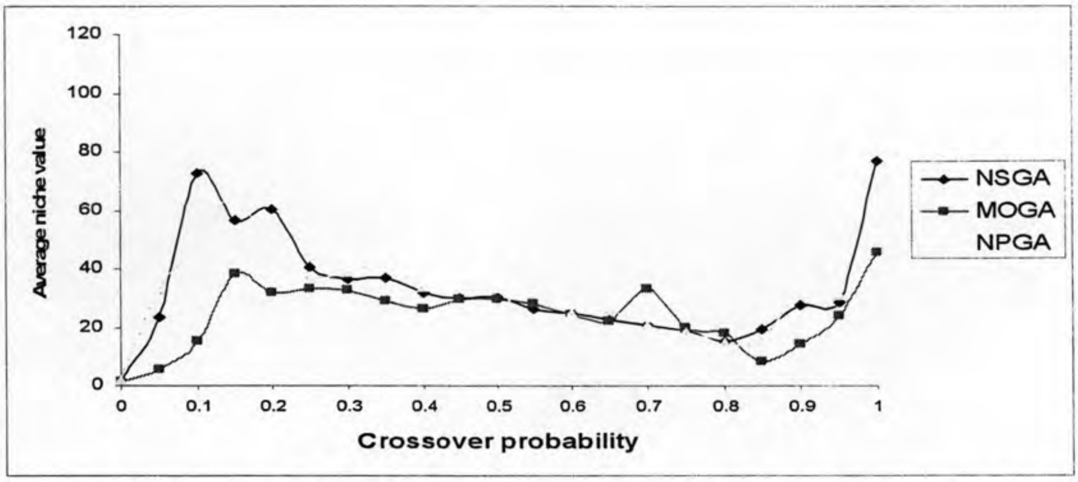

Figure 5.45: Relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability with MOGA, NSGA and NPGA on problem No.4.

Case Elitist Multi-Objective Evolutionary Algorithms

Next, we applied the NSGA-II and SPEA on problem No.4. Both algorithms without the mutation operator cannot find the true Pareto-optimal solutions as shown in Fig. 5.46. When we used algorithms with mutation operator, the mutation probability value has a small effect to find a good spread of Pareto-optimal solutions as shown in Fig. 5.47.

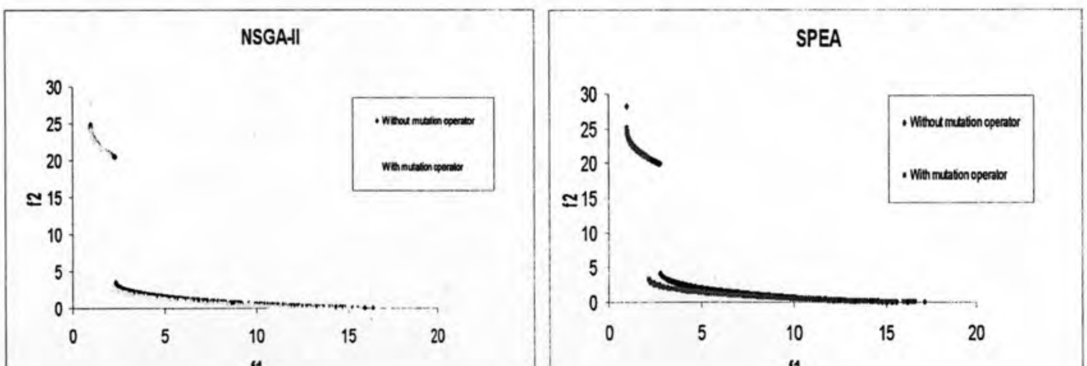

Figure 5.46: Pareto optimal solution obtained of problem No.4 after $100^{\text{-th}}$ generation with an NSGA-II (left) and an SPEA (right) for compare both algorithms with mutation operator and without mutation operator.

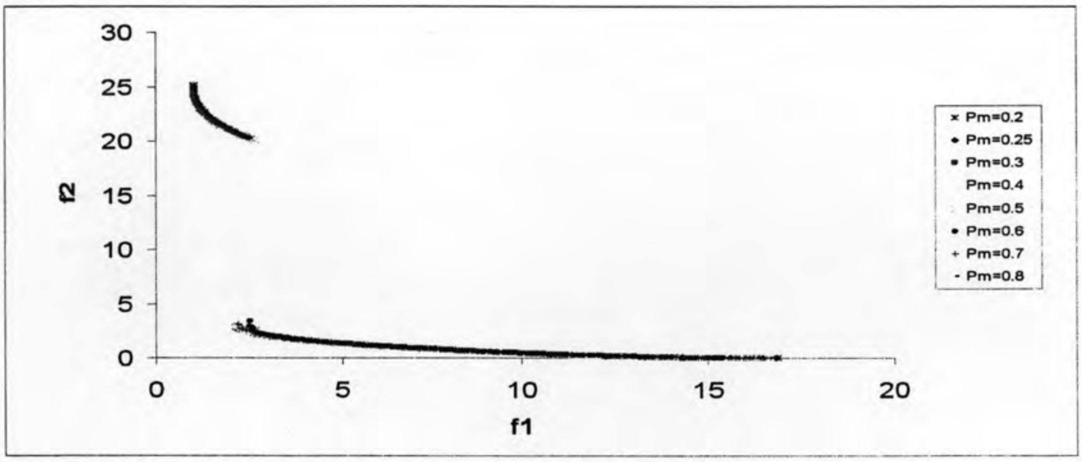

Figure 5.47: Effect of the value of mutation probability on problem No.4 after 100^{-th} generation with an SPEA.

The effect of crossover probability was analyzed. When the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front for both algorithms. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations.

-In NSGA-II, the value of crossover probability between the ranges of 0.15-1 can produce a good Pareto-optimal solution. The Pareto front obtained is close to the true Pareto optimal front. Figure 5.48 shows the obtained Pareto optimal front.

-In SPEA, the value of crossover probability between the ranges of 0.15-1 can produce a good Pareto-optimal solution. The Pareto front obtained shows in Fig. 5.49.

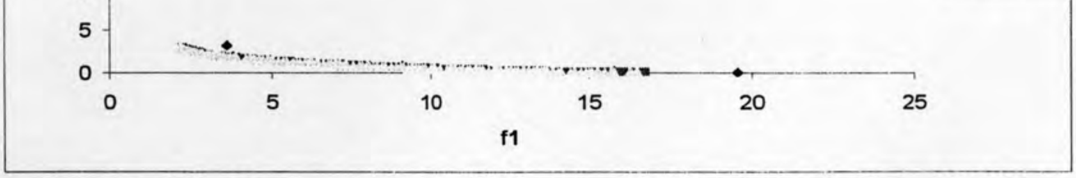

Figure 5.48: Pareto optimal solution obtained of problem No.4 after 100^{-th} generation with an NSGA-II (effect of crossover probability).

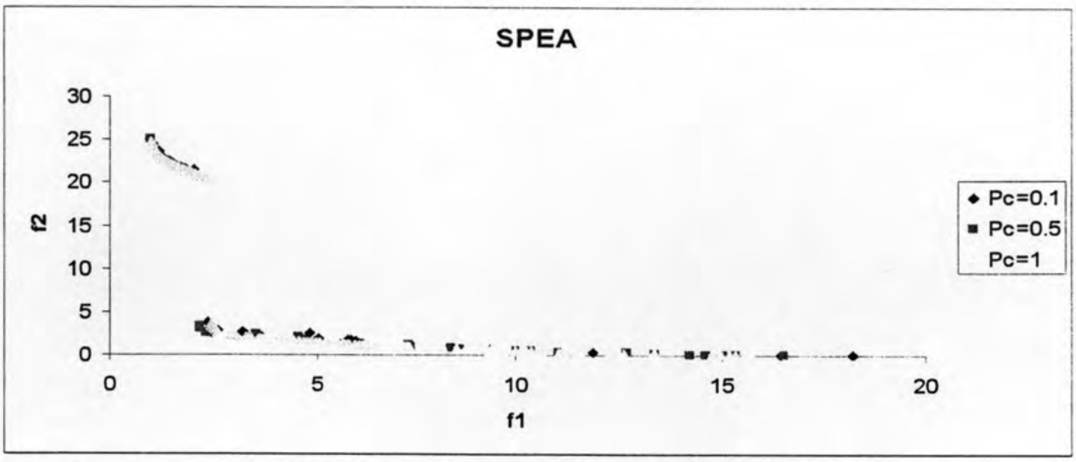

Figure 5.49: Pareto optimal solution obtained of problem No.4 after 100-th generation with an NSGA-II (effect of crossover probability).

Next, we determined distribution of solutions on the obtained Pareto front. In NSGA-II, the value of crossover probability between the ranges of 0.15-1 can produce the obtained Pareto optimal front with a similar average distance of solution on obtained Pareto optimal front. NSGA-II with crossover probability in this rank can create all solutions converged to true Pareto optimal solution. In SPEA, the value of crossover between the ranges of 0.55-1 can create the size of external population more than the population size.

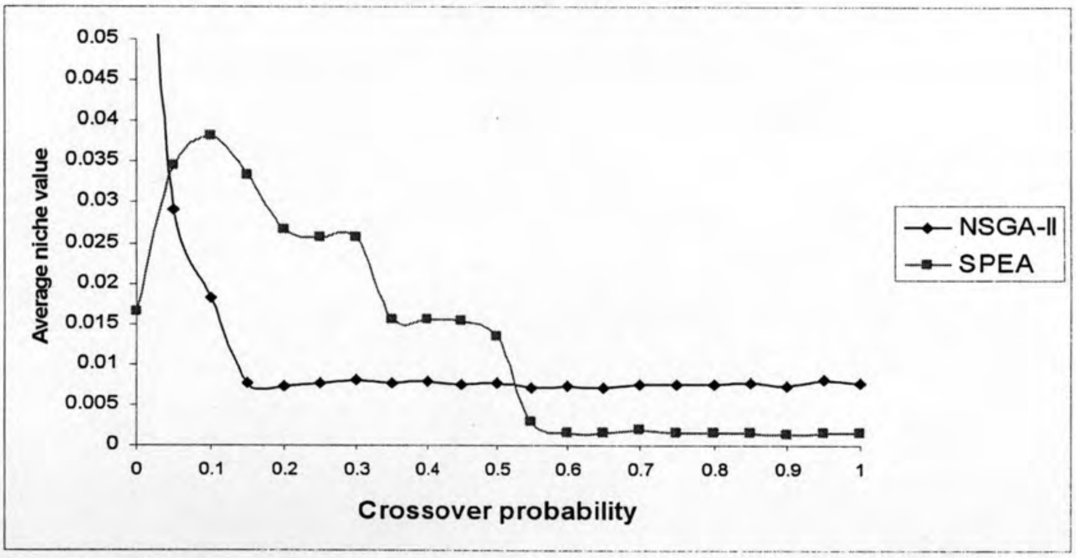

Figure 5.50: Relationship between average crowding distance and crossover probability of problem No.4.

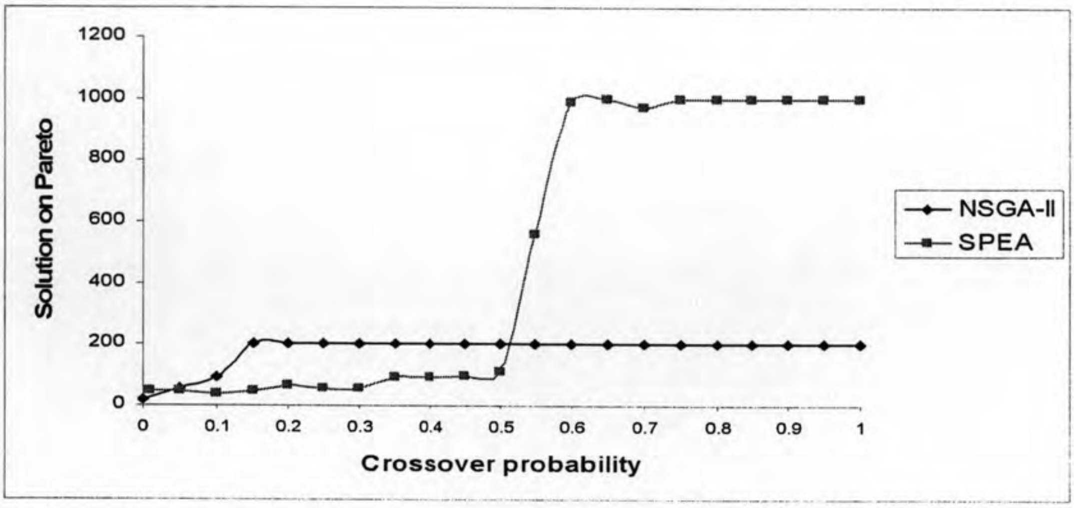

Figure 5.51: Relationship between number of solution on Pareto optimal front and crossover probability of problem No.4.

Hence, in this problem, we conclude that the optimal value of mutation probability is between 0.02 - 0.1 for Non-Elitist Multi-Objective Evolutionary Algorithms and the optimal value of mutation probability is more than 0.02 for Elitist Multi-Objective Evolutionary Algorithms. While the suitable value of crossover probability for MOGA, NPGA and NSGA is between 0.4 and 0.6, and the suitable value of crossover probability for NSGA-II is between 0.15-1 and between 0.55-1 for SPEA.

5.1.5 Problem No 5:

$$\text{Minimize} \quad f_1(x) = \sum_{i=1}^{n-1} (-10\exp(-0.2\sqrt{x_i^2 + x_{i+1}^2})$$

$$\text{Minimize} \quad f_2(x) = \sum_{i=1}^{n} (|x_i|^{0.8} + 5\sin(x_i)^3 \qquad (5.5)$$

$$\text{Subject to} \quad -5 \le x_i \le 5$$

The characteristic of feasible objective space and true Pareto optimal solution of problem No.5 are shown in Fig. 5.52. This problem is an exponential function in objective 1 and is a sinusoidal in objective 2. This problem has many local Pareto optimal fronts. So that it is very difficulty to find the global Pareto optimal solution.

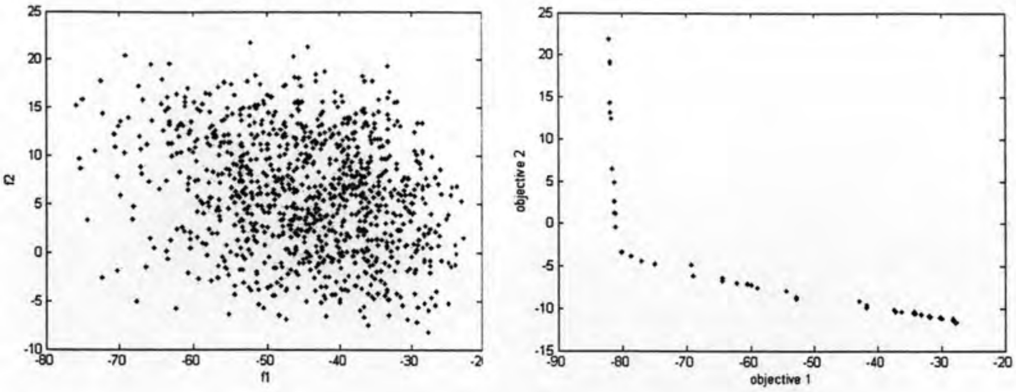

Figure 5.52: Feasible objective space of problem No.5 shown in the left hand side and the true Pareto optimal solution of problem No.5 shown in the left hand side.

Case Non-Elitist Multi-Objective Evolutionary Algorithms

We applied the MOGA, NSGA and NPGA on problem No.5. First, we used the algorithms without a mutation operator.. The results show that three algorithms without the mutation operator cannot find the true Pareto optimal solution. It is shown in Fig. 5.53. When three algorithms are applied to non-uniform mutation with mutation probability more than 0.04, a different scenario emerges. The algorithms can find more solution near the true Pareto optimal solution, but when the value of mutation probability is larger than 0.1, algorithms show inability to find a good spread of the Pareto-optimal solutions.

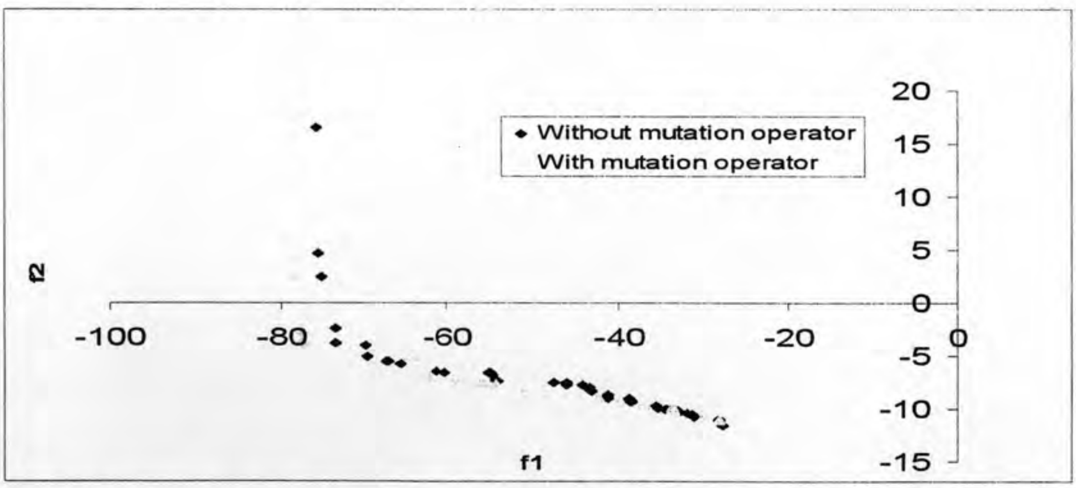

Figure 5.53: Pareto optimal solution obtained after 100-th generation with an NSGA on problem No.5.

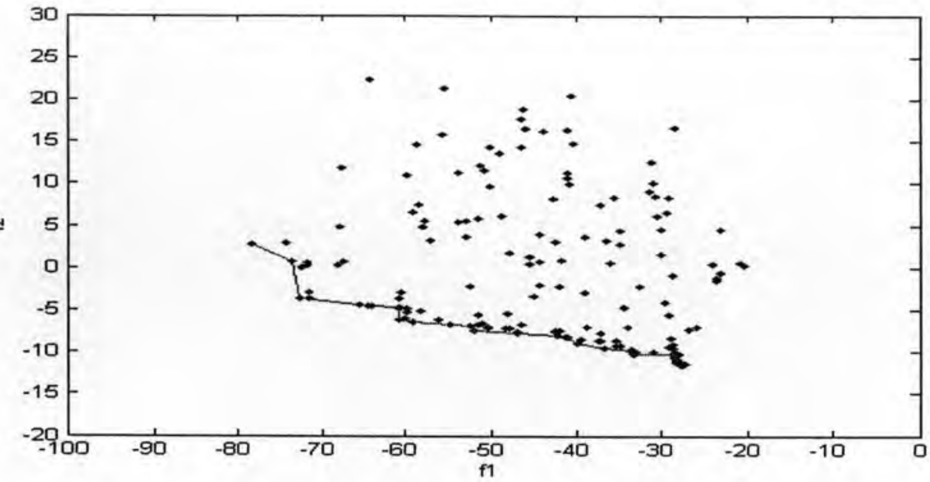

Figure 5.54: Population after 100^{-th} generation obtained with an NSGA (mutation probability 0.12 and crossover probability 0.5) on problem No.5

The influence of crossover probability was analyzed. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations but if too large crossover probability is used, the population quickly loses its diversity. Consequently, the population converges to local Pareto-optimal front. But when the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front.

- In MOGA, the value of crossover probability between the ranges of 0.4-0.6 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.6, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.4 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an MOGA is shown in Fig. 5.55.
- In NSGA, the value of crossover probability between the ranges of 0.4-0.6 can produce a good Pareto-optimal solution. In the case that the crossover probability was set larger than 0.6, the algorithm is unable to obtain a whole Pareto-optimal front. If crossover probability smaller than 0.4 is used, the obtained Pareto-optimal solution is away from the true Pareto-front. Pareto optimal solution obtained with an NSGA is shown in Fig. 5.56.

- In NPGA, the value of crossover probability between the ranges of 0.4-0.6 can produce a good Pareto-optimal solution. If the value of crossover probability was set lager than 0.6, the algorithm cannot find a good spread of Pareto-optimal front. The true Pareto-optimal front cannot be obtained, if the crossover probability was set lower than 0.4. Pareto optimal solution obtained with an NPGA is shown in Fig. 5.57.

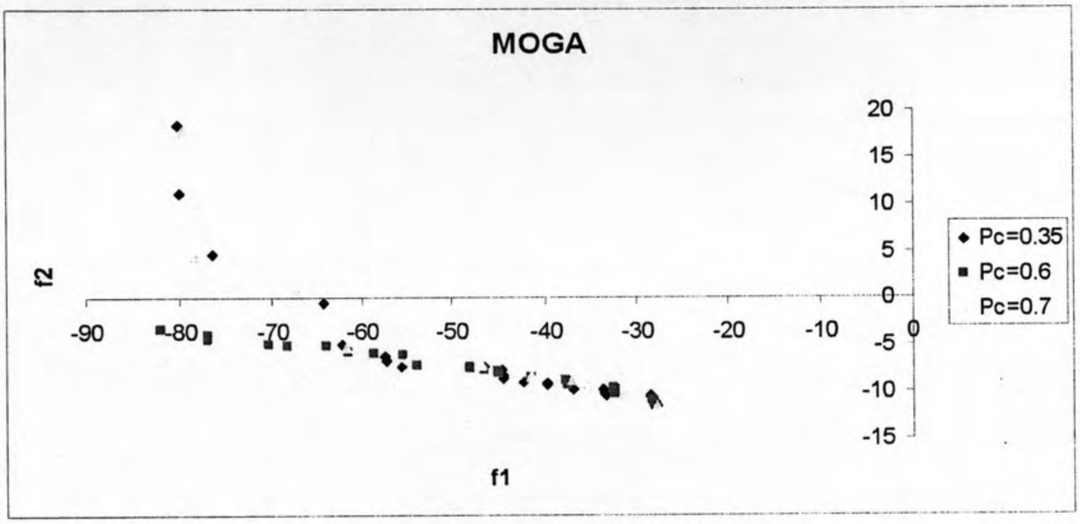

Figure 5.55: The Pareto-optimal solution for problem No.5 after 100^{-th} generation obtained with an MOGA (mutation probability 0.05).

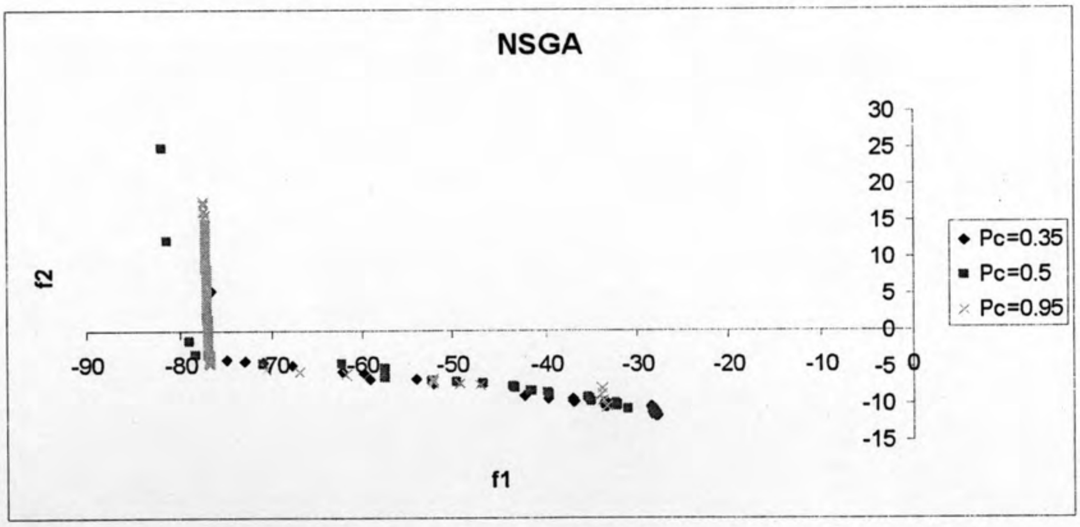

Figure 5.56: The Pareto-optimal solution for problem No.5 after 100^{-th} generation obtained with an NSGA (mutation probability 0.05).

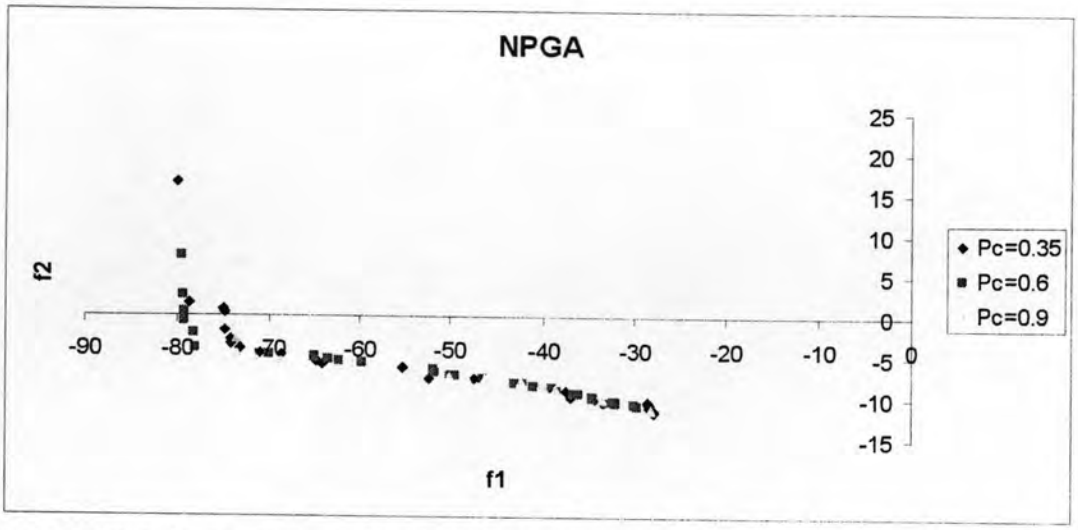

Figure 5.57: The Pareto-optimal solution for problem No.5 after 100^{th} generation obtained with an NPGA (mutation probability 0.05).

The distribution of solutions on the obtained Pareto optimal front was determined. In MOGA NSGA and NPGA, the value of crossover probability between the ranges of 0.4-0.6 can produce a Pareto-optimal solution with a similar average niche value. The relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability is shown in Fig. 5.58.

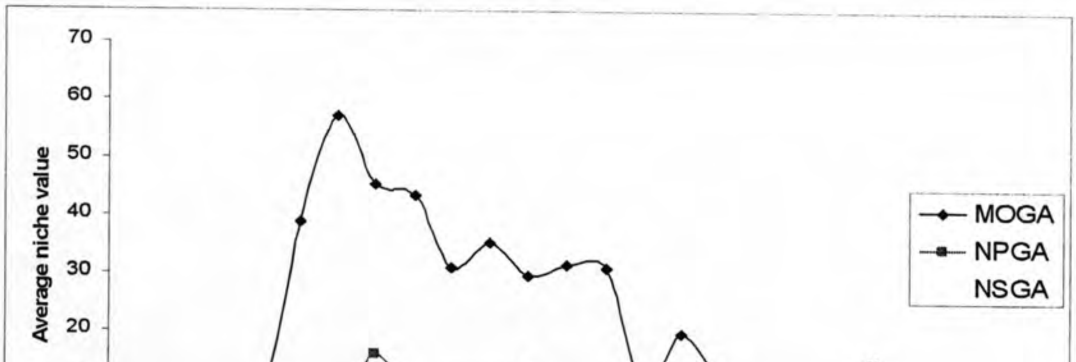

Figure 5.58: Relationship between average niche value of solution on Pareto optimal solution obtained and crossover probability with MOGA, NSGA and NPGA on problem No.5.

Case Elitist Multi-Objective Evolutionary Algorithms

Next, we applied the NSGA-II and SPEA on problem No.5. Both algorithms without the mutation operator cannot find the true Pareto-optimal solutions as shown in Fig. 5.59. When we used algorithms with mutation operator, the mutation probability value has a small effect to find a good spread of Pareto-optimal solutions as shown in Fig. 5.60.

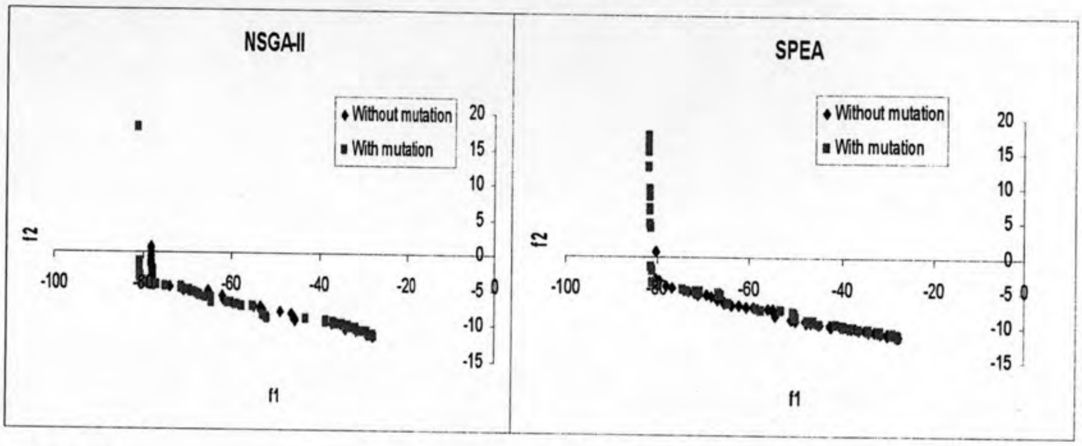

Figure 5.59: Pareto optimal solution obtained of problem No.5 after $10^{0\text{-th}}$ generation with an NSGA-II (left) and an SPEA (right) for compare both algorithms with mutation operator and without mutation operator.

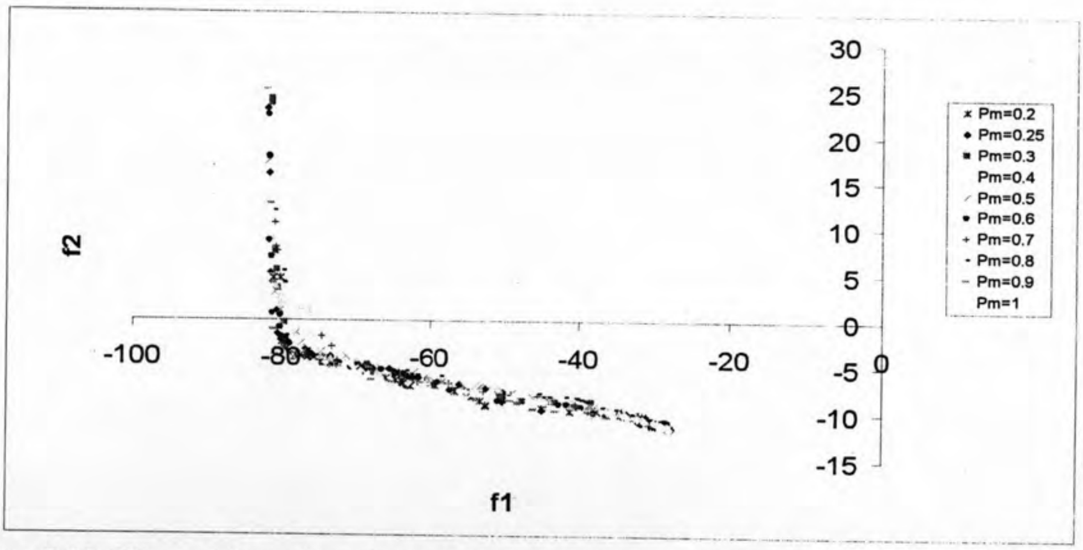

Figure 5.60: Effect of the value of mutation probability on problem No.5 after $100^{\text{-th}}$ generation with an NSGA-II.

The effect of crossover probability was analyzed. When the value of crossover probability is too low, the obtained Pareto-optimal solution is away from the actual Pareto-front for both algorithms. When the value of crossover probability is large, crossover operator can create offspring populations which are fairly different from the parent populations.

-In NSGA-II and SPEA, the value of crossover probability between the ranges of 0.4-1 can produce a good Pareto-optimal solution. The Pareto front obtained is close to the true Pareto optimal front. Figure 5.61 shows the obtained Pareto optimal front.

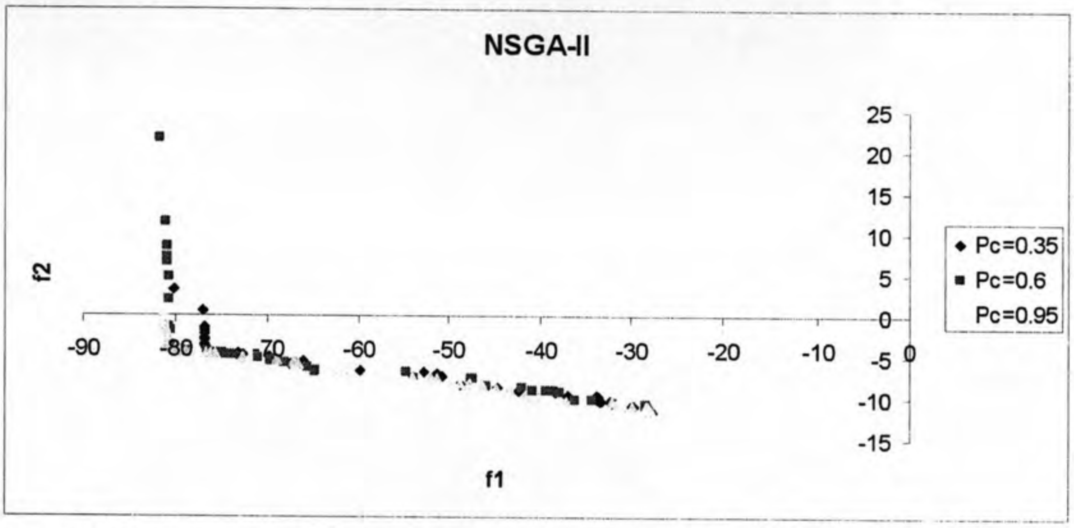

Figure 5.61: Pareto optimal solution obtained of problem No.5 after 100-th generation with an NSGA-II (effect of crossover probability).

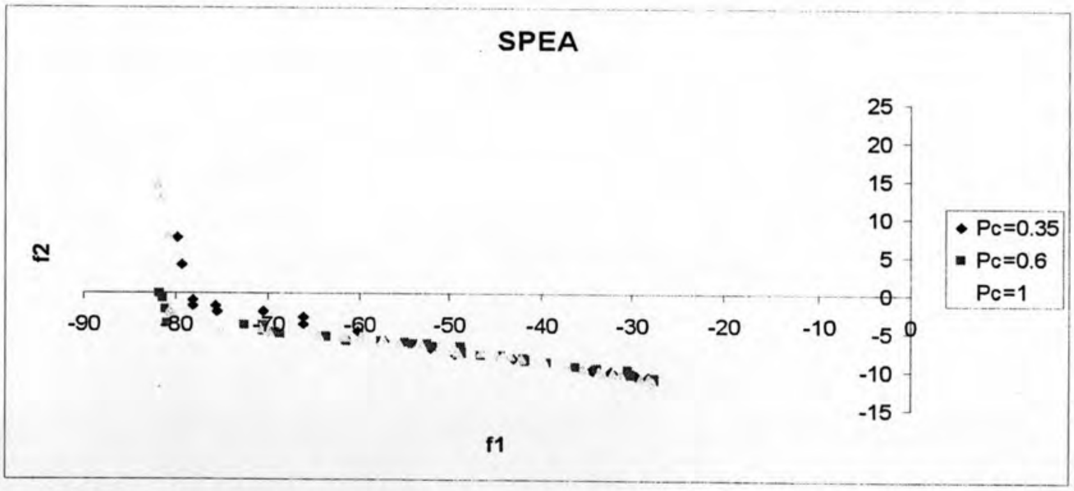

Figure 5.62: Pareto optimal solution obtained of problem No.5 after 100 generation with an SPEA (effect of crossover probability).

Next, we determined distribution of solutions on the obtained Pareto front. In NSGA-II, the value of crossover probability between the ranges of 0.45-1 can produce the obtained Pareto optimal front with a similar average distance of solution on obtained Pareto optimal front. NSGA-II with crossover probability in this rank can create all solutions converged to true Pareto optimal solution. In SPEA, the value of crossover between the ranks of 0.45-1 can create the size of external population more than the population size.

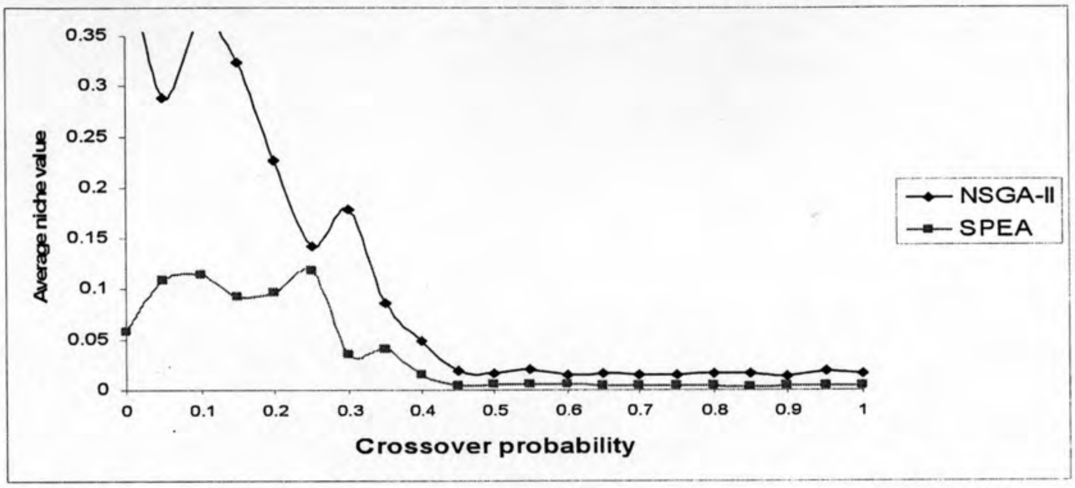

Figure 5.63: Relationship between average crowding distance and crossover probability of problem No.5.

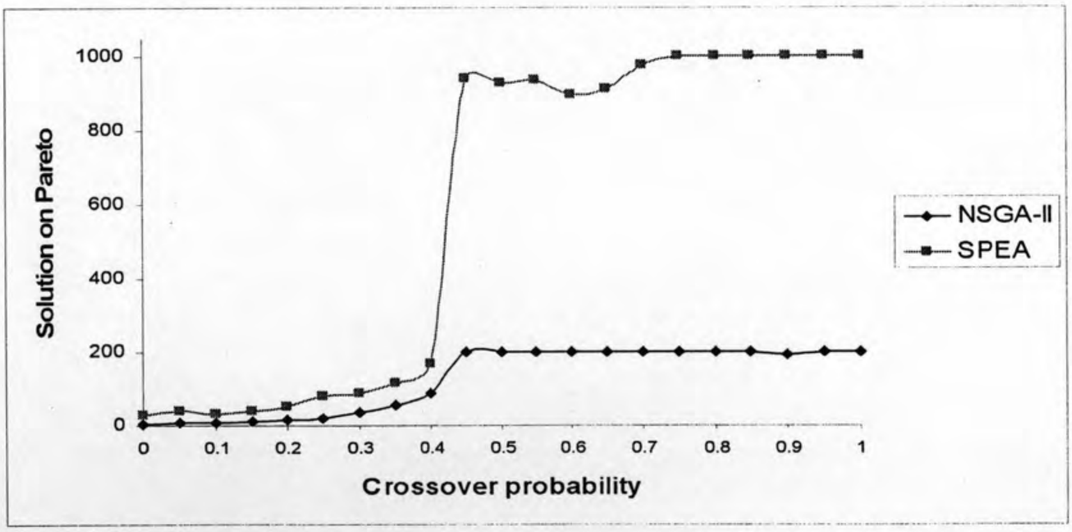

Figure 5.64: Relationship between number of solution on Pareto optimal front and crossover probability of problem No.5.

Hence, in this problem, we conclude that the optimal value of mutation probability is between 0.04 - 0.1 for Non-Elitist Multi-Objective Evolutionary Algorithms and the optimal value of mutation probability is more than 0.04 for Elitist Multi-Objective Evolutionary Algorithms. While the suitable value of crossover probability for MOGA, NPGA and NSGA is between 0.4 and 0.6, and the suitable value of crossover probability for NSGA-II is between 0.45-1 and between 0.45-1 for SPEA.

5.2 Summary

In this chapter, we analyze the effect of genetic parameters using in five algorithms that is MOGA, NSGA, NPGA, NSGA-II and SPEA on five optimization problems.

Case Non-Elitist Multi-Objective Evolutionary Algorithms

Mutation probability

- The optimal value of mutation probability is between the rank of 0.04- 0.1.
- Too large mutation probability is used, the algorithms show inability to find a good spread of the Pareto-optimal solutions.
- Without the mutation operator, algorithms cannot find true Pareto-optimal solutions.

Crossover probability

- The optimal value of crossover probability is between the ranks of 0.4-0.6.
- Too large crossover probability is used, the population lose diversity and converge to a local of the Pareto-optimal front.
- Too low crossover probability is used, the obtained Pareto-optimal solution is away from the true Pareto-front.

Case Elitist Multi-Objective Evolutionary Algorithms

Mutation probability

- Almost mutation probability value have small effect for Elitist Multi-Objective Evolutionary Algorithms
- Without the mutation operator, algorithms cannot find true Pareto-optimal solutions.

Crossover probability

- The optimal value of crossover probability is between the rank of 0.55-1
- Too low crossover probability is used, the obtained Pareto-optimal solution is away from the true Pareto-front