สำนักงานวิทยทรัพยากร จุฬาลงกรณ์มหาวิทยาลัย

Chapter III

Formulation of 3 problems

In this chapter, we are describing how to formulate our problems from the conjectures. Moreover; we also show that the problems are equivalent to minimizing convex functions.

1. An equilateral Triangle

We suppose that there is a unit arc γ that can not be covered by an equilateral triangle in any orientations. Thus, the arc γ must not be covered by the triangle in the standing position and its reflection illustrated by Figure 3.1.

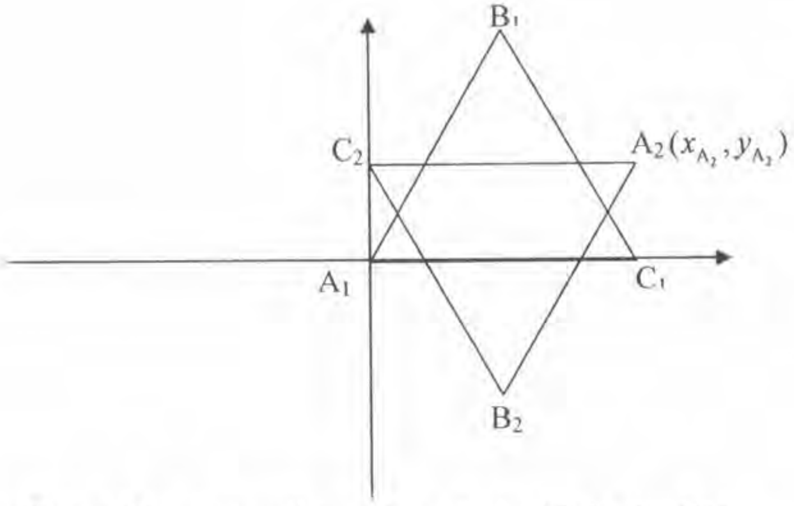

Figure 3.1 : An equilateral triangle $A_1B_1C_1$in standing position and is its reflection $A_2B_2C_2$

According to the Figure 3.1, $A_1B_1C_1$ is an equilateral triangle in standing position and $A_2B_2C_2$ is its 180° rotation. Since γ cannot be covered by the triangle. We can translate γ so that it touches the conner of $C_1\hat{A}_1B_1$ and crosses $\overline{B_1C_1}$ .Moreover, it touches the conner of $C_2\hat{A}_2B_2$ and crosses $\overline{C_2B_2}$. We define $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, $P_4(x_4,y_4)$, and $P_5(x_5,y_5)$ as points on the sides $\overline{A_1C_1}$, $\overline{A_1B_1}$, $\overline{A_2C_2}$, and $\overline{A_2B_2}$ of the triangle that touch γ, respectively. Moreover, $P_3(x_3,y_3)$ and $P_6(x_6,y_6)$ are defined as points which are not on the triangles. Thus they are not on the sides $\overline{B_1C_1}$ and $\overline{C_2B_2}$, respectively.

In particular $y_1 = 0$,

$$y_2 = \tan(\frac{\pi}{3})x_2,$$

$$y_3 \geq -\tan(\frac{\pi}{3})(x_3 - 1),$$

$$y_4 = y_{A_2},$$

$$y_5 = \tan\frac{\pi}{3}(x_5 - x_{A_2}) + y_{A_2}, \text{ and}$$

$$y_6 \geq -\tan(\frac{\pi}{3})(x_6 - x_{A_2} + 1) + y_{A_2}.$$

Example 3.1 Suppose $\prod$ be the polygonal arc $P_1\ P_2\ P_3\ P_4\ P_5\ P_6$

Let $L = P_1P_2 + P_2P_3 + P_3P_4 + P_4P_5 + P_5P_6$ be the length of $\prod$.

We want to minimize the length function.

$$L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} + \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2} + \sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2}$$
$$+ \sqrt{(x_4 - x_5)^2 + (y_4 - y_5)^2} \sqrt{(x_5 - x_6)^2 + (y_5 - y_6)^2}$$

subject to the constrains

$$y_1 = 0,$$

$$y_2 = \tan(\frac{\pi}{3})x_2,$$

$$y_3 \geq -\tan(\frac{\pi}{3})(x_3 - 1),$$

$$y_4 = y_{A_2},$$

$$y_5 = \tan\frac{\pi}{3}(x_5 - x_{A_2}) + y_{A_2}, \text{ and}$$

$$y_6 \geq -\tan(\frac{\pi}{3})(x_6 - x_{A_2} + 1) + y_{A_2}.$$

According to the Corollary 1, the length L is convex with linear constraints. Hence, this problem is a convex programming.

Note : This problem is formed by the idea that if the poly-segment can not be covered, it must pass through these six points. Moreover; the poly-segment satisfied example 3.1 is considered to pass through $(x_1,y_1),(x_2,y_2),(x_3,y_3),(x_4,y_4),(x_5,y_5)$ and (x_6,y_6) respectively. A lot of more similar problems will be formed by changing the order of passing points.

2. A right-angled isosceles triangle

We suppose that there is a unit arc γ that can not be covered by an isosceles right-angled triangle in any orientations. Thus, the arc γ must not be covered by the triangle in the standing position and its reflection illustrated by Figure 3.2

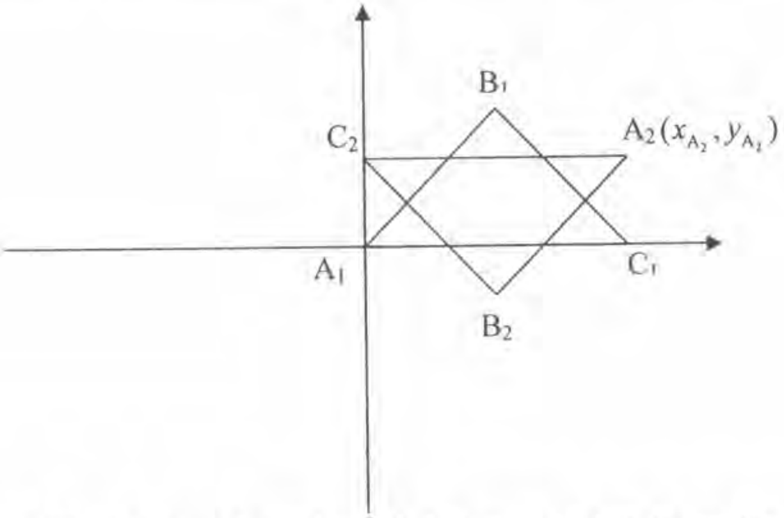

Figure 3.2 : An isosceles right-angled triangle $A_1B_1C_1$ in standing position and its reflection $A_2B_2C_2$

According to the Figure 3.2, $A_1B_1C_1$ is an isosceles right-angled triangle in standing position and $A_2B_2C_2$ is its 180° rotation. Since γ cannot be covered by the triangle. We can translate γ so that it touches the conner of $C_1\hat{A}_1B_1$ and crosses $\overline{B_1C_1}$.Moreover, it touches the conner of $C_2\hat{A}_2B_2$ and crosses $\overline{C_2B_2}$. We define $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, $P_4(x_4,y_4)$, and $P_5(x_5,y_5)$ as points on the sides $\overline{A_1C_1}$, $\overline{A_1B_1}$, $\overline{A_2C_2}$, and $\overline{A_2B_2}$ of the triangle that touch γ, respectively. Moreover, $P_3(x_3,y_3)$ and $P_6(x_6,y_6)$ are defined as points which are not on the triangles. Thus they are not on the sides $\overline{B_1C_1}$ and $\overline{C_2B_2}$, respectively.

In particular $y_1 = 0$,

$y_2 = \tan(\frac{\pi}{4})x_2$,

$y_3 \geq -\tan(\frac{\pi}{4})(x_3 - 1)$,

$y_4 = y_{A_2}$,

$y_5 = \tan\frac{\pi}{4}(x_5 - x_{A_2}) + y_{A_2}$, and

$y_6 \geq -\tan(\frac{\pi}{4})(x_6 - x_{A_2} + 1) + y_{A_2}$.

Example 3.2 Suppose $\prod$ be the polygonal arc $P_1\ P_2\ P_3\ P_4\ P_5\ P_6$

Let $L = P_1P_2 + P_2P_3 + P_3P_4 + P_4P_5 + P_5P_6$ be the length of $\prod$.

We want to minimize the length function

$$L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} + \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2} + \sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2}$$
$$+ \sqrt{(x_4 - x_5)^2 + (y_4 - y_5)^2}\sqrt{(x_5 - x_6)^2 + (y_5 - y_6)^2}$$

subject to the constrains

$y_1 = 0$,

$y_2 = \tan(\frac{\pi}{4})x_2$,

$y_3 \geq -\tan(\frac{\pi}{4})(x_3 - 1)$,

$y_4 = y_{A_2}$,

$y_5 = \tan\frac{\pi}{4}(x_5 - x_{A_2}) + y_{A_2}$, and

$y_6 \geq -\tan(\frac{\pi}{4})(x_6 - x_{A_2} + 1) + y_{A_2}$.

Note : The poly-segment satisfied the example 3.2 is considered to pass through $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), (x_5, y_5)$ and (x_6, y_6) respectively. A lot of more similar problems will be formed by changing the order of passing points

3. A 30°- 60°-90° triangle

We suppose that there is a unit arc γ that can not be covered by a 30°- 60°-90° triangle in any orientations. Thus, the arc γ must not be covered by the triangle in the standing position and its reflection illustrated by Figure 3.3.

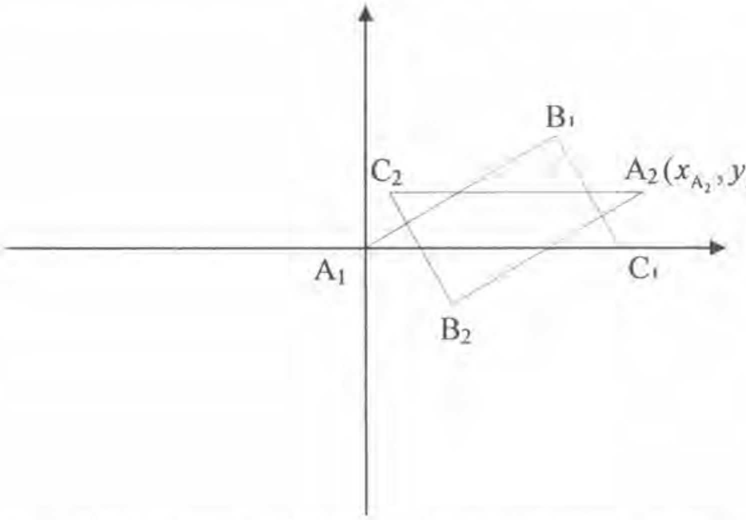

Figure 3.3 : A 30°- 60°-90° triangle $A_1B_1C_1$ in standing position and its reflection $A_2B_2C_2$

According to the Figure 3.3, $A_1B_1C_1$ is a 30°- 60°-90° triangle in standing position and $A_2B_2C_2$ is its 180° rotation. Since γ cannot be covered by the triangle. We can translate γ so that it touches the conner of $C_1\hat{A}_1B_1$ and crosses $\overline{B_1C_1}$ .Moreover, it touches the conner of $C_2\hat{A}_2B_2$ and crosses $\overline{C_2B_2}$. We define $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, $P_4(x_4,y_4)$, and $P_5(x_5,y_5)$ as points on the sides $\overline{A_1C_1}$, $\overline{A_1B_1}$, $\overline{A_2C_2}$,and $\overline{A_2B_2}$ of the triangle that touch γ, respectively. Moreover, $P_3(x_3,y_3)$ and $P_6(x_6,y_6)$ are defined as points which are not on the triangles. Thus they are not on the sides $\overline{B_1C_1}$ and $\overline{C_2B_2}$, respectively.

In particular $y_1 = 0$,

$$y_2 = \tan(\frac{\pi}{6})x_2 ,$$

$$y_3 \geq -\tan(\frac{\pi}{3})(x_3 - 1) ,$$

$$y_4 = y_{A_2} ,$$

$$y_5 = \tan\frac{\pi}{6}(x_5 - x_{A_2}) + y_{A_2} \text{, and}$$

$$y_6 \geq -\tan(\frac{\pi}{3})(x_6 - x_{A_2} + 1) + y_{A_2} .$$

Example 3.3 Suppose $\prod$ be the polygonal arc $P_1\ P_2\ P_3\ P_4\ P_5\ P_6$

Let $L = P_1P_2 + P_2P_3 + P_3P_4 + P_4P_5 + P_5P_6$ be the length of $\prod$.

We want to minimize the length function

$$L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} + \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2} + \sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2}$$
$$+ \sqrt{(x_4 - x_5)^2 + (y_4 - y_5)^2}\sqrt{(x_5 - x_6)^2 + (y_5 - y_6)^2}$$

subject to the constrains

$y_1 = 0$,

$y_2 = \tan(\frac{\pi}{6})x_2$,

$y_3 \geq -\tan(\frac{\pi}{3})(x_3 - 1)$,

$y_4 = y_{A_2}$,

$y_5 = \tan\frac{\pi}{6}(x_5 - x_{A_2}) + y_{A_2}$, and

$y_6 \geq -\tan(\frac{\pi}{3})(x_6 - x_{A_2} + 1) + y_{A_2}$.

Note : The poly-segment satisfied the example 3.3 is considered to pass through $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), (x_5, y_5)$ and (x_6, y_6) respectively. A lot of more similar problems will be formed by changing the order of passing points