## **CHAPTER 5**

## DISCUSSIONS AND CONCLUSIONS

The straightforward finite element (FE) formulation proposed here, which is derived directly from governing equations of electromagnetic field for TE and TM modes, succeeds to approximate the band gap characteristics of 2D PCs. By imposing the Bloch's functions on the boundary of the unit cell, the periodicity of the structure can be modeled. Thus, only one unit cell is enough for analysis of the structure. Imposing the periodic modeling on the boundary yields the periodic boundary conditions (PBC). The existing photonic band gap can be approximated sufficiently where this is the most important property of any PCs. The validity of the method is observed by calculating the normalized error norm that measures the estimate error between the approximate solutions and the reference solutions.

Dealing with polygonal FEM, some parameters must be considered first in order to reduce the error or to increase the accuracy such as the numbers of used gauss points, number of unknowns (nodes in polygonal mesh), and number of elements. The investigation of gauss points used in the numerical integration is important in order to find the optimum number of integration points such that the error can be reduced. The analysis of the band gap characteristic of 2D PCs using polygonal FEM with less number unknowns and less number of elements can be obtained compared with which using linear triangular FEM. Based on the band gap characteristics, the lower bands can converge quickly than the higher bands due to the dynamism of the higher frequency modes. Since the band gap will be on the lower bands, the existence of the band gap can be detected or be captured earlier. According to the results, the polygonal FEM can approximate the upper and lower frequencies of the gap quicker than the linear triangular FEM. Therefore, when the PC is designed to have a certain working frequency inside the captured band gap as either a resonant cavity or a waveguide, this method is enough to calculate the desired band gap characteristic such as the defect modes inside the photonic band gap. Hence, in this case ignoring the accuracy of the higher order modes will not affect the investigation of the next function of the 2D PCs.

The polygonal FEM using the Wachspress shape function can build the conformity among arbitrary elements. It is known that the 3-node triangular elements are linear elements that yield first order interpolation functions while the larger number of *n*-node in elements yield higher order elements with higher order interpolation functions. The higher order elements, such as pentagons and hexagons, must be placed properly while considering the shape of the elements otherwise the approximate solutions will give low accuracy. However, the combination of linear and higher order elements are placed properly on the right parts of the calculation domain. This is why the polygonal FEM can give greater flexibility and the higher accuracy of the solutions.

The combination between the linear elements and the higher order elements should be designed properly in order for the optimum mesh to be obtained. This is closely related to the behavior of the systems. In order to investigate the dynamism of behavior of the system, it is enough to test the system using the linear triangular elements, which is easy to generate. After the behavior of the system is learnt, the more fluctuating area need the higher accuracy in the approximation such that the higher order polygonal elements are suitable to assign while the more idle area will be enough to be assigned by the lower order elements such as linear triangular elements.

The approximation using hybrid polygonal elements might give results, which are more sensitive. However, they are possible to give the higher accuracy shown by the average errors used to select the good meshes generally. The sensitivity of the hybrid elements is on the parts of linear elements. Here, the linear elements are preferred to approximate the areas that provide less dynamic behavior where it normally occurs with lower frequency. In the unit cell, the lower frequency parts are in the higher dielectric medium where the fields are more concentrated in. The model of the unit cell here employs the dielectric rod embedded in the air medium. When the linear elements are generated with less accuracy, the errors provided by the hybrid elements can be high while the higher order polygonal elements remain more stable than the hybrid ones. Hence, more efficiency in calculations can be obtained using less number of unknowns, less number of elements, and higher accuracy of the solutions.

When the PC is designed to have a certain working frequency inside the captured band gap as either a resonant cavity or a waveguide, the polygonal FEM is enough to calculate the desired frequency band gap where it can be approximated sufficiently.

Analyzing the distorted model of the unit cell using this formulation is done sufficiently where the main behavior can be figured out. In terms of existing cracked rod and deformed rod, the behaviors of the system tend to shift the frequency bands from the ideal band gap characteristics. When the cracked rod, which is filled by the air, is getting larger, the frequency bands tend to move to the higher parts. In contradiction, when the deformed rod makes the rod to have larger portion in the unit cell, the frequency bands move to the lower parts of the frequency. This work has proved the ability of this numerical methods in order to investigate the behavior of the system due to the distorted geometry of the structures in terms of the volume of the distorted region in the structure. More investigation is still needed in order to analyze the nature of light under the distorted geometry of the structures.

As a summary, here is the comparison between the polygonal FEM and linear triangular FEM.

## Linear triangular FEM:

- Pro: 1. easy to generate triangular mesh
  - 2. more stable
  - 3. analytical integration

Con: 1. need more elements

- 2. Need more unknowns
- 3. linear polynomial interpolation
- 4. lower accuracy
- 5. one shape of elements (less flexible)

## Polygonal FEM:

- Pro: 1. Less elements
  - 2. less unknowns
  - 3. can be higher order polynomial interpolation
  - 4. higher accuracy
  - 5. arbitrary shape of polygonal elements (more flexible)
- Con: 1. More effort to generate polygonal mesh
  - 2. More sensitive
  - 3. Numerical integration

The next application of this polygonal FEM can be taken for analyzing several PC devices or other device in of electromagnetic fields as well as optical communication devices.