Decomposing Human Capital: Education and Health Investments in the Time of Population Aging

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เหตุใดอายุเกษียณจึงยังถูกกำหนดแบบตายตัวในขณะที่อายุขัยเฉลี่ยยืนยาวขึ้นเรื่อยๆ อะไรคือคำอธิบายสำหรับค่าใช้จ่ายด้านสุขภาพที่มีแนวโน้มสูงขึ้นเรื่อยๆในขณะที่ค่าใช้จ่ายเพื่อการศึกษาเริ่มหยุดนิ่ง การไปด้วยกันระหว่างทุนด้านการศึกษาและทุนด้านสุขภาพส่งเสริมให้เกิดการลงทุนในการศึกษาและสุขภาพ นำมาซึ่งการขยายตัวทางเศรษฐกิจและความยืนยาวของชีวิตที่สูงขึ้นในกลุ่มประเทศที่มีการพัฒนาทางเศรษฐกิจที่สูง อย่างไรก็ตามความสอดคล้องกันระหว่างทุนด้า การศึกษาและทุนด้านสุขภาพได้หายไปในศตวรรษที่ผ่านมา กลายเป็นการแลกเปลี่ยนทดแทนระหว่างกัน ซึ่งนำไปสู่การเติบโตทางเศรษฐกิจที่ต่ำและการพึ่งพิงวัยชราในระดับสูงหรือที่เรียกว่าภาวะประชากรผู้สูงอายุนั่นเอง งานวิจัยนี้ได้พัฒนาแบบจำลองการเหลื่อมเวลาข้ามรุ่นของคนสามรุ่นอายุโดยการรวมการสะสมทุนด้านการศึกษาและทุนด้านสุขภาพเข้าไปในแบบจำลองด้วย เพื่อที่จะศึกษาความไม่ยืดหยุ่นของอายุเกษียณ ความไม่ยั่งยืนทางการเงินของระบบประกันสังคมและการล่มสลายของระบบครอบครัวแบบดั้งเดิม ข้อค้นพบหลักของงานวิจัยนี้มีดังนี้ ประการแรก เมื่อนวัตกรรมเร่งให้เกิดการเสื่อมของผลิตภาพอย่างรวดเร็ว ผู้สูงอายุจะเลือกเกษียณอายุเร็วแม้ว่าพวกเขาคาดว่าจะมีชีวิตอยู่ต่อไปอีกนาน ประการที่สอง ระบบประกันสังคมแบบได้มาจ่ายไปบิดเบือนการตัดสินใจในการลงทุนทั้งทางด้านการศึกษาและด้านสุขภาพ ปัญหาจริยาวิบัติก่อให้เกิดการลงทุนด้านการศึกษาที่ต่ำและการลงทุนด้านสุขภาพที่สูงกว่าระดับที่เหมาะสมที่สุด ประการที่สาม การขยายตัวทางเศรษฐกิจและความยืนยาวของชีวิตขึ้นอยู่กับระดับความเห็นแก่ผู้อื่นทั้งเห็นแก่ลูกและเห็นแก่พ่อแม่ หากคนมีระดับความเห็นแก่ผู้อื่นทั้งเห็นแก่ลูกและเห็นแก่พ่อแม่ที่แตกต่างกันแล้ว การเปลี่ยนแปลงนโยบายต่างๆจะส่งผลกระทบต่อสวัสดิการของพวกเขาแตกต่างกัน ในภาพรวม แบบจำลองในงานวิจัยนี้ประสบความสำเร็จในการเลียนแบบการแลกเปลี่ยนทดแทนระหว่างกันระหว่างทุนด้านการศึกษาและทุนด้านสุขภาพ และชี้ให้เห็นว่ากลุ่มประเทศที่พัฒนาแล้วได้เกิดการสูญเสียการลงทุนในการศึกษาไปโดยได้การเพิ่มขึ้นของการลงทุนในสุขภาพกลับมาแทน

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Why is the retirement age rigid while longevity improves? What are the plausible explanations for the recent upsurge in health expenditure while education expenditure has been stalemated? Complementarity between education and health capital, which encouraged education and health investments, enabled rapid economic growth and greater life expectancy in the developed economies. However, it was lost at some point in the last century, and the subsequent rise of trade-off between education and health capital has brought about low economic growth and high old-age dependency, namely the problems of population aging. By endogenizing both education and health capital accumulations, this dissertation develops the three-period overlapping generations models in order to investigate the issues such as rigidity of retirement age, financially unsustainable social security systems, and collapse of traditional family. The main findings of this dissertation are as follows: First, when highly innovative environments accelerate depreciation of productivity at old-age, people chose early retirement even if they are expected to live longer. Second, Pay-As-You-Go (PAYG) social security distorts both education and health investment decisions. The double moral hazards induce lower education and higher health investments than their optimal levels. Third, economic growth and longevity depend on the degrees of parental and filial altruism, respectively. If people are parentally and filially altruistic at varied degrees, policy changes affect their welfare differently. On the whole, the models in this dissertation succeed in replicating the trade-off between education and health capital and reveal that, in the advanced societies, investments in health tend to increase at the cost of those in education.

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Chapter 1: Introduction

Education capital has been the key element of endogenous growth theory since its earliest development, and the incorporation of health capital into the theory follows as longevity in the developed countries continues to improve throughout the last century. While education capital is the indicator of quality of life due to its effects on workers' productivity, health capital is introduced as the critical determinant of quantity of life (Ehrlich & Chuma, 1990; Grossman, 1972b). Provided that education capital, or knowledge, is embedded in each worker, health capital can be taken as the necessary condition for education capital to be effective. Accordingly, complementarity between the two major components of human capital is emphasized in the microeconomic literature. Fuchs (1982) suggests that if people invest more in their health they can live and work longer, hence the rate of return on their education investments increases. In the early to mid-twentieth century, when public health expenditure, which aimed at the reduction of infant mortality and brought about the significant rises in life expectancy at birth, went hand in hand with the achievements of compulsory education, the productivity of young workers grew at the unprecedented pace. Though this historical event is the sound proof of complementarity between education and health capital, in more recent years the two elements of human capital are not functioning as coordinately as before.

On the one hand, Fuchs (1982) assumes that people's labor force participation depends on their health condition. Namely, in both cases that people work until they die and choose to retire when they become unfit for physically demanding tasks, their working life can be extended as their health condition improves. These two scenarios of no retirement and retirement due to poor health conditions of the elderly were highly applicable for the developing economies but became less and less so for the developed economies, where mechanization had drastically reduced physically demanding tasks. If we look at the case of the United States, the normal retirement age was set to 65 in 1935 and gradually increasing to 66 by 2013 whereas life expectancy at birth increased 18 years during the same period ("OECD Factbook 2014: Economic, Environmental and Social Statistics," 2014; "Social Security Programs Throughout the World: The Amaricas, 2013," 2014). Why is the retirement age in the US significantly rigid and not adjusted to the large improvements in longevity? The age-efficiency profile evidenced in the labor literature sheds light on this question. The peak ages of efficiency shift from 45 in 1950 to 55 in 2000, and the peak age in 2000 attains higher efficiency than that in 1950. Nevertheless, the two curves converge at the age of 65 and are almost identical until they reach zero at age 91 (Ferreira & dos Santos, 2013). The above findings imply that, after reached the peak levels, efficiency of elder workers is depreciating faster in 2000 than in 1950, and their improved longevity seems to have little effects on their productivity at old-age. If the level of efficiency at the age of 65 is the minimum requirement for productive employment, the retirement age cannot go up beyond this age. Contrary to the above mentioned assumption in Fuchs (1982), it can be assumed that labor force participation of the elders in the advanced societies depends on their remaining stock of education capital. As Sala-I-Martin (1996) suggests, the elders in the highly advanced economies retire not because their health condition worsens but because they have increasingly tough time to catch up with new technology. Once people's life expectancy surpasses the age, at which their efficiency reduces to the minimum requirement level for productive employment, complementarity between education and health capital is lost, and the large improvements in longevity will simply translate into the longer period of retirement.

On the other hand, both provisions of education and health require resources. When investments in education and health capital have reached considerable portions of total expenditure, there should be the trade-off, or substitutability, between the two human capital investments. Total health expenditure in the United States doubled from the 1980's level and reached 17.4% of GDP in 2009 while its total education expenditure was 7.2% of GDP and nearly stagnant for the last two decades ("OECD Factbook 2014: Economic, Environmental and Social Statistics," 2014). The data clearly show that the US economy puts the higher weight on investments in health than education. This increases old-age dependency and slows down economic growth and hence brings about the problems of population aging. What are the reasons behind the steep increase in health expenditure and the stagnation of education expenditure? In the first place, it has to be noted that the trade-off between education and health investments, or the quality and quantity of life, has not been thoroughly discussed in the existing literature.[1] At the early phases of demographic transition, the decreases in fertility were associated with greater human capital investments per child, and the phenomena were called the trade-off between the quality and quantity of children. The rapid economic growth accompanied by the fall in young-age dependency rates was explained very well by the theoretical studies that endogenized both child bearing and education investments decisions (Becker, Murphy, & Tamura, 1990; Ehrlich & Lui, 1991). In the same period, life expectancy was increasing due mostly to reductions in child mortality, which was achieved through improvements in basic sanitary conditions. For that reason, longevity has been customarily assumed to be exogenous to households' decisions (Ferreira & Pessoa, 2007). However, in the developed economies, people almost certainly survive to old-age, and the recent improvements in longevity are the result of decreases in adult mortality. As Ehrlich and Chuma (1990) suggest, individuals' purposeful investments in health should be the major determinant of rising life expectancy beyond the retirement ages. Analogues to the above case, the trade-off between the quality and quantity of life, which causes the problems of population aging, should be examined by endogenizing both education and health investments decisions.

High life expectancy and low fertility accompanied by the rigid retirement ages have brought about the significant increases in old-age dependency in the developed countries. According to Li, Zhang, and Zhang (2007), though both improved longevity and high old-age dependency are the conspicuous outcomes of population aging, they play rather opposing roles as the determinants of national savings and economic growth. Their findings reveal that exogenous improvements in longevity increase saving and growth rates of the economy, whereas higher old-age dependency reduces aggregate savings and slow economic growth. Moreover, the

[1] van Zon and Muysken (2001) are the exceptional case as they discuss the trade-off between education and health investments. Studies on endogenous longevity, such as Ehrlich and Chuma (1990) and Jie Zhang, Zhang, and Leung (2006), tend to assume away from human (education) capital investments, hence economic growth, for simplicity.

latter worsens financial sustainability of Pay-As-You-Go (PAYG) social security systems. The developed nations have instituted the PAYG systems during the early to middle parts of the last century. At the inception of social security, the large number of working adults supported the smaller number of elders, whose retirement period was still short, and the social security trust funds accumulated large surpluses. As old-age dependency rises, the contributions from workers are decreasing, and the benefits paid to retirees are increasing. The slowing down of economic growth also makes the contributions less than expected at the introduction of PAYG systems. The majority of PAYG social security systems were deemed to be financially unsustainable by the end of the last century. This is the one of the most urgent problems of population aging and should be examined from the standpoint of the trade-off between the quality and quantity of life because sustainability of PAYG systems depends both economic growth and old-age dependency.

While the quality and quantity of life are augmented through education and health investments, respectively, neither life-cycle models nor models with parental altruism motivates both of the investments. The family insurance mechanism introduced in Ehrlich and Lui (1991) connects family members through material dependency, and young agents educate their children because it increases old-age supports that they expect to receive during their retirement period. If this mechanism is added to life-cycle models, even selfish agents invest in their children's education and own health capital as the latter increases the old-age supports further by extending their retirement period.[2] However, in reality family members are connected not only materially but also emotionally. According to Horioka (2002), Japanese survey suggests that 92% of respondents would help their children without any rewards, and 87% of them would help their old parents out of love. As in the case that parental altruism encourages education investments in children, health investments are motivated by filial altruism of agents who care about their old parents' well-being. Investigating how the trade-off between the quality and quantity of life is influenced by the hypotheses on family relationships, such as self-interest, filial altruism, parental altruism, and rarely studied reciprocal (two-sided) altruism should be worthwhile as the advances of urbanization have been making the family insurance mechanism less practicable in the developed countries.

This dissertation decomposes human capital into two parts, education and health, and tackles the problems of population aging. The summary of its three chapters discussing the optimal retirement age, financial sustainability of social security programs, and the four hypotheses of family relationships is provided in the following paragraphs.

Even at the advanced stage of demographic transition, an economy's old-age dependency rate does not necessarily rise if its elders choose to work longer. Nevertheless, in developed countries, retirement ages have failed to keep pace with increasing life expectancy. The resulting rises in old-age dependency put pressure on the financial capacity of Pay-As-You-Go (PAYG) social security systems, and pension eligibility ages have been raised in many countries. A body of studies discuss whether the increases in eligibility ages are compatible with interests of the elderly or not, and the results are inconclusive (Aisa, Pueyo, & Sanso, 2012; Bloom, Canning,

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[2] As longevity is exogenous in Ehrlich and Lui (1991), agents invest only in human (education) capital of their children.

Mansfield, & Moore, 2007; Ferreira & Pessoa, 2007). Though the studies differ in their assumptions about how the elders decide to work or retire, one assumption common to all is exogenous longevity. The second chapter of this dissertation, "Endogenous Retirement, Education and Health Capital of the Elderly," develops the three period overlapping-generations model examining the joint determination of retirement and investments in education and health.[3] The model's key assumption is as follows: Retirement decisions in developing societies are outcomes of worsening health conditions, but in advanced economies, where the more people educate their children the faster their past education become obsolete, healthy elders choose to retire when their education capital stocks are depreciated. The social planner compares lifetime utilities of an agent derived for each scenario of working or retirement at old-age and chooses one that yields the higher utility. As a result, at the low levels of development labor force participation of the elders rises unambiguously with their health level. The higher average education level in the developed economies accelerates depreciation of elder workers' productivity and causes the rise of retirement. The numerical analyses show that higher life expectancy and social welfare are attained if the elders retire than in case that they continue to work. Moreover, in the latter case, social welfare decreases with the level of development. These results suggest that policy changes of raising the pension eligibility ages would increase unemployment at old-age.

In order to secure old-age consumption, PAYG social security programs facilitate intergenerational transfers from working generations to retired ones. While the return to PAYG programs depends positively on wage income growth and negatively on population growth of retirees relative to that of workers, the stagnant economic growth and high old-age dependency rates in Japan have been detrimental to its social security financing. Though the large literature has been discussing feedback effects of social security expansions on household savings, fertility, and economic growth, their joint effects on education and health investments have not studied yet (Junsen Zhang, Zhang, & Lee, 2001; Jie Zhang et al., 2006). The third chapter of this dissertation, "Pay-As-You-Go Social Security with Endogenous Longevity," examines the feedback effects of the PAYG programs on both longevity and economic growth.[4] In the three-period overlapping-generations setup, three options of securing old-age consumption, family insurance and fully funded (FF) and PAYG social security, are examined. Family insurance motivates agents to invest in both education and health, but FF programs affect neither of two investments. While earnings-related PAYG programs prompt agents to invest in health alone, if the social planner solution was concerned, PAYG social security would encourage only education investments. Consequently, the earnings-related programs distort the key family-based choices via two channels of moral hazards. One results in lower economic growth, and the other induces higher old-age dependency. This study suggests that the expansion of the PAYG system worsen the financial situation of the very system, or it probably is the social security system that has brought about the particular demographic and macroeconomic trends currently observed in Japan. The

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[3] Chapter 2 article was presented at the 79th International Atlantic Economic Conference, Milan, March 2015.

[4] Chapter 3 article was presented at the 14th International Convention of the East Asian Economic Association, Bangkok, November 2014.

simulation results reveal that the increases in PAYG contribution rate raise the replacement rate less than proportionally due to the feedback effects and reduce social welfare.

In the recent past, most of developed economies have undergone the demographic transition, and rapid population aging is accompanied by changes in intergenerational relationships among family members. Though how individuals' altruistic traits interact with longevity and economic growth should be of particular interest for studies of population aging, there is little agreement on which type of family relationships fits well to the real societies. If we look at existing studies on endogenous fertility, one body of studies assumes the hypothesis of parental altruism, and the other assumes that of filial altruism. The latter criticizes parental altruism models, which fail to explain the process of demographic transition (Barro & Becker, 1989; Becker & Barro, 1988; Boldrin & Jones, 2002). The fourth chapter of this dissertation, "Altruism and Four Shades of Family Relationships," develops an overlapping-generations model of the joint determination of education and health investments under four hypotheses of self-interest, filial altruism, parental altruism, and reciprocal altruism.[5] The study suggests that the hypotheses of self-interest and reciprocal altruism seem to fit better to the real economies because under these hypotheses the models predict the realistic levels of economic growth and life expectancy. The simulation analysis reveals that social welfare decreases by an expansion of PAYG program under the hypothesis of filial altruism but increases under other hypotheses. Accordingly, policy makers have to know which hypothesis of household behavior is applicable for their society so as to ensure the expansion of PAYG social security is welfare improving.

On the whole, improvements in longevity are good without doubt, and the elderly who worked hard for decades should be able to enjoy their retirement life. As long as the retirement ages go up proportional to the increases in life expectancy, old-age dependency should be kept within manageable ranges. If this is the case, there will be no issue such as financial sustainability of PAYG social security. In fact, the problems of population aging have only one root, that is, the rigidity of retirement age. This dissertation argues that maintaining high productivity at old age and providing elders with incentives to stay in workforce by way of social security designs are the key to dissolve the rigidity because the elders are healthy enough to work longer and to enjoy retirement leisure at a later date.

The rest of paper is organized as follows. Chapter 2, Chapter 3, and Chapter 4 present the three articles outlined above. Each article includes the review of existing literature on the respective subject, the theoretical models, and the numerical analyses. Chapter 5 provides concluding remarks.

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[5] Chapter 4 article was presented at the 15th Eurasia Business and Economics Society Conference, Lisbon, January 2015 and published in *Eurasian Economic Review*, Volume 5, 345-365.

Chapter 2: Endogenous Retirement, Education and Health Capital of the Elderly

Abstract

Elderly people retire because their productivity declines with age. In the past, agents' productivity was strongly affected by their health condition. Today, it is defined by their education capital stock. In this study, we develop a model of the joint determination of labor force participation and human capital investments, which affect depreciation of productivity at old-age. Our optimization problem is solved in two stages. First, the social planner optimally chooses levels of education and health investments and a contribution rate of Pay-As-You-Go (PAYG) social security for an agent. Second, given the optimal decisions, the social planner compares lifetime utilities derived for two cases of working at old-age and retirement and chooses one yields a higher utility. Our endogenous longevity model shows that, in a less developed society, the better health condition, hence higher productivity, of elders strengthens the stimulus for working at old-age. On the other hand, the stimulus for working at old-age weakens with levels of development because productivity of old workers depreciates faster in highly innovative environments. Consequently, in developed economies, policy changes that encourage elders to stay in the labor market longer would increase unemployment at old-age. The model is calibrated to fit Japanese economy to quantitatively examine the effects of changes in levels of development on key choice variables. Our simulation results suggest that life expectancy and social welfare when agents retire are higher than those if elder agents continue to work. In the latter case, social welfare becomes smaller as the economy develops. Therefore, the welfare gap between the two scenarios is wider in more advanced economies.

JEL Classification Codes: H55, I15, I25, J26, O15
Keywords: Social security, Health capital, Education capital, Retirement, Growth

2.1 Introduction

In the course of the twentieth century, more and more people choose to retire at the end of their life. Early in the century, when people's health condition was the crucial requirement for their labor force participation, elder workers might retire when they were not strong enough to perform physically demanding tasks.[6] If this was the case, retirement ages should be rising with improving longevity. Kalemli-Ozcan and Weil (2010) call the positive correlation between retirement ages and longevity as the horizon effect. Theoretically, the horizon effect can be captured by disutility of

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[6] It has to be noted that, in the more recent past, mechanization of the physically demanding tasks has made agents' health condition a less essential aspect of their productivity. As discussed below, this situation corresponds to $\phi = 0$ in our model.

working in agents' preference, which decreases with life expectancy. According to Bloom et al. (2007), as workers' health level improves, disutility of working decreases, hence they endogenously choose later retirement unless there are institutional distortions, such as social security programs. However, as the century rolled on, the rigidity of retirement age along with rapidly increasing life expectancy became apparent. Since the age of Social Security benefit entitlement was set to 65 in 1935, it has become the normal retirement age in the United States, and several countries have followed suit. While policy changes mandated by Congress have gradually and marginally increased the retirement age to 66 by 2011, life expectancy at birth in the United States has increased 17 years for the same period of time ("OECD Factbook 2014: Economic, Environmental and Social Statistics," 2014; "Social Security Programs Throughout the World: The Amaricas, 2013," 2014). These historical facts suggest that the current retirement age in developed countries is significantly sticky and not proportionally adjusting to the large improvements in life expectancy. The natural consequence of the rigid retirement age and increasing life expectancy is the rise of retirement as people spend longer time in retirement. Considering that the horizon effect should exist in reality as well as in theory, the rise of retirement implied that some divergent effects had prevented the retirement age from rising for the past several decades.

What can be possible candidates for the divergent effects, which have brought about the reduction in elder workers' labor force participation? On the one hand, from the perspective of agents' preferences, we have to consider the rise of utility from retirement leisure, which is directly opposing disutility of working discussed above. Utility from old-age leisure increases with the parameter value measuring its intensity relative to utility from consumption, and the leisure effect decreases labor force participation of the elderly. On the other hand, there are several external factors that induce workers to retire at an earlier age. If we look at the existing literature, there are three commonly accepted explanations for the rise of retirement.[7] First, social security systems implemented by a good number of governments in the early twenties century should have large impacts on elder workers' retirement decisions because most of the systems provide them with attractive monetary incentives to retire at designated ages. Though the pension effect is one of the most evident reasons of the rise of retirement, some studies report small impacts of changes in public pension benefits on agents' retirement decisions (Duval, 2004; Kalemli-Ozcan & Weil, 2010). Second, the income effect due to the concurrent increases in wages explains the retirement behavior since retirement leisure is a normal good. This explanation assumes that the income effect dominates a possible substitution effect from higher wages, which could lead to the fall of retirement due to higher opportunity costs of retirement leisure. The third explanation of the rise in retirement is the fast change in production technology. If the rate of changes in technology, not necessarily its level, is higher, productivity of elder workers would be relatively lower than that of younger ones. That is, depreciation of productivity at old-age rises with the rate of technological changes. The technology effect suggests that higher education investments would reduce old workers' labor force participation rates. However, it has to be noted that in the existing literature there is no clear consensus in terms of the effects of education

[7] See our literature review in the following section. Also see Sala-I-Martin (1996) and Kalemli-Ozcan and Weil (2010). The latter paper suggests the fourth explanation, the uncertainty effect.

investments on agents' retirement behavior. For example, the results in Aisa et al. (2012) contradict the third explanation as they find that the higher productivity at young-age becomes the stronger stimulus for labor force participation at old-age. In order to reconcile the inconsistency, our assumptions are as follows: In the less developed countries, where productivity of workers are determined mostly by their physical strength, there would be the positive effect of increasing health investments on labor force participation rates as the horizon effect suggest. In the more advanced economies, where the majority of workers are healthy enough to perform their deskwork, the faster pace of innovation due to higher education investments would have negatively affect labor force participation of the elderly. In the subsequent sections, we take both of the cases into account. To sum up, there are the four factors, namely, the leisure effect, the pension effect, the income effect, and the technology effect, that would compensate the opposing horizon effect and prevent the retirement age from rising in accordance with the large increases in life expectancy.

Our discussion so far suggests that how deeply retirement behavior of the elderly is linked with their education and health investments decisions when young. In this study, we develop and analyze a model of the joint determination of education, health, and retirement decisions in order to examine the relationship between the two components of human capital and labor force participation of the elderly. Following the seminal work of Grossman (1972b) and Ehrlich and Chuma (1990), our model endogenizes not only the investment decision on education capital but also that on health capital. Once health capital investment, or equivalently a fraction of time spent for health maintenance activities, becomes individuals' choice variable, longevity is endogenized. As a consequence, utility from retirement leisure is positively related to agents' health capital investments and life expectancy. Increases in life expectancy strengthen the leisure effect because the elderly can enjoy longer and healthier retirement life. We also assume that deterioration of elder workers' productivity is affected by both education and health capital investments. The deterioration is supposed to decrease with health investments in the developing countries, but the effect becomes negligible in the developed countries because the elder workers are healthy enough to perform their white collar jobs. According to our argument on the technology effect, the higher education investments in their children should affect the agents' productivity at old-age negatively because the younger competitors are equipped with newer and larger education capital. Our theoretical model, which incorporates all the effects discussed above except the pension effect, is intended to reveal that agents' education and health investments decisions have significant and distinctive impacts on labor force participation of the elderly.

The rest of chapter is organized as follows. Section 2.2 reviews theoretical literature discussing subjects such as endogenous longevity and endogenous retirement. Section 2.3 introduces the model, which enables us to examine the income, leisure, and technology effects of preventing the retirement age from rising as well as the horizon effect of extending working life on agents' retirement behavior. In Section 2.4, we calibrate the model to fit Japanese economy in order to quantitatively analyze the effects of development levels on individuals' human capital investments and labor force participation. Section 2.5 provides concluding remarks.

2.2 Literature Review

2.2.1 Endogenous Longevity

Grossman (1972a) is the seminal work on the demand for health. In order to differentiate health capital from other forms of human capital, Grossman defines agents' health level as a determinant of the total amount of time or the "quantity of life" that they can provide for market and non-market activities. It is assumed that the stock of health is augmented by purposeful investments but depreciates over time at increasing rates. Because of this depreciation, death of agents occurs once their stock of health is reduced to its minimal level. In his study, longevity depends on agents' stock of health capital that maximizes their utility and hence becomes an endogenously determined variable. From their observation that life expectancies at birth differ substantially across population groups and over time, Ehrlich and Chuma (1990) suggest that these differences result not only from exogenous factors such as technological and biological constraints but also from people's demands for longevity. Their intertemporal setting employing the technology of health capital accumulation is a modified continuous-time version of Grossman (1972b), and their optimization analysis yields the value of health capital as a ratio of a shadow price of health capital to that of nonhuman capital. Their results show that the value of health capital is consisted of two terms, the value of life extension and the value of healthy life. The former is rising with agents' age at an increasing rate, and the latter is falling towards the end of their life. It has to be noted that, these arguments assume that the agents work until the day of their death. Because, in reality, people spend a significant portion of their life in retirement, we will turn to a body of studies, which have incorporated retirement into their analyses.

While Grossman (1972b) and Ehrlich and Chuma (1990) focus on individual's demand for health services, van Zon and Muysken (2001) show that health is one of the engines of economic growth. According to them, if economic growth is determined by an accumulation of knowledge within agents as shown in Lucas (1988), their health level should also be an important determinant of economic growth. Following Grossman (1972b), van Zon and Muysken assume that life expectancy is proportional to the stock of health capital, and a good health directly influences agents' utility. The production characteristics of their health sector are based on the specialization argument of Romer (1990), in which knowledge within the health sector grows at the same rate as the growth rate of education capital and is proportionally related to the fraction of effective labor spent on health maintenance activities. Though people in their model retire so that there is a distinction between active and inactive population, the formal retirement age is fixed and exogenously given.

2.2.2 Endogenous Retirement

The more recent framework on endogenous retirement in the discrete-time overlapping generations economy is developed by Matsuyama (2008) as an extension to the one-sector neoclassical growth model of Diamond (1965). While in Diamond's model all workers are forced to retire once they enter their old period, Matsuyama's agents are allowed to make a decision of whether to continue working at old-age or to retire.[8] Subsequently, the study's main finding is negative interdependency between wage growth and old workers' labor force participation. Matsuyama argues that the income effect resulting from wage growth of young agents induces most of them to retire when they are old. A common key assumption in Diamond (1965) and Matsuyama (2008) is that agents' survival to old-age is certain, which implies that their life expectancy is constant. However, the changes in life expectancy, which have been large for the last decades as discussed in our introduction, surely affect elder workers' retirement decisions. Aisa et al. (2012) point this out and incorporate survival probability of old agents into their analysis.[9] They assert that impacts from exogenous improvements in life expectancy can be large enough to offset the income effect. More specifically, longevity affects agents' retirement decisions via two diverging channels. The first channel is the leisure effect. It says that the utility of leisure rises with the survival probability, therefore longer life is the stimulus for retirement. The second channel is the horizon effect. Better health enables the elderly to maintain their productivity relatively high, hence it is the stimulus for working at old-age. Aisa et al. (2012) assert that the horizon effect is large so as to dominate the leisure effect. It has to be noted that their assumption is more suitable for developing economies, where workers' physical condition of workers strongly affects their labor force participant decision.

There is the larger literature on endogenous retirement in continuous-time setups. Bloom et al. (2007) follow the finite horizon formulation of Blanchard (1985), in which agents face a constant probability of death throughout their lifetime, for developing their model of optimal retirement decisions. In order to endogenize retirement decisions, they specify the lifetime utility function which includes not only utility from consumption but also disutility of working. The condition derived from their optimal control problem implies that elder agents work only if additional utility from higher consumption associated with longer working life exceeds disutility of working. Their key assumption is that disutility of working increases with agents' age and decreases with their survival probability. Thus, in their analysis the horizon effect is prevailing without the leisure effect.[10] As in the case of Aisa et al. (2012), assuming dominance of the horizon effect is reasonable at certain stages of lower development. However, as the economy further develops disutility of working must be affected not only by agents' survival probability but also by their efficiency. Ferreira and Pessoa (2007) incorporate experience-earning profiles, which reveal that agents' productivity starts to decline towards the end of their working life, into their analysis of retirement behavior. According to them, when old agents' productivity is decreasing faster today than in the past, their retirement age can fall even if they expect to live longer. In contrast to the horizon effect in Bloom et al. (2007), Ferreira and Pessoa (2007) predict the existence of the technology effect. It says that in advanced economies the

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[8] As Matsuyama (2008) noted, if the agent is forced to retire, or equivalently if $e_{t+1} = 0$, his model is identical to Diamond's original model.

[9] If the agents' survival to old-age is certain, $p = 0$, and the factor giving the remaining productivity at old-age is set to zero, $\delta = 0$, the model of Aisa et al. (2012) becomes identical to Matsuyama's original model.

[10] Bloom et al. (2007) assert that the pension effect would be strong enough to outweigh the horizon effect. Heijdra and Romp (2009) extend the "perpetual-youth" model in Bloom et al. (2007) by replacing the latter's constant death rate with a more realistic mortality process.

technology effect dominates the horizon effect, and increases in education investments, which are motivated by significant improvements in longevity, induce the rise of retirement. Kalemli-Ozcan and Weil (2010) examine the effect of declining uncertainty about the day of agents' death on their retirement decisions and assert that as life expectancy improves the agents plan and save for early retirement. Their analysis shows that this uncertainty effect would like to dominate the horizon effect.

Through this literature review, we find that elderly workers' productivity and survival probability are the important determinants of their retirement behavior. However, in the above studies dealing with endogenous retirement, both productivity and longevity of agents are exogenously given while these should depend on their education and health capital investment decisions, respectively. In order to examine the joint determination of human capital investments and labor force participation of the elderly, we endogenize education and health investments decisions in the following sections.

2.3 The Model

The process of health capital accumulation is integrated into the baseline model of Ehrlich and Lui (1998), which has already employed the education capital accumulation process. While their main focus is on the trade-off between the quality and quantity (fertility) of children, we are interested in an alternative trade-off between the quality and quantity (longevity) of the elderly in an advanced economy with aging population. As in such an economy fertility is close to the replacement level, and child mortality is constant and very low, the simplifying assumptions are that a young agent always has one child, and the probabilities of survival from child to middle-age and from middle-age to old-age equal to one. We also assume that in the advanced economy traditional family insurance in Ehrlich and Lui (1998) has been replaced by Pay-As-You-Go (PAYG) social security. Lastly, in this overlapping generation model, a social planner controls all resources that the young agent spends on educating her sole child and on maintaining her own health level in preparation for old-age needs. The social planner chooses the optimal contribution rate of the PAYG social security system as well.

2.3.1 Preferences

The economy consists of generations of agents and the social planner with perfect foresight. Each agent goes through three periods of lifetime: child, young parent, and old parent. For an agent born in period $t-1$, her lifetime utility is given by:

$$U^{t-1}(u_t, v_t, e_{t+1}) = [c_t^y][c_{t+1}^o]^\beta [l_{t+1}^o]^\eta, \qquad 0 < \beta < 1, \qquad \eta > 0, \qquad (2\text{-}1)$$

where the agent derives utility from her young-age and old-age consumption c_t^y and c_{t+1}^o along with old-age leisure l_{t+1}^o. This specification is one of the monotonic transformations of standard log utility function given as Equation (2-2), and the similar preferences are employed in the endogenous retirement models in Aisa et al.

(2012) and Matsuyama (2008). By taking log of Equation (2-1), the agent's lifetime utility is transformed to:

$$\ln U^{t-1} = \ln c_t^y + \beta \ln c_{t+1}^o + \eta \ln l_{t+1}^o. \tag{2-2}$$

This specification guarantees interior solutions for the agent's life-cycle consumption and leisure in her old period. Utility from old-age consumption is discounted for the subjective rate of time preference ρ, where $\beta = \left(1/(1+\rho)\right)^{30}$. Attached to the third terms in the right-hand side of Equation (2-2), the constant parameter η is measuring the intensity of utility from old-age leisure.

2.3.2 Human Capital Accumulation

We introduce the endogenous health maintenance process in this study. Suppose that each agent is born with perfect health and can maintain the same health level throughout her young parenthood, the health level of the young agent H_t^y at time t is normalized to one. The young parent devotes a fraction of her time v_t to maintain her own health level preparing for when she is old. The average health level of old parent at time $t+1$ is given by:

$$H_{t+1}^o = \psi v_t H_t^y = \psi v_t, \qquad \psi > 0, \tag{2-3}$$

where ψ is a constant parameter measuring the productivity of health capital investment. The old parent's health level represents the length of her old-age period as a portion of one generation.[11]

The young parent acts as an educator of her child. The intergenerational transfer of education capital is directed from the young parent, who possesses education capital stock E_t^y and spends her time u_t at time t to her child. Consequently, the child maintains the education level E_{t+1}^y during her young parenthood at time $t+1$. The technology of imparting education capital to next generation is given by:

$$E_{t+1}^y = \gamma u_t E_t^y, \qquad \gamma > 1, \tag{2-4}$$

where γ is a constant parameter measuring the productivity of education capital investment.[12]

The young parent's productivity E_t^y depreciates as she gets old, and the fraction of young-age productivity maintained in the old-age period depends on the exogenous component $\bar{\delta}$ multiplied by the endogenous component $(\gamma u_t)^{\mu}(\alpha\psi v_t)^{\phi}$, where we assume that the health level of old parent ψv_t always works to alleviate the

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[11] In this study, one generation is thirty years, and the length of the agent's old-age period is given by $\psi v_t = \left(\overline{le} - 59\right)/30$, where $\overline{le}$ is the average life expectancy at birth.

[12] The thirty years period growth rate is given by $\gamma u_t = (1+g)^{30}$, where g is the average annual growth rate of the economy. See our numerical analyses in the following section.

depreciation of productivity, hence we set the constant parameter ϕ to be non-negative. As discussed in the previous sections, there is no clear consensus in terms of the effect of time devoted to education investment u_t on the depreciation of productivity. In this study, we assume that the higher the education level of young adults is attained the smaller the old agents' productivity becomes relatively because the elder agents have to compete with their well-educated children in the labor markets. Therefore, we set the constant parameter μ to be non-positive. The intertemporal transition of education capital from the young parent at time t to her old parenthood at time $t+1$ is given by:[13]

$$E_{t+1}^{o} = \bar{\delta}(\gamma u_t)^{\mu}(\alpha\psi v_t)^{\phi}E_t^{y}, \qquad 0 \leq \bar{\delta} \leq 1, \qquad \mu \leq 0, \qquad \phi \geq 0, \tag{2-5}$$

where α is a scalar to make the term $\alpha\psi v_t$ larger than one. As a result, the effects of elder's education and health levels on her productivity increase with the absolute values of parameters μ and ϕ, respectively.[14]

2.3.3 Time Allocation

The social planner makes all decisions for the agent born in period $t-1$. The total amount of time endowed to the young parent at time t is H_t^y, which is her current health level and normalized to one. On behalf of the young parent, the social planner chooses time devoted to education of her child u_t, time devoted to invest in her own health v_t, and hence time spent on work $1-u_t-v_t$. The young parent combines her earning capacity E_t^y with the time allocation for working and earns wage income.[15] The social security, which is going to be added to this model, is the conventional PAYG system. Contributions to the social security are collected from the young parent based on her education capital stock and her health level that is perfect and normalized to one, and in the same period the social security benefit is paid to her parent as an intergenerational transfer without constituting any funds. At time t, the government sets the contribution rate τ so as to balance its budget constraint. The balanced PAYG benefit S_t is defined as:

$$S_t = \tau E_t^y H_t^y. \tag{2-6}$$

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13 If the agent invests in her own education, this should help her to maintain higher productivity at old-age. However, the rate of return from investments in her own education is lower than that from investments in her own health because health investments increase both productivity at old-age and length of old-age period. Therefore, she does not invest in her own education as the optimization problem leads to corner solution. In order to investigate the tradeoff between education and health capital, which are the determinants of economic growth and longevity, respectively, as clearly as possible, the other types of investments such as private savings are not examined in this study.

14 The term γu_t is supposed to be larger than one as long as the economy's growth rate is positive, whereas the term ψv_t is always less than one as it represents a portion of one generation.

15 The earning capacity of the young parent at time t is $wE_t^y H_t^y$, where w is the wage rate per effective labor, to be precise. As we normalized both w and H_t^y equal to one, the earning capacity is simplified to E_t^y.

The young parent earns wage income and pays the PAYG contribution $\tau E_t^y H_t^y$ to the government. Then, her consumption at time t is determined by:

$$c_t^y = E_t^y H_t^y (1 - u_t - v_t - \tau) = E_t^y (1 - u_t - v_t - \tau). \tag{2-7}$$

Upon reaching her old parenthood, the same agent receives the social security benefit from the government $S_{t+1} = \tau E_{t+1}^y H_{t+1}^y$ and also earns wage income from her old-age work $H_{t+1}^o E_{t+1}^o$ if she continues to work. The consumption of the old parent at time $t + 1$ is given by:[16]

$$\begin{aligned} c_{t+1}^o = E_{t+1}^o H_{t+1}^o e_{t+1} + \tau E_{t+1}^y H_{t+1}^y \\ = E_t^y \left[\bar{\delta}(\gamma u_t)^\mu (\alpha \psi v_t)^\phi \psi v_t e_{t+1} + \tau \gamma u_t\right], \end{aligned} \tag{2-8}$$

where e_{t+1} is the indicator function taking the value 0 if the old agent retires and 1 if she continues to work. The leisure of the old parent is stated as:

$$l_{t+1}^o = 1 + H_{t+1}^o (1 - e_{t+1}) = 1 + \psi v_t (1 - e_{t+1}), \tag{2-9}$$

where, in case that she continues to work ($e_{t+1} = 1$), she still enjoy the base level of leisure that is normalized to 1 and, in case that she retires ($e_{t+1} = 0$), her retirement leisure depends on her health level in addition to the base leisure and amounts to $1 + \psi v_t$.

2.3.4 The Optimization Problem

In this study, we concentrate on the social planner solution in order to maintain the rate of return on education capital investment positive,[17] and the problem will be solved through the two-stage optimization process. For the first stage, the social planner maximizes the logarithmic utility function (2-2) with respect to time devoted to education and health investments u_t and v_t and the PAYG social security contribution rate τ subject to budget constraints (2-7), (2-8), and (2-9). The necessary optimal conditions are as follows:

$$u_t: \quad \frac{1}{c_t^y} = \frac{\beta}{c_{t+1}^o}\left[\bar{\delta}\mu\gamma(\gamma u_t)^{\mu-1}(\alpha\psi v_t)^\phi \psi v_t e_{t+1} + \tau\gamma\right], \tag{2-10}$$

$$v_t: \quad \frac{1}{c_t^y} E_t^y = \frac{\beta}{c_{t+1}^o} E_t^y \bar{\delta}(1+\phi)(\gamma u_t)^\mu (\alpha\psi v_t)^\phi \psi e_{t+1} + \frac{\eta}{l_{t+1}^o}\psi(1 - e_{t+1}), \tag{2-11}$$

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[16] The social planner recognizes the PAYG benefit as a function of the old agent's own health capital stock and her child's education capital stock. However, the agent herself takes the defined-benefit S_{t+1} as given. This leads to two moral hazards in a decentralized solution.

[17] In the case of a selfish agent, such as our current model, only two ways making the rate of return on education investment positive are specifying family insurance system and PAYG social security with a social planner solution (Ehrlich & Lui, 1998).

$$\tau: \qquad \frac{1}{c_t^y} = \frac{\beta}{c_{t+1}^o}\gamma u_t. \tag{2-12}$$

Equation (2-10) says that the loss in utility from earning less and investing more for her child's education is compensated by the utility gain obtained from higher old-age consumption due to the increase in the PAYG social security benefit due to the second term in the square bracket, but the gain decreases due to her lower old-age wage income when she chooses to work due to the first term in the square bracket.[18] Equation (2-11) implies that utility foregone from reducing one more unit of the young parent's consumption for investing in her own health capital is equal to utility obtained from higher old-age consumption due to the longer working life-span and higher old-age productivity, if she chooses to work at old-age due to the first term on the right-hand side, or from her longer retirement leisure, if she retires due to the second term. Equation (2-12) suggests that the rate of return from the PAYG social security equals to the growth rate of economy γu_t. By rearranging Equation (2-12), the optimal social security contribution rate is given by:

$$\tau = \frac{1}{1+\beta}\left[\beta(1-u_t-v_t) - \bar{\delta}(\gamma u_t)^{\mu}(\alpha\psi v_t)^{\phi}\frac{\psi v_t}{\gamma u_t}e_{t+1}\right], \tag{2-13}$$

which shows that if the agent continues to work when she is old ($e_{t+1} = 1$) the optimal PAYG social security contribution rate τ is lower than the other case ($e_{t+1} = 0$) because in the former case she relies on the social security system less for her old-age consumption. The longer life expectancy when she reaches old-age period ψv_t decreases the optimal contribution rate whereas the higher productivity of future generation γu_t increases it. Using Equation (2-13), Equations (2-6) and (2-7), young-age and old-age consumption can be restated as:

$$c_t^y = \frac{1}{1+\beta}E_t^y\left[1-u_t-v_t+\bar{\delta}(\gamma u_t)^{\mu}(\alpha\psi v_t)^{\phi}\frac{\psi v_t}{\gamma u_t}e_{t+1}\right], \tag{2-14}$$

$$c_{t+1}^o = \frac{\beta}{1+\beta}\gamma u_t E_t^y\left[1-u_t-v_t+\bar{\delta}(\gamma u_t)^{\mu}(\alpha\psi v_t)^{\phi}\frac{\psi v_t}{\gamma u_t}e_{t+1}\right]. \tag{2-15}$$

In the second stage of this optimization problem, the social planner decides if the agent continues to work or retires when she is in her old period at time $t+1$. By substituting Equations (2-14) and (2-15) into the lifetime utility function (2-1), the social planner derives the agent's respective lifetime utilities for two cases of working at old-age and retirement and compares them to find which one yields a higher level of utility. The level of utility if the agent decides to work when she is old is given by:

[18] The parameter μ is assumed to be non-positive.

$$U^{t-1}(e_{t+1}=1) = [\beta\gamma u_1]^{\beta}\left[\frac{1}{1+\beta}E_t^y\right]^{1+\beta}\left[1-u_1-v_1 + \bar{\delta}(\gamma u_1)^{\mu}(\alpha\psi v_1)^{\phi}\frac{\psi v_1}{\gamma u_1}\right]^{1+\beta}[1]^{\eta}, \qquad (2\text{-}16)$$

and the level of utility if she retires is given by:

$$U^{t-1}(e_{t+1}=0) = [\beta\gamma u_0]^{\beta}\left[\frac{1}{1+\beta}E_t^y\right]^{1+\beta}[1-u_0-v_0]^{1+\beta}[1+\psi v_0]^{\eta}, \qquad (2\text{-}17)$$

where the subscripts 1 and 0 indicate the different values of education and health investments u and v when the agent work at old-age and retire, respectively. Comparing Equations (2-16) and (2-17), the condition for the agent to prefer working to retiring in her old-age, or equivalently to choose $e_{t+1}=1$, is stated as:

$$[u_1]^{\beta}\left[1-u_1-v_1+\bar{\delta}(\gamma u_1)^{\mu}(\alpha\psi v_1)^{\phi}\frac{\psi v_1}{\gamma u_1}\right]^{1+\beta} \geq [u_0]^{\beta}[1-u_0-v_0]^{1+\beta}[1+\psi v_0]^{\eta}. \qquad (2\text{-}18)$$

The immediate implications of the condition (2-18) are as follows: The part $(\gamma u_1)^{\mu}$, where $\mu \leq 0$, in the left-hand side term represents the technology effect as the stimulus for working at old-age decreases when the agent invests more in her child's education. The adjacent part $(\alpha\psi v_1)^{\phi}$, where $\phi \geq 0$, represents the horizon effect, and the stimulus for working at old-age increases when the agent invests more in her own health. The last part of the right-hand side term $1+\psi v_0$ is utility from retirement leisure, which increases with health investments or life expectancy. This part represents the leisure effect as increases in the part induce the agent to retire.[19] It has to be noted that, in exogenous growth and longevity models, the above three effects on the agent's retirement behavior do not depend on the key choice variables, whereas the three effects are endogenously given in our model.

When the agent chooses to retire, by setting $e_{t+1}=0$ in Equations (2-10), (2-11), and (2-13) and equalizing the rates of return from education and health investments with that from the PAYG social security so that interior solutions are derived, we can analytically solve for the optimal levels of education and health investments and PAYG contribution rate as follows:

$$u_0{}^* = \tau_0{}^* = \frac{\beta(1+\psi)}{(1+2\beta+\eta)\psi}, \qquad v_0{}^* = \frac{\eta\psi-(1+2\beta)}{(1+2\beta+\eta)\psi}.$$

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[19] The contribution rate of PAYG social security τ does not appear in the condition (2-18). As we obtain the social planner solution, the pension effect is not well defined, and we do not discuss it in this study.

$v_0{}^*$ increases with the value of parameter ψ or η, which measures the productivity of health investments or the intensity of utility from leisure, respectively, but $u_0{}^*$ and $\tau_0{}^*$ decrease with these parameter values. The parameter γ measuring the productivity of education investments does not appear in none of the optimal values. Consequently, $1 - u_0{}^* - v_0{}^*$ in the right-hand side of the condition (2-18) equals to $(1+\beta)(1+\psi)/(1+2\beta+\eta)\psi$, and $1 + \psi v_0{}^*$ equals to $\eta(1+\psi)/(1+2\beta+\eta)$.

In case that the agent chooses to work at old-age, that is, if $e_{t+1} = 1$, after setting the rates of return from human capital investments and that from social security to be equal, we can show that the optimal levels of $u_1{}^*$, $v_1{}^*$, and $\tau_1{}^*$ are derived by solving Equation (2-13) and the two equations below simultaneously.

$$u_1 = \bar{\delta}\mu(\gamma u_1)^{\mu}(\alpha\psi v_1)^{\phi}\frac{\psi v_1}{\gamma u_1} + \tau_1, \tag{2-19}$$

$$v_1 = \bar{\delta}(1+\phi)(\gamma u_1)^{\mu}(\alpha\psi v_1)^{\phi}\frac{\psi v_1}{\gamma u_1}. \tag{2-20}$$

Though there are no analytical solutions for this problem in general, in case that ϕ is zero, we can solve the problem analytically and express the optimal levels of $u_1{}^*$, $v_1{}^*$, and $\tau_1{}^*$ by the preference and productivity parameters. In the following sub-sections, we examine implications of the condition (2-18) further by distinguishing three scenarios, where depreciation of productivity at old age is exogenous to education and health investments ($\mu = \phi = 0$), alleviated by health investments ($\mu = 0$, $\phi > 0$), or aggravated by education investments ($\mu < 0$, $\phi = 0$).

(i) The depreciation is exogenous to human capital investments: $\mu = \phi = 0$

By setting both μ and ϕ to zero, this scenario implies that depreciation of the agent's education capital stock is independent of her human capital investments decisions as Equation (2-5) becomes $E^o_{t+1} = \bar{\delta}E^y_t$. When the fraction of young-age productivity maintained in the old-age period depends only on an exogenous parameter $\bar{\delta}$, the condition for working at old-age (2-18) simplifies to:

$$[u_1{}^*]^{\beta}[1-u_1{}^*]^{1+\beta} \geq [\beta]^{\beta}[1+\beta]^{1+\beta}[\eta]^{\eta}\left[\frac{1+\psi}{1+2\beta+\eta}\right]^{1+2\beta+\eta}\left[\frac{1}{\psi}\right]^{1+2\beta}, \tag{2-21}$$

where the right-hand term is evaluated by using the optimal levels of $u_0{}^*$, $v_0{}^*$, and $\tau_0{}^*$. As both μ and ϕ are zero in this scenario, the technology and horizon effects are absent in the condition (2-21). Using Equations (2-13), (2-19), and (2-20), we can analytically solve for the optimal levels of $u_1{}^*$, $u_1{}^*$, and $\tau_1{}^*$ as follows:

$$u_1{}^* = \tau_1{}^* = \frac{\bar{\delta}\psi}{\gamma}, \qquad v_1{}^* = \frac{\beta\gamma - (1+2\beta)\bar{\delta}\psi}{(1+\beta)\gamma}.$$

On the contrary to the case that the agent chooses to retire, $v_1{}^*$ decreases with the value of parameter ψ, but $u_1{}^*$ and $\tau_1{}^*$ increase with this value. Increases in the value of γ have the opposite effects on the three values. By substituting these optimal values in, the above condition for the agent to work in her old-age (2-21) is expressed only by the preference and productivity parameters in the model and given by:

$$[\bar{\delta}\psi]^{\beta}[\gamma - \bar{\delta}\psi]^{1+\beta}\left[\frac{1}{\gamma}\right]^{1+2\beta} \geq [\beta]^{\beta}[1+\beta]^{1+\beta}[\eta]^{\eta}\left[\frac{1+\psi}{1+2\beta+\eta}\right]^{1+2\beta+\eta}\left[\frac{1}{\psi}\right]^{1+2\beta}. \quad (2\text{-}22)$$

The implications of the condition (2-22) for the plausible ranges of parameter values are as follows: If the parameter γ, ψ, or η becomes larger, the stimulus for retirement strengthens. Whereas, the increases in the parameter β or $\bar{\delta}$ weaken the stimulus for retirement.[20] As a result, this scenario of exogenous deterioration of productivity at old-age confirms that the income and leisure effects exist because wages and utility from leisure increase with γ and η, respectively. The effect of ψ on retirement decision deserves a special attention. It is a part of the leisure effect because increases in ψ, which are associated with longer lifespan, enable the agent to enjoy her longer and healthier retirement life and hence strengthen the stimulus for retirement. In the existing endogenous retirement models, in which longevity is exogenously given, and hence ψ is absent, the leisure effect depends only on η.

(ii) The depreciation is alleviated by health investments - Low development: $\mu = 0, \phi > 0$

The positive value of ϕ suggests that the healthier the agent is the higher her old-age productivity becomes, whereas her education level does not affect it because μ is zero. This scenario signifies that the agent engages in physically demanding tasks in a society at a low level of development. When the parameter μ equales zero, the labor participation condition (2-18) is simplified to:

$$[u_1{}^*]^{\beta}\left[1 - u_1{}^* - \frac{\phi}{1+\phi}v_1{}^*\right]^{1+\beta} \geq [\beta]^{\beta}[1+\beta]^{1+\beta}[\eta]^{\eta}\left[\frac{1+\psi}{1+2\beta+\eta}\right]^{1+2\beta+\eta}\left[\frac{1}{\psi}\right]^{1+2\beta}. \quad (2\text{-}23)$$

There exists the horizon effect due to the positive value of ϕ, but the technology effect is absent in this low development economy. This particular scenario is comparable to the model in Aisa et al. (2012) as they also assume that productivity of elder workers increases with their life expectancy. They suggest that as far as (exogenous) improvements in health level of workers alleviate depreciation of productivity at old-age it motivates the workers to stay in the labor market longer, and this horizon effect would dominate the income and leisure effects. Nevertheless, their study does not include the technology effect as in this scenario.

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[20] The values of parameters are discussed in the following section in detail.

Because there are no analytical solutions for the optimal levels of $u_1{}^*$, $u_1{}^*$, and $\tau_1{}^*$ when ϕ is not zero, we cannot express the above condition solely by the preference and productivity parameters. It would be safe to say that the increase in η was still the stimulus for retirement, but because the optimal levels $u_1{}^*$ and $v_1{}^*$ were dependent of the other parameters we could not predict their effects on the agent's labor force participation decision. In the following section, we will perform numerical analyses in order to find these effects.

(iii) The depreciation is aggravated by education investments – High development: $\mu < 0, \phi = 0$

In this scenario, the agent's health investment does not alleviate depreciation of her productivity anymore as ϕ is zero. At the same time, the negative value of μ implies that the more the agent invests in her child's education the faster her own productivity at old-age deteriorates relatively. These contexts signify that the agent resides in a developed economy, where she may retire due to her depreciated productivity even if she is physically fit enough to work. Under this scenario, the condition for the agent's working at old-age (2-18) is simplified to:

$$[u_1{}^*]^\beta[1-u_1{}^*]^{1+\beta} \geq [\beta]^\beta[1+\beta]^{1+\beta}[\eta]^\eta\left[\frac{1+\psi}{1+2\beta+\eta}\right]^{1+2\beta+\eta}\left[\frac{1}{\psi}\right]^{1+2\beta}, \qquad (2\text{-}24)$$

where the horizon effect is absent, but the technology effect exists. The analytical solutions for the optimal levels of $u_1{}^*$, $u_1{}^*$, and $\tau_1{}^*$ when the agent works at old-age in the developed economy are given by:

$$u_1{}^* = \frac{(\bar{\delta}\psi)^{\frac{1}{1-\mu}}}{\gamma}, \qquad v_1{}^* = \frac{\beta\gamma-(1+2\beta)(\bar{\delta}\psi)^{\frac{1}{1-\mu}}}{(1+\beta)(1-\mu)\gamma},$$

$$\tau_1{}^* = \frac{(1+\beta(1+\mu))(\bar{\delta}\psi)^{\frac{1}{1-\mu}}-\beta\mu\gamma}{(1+\beta)(1-\mu)\gamma}.$$

Consequently, the condition for the agent to work in her old-age (2-18) is expressed only by the preference and productivity parameters in the model and given by:

$$\left[(\bar{\delta}\psi)^{\frac{1}{1-\mu}}\right]^\beta\left[\gamma-(\bar{\delta}\psi)^{\frac{1}{1-\mu}}\right]^{1+\beta}\left[\frac{1}{\gamma}\right]^{1+2\beta} \geq [\beta]^\beta[1+\beta]^{1+\beta}[\eta]^\eta\left[\frac{1+\psi}{1+2\beta+\eta}\right]^{1+2\beta+\eta}\left[\frac{1}{\psi}\right]^{1+2\beta}. \qquad (2\text{-}25)$$

The only difference between the labor force participation conditions (2-22) and (2-25) is whether the terms $\bar{\delta}\psi$ is powered to 1 or $1/(1-\mu)$. Therefore, the same predictions on the effects of the parameters other than μ in the sub-section (i) apply under this scenario as well. If the additional parameter μ decreases (or the absolute value of μ increases), the value of right-hand side term in the condition (2-25) increases, or equivalently, the stimulus for retirement strengthens. This result confirms that the technology effect exists and becomes stronger when the absolute value of μ increases. As discussed in Ferreira and Pessoa (2007), in an economy experiencing rapid technological changes, there is the higher chance of people choosing retirement over working at old-age.

In the following section, we will quantitatively examine the effects of changes in the two parameter values μ and ϕ on the agent's human capital investments, her labor force participation, and social welfare.

2.4 Numerical Analyses

In order to quantify the predicted effects of development levels on the optimal education and health investments and PAYG social security contribution, the parameters of our model are calibrated using actual Japanese data and consensus estimates from the existing literature. Following Blackburn and Cipriani (2005), the time preference parameter ρ is set to be 0.023, so the discount factor is $\beta = (1/1.023)^{30} \cong 0.5055$ as the one period in this study spans thirty years.[21] The average length of old-age period of Japanese elders from age 60 to 90 is calculated to be $(80.4 - 59)/30 \cong 0.7133$ from the last thirty years' average life expectancy, 80.4, and the long-run one period growth of $(1 + 0.0158)^{30} \cong 1.6005$ is derived from the average annual growth rate of the same period, 1.58% ("World Development Indicators 2013," 2013). As the majority of Japanese elders currently choose to retire, from Equations (2-3) and (2-4), we can specify that $\psi {v_0}^* \cong 0.7133$, and $\gamma {u_0}^* \cong 1.6005$, where ${v_0}^*$ and ${u_0}^*$ are the optimal levels of education and health investments in case that the agent chooses to retire, respectively. Together with the above two values and the actual PAYG social security contribution rate ${\tau_0}^* = 0.18$ in Japan, the preference parameter η and the productivity parameters γ and ψ are solved by combining the optimal levels of education and health investments ${u_0}^* = {\tau_0}^* = \beta(1+\psi)/(1+2\beta+\eta)\psi$ and ${v_0}^* = \big(\eta\psi - (1+2\beta)\big)/(1+2\beta+\eta)\psi$ ("Social Security Program throughout the World: Asia and Pacific 2012," 2013). The solution yields the following calibrated parameter values: $\eta \cong 1.9152$, $\gamma \cong 8.8915$ and $\psi \cong 2.5124$. Under the baseline scenario (i) where the agent chooses to work at old-age $e_{t+1} = 1$, and the parameters μ and ϕ equal to zero, the parameter $\bar{\delta}$, which measuring the exogenous component of depreciation of productivity at old-age, is calculated to be 0.6370 from ${u_1}^* = \bar{\delta}\psi/\gamma$. The remaining scalar parameter $\alpha \cong 6.6820$ is chosen so as to make $\gamma {u_1}^* = \alpha\psi {v_1}^*$ under the baseline scenario (i). These parameter values are used for the other two scenarios, where either $\mu = 0$, $\phi > 0$ or $\mu < 0$, $\phi = 0$, examined in this section as well. We then perform comparative static analyses numerically.

Table 1 reports the simulated effects of changes in the parameter values μ and ϕ, which represent five levels of development, on the agent's key choice variables

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[21] While Blackburn and Cipriani (2005) do not specify to which economy their model is calibrated, the values of discount factor should be similar among the developed economies (see e.g. International Monetary Fund, Interest Rates, Discount Rate for Japan and United States).

when she chooses to work at old age ($e_{t+1} = 1$). As discussed in the previous section, the larger value of ϕ signifies lower development, whereas the larger absolute value of μ is associated with higher development. Consequently, in *Table 1*, moving from the second column to the sixth column, in which the agent works at old-age, the level of development increases. The optimal levels of key choice variables when she chooses to retire ($e_{t+1} = 0$) are reported in the first column of *Table 1*.[22] The optimal levels of education investment and PAYG contribution rate are same (${u_0}^* = {\tau_0}^* = 0.18$). It has to be noted that the optimal level of health investment (${v_0}^* = 0.2839$) in the first column is higher than those in the other columns. This implies that the retiring agent's old-age period is significantly longer than that of working elders. Considering that the length of old-age period is given by $\psi v \times 30$, while the old-age periods of working elder in the second to sixth columns span 7.2 years or less, in the first column the agent lives for 21.4 years after her retirement.[23] As for the five cases, in which the agent chooses to work at old-age ($e_{t+1} = 1$), the higher level of development are associated with the lower education investment of the agent ${u_1}^*$ and hence the lower growth rate of economy γu. In contrast, the higher the level of development gets the larger the health investment ${v_1}^*$ and life expectancy ψv becomes. These predictions accord well with empirical observations, namely, as the economy develops, economic growth slows down, while life expectancy rises.

Now, we turn to the agent's retirement decision. The seventh row, $Work\ or\ Retire$, displays the stimuli for retirement and working at old-age in the first column and the other five columns, respectively. The stimulus for retirement equals to the right-hand side terms in the condition for working at old-age (2-18), and that for working at old-age equals to its left-hand side term. When the model is calibrated to Japanese economy, the stimulus for retirement in the first column, 0.4610, is greater than any one of the stimuli for working at old-age, which range between 0.3134 and 0.3071, and the agent chooses retirement under all of the scenarios. Nevertheless, we can point out that the stimulus for working at old-age is smaller as an economy develops. On the one hand, in the scenario (ii) of the less developed economy, the second and third columns of *Table 1*, for which we do not have analytical solutions, our numerical analyses confirm that the horizon effect exists as the stimuli for working at old-age under this scenario are larger than those under the other two scenarios shown in the fourth to sixth columns.[24] Therefore, as Aisa et al. (2012) suggest, when the agent's better health condition alleviates depreciation of her productivity at old-age, the stimulus for working at old-age is stronger as the value of ϕ increases. On the other hand, our analytical and numerical results accord with the prediction in Ferreira and Pessoa (2007) as in a highly innovative economy the technology effect becomes dominant, and people tend to choose early retirement. Thus, the stimulus for working at old-age is weaker as the absolute value of μ increases.

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[22] The increases in the level of development do not affect the key choice variables of retiring agent shown in the first column.

[23] As the one period in this model spans 30 years, the annual growth rate is given by: $(\gamma u)^{\frac{1}{30}} - 1$, and the life expectancy at birth of the agents is given by: $30(1 + 1 + \psi v)$.

[24] The horizon effect exists but is not dominant, as the agent chooses retirement.

Lastly, we report the effects of changes in level of development on social welfare of the economy for each scenario. Letting $ln\,\bar{U}^{t-1}$ denote the utility level of the agent born in $t-1$ evaluated at the steady-state values of the choice variables, we can now express the social welfare function as:

$$\Omega = \frac{1}{1-\omega}\sum_{t=1}^{\infty}\omega^{t-1}\ln\bar{U}^{t-1}, \qquad 0<\omega<1, \tag{2-26}$$

where ω is the generational discounting factor and set to be equal to β. The initial level of young agent's earning capacity E_1^y is set to be the average per capita GDP value from 1995 to 2014 expressed in US Dollar using purchasing power parity to convert Japanese currency ("World Development Indicators 2013," 2013). The highest level of social welfare is achieved when the agent chooses to retire (e_{t+1} =0) in the first column. If she continues to work when old (e_{t+1} =1), the less the economy is developed the higher the level of social welfare is as shown in the eighth row of *Table 1*. This result suggests that, if the government tries to raise the pension eligibility age of its social security program, to encourage the elders to stay in the labor market requires the larger sacrifice in terms of social welfare as the economy develops.

Table 1: Simulation results: quantitative impacts of changes in the parameters μ and ϕ

	Retiring	$\mu = 0$ $\phi = 0.2$	$\mu = 0$ $\phi = 0.1$	$\mu = 0$ $\phi = 0$	$\mu = -0.1$ $\phi = 0$	$\mu = -0.2$ $\phi = 0$
u: Education investment (%)	0.18	0.2137	0.2015	0.18	0.1725	0.1664
γu: Growth rate (%)	1.6005	1.9005	1.7915	1.6005	1.5335	1.4798
v: Health investment (%)	0.2839	0.0565	0.0709	0.0953	0.0958	0.0946
$\psi v * 30$: Longevity (years)	21.399	4.26	5.346	7.185	7.221	7.125
τ: PAYG contribution rate (%)	0.18	0.2137	0.2015	0.18	0.1820	0.1853
$\bar{\delta}(\gamma u_1)^{\mu}(\alpha\psi v_1)^{\phi}$: Depreciation (%)	N/A	0.6304	0.6482	0.6370	0.6104	0.5890
$Work\ or\ Retire$	0.4610	0.3134	0.3132	0.3117	0.3093	0.3071
Ω: Social welfare	64.5137	64.0170	63.6427	62.9133	62.6121	62.3589

Source: The authors' simulation.
Notes: Parameter values used are: $\beta = 0.5055$, $\eta = 1.9152$, $\gamma = 8.8915$, $\psi = 2.5124$, $\bar{\delta} = 0.6370$, $\alpha = 6.6820$ $E_1^y = 33392$, and $\omega = \beta$.

2.5 Conclusion

In this study, we develop the three-period overlapping generations model examining the joint determination of human capital investments and old agents' labor force participation. We assume that the basis of retirement behavior has undergone the transition during the last half of the twentieth century. While in the past people's retirement decisions were outcomes of their worsening health conditions, in current and more advanced economies people would choose to retire because of their depreciated stocks of education capital. We try to capture this transition in our model, which is solved in two stages. In the first stage of our optimization problem, the social planner chooses the optimal levels of investments in education and health and postulates the optimal contribution rate of the PAYG system as the function of the two human capital investments. In the second stage, the social planner compares lifetime utility derived for the cases of retirement and working at old-age and chooses one that yields higher utility for the agent. More specifically, in this stage, we are examining the relative strength of the stimuli for retirement and working at old-age. The former includes the income, leisure, and technology effects, and the latter comprises of the horizon effect. The magnitudes of the technology and horizon effects depend on the parameters μ and ϕ and are endogenous to education and health investments of the agent.[25] At the low level of development, the positive values of ϕ implies that depreciation of productivity at old-age is alleviated by better health conditions of elders, and their labor force participation rises with the value of ϕ. As the economy grows, jobs become less demanding physically, and rapid innovation makes it harder for elder workers to keep up with new technology. The negative value of μ signifies that depreciation of productivity at old-age is aggravated by higher and more updated education capital of their children. First time at this phase of development, the chance that increases in education investment result in the rise of retirement emerges. Our numerical analysis confirms that the higher the level of development is achieved the weaker the stimulus for working at old-age becomes. If this is the case, the recent pension reforms in the developed countries, which had raised their social security programs' pension eligibility ages, would go against the interest of elders, and the raise in retirement ages might lead to unemployment at old-age. The simulation results also reveal that in an advanced economy there would be the very high costs of social welfare if the elders had to switch from retiring to continuing to work at old-age.

[25] According to Graebner (1980), the higher agents' education level gets the faster their education capital stocks depreciate.

Chapter 3: Pay-As-You-Go Social Security with Endogenous Longevity

Abstract

This study develops a three-period overlapping generations model that includes a health maintenance sector in addition to a child education sector. A young parent invests in her child's education in order to secure her old-age supports and in her own health to reap higher old-age consumption. We find that Pay-As-You-Go (PAYG) social security distorts the key family-based choices via two channels of moral hazards. One results in lower investments in education, and another causes higher investments in health. With the rigidity in formal retirement age, these distortions could retard economic growth and raise old-age dependency. The model is calibrated to fit Japanese economy to examine the effect of the recent social security reform required to make the PAYG system sustainable on economic growth and old-age dependency rates. Our numerical analyses reveal that an increase in the contribution rate can sustain the system in the short-run. However, the financial sustainability of the PAYG system will disappear in the long-run when the old-age dependency rate becomes drastically high and the economic growth stays low as the moral hazard effects accumulate.

JEL Classification Codes: H55, I15, I25, J14, O15
Keywords: Social security, Health, Education, Old-age dependency, Growth

3.1 Introduction

Throughout history, family or community insurance mechanisms were the prevailing means of old-age supports until developed nations introduced social security programs in the early twenties century. While there is no doubt that children provide their elderly parents with intergenerational transfers out of love, Ehrlich and Lui (1991) portray and model the family insurance mechanism in the case of selfish agents connected via material interdependencies.[26] In each family unit, parents raise and educate their children, who upon reaching young adulthood repay their parents by offering them old-age supports, and self-enforcing implicit contracts between generations determine how these intergenerational transfers are carried out. By the implicit contracts, an endogenously determined rate of compensation specifies the amount of old-age supports to be proportional to the stock of education capital produced in the children. As a result, the process of accumulating education capital has been motivated through the mutual insurance mechanism within the family unit. Their model also shows that optimal intergenerational trades will maximize not only the joint utility of overlapping generations but also the economy's attainable growth

[26] This refers to their benchmark model. Ehrlich and Lui (1991) further investigate the altruistic agent models.

rate. Under an extended family or communal arrangement, the safe family-insurance mechanism could have been an equivalent of socially efficient and actuarially fair security system. However, as Ehrlich and Lui (1991) suggest, a higher level of education capital reduces the rate of return on children's quantity relative to that on their quality, and fertility rates in developed countries have fallen to near its lowest permissible level by the end of the twenties century. This combined with the other factors such as urbanization leads to the transition of social norms, that is, more people start to live in a nuclear family set-up than stay with the traditional extended family. Then, if a family insurance system is operated within a nuclear family, portfolio risks of the system are no longer diversifiable because there is a possibility of children not to survive to adult or a case of non-compliance of bad children with the family contracts.

Due to the default risks of the family based insurance system inherent in the nuclear family set-up, larger societies have adopted public security systems. According to Barr and Diamond (2006), while from an individual's point of view income security in old age requires consumption smoothing and insurance, public policies generally aim at additional objectives of poverty relief and redistribution. The first two goals are attainable through fully funded (FF) or earning-related Pay-As-You-Go (PAYG) social security systems, but the latter two are better addressed by universal or defined-benefits social security programs, which provide flat-rate cash benefits to residents. All over the world, PAYG social security systems are chosen by a majority of countries because management costs of the systems are significantly low compared to FF social security systems. On the other hand, Singapore, Malaysia, and some countries in the South America have decided on FF social security systems. As discussed in several studies, FF schemes can be taken as a kind of mandatory annuitized savings, and its social security fund holds financial assets in order to pay for the future claims of the retirees, whereas future benefits of PAYG social security systems are based on the governments' promises (Bloom et al., 2007; Junsen Zhang et al., 2001). PAYG social security has been introduced in many countries as a state-provided alternative to the family insurance system discussed above. Ehrlich and Lui (1998) extend the family insurance model of Ehrlich and Lui (1991) by introducing the conventional PAYG type social security. In their extended model, young parents pay a proportional social security tax and continue to invest in their children because both social and family insurance systems cannot exist without children. Once the young parents survive to old age, they will receive defined-benefits from the government and old-age supports from their children. The benefits paid to old parents are uniformly set and independent of their children's contribution and hence of their past investments in children's education capital. Consequently, the young parents cannot realize the social gains from a marginal increase in their investments in children. Ehrlich and Lui (1998) point out that this moral hazard adversely affects the young parents' key family-based decision. If the economy has already reached the minimal fertility level, an exogenous increase in the PAYG proportional tax unambiguously reduces the parents' investment in their children's education capital and the economy's growth rate.

While survival probabilities are exogenously determined in Ehrlich and Lui (1998) and many other studies on the problems of aging population (Ehrlich & Kim, 2007; Sala-I-Martin, 1996; Junsen Zhang et al., 2001),[27] in this study we extend their selfish agent models by introducing an endogenous health accumulation process. Under Japan's earnings-related PAYG social security system, when longevity becomes an individual's choice variable, young parents may allocate more resources to invest in their own health capital instead of their children's education capital because they can enjoy longer retirement life without affecting the per unit period amount of social security benefits already set by their previous earnings.[28] Thus, young parents cannot realize social losses from a marginal increase in the investments in their own health capital. This extra moral hazard positively affects the young parents' incentive to invest in their own health capital and, subsequently, increases the economy's old-age dependency rate. Then, the main purpose of this study is to investigate how the double moral hazards affect financial sustainability of PAYG social security systems and to compare the results with those of FF social security systems.

The rest of chapter is organized as follows. Section 3.2 introduces basic concepts and rules of Japanese public pension systems and discusses the prospects of their future financial sustainability using macroeconomic and demographic data. Section 3.3 introduces the models, which enable us to compare the two types of social security systems. In Section 3.4, we calibrate the models to fit Japanese economy and analyze the effects of increases in the contribution rates on the replacement rates and social welfare in the long-run. Section 3.5 provides concluding remarks.

3.2 Facts and Data: Pay-As-You-Go Social Security in Japan

Two kinds of pubic old-age pension systems are currently effective in Japan: National Pension Program, a universal program covering all Japanese citizens aged 20 to 59, and Employees' Pension Insurance, an earnings-related PAYG system for employees younger than age 70 excluding self-employed persons.[29] There are no direct contributions to National Pension Program, and a part of contributions from Employees' Pension Insurance are transferred to the former in order to pay contributions for low-income residents and dependent spouses of insured persons. As for Employees' Pension Insurance, employees who are working in covered firms contribute 8.338% of their monthly wage class earnings, and employers contribute the same percentage of the employees' monthly payroll. While Employees' Pension Insurance is meant for securing or smoothing consumption of the retired people, National Pension Program is aiming at the objectives of poverty relief and redistribution. Once insured people reach age 65, National Pension Program pays the

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[27] van Zon and Muysken (2001), Blackburn and Cipriani (2002), Chakraborty (2004), and Barro (2013) are notable exceptions as they include both education and health as the components of human capital. Grossman (1972b) and Ehrlich and Chuma (1990) are the seminal studies on endogenous longevity.

[28] Though Ehrlich and Lui (1998) model the defined-benefit PAYG system, in this study, our choice of PAYG social security is the earning-related benefit type as it is selected in Japan.

[29] In Japan, there is one more system, Mutual Aid Association Pension System, which is for government workers. We do not show the details of this system because it is used by relatively small number of people and similar to National Pension System.

flat-rate full pension of 64,000 yen per month with 40 years of contributions, and the benefits from Employees' Pension Insurance are based on the insured persons' average monthly salary multiplied by the number of months of coverage and by a coefficient determined by the insured persons' date of birth (Gruber & Wise, 1998; "Social Security Programs Throughout the World: Asia and the Pacific, 2012," 2013). While National Pension Program is very important in terms of the larger population base it covers and social welfare it provides elder people with through its redistribution or poverty-relief nature, in this study, we will focus on Employees' Pension Insurance because of its straightforward PAYG social security characteristics. Generally speaking, the PAYG systems only hold claims on the government for future benefit payment and do not generate national savings. In case of Japanese Employees' Pension Insurance, the government collects contributions on earnings of employed workers and pays earnings-related benefits to retired people of the same period so as to maintain its social security fund in balance.[30] As shown in the following sections, because of this rule of contributions and payments, the system's earnings replacement rate, which is a ratio of pension benefits to retirees' past earnings, would be higher if per capita GDP growth was higher, and old-age dependency was lower at a given contribution rate.

The current law of Employees' Pension Insurance has become effective in 1954 when Japan's per capita GDP growth rate was increasing, and its old-age dependency rate was almost negligible at less than 10%. However, the past data from 1960 to 2012 in *Figure 1* and *2* clearly show that its growth rate is trending downward and approaching zero growth whereas its old-age dependency rate is rising at the increasing rates since the 1990s. Thus, the both trends alerted Japanese government to the imminent financial collapse of Employees' Pension Insurance, and in 2004 the government amended the law and started to gradually raise the contribution rate about 10% from current 8.338% to 9.15% by 2017 in order to maintain the replacement rate above the promised rate of 50% (Horioka, 1999; Horioka, Suzuki, & Hatta, 2007). According to Results of the 2014 Actuarial Valuation, Employees' Pension Insurance can maintain the replacement rate of 50% in the long-run (beyond the year of 2043) only if the country would attain the positive growth rate of at least 0.4%, the total fertility rate of 1.35, and the certain level of old-age dependency rate (working age population is expected to decrease 10% from 66 million to 59 million) ("Results of the 2014 Actuarial Valuation," 2014). *Figure 3* shows that the total fertility rate in Japan is kept mostly at1.35 for the last 20 years.

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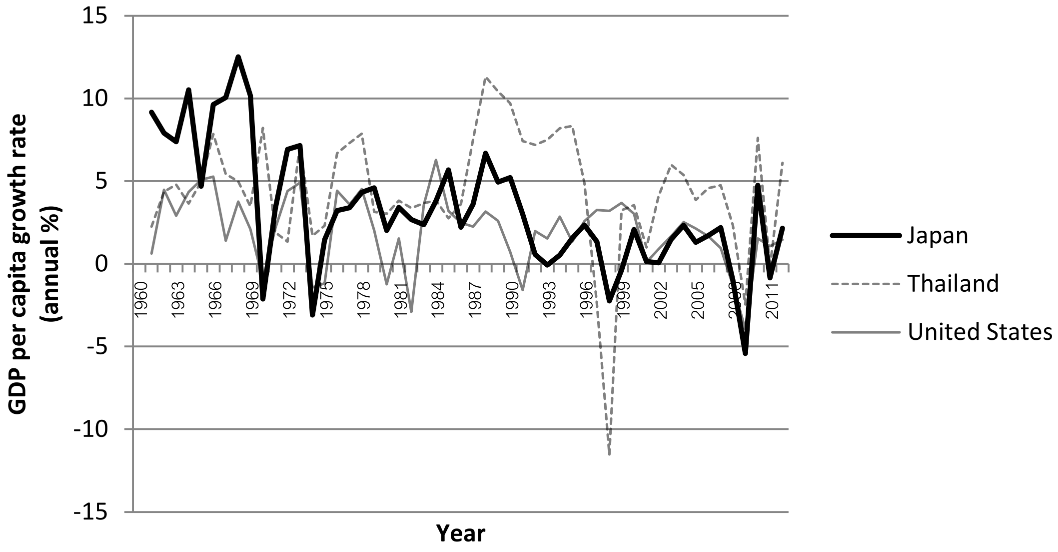
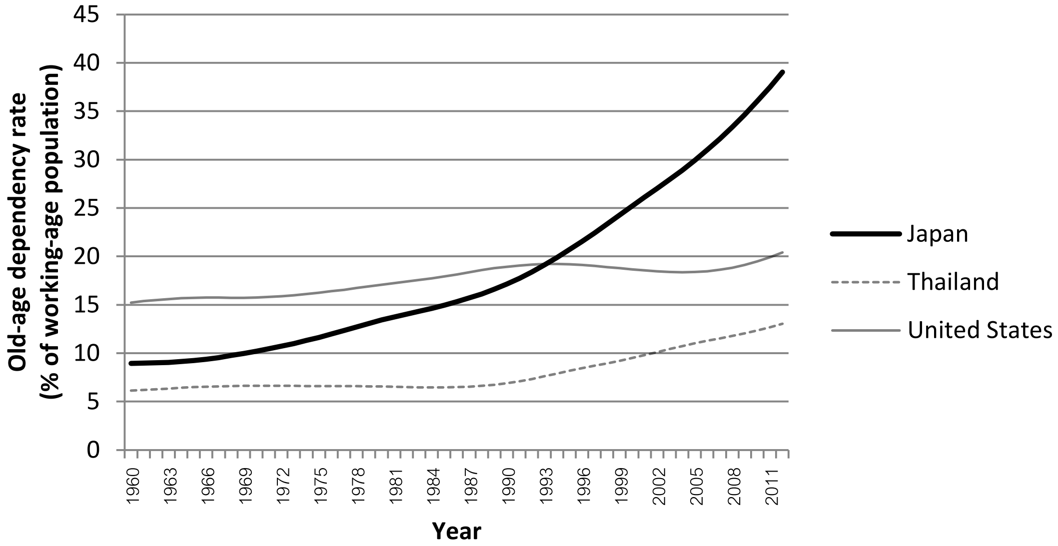
[30] In this study, we call this type of social security as the earnings-related PAYG system.



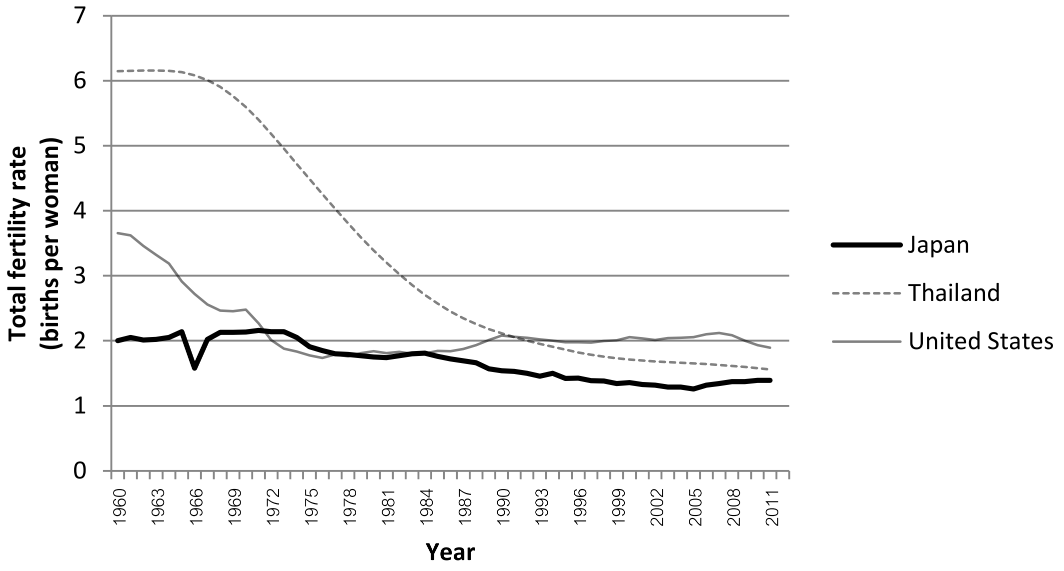
Source: World Development Indicators (2013).

Fig. 1: GDP per capita Growth Rate



Source: World Development Indicators (2013).

Fig. 2: Old-Age Dependency Rate



Source: World Development Indicators (2013).

Fig. 3: Total Fertility Rate

While the Japanese government has to keep economic growth high and old-age dependency low in order to maintain the promised level of earnings replacement rate, the above mentioned moral hazards may operate in a detrimental manner. As our theoretical models suggest shortly, the earnings-related PAYG benefits are not linked to individuals' investments in children's education but linked to their investments in own health. The increases in social security contribution rate keep its growth rate lower and its old-age dependency rate higher than their optimal levels, which can be attainable if individuals recognize the social benefits from their education investments and the social losses from their health investments. Consequently, the contribution rate hike in Japan introduced to keep the earnings replacement rate above 50% might be effective only in the short-run. Our model presented in the next section predicts that the lower growth rate and the higher old-age dependency rate resulting from the double moral hazards would cancel out the effects of the contribution rate hike in the long-run.

3.3 The Models

3.3.1 The model with Pay-As-You-Go Social Security

The process of health capital accumulation is integrated into the baseline intergenerational trade model of Ehrlich and Lui (1998), which examines the effects of introducing a social security system on key family-based choices. As their baseline model deals with the case of selfish parents and hence assumes away altruism, the representative young parent always chooses the lower bound fertility level. Accordingly, the simplifying assumptions in this study are that each young parent always chooses to have one child, and the probability of survival through child to old is exogenous to the model and very close to one as it is so in advance economies. In this overlapping generations model, a young parent controls resources spent on educating her sole dependent child and on maintaining her own health level and forms an implicit contract with the child. By this contract, the young parent stands to receive from her grown-up child an amount of old-age supports, which is the child's wage income $E_{t+1}^{y}(1 - u_{t+1} - v_{t+1})$, of which her earning capacity was bestowed by her parent, multiplied by a fixed compensation rate $\bar{z}$.[31] Each representative old parent belongs to different generations has her specific period lifespan H_{t+1}^{o} depending on her own health investment when she was young. Hence, the intra-family transfer to the old parent is $\bar{z}E_{t+1}^{y}(1 - u_{t+1} - v_{t+1})H_{t+1}^{o}$, and the cost to each young parent is given by $\bar{z}E_{t}^{y}(1 - u_{t} - v_{t})H_{t}^{o}$.

The young parent acts as an educator of her child and devotes a fraction of her time endowment u_t to the child's education capital accumulation. The technology of imparting education capital to the child is given by:

$$E_{t+1}^{y} = \gamma u_t E_t^{y}, \qquad \gamma > 1, \tag{3-1}$$

where γ is the constant parameter measuring the productivity of education capital investments. As the young parent spends a fraction of her time endowment v_t for her own health investment, the net increase in the average health level is given by:

$$H_{t+1}^{o} = \psi v_t H_t^{y} = \psi v_t, \qquad \psi > 0, \tag{3-2}$$

where ψ is the constant parameter measuring the productivity of health capital investments. As the agent maintains perfect health from her childhood through young parenthood, the young parent's stock of health capital at time t is set to 1, and each agent lives to her old-age period with certainty. While the lengths of the first two periods are fixed to be 30 years which are also normalized to 1, that of the last period depends on her health investment v_t when young.

The social security system, which is going to be added to the family insurance setup, is the earnings-related PAYG type. Following Japanese social security system after the 2004 reform, the contribution rate τ is fixed, and the earning replacement rate ϕ_{t+1} would be adjusted so as to balance the government budget. Contributions are collected from the young parent based on her wage income, which is given by her education capital stock E_t^{y} multiplied by labor time $1 - u_t - v_t$, and the social security benefit S_{t+1} is paid to the same agent when she becomes the old parent. At time $t + 1$, the lifetime benefit of old agent S_{t+1} equals the lifetime contribution of young agent. The balanced PAYG benefit at time $t + 1$ is defined as:

$$S_{t+1} = \tau E_{t+1}^{y}(1 - u_{t+1} - v_{t+1})H_{t+1}^{y}, \tag{3-3}$$

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[31] In Ehrlich and Lui (1998), the young parent stands to receive from her grown-up child an amount of old-age supports, which is the child's stock of education capital E_{t+1}^{y} bestowed by the young parent multiplied by the compensation rate $\bar{z}$.

where $H_{t+1}^{y} = 1$, due to perfect health of the young agent. As the earnings-related PAYG social security system is practiced in Japan, the amount paid to the old parent at time $t + 1$ is her previous wage income multiplied by the earnings replacement rate ϕ_{t+1}, and her retirement period is given by her old-age health capital H_{t+1}^{o}. Then, the government budget constraint at time $t + 1$ can be restated as:[32]

$$S_{t+1} = \phi_{t+1} E_t^y (1 - u_t - v_t) H_{t+1}^o. \tag{3-4}$$

Combining Equations (3-1), (3-2), (3-3) and (3-4), the earnings replacement rate at time $t + 1$ as a function of the contribution rate τ is given by:

$$\phi_{t+1} = \tau \frac{\gamma u_t}{\psi v_t} \frac{1 - u_{t+1} - v_{t+1}}{1 - u_t - v_t}. \tag{3-5}$$

Bearing in mind that numbers of young and old agents are same in this study, at time t, the costs to the young parent are the old-age support paid to its old parent $\bar{z} E_t^y (1 - u_t - v_t) H_t^o$ and the PAYG contribution $\tau E_t^y (1 - u_t - v_t) H_t^y$ paid to the government. The young parent's total consumption at time t is given by:

$$C_t^y = E_t^y (1 - u_t - v_t)(1 - \tau - \bar{z} H_t^o) H_t^y. \tag{3-6}$$

The old parent receives the old-age support from its grown-up child $\bar{z} E_{t+1}^y (1 - u_{t+1} - v_{t+1}) H_{t+1}^o$ and the PAYG benefit S_{t+1} from the government. The total consumption of the old parent at time $t + 1$ is given by:[33]

$$C_{t+1}^o = \bar{z} E_{t+1}^y (1 - u_{t+1} - v_{t+1}) H_{t+1}^o + S_{t+1}. \tag{3-7}$$

Taking into account that the young and old periods last H_t^y and H_{t+1}^o, respectively, the young and old parents' consumptions per unit period c_t^y and c_{t+1}^o are stated as follows:

[32] The social security benefits are promised by the government at time t as shown in Equation (3-4). Though the actual lifetime benefit at time $t + 1$ can be stated as $S_{t+1}{}^* = \varphi_{t+1} E_{t+1}^y (1 - u_{t+1} - v_{t+1}) H_{t+1}^o$, this amount is not realized when the then young agent contributes to the social security program at time t.

[33] In case that a probability of survival from childhood to young adult π (due to random and exogenous events) is less than one, the old age consumption is stated as $c_{t+1}^o = \pi \bar{z} E_{t+1}^y (1 - u_{t+1} - v_{t+1}) H_{t+1}^o + S_{t+1}$. In the case of the extended family, π equals one and there is no need to have any social security. However, in the other case of the nuclear family as seen in large cities, π should be less than one because of accidental deaths or non- compliances of children, and these would be the main reason for social security system's existence. While we assume that the probability of survival trough child to old is equal to one, this issue will be in the further development of this model (Ehrlich & Lui, 1998).

$$\frac{C_t^y}{H_t^y} = c_t^y = E_t^y(1 - u_t - v_t)(1 - \tau - \bar{z}H_t^o), \tag{3-8}$$

and

$$\frac{C_{t+1}^o}{H_{t+1}^o} = c_{t+1}^o = \bar{z}E_{t+1}^y(1 - u_{t+1} - v_{t+1}) + \frac{S_{t+1}}{H_{t+1}^o}. \tag{3-9}$$

The lifetime utility of an agent born in $t - 1$ is written as:

$$U^{t-1} = H_t^y \frac{c_t^{y\,1-\sigma}}{1 - \sigma} + \beta H_{t+1}^o \frac{c_{t+1}^{o\;\;1-\sigma}}{1 - \sigma}, \tag{3-10}$$

where $1/\sigma$ is the intertemporal elasticity of substitution in consumption. Utility from the agent's young-age per unit period consumption is multiplied by her health level H_t^y $(= 1)$, which is also the length of young-period. Utility from the agent's old-age per unit period consumption, the second term, is discounted by $\beta = \left(1/(1 + \rho)\right)^{30}$, where ρ is the subjective rate of time preference, and multiplied by her health level H_{t+1}^o $(= \psi v_t < 1)$.

The decentralized equilibrium solution is derived by setting $S_{t+1} = \phi_{t+1}E_t^y(1 - u_t - v_t)H_{t+1}^o$ in Equation (3-9) as an individual recognizes the PAYG benefit as a certain ratio of their previous wage rate adjusted to old period life expectancy at time t. Using Equation (3-1), the per unit period old age consumption is restated as:

$$c_{t+1}^o = \bar{z}\gamma u_t E_t^y(1 - u_{t+1} - v_{t+1}) + \phi_{t+1}E_t^y(1 - u_t - v_t).$$

The representative young parent chooses the equilibrium values of her child's education capital stock E_{t+1}^y (u_t) and her own health capital stock H_{t+1}^o (v_t) by maximizing her utility function (3-10) subject to Equations (3-1), (3-2), (3-8), and (3-9) taking $\bar{z}$, E_t^y, H_t^o, and ϕ_{t+1} as given. The first-order conditions of this decentralized problem are stated as:[34]

$$u_t: \quad \left(\frac{c_{t+1}^o}{c_t^y}\right)^\sigma \geq \frac{\beta\gamma(1 - u_{t+1} - v_{t+1})\left(\bar{z}\psi v_t - \tau\dfrac{u_t}{1 - u_t - v_t}\right)}{1 - \tau - \bar{z}\psi v_{t-1}} = \beta R_u^{PYDC}, \tag{3-11}$$

$$v_t: \quad \left(\frac{c_{t+1}^o}{c_t^y}\right)^\sigma \geq \frac{\beta\gamma(1 - u_{t+1} - v_{t+1})\left[\bar{z}\psi v_t + \tau\left(\dfrac{1 - u_t - (2 - \sigma)v_t}{1 - u_t - v_t}\right)\right]\dfrac{u_t}{v_t}}{(1 - \sigma)(1 - \tau - \bar{z}\psi v_{t-1})} = \beta R_v^{PYDC}, \tag{3-12}$$

where Equation (3-5) is applied to eliminate ϕ_{t+1}, and R_u^{PYDC} and R_v^{PYDC} are the decentralized equilibrium rates of return on the investments in child's education and in young parent's own health under PAYG social security, respectively. If there is

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[34] See Appendix *E* for the detail.

only the family insurance system as in the case of Ehrlich and Lui (1991),[35] the decentralized equilibrium rates of return coincide with the social planner equilibrium rates. However, it is not the case in this model due to the existence of two externalities caused by the nature of earnings-related PAYG social security system as discussed shortly.

Because the social planner recognizes the PAYG benefit as a certain ratio of the young parent's children's earning capacity, the social planner solution is derived by setting $S_{t+1} = \tau E^y_{t+1}(1 - u_{t+1} - v_{t+1})H^y_{t+1}$ as shown in Equation (3-3). Using Equations (3-1) and (3-2), the per unit period old age consumption is restated as:

$$c^o_{t+1} = \bar{z}\gamma u_t E^y_t (1 - u_{t+1} - v_{t+1}) + \tau\gamma u_t E^y_t (1 - u_{t+1} - v_{t+1}) \frac{1}{\psi v_t}.$$

It has to be noted that the social security benefit, the second term, gets smaller as the agent live longer in retirement. The social planner chooses the optimal values of the young parent's education capital stock E^y_{t+1} (u_t) and health capital stock H^o_{t+1} (v_t) by maximizing Equation (3-10) taking $\bar{z}$, E^y_t, H^o_t, and τ as given. The first-order conditions of this social planner problem are stated as:[36]

$$u_t: \qquad \beta R_u^{PYSP} = \left(\frac{c^o_{t+1}}{c^y_t}\right)^\sigma \geq \frac{\beta\gamma(1 - u_{t+1} - v_{t+1})(\bar{z}\psi v_t + \tau)}{1 - \tau - \bar{z}\psi v_{t-1}}, \qquad (3\text{-}13)$$

$$v_t: \qquad \beta R_v^{PYSP} = \left(\frac{c^o_{t+1}}{c^y_t}\right)^\sigma \geq \frac{\beta\gamma(1 - u_{t+1} - v_{t+1})(\bar{z}\psi v_t + \sigma\tau)\frac{u_t}{v_t}}{(1 - \sigma)(1 - \tau - \bar{z}\psi v_{t-1})}, \qquad (3\text{-}14)$$

where R_u^{PYSP} and R_v^{PYSP} are the socially optimal rates of return on the investments in child's education and in young parent's own health under PAYG social security, respectively.

The socially optimal rate of return on the investment in child's education R_u^{PYSP} reflects the full compensation that the old parent is entitled to receive through both direct transfers from her child and the child's contributions to the social-security fund. By contrast, under the earnings-related PAYG system, the young agent recognizes the old-age benefit paid as $S_{t+1} = \phi_{t+1} E^y_t (1 - u_t - v_t) H^o_{t+1}$, which is independent of her child's contribution to the social security fund. This causes the moral hazard concerning the young parent's incentive to invest in her child's education capital, and the decentralized equilibrium rate of return on her education investment R_u^{PYDC} in Equation (3-11) is unambiguously lower than the social planner equilibrium level R_u^{PYSP} in Equation (3-13) as:[37]

$$\bar{z}\psi v_t - \tau \frac{u_t}{1 - u_t - v_t} < \bar{z}\psi v_t + \tau.$$

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[35] If the tax rate τ is set to zero, this model becomes the equivalent of the family insurance system model in Ehrlich and Lui (1991).

[36] See Appendix *E* for the detail.

[37] This first moral hazard was already suggested in Ehrlich and Lui (1998).

Accordingly, an exogenous increase in the contribution rate τ necessarily reduces the young parent's investment in her child's education capital and slows the economy's growth rate.

Analogous to the case of education investment, the socially optimal rate of return on the investment in own health R_v^{PYSP} reflects the full compensation from her child and the social-security system. The comparison of this rate of return with the decentralized equilibrium rate under the PAYG system R_v^{PYDC} in Equation (3-12) reveals that the decentralized equilibrium rate would be higher than the social planner equilibrium level R_v^{PYSP} in Equation (3-14) for $\sigma = 0.5$ and plausible values of u_t and v_t as:[38]

$$\bar{z}\psi v_t + \tau\left(\frac{1 - u_t - (2-\sigma)v_t}{1 - u_t - v_t}\right) > \bar{z}\psi v_t + \sigma\tau.$$

This causes the "extra" moral hazard concerning the young parent's incentive to invest in her own health capital. A higher PAYG contribution rate τ induces greater health capital accumulation of the young parent because the young parent has no incentive to consider the marginal social losses caused by the improvement in her own improved longevity. As a result, the young parent allocates more time for investment in her own health, and upon reaching old age the larger stock of her health capital forces the subsequent generation to contribute more to the PAYG social security fund.

All aspects combined, the increase in the PAYG social security contribution rate slows education capital accumulation and accelerates health capital accumulation. From the perspective of demographic transition, the increase in the contribution rate reduces the economy's long-run growth rate and raises its old-age dependency rate while fertility tends to fall to the minimal level, and life expectancy converges to the distortedly high equilibrium level. As a result, financial sustainability of the earnings-related PAYG social security system would differ significantly between the decentralized and social planner solutions because PAYG's sustainability depended on both the economy's growth rate and old-age dependency rate.

For the PAYG social planner problem, assuming that an interior solution in u_t and v_t exists, we equate the two rates of return, that is, $R_u^{PYSP} = R_v^{PYSP}$. By combining this and Equations (3-1), (3-8), and (3-9), the explicit social planner solution for the law of motion of education capital accumulation is given by:[39]

PYSP:

$$E_{t+1}^{y} = (1 - u_t - v_t)\psi v_t(\beta\gamma)^{\frac{1}{\sigma}}\left(\frac{(1 - u_{t+1} - v_{t+1})(\bar{z}\psi v_t + \tau)}{1 - \tau - \bar{z}\psi v_{t-1}}\right)^{\frac{1}{\sigma}-1} E_t^{y}. \quad (3\text{-}15)$$

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[38] Existing studies such as Ehrlich and Kim (2007) set $\sigma = 0.5$. See *Table 2* below for the plausible values of u_t and v_t.

[39] See Appendix *E* for the details.

The first-order conditions (3-13) and (3-14) also give us the relationship between the two choice variables:

PYSP:

$$u_t = \frac{(1-\sigma)(\bar{z}\psi v_t + \tau)}{\bar{z}\psi v_t + \sigma\tau} v_t. \tag{3-16}$$

Via the analogical processes, the explicit decentralized solution for the respective law of motion for the PAYG decentralized problem is given by:

PYDC: (3-17)

$$E_{t+1}^{y} = (1-u_t-v_t)\psi v_t \frac{\left[\beta\gamma\left(\bar{z}\psi v_t - \tau\frac{u_t}{1-u_t-v_t}\right)\right]^{\frac{1}{\sigma}}}{\bar{z}\psi v_t + \tau}\left(\frac{1-u_{t+1}-v_{t+1}}{1-\tau-\bar{z}\psi v_{t-1}}\right)^{\frac{1}{\sigma}-1} E_t^{y}.$$

In this case, the relationship between times spent on education and health investments is given by:

PYDC:

$$u_t = \frac{(1-\sigma)\bar{z}\psi v_t}{\bar{z}\psi v_t + \tau} v_t. \tag{3-18}$$

An immediate implication of Equations (3-15) and (3-16) is that the higher the PAYG contribution rate, the more time agents would spend for their children's education therefore the economy would grow faster in case of the social planner solution. However, as for the decentralized solution, Equations (3-17) and (3-18) suggest that the contribution rate hike would slow down the economy when $\sigma = 0.5$. Then, what are the net effects of raising the PAYG contribution rate on sustainability of the PAYG system when the models are calibrated to Japanese economy? Because we do not have analytical solutions for the both solutions of the PAYG social security problem we simulate the system of equations in Section 3.4. Before that, in the next sub-section, financial sustainability of FF social security system is analyzed to enable us to compare the results of this sub-section with the next ones.

3.3.2 The Model with Fully Funded Social Security

In this sub-section, we introduce fully funded (FF) type social security into the family insurance setup. As Junsen Zhang et al. (2001) assert that the FF system can be modeled as a mandatory annuitized saving, the basic assumptions in this study are that benefits from FF social security are annuitized and affected by agents' health levels when old H_{t+1}^{o}. The technology of imparting education capital to the child, Equation (3-1), the net increase in the average health level, Equation (3-2), and the lifetime utility of an agent born in $t-1$, Equation (3-7), are the same as the ones in the case of PAYG social security.

The costs to the young parent are the old-age support paid to her old parent $\bar{z}E_t^{y}(1-u_t-v_t)H_t^{o}$ and the mandatory contribution $\varsigma E_t^{y}(1-u_t-v_t)H_t^{y}$. Then, the young parent's total consumption at time t is given by:

$$C_t^y = E_t^y(1-u_t-v_t)(1-\varsigma-\bar{z}H_t^o)H_t^y. \tag{3-19}$$

In the case of family insurance plus FF social security, the old parent receives the transfer from her grown-up child $\bar{z}E_{t+1}^y(1-u_{t+1}-v_{t+1})H_{t+1}^o$ and the FF social security benefits $S_{t+1}=(1+r_{t+1})\varsigma E_t^y(1-u_t-v_t)H_{t+1}^o/H_t^o$ from the government. Then, the total consumption of the old parent at time $t+1$ is:

$$C_{t+1}^o = \bar{z}E_{t+1}^y(1-u_{t+1}-v_{t+1})H_{t+1}^o + (1+r_{t+1})\varsigma E_t^y(1-u_t-v_t)\frac{H_{t+1}^o}{H_t^o}. \tag{3-20}$$

Contrary to the previous case of PAYG social security, individuals can recognize that the benefits from FF social security are related to their period lifespan H_{t+1}^o, hence the decentralized equilibrium and social planner solutions of the FF social security model coincide.

Taking into account that the young and old periods last H_t^y and H_{t+1}^o, respectively, the young and old parents' consumptions per unit period are stated as follows:

$$\frac{C_t^y}{H_t^y} = c_t^y = E_t^y(1-u_t-v_t)(1-\varsigma-\bar{z}H_t^o), \tag{3-21}$$

and

$$\frac{C_{t+1}^o}{H_{t+1}^o} = c_{t+1}^o = \bar{z}E_{t+1}^y(1-u_{t+1}-v_{t+1}) + (1+r_{t+1})\varsigma E_t^y(1-u_t-v_t)\frac{1}{H_t^o}. \tag{3-22}$$

The representative young parent chooses the equilibrium values of her child's education capital stock E_{t+1}^y (u_t) and the her own health capital stock H_{t+1}^o (v_t) by maximizing Equation (3-10) taking $\bar{z}$, E_t^y, H_t^o, $1+r_{t+1}$, and ς as given. The first-order conditions of the FF social security problem are stated as:

$$u_t: \quad \beta R_u^{FF} = \left(\frac{c_{t+1}^o}{c_t^y}\right)^\sigma \geq \frac{\beta\gamma(1-u_{t+1}-v_{t+1})\left(\bar{z}\psi v_t - \frac{(1+r_{t+1})\varsigma}{\gamma v_{t-1}}\frac{v_t}{1-u_{t+1}-v_{t+1}}\right)}{1-\varsigma-\bar{z}\psi v_{t-1}}, \tag{3-23}$$

$$v_t: \quad \beta R_v^{FF} = \left(\frac{c_{t+1}^o}{c_t^y}\right)^\sigma \geq \frac{\beta\gamma(1-u_{t+1}-v_{t+1})\left[\bar{z}\psi v_t + \frac{(1+r_{t+1})\varsigma}{\gamma v_{t-1}}\frac{1-u_t-(2-\sigma)v_t}{1-u_{t+1}-v_{t+1}}\frac{v_t}{u_t}\right]\frac{u_t}{v_t}}{(1-\sigma)(1-\varsigma-\bar{z}\psi v_{t-1})}, \tag{3-24}$$

where R_u^{FF} and R_v^{FF} are the rates of return on the investments in child's education and in young parent's own health under FF social security, respectively. Though there is no externality in this model, the comparison of Equation (3-23) with Equation (3-13) shows that R_u^{FF} is unambiguously lower than R_u^{PYSP} as:

$$\bar{z}\psi v_t - \frac{(1+r_{t+1})\varsigma}{\gamma v_{t-1}} \frac{v_t}{1-u_{t+1}-v_{t+1}} < \bar{z}\psi v_t + \tau,$$

for any positive values of τ and ς because FF social security system does not provide agents with incentives to invest in children's education.

For the FF social security problem, assuming that an interior solution in u_t and v_t exists, we equate the rates of return, that is, $R_u^{FF} = R_v^{FF}$. By combining this and Equations (3-1), (3-21), and (3-22), the FF problem's explicit solution for the law of motion for education capital accumulation is given by:

FF: (3-25)

$$E_{t+1}^{y} = \frac{1-u_t-v_t}{1-u_{t+1}-v_{t+1}} \left[\frac{\left[\beta\gamma(1-u_{t+1}-v_{t+1})\left(\bar{z}\psi v_t - \frac{(1+r_{t+1})\varsigma}{\gamma v_{t-1}} \frac{v_t}{1-u_{t+1}-v_{t+1}}\right)\right]^{\frac{1}{\sigma}}}{\bar{z}(1-\varsigma-\bar{z}\psi v_{t-1})^{\frac{1}{\sigma}-1}} - \frac{(1+r_{t+1})\varsigma}{\bar{z}\psi v_{t-1}} \right] E_t^{y}.$$

The first-order conditions (3-23) and (3-24) above also give us the relationship between the two choice variables:

FF: $$u_t = (1-\sigma)v_t - \frac{(1+r_{t+1})\varsigma}{\gamma\bar{z}\psi v_{t-1}} \frac{1-u_t-v_t}{1-u_{t+1}-v_{t+1}}. \quad (3\text{-}26)$$

Under the FF social security system, if the mandatory saving rate is raised, time spent for the child's education and the economy's growth rate decrease, though the effect should be weaker than the case of PAYG decentralized solution. However, note that we do not have analytical solutions for the FF social security problem. We simulate the models in order to examine the net effects of raising the mandatory contribution rate on times spent for education and health investments and report the numerical results in the next section.

3.4 Quantifying and Comparing the Results by Numerical Simulations

To quantify and compare the net effects of raising the social security contribution rates on the house-hold choice variables in the three models discussed so far, the basic parameters of the models are calibrated using actual Japanese data and consensus estimates from the existing literature. Following Ehrlich and Kim (2007), the intertemporal elasticity of substitution in consumption $1/\sigma$ is set to be 2 (i.e. $\sigma = 0.5$), and the time preference parameter ρ is set to be 0.01, so the discount factor is $\beta = (1/1.01)^{30}$ as the one period of the models in this study spans thirty years.[40] From their numerical analysis in Ehrlich and Lui (1991), in which the compensation rate of the family insurance system is an endogenous variable, the compensation rate $\bar{z}$ exogenously given in this study without private savings is calculated to be 0.27.[41] For the earnings-related PAYG decentralized solution, which represents the current situation in Japan, the remaining productivity parameters γ and ψ are solved by combining the two first-order conditions concerning u and v at the growth equilibrium steady-state. To do so, we use the survival probability of Japanese population from age 60 to 90, which is calculated to be $2/3$ from the last thirty years' average life expectancy of 80, and the long-run one period growth of $(1 + 0.019)^{30}$ from the average annual growth rate of the same period. Setting the contribution rate at $\tau = 0.08$, which is close to Japan's actual contribution rate in 2015, the solutions yield the following calibrated parameter values: $\gamma = 12.9566$ and $\psi = 1.7000$ ("Social Security Programs Throughout the World: Asia and the Pacific, 2012," 2013; "World Development Indicators 2013," 2013).[42]

In *Table 2*, for each type of social security programs, the simulated effects of increases in the PAYG and FF contribution rates τ and ς from 0.04 to 0.12 per capita (from 0.08 to 0.24 per household) at the growth equilibrium steady-state are shown. The increases in τ and ς lower u hence the economy's growth rate γu in the cases of the PAYG decentralized and FF models, whereas the opposite happens in the case of the PAYG social planner model. As discussed in the previous section, it is found that the earnings-related PAYG program is the most detrimental to economic growth. As for the health investment v, the same raises in τ and ς always increase time spent for health investment v hence the agents' period life expectancy ψv though the effect is the weakest for the PAYG social planner solution and the strongest for the PAYG decentralized solution. Then, the simulated levels of replacement rate ϕ, which from Equation (3-5) can be stated as $\phi = \tau\gamma u/\psi v$ in steady-state, diverge among the three models significantly. When the contribution rate τ reaches to 0.12, the replacement rate under the PAYG decentralized model is as low as 0.2799, which is roughly half

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[40] While Ehrlich and Kim (2007) calibrate their model's parameter values using actual data in the United States, the values of intertemporal elasticity of substitution and discount factor should be similar among the developed economies.

[41] In Ehrlich and Lui (1991), the optimal compensation rate is 0.18, and the equilibrium value of savings rate is 0.16. Hence, the sum of these two means is available for the representative agent to consume in old-age period. In the absence of private savings, the compensation rate must be raised to a level close to the sum to provide the elder agent with sufficient old-age supports. We add up the above two rates and subtract the current social security contribution rate to derive our compensation rate, which is exogenously given in this study without private savings.

[42] See Appendix *F* for the details.

of the respective rate 0.5685 under the PAYG social planner model. The replacement rate under FF social security is predicted to be 0.3793 if $\tau = 0.12$, clear improvement over the case of PAYG decentralized model. Consequently, this study finds that the raises in the contribution rate will not improve sustainability of the earnings-related PAYG system in the long-run because the lower replacement rate of the decentralized solution requires the further increases in the contribution rate. Lastly, we report the respective changes in social welfare of the economy. Social welfare Ω is given by:

$$\Omega = \frac{1}{1-\omega}\sum_{t=2}^{\infty}\omega^{t-1}\overline{U}^{t-1}, \qquad 0 < \omega < 1, \tag{3-27}$$

where ω is the generational weight and equated to 0.95^{30}, and $\overline{U}^{t-1}$ is the steady-state utility value of the generation born at time $t-1$. The changes in social welfare from the increases in τ and ς are positive only in the case of PAYG social planner solution but negative in the cases of PAYG decentralized solution and FF model. Social welfare is improved from the initial level of 3.2735 attained under the family insurance system only in case of PAYG social planner model, which attain 3.5257 when $\tau = 0.12$, but worsened if either the earnings-related PAYG or FF social security system is introduced.

Table 2: Simulation results: quantitative impacts of changes in the PAYG/FF social security contribution rates

	Family insurance	Family insurance + PAYG decentralized			Family insurance + PAYG social planner			Family insurance + Fully funded		
τ, ς: Contribution rates	0	0.04	0.08	0.12	0.04	0.08	0.12	0.04	0.08	0.12
u: Time for education	0.1824	0.1540	0.1357	0.1231	0.2108	0.2336	0.2528	0.1640	0.1470	0.1310
$(1 + g = \gamma u)$	(2.3630)	(1.9949)	(1.7588)	(1.5950)	(2.7312)	(3.0266)	(3.2757)	(2.1248)	(1.9049)	(1.7013)
v: Time for health	0.3648	0.3788	0.3922	0.4051	0.3825	0.3958	0.4067	0.3788	0.3922	0.4051
$(l/30 = \psi v)$	(0.6201)	(0.6439)	(0.6667)	(0.6887)	(0.6502)	(0.6728)	(0.6914)	(0.6439)	(0.06667)	(0.6887)
ϕ: Replacement rates $\phi = \tau\gamma u/\psi v$	0	0.1239	0.2111	0.2779	0.1680	0.3599	0.5685	0.0888	0.1716	0.2492
Ω: Social welfare in $E_1^{y\,0.5}$	3.2735	3.1863	3.1061	3.0313	3.3845	3.4660	3.5257	3.1965	3.1193	3.0419

Source: The authors' simulation.
Notes: Parameter values used are: $\sigma = 0.5$, $\beta = (1/1.01)^{30}$, $\bar{z} = 0.27$, $\gamma = 12.9566$, $\psi = 1.7000$, and $\omega = 0.95^{30}$.

3.5 Conclusion

In pre-modern societies, the sole way of securing old-age consumption has been the family insurance system, which maximizes the economy's growth rate by motivating education investments. It also provided agents with an incentive to improve their health level so as to enjoy longer retirement. However, as Ehrlich and Lui (1991) suggest, the family insurance system tends to reduce fertility rates as a consequence of trade-off between the quality and quantity of children. The lower societies' fertility rates get the larger default risks of the family insurance system become due to the transformation towards nuclear families. Then, the governments have recognized needs for publicly provided old-age security programs for smoothing consumption among generations and redistributing wealth among citizens. While there are a couple of options of securing old-age consumption, namely, family insurance, private savings, FF social security, and PAYG social security, the governments adopt either of the last two systems. As discussed in this study, each option provides individuals with a distinct set of incentives to invest in education and health. Family insurance has been motivated agents to invest in their children's education and their own health. It should be noted that this was the only option, which motivated individuals to spend time to enhance children's education level, if we excluded the hypothetical case of PAYG social planner solution. Private savings would affect neither of two investments, whereas the earnings-related PAYG social security and the FF social security encourage only health investment without motivating education investment. Because the earnings-related PAYG social security program provides agents with benefits that are proportional to their previous earnings, when they are young the agents fail to recognize both the marginal social benefits of investment in children's education and the marginal social losses of investment in their own health. This results in the double moral hazards, and the decentralized and social planner solutions for the earnings-related PAYG social security system differ.

In Japan, where the traditional family insurance system and the earning-related PAYG system coexist, its economic growth comes solely from the family insurance system as far as its young workers do not recognize the social benefits of the PAYG system. Then, Japanese economy has been growing slower than the case without the moral hazard. On the other hand, both family insurance and social security systems have contributed to its drastic increases in life expectancy. Due to the rigidity of formal retirement age and the near replacement level fertility, the society's old-age dependency rate has inevitably increased. The high old-age dependency together with the low economic growth puts significant burden on financial sustainability of Japanese PAYG system. This study suggests that the planned increases in the social security contribution rate in order to sustain the system's replacement rate at the promised level might be effective only in the short-run because the double moral hazards would further depress the earnings replacement rate in the long-run. Though the PAYG social security system in general was often criticized for the vulnerability to possible financial collapses, the same system potentially possesses the very favorable characteristics of encouraging additional education investments and suppressing too much health investments only if the social security benefits were not related to retirees' previous earnings but to their children's earnings and their own life expectancy.

Chapter 4: Altruism and Four Shades of Family Relationships

Abstract

We develop a model of the joint determination of education and health investments under four hypotheses of self-interest, filial altruism, parental altruism, and reciprocal altruism. Three-period-lived agents in our overlapping generations model optimally choose fractions of time devoted to investments in children's education and old parents' health. The agents with parental altruism spend the longest time in education, hence the economy's growth rate is the highest among the four hypotheses. However, their life expectancy is predicted to be the second lowest. The agents with filial altruism invest in health the most and enjoy the longest lifespan. Under this hypothesis, the economy grows at the slowest rate because they substitute away from education investments. The self-interest and reciprocal altruism hypotheses yield the results with more balanced investments between the two kinds of human capital. The models are calibrated to fit Japanese economy to examine effects of an expansion of Pay-As-You-Go (PAYG) social security on the macroeconomy. We find that raises in the contribution rate make the economy grow faster but negatively affect life expectancy of old agents under all hypotheses. Social welfare increases by the expansion of social security under the hypotheses of self-interest, parental altruism, and reciprocal altruism but decreases under the hypothesis of filial altruism.

JEL Classification Codes: D64, H55, I15, I25, O15
Keywords: Altruism, Social security, Health, Education, Growth

4.1 Introduction

In this study, we develop and analyze a simple model of human capital investments decisions by utility maximizing households under four hypotheses of self-interest, filial altruism, parental altruism, and reciprocal altruism. If we look at studies on households' fertility decisions, the majority of these studies have assumed that members of households are connected via altruism from parents to children as this leads to a reason for bearing children (Barro & Becker, 1989; Becker & Barro, 1988). More recent papers, which employ altruism from children to parents, argue that parents procreate because they expect to get old-age supports from their grown-up children who care well-being of their elderly parents (Boldrin & Jones, 2002). It seems that authors of these studies have chosen one from possible hypotheses on household preferences, which fits best for their theoretical proposition, but pay not so much attention to whether the particular hypothesis replicates the real economy well or not. According to Horioka (2002), who discusses the problems of bequest motives and bequest division for the cases of selfish, altruistic, and dynastic individuals, there is little agreement on which type of family relationships fits well to the real economies. Horioka (2014) shows that for Japanese and Chinese economies the selfish model is most applicable, whereas the (parental) altruism model is most applicable for American and Indian economies. The results go against the common believes that Americans are selfish as they appreciate individualism, and Japanese people are highly altruistic (Hayashi, 1986). Under three hypotheses of self-interest, forward (parental) altruism, and backward (filial) altruism, Cigno and Rosati (1996) theoretically derive comparative-static predictions on households' savings and fertility decisions and compare the predictions with empirical data from Germany, Italy, the United Kingdoms, and the United States. They conclude that the self-interest model with endogenous fertility is the only one consistent with their data while altruistic models are categorically rejected. The latter two studies are different from the former groups of theoretical works as they do not assume that household preferences can be chosen arbitrarily and have been contributing to the discussion on the most appropriate model of household behavior either on bequest motives and bequest division or on saving and fertility. At this point, it has to be noted that our fourth hypothesis, reciprocal altruism, is missing from the above arguments.[43] Horioka (2002) suggests that reciprocal (two-sided) altruism might be a highly plausible hypothesis as individuals were altruistic not only towards their children but also towards their parents.[44] Nevertheless, he does not include this hypothesis in his study. His explanation for the exclusion is that family members would help each other due to risk-sharing rather than purely altruistic considerations, and as Horioka (2002) analyzes data based on public opinion surveys it would have been difficult for him to separate the two considerations. Because there is no confusion between the two considerations in our theoretical framework, reciprocal altruism is included in this study. Preliminarily, we expect that individuals in households are at least partially connected via both filial and parental altruism therefore the rarely examined hypothesis of reciprocal altruism should be a strong candidate for the most appropriate model of household preferences.[45] However, to actually find a model that fits best for a specific economy requires empirical studies (i.e. two parameters measuring degrees of filial and parental altruism α^F and α^P in Equation (4-1) have to be estimated). In this study, we concentrate on showing that predictions of theoretical models depend heavily on the models' assumption on household preferences. Then, our main research questions are as follows: How are a model's key household decisions modified if an assumption on household preferences is replaced by another without changing any other parts of its theoretical framework? What happens if policy makers introduce a government policy, which is known to improve social welfare under a particular hypothesis of family relationships, in a society under a different hypothesis?

This paper's focus is placed on advanced economies, which have gone through the demographic transition and are experiencing problems of aging population. In

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[43] Blackburn and Cipriani (2005) is the only theoretical paper discussing two-sided (reciprocal) altruism.

[44] One-sided (parental) altruism is frequently employed in the theoretical literature, but the idea that parents unconditionally care well-being of their children, who do not care that of their parents at all, is rather odd.

[45] As shown in our results, households devote similar fractions of time to education investments under two hypotheses of self-interest and reciprocal altruism because two diverging effects from individuals' filial and parental altruistic traits tend to cancel each other (see Equation (4-24)). This similarity may disguise households connected via reciprocal altruism as selfish ones.

such economies, fertility rates tend to be around the replacement level, and individuals maintain nearly perfect health when young and are still healthy enough to work when old. Considering these stylized facts, studies on households' fertility decisions have become less and less relevant whereas those on their health capital investments are gaining significance. Following the seminal works of Grossman (1972b) and Ehrlich and Chuma (1990), our model endogenizes households' investments decisions on health capital in addition to those on education capital. Namely, middle-age agents choose fractions of time spent for education of their children and time devoted to old-age care services for their elderly parents. Though it is relatively straightforward to show that altruistic individuals have incentives to invest in family members' human capital, to provide selfish ones with these incentives requires an intricate theoretical framework, which defines material interdependencies within their family. For this reason, our model also follows basic set-ups of Ehrlich and Lui (1991), whose notable feature is a self-enforcing family insurance system. Under this system, middle-age agents spend a fraction of time to educate their children, hence upon reaching old parenthood the same agents are entitled to financial supports from their grown-up children.[46] As an amount of financial supports for the elderly is set to be proportional to their children's education capital stock, this mutual insurance mechanism provides selfish agents with incentives not only to have children but also to invest in their human (education) capital. The baseline model in this study adopts a modified version of the family insurance system. Under the modified system, an amount of financial supports available for old-age agents is agreed to be proportional to their children's education capital stock and their own health capital stock.[47] This family insurance system together with norms of filial care of the elderly, whose details we will elucidate shortly, motivates selfish agents to invest in both education capital of children and health capital of old parents. It has to be noted that our study on endogenous longevity and economic growth differs from Ehrlich and Lui (1991), the studies on endogenous fertility and economic growth, at two particularly important points. In terms of key household choices, though individuals in their studies face a trade-off between the quantity (fertility) and quality of children, those in this study confront an alternative trade-off between the quality of children and the quantity (longevity via health investments) of old parents. In terms of household preferences, the baseline models in Ehrlich and Lui (1991) assume the hypothesis of self-interest, and in later sections they augment these models by adding the assumption of parental altruism, which further facilitates investments in both the quality and quantity of children. The model in this study, which deals with both problems of children and the elders, derives predictions on key household decisions under the four hypotheses of family relationships and compare them. The family insurance system motivates selfish agents to invest in education and health capital, and these investments are enhanced by adding assumptions of parental and filial

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[46] According to Ehrlich and Lui (1991), various empirical evidences support this view of the family relationships in the case of the United States (see their note 1). Analogous to our previous argument that family members should be connected via both filial and parental altruism, they are likely to be connected via not only emotional but also material interdependencies.

[47] An amount of financial supports is proportional to beneficiaries' health level because in this study the health level of old agents is an equivalent of their consumption capacity or period lifespan (see our note 60).

altruism, respectively (or by adding that of reciprocal altruism). In reality, health capital investments flowing from children to old parents should be at least partially motivated by filial altruism, but this seeming relationship has not been examined in theoretical studies.[48]

Once individuals' health capital investment becomes their choice variable, social norms dictating how they should behave towards their elderly parents become important. On one hand, education capital is transmitted from parents to children. Assuming that the family insurance system is in place, individuals educate their children and are compensated for the education efforts by the same children. Ehrlich and Lui (1991) discuss that, with some additional assumptions such as a penalty for violating the family contract, the system is self-enforcing under hypotheses with or without parental altruism. On the other hand, health capital is transmitted from children to parents. If there are no norms of filial care for the elderly, young parents have to provide their old parents with health care services not knowing whether they can get equivalent treatments from their children. Then, the mere existence of the family insurance system does not guarantee that young parents will be compensated for their past health care efforts by their children without filial altruism. Fortunately, according to Ogawa and Retherford (1993), social norms of filial care do exist, and people in the real world can expect old-age supports from their children. They report that even though Japan's socioeconomic (e.g. real per capita income) and demographic (e.g. fertility and mortality) conditions are changing at extraordinarily fast rates during its post war period, norms of filial care for the elderly remain stable until recently. More specifically, their examination of the National Survey on Family Planning suggests that, before 1986, nearly 80% of Japanese married women below age fifty consider norms of filial care as socially accepted standards. If individuals spend a certain fraction of time for taking care of elderly parents knowing that their grown-up children mimic what parents did for grandparents, the family insurance system becomes self-enforcing under assumptions with or without filial altruism. Therefore, our theoretical model is meant to capture the salient features of Japanese economy for this particular period and incorporates the concept of norms of filial care for the elderly.[49]

Ogawa and Retherford (1993) discuss further that, after 1986, as the burden of old-age supports on adult children increases due to high old-age dependency rates in Japan, a major normative shift occurs, and only 50% of respondents of the same National Survey give pro-care responses. Though we have to carefully distinguish social norms from altruistic traits of individuals, these two factors defining family relationships may affect each other.[50] As the major normative shift concerning filial

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[48] Filial altruism is discussed in Cigno and Rosati (1996), Boldrin and Jones (2002), and Blackburn and Cipriani (2005). Endogenous longevity is discussed in van Zon and Muysken (2001), van Zon and Muysken (2005), Blackburn and Cipriani (2002), Chakraborty (2004), and Barro (2013). However, the two subjects have not been brought together in one study.

[49] The concept of social norms is expressed by setting time devoted to health investments in different periods equal, $v_t = v_{t+1} = v_{t+2}$ (see Equation (4-3) and note 55). It has to be noted that norms of filial care do not specify a fraction of time that agents spend on old-age care services.

[50] In contrast to the concept of social norms mentioned in note 10, we measure agents' degrees of filial and parental altruism by parameters attached to utilities derived from their parents' and children's old-age consumption, respectively (see Equation (4-1)).

care has observed after 1986, chances are that household preferences in Japan might undergo transformation from one hypothesis to the other as well.[51] Accordingly, the contribution of this study on endogenous longevity and economic growth would be the presentation of theoretical framework for future empirical studies in order to find the most appropriate model of household preferences for an economy in the time of rapid population aging. As the proportion of individuals who take norms of filial care as socially accepted standards decreases, a substantial part of the burden of old-age supports has been shifted from family insurance systems to public pension systems. By introducing Pay-As-You-Go social security into our baseline model, our numerical analyses find that impacts of the government policy on the macroeconomy depend on the choice of hypotheses of family relationships.

The rest of chapter is organized as follows: Section 4.2 reviews theoretical studies which assume one or more of the hypotheses of household preferences. Section 4.3 introduces the model which enables us to compare implications of the four hypotheses analytically. In Section 4.4, we introduce Pay-As-You-Go social security and calibrate the extended model to fit Japanese economy in order to analyze effects of increases in the PAYG contribution rate on key household decisions and social welfare numerically. Section 4.5 provides concluding remarks.

4.2 Literature Review

In a three-period-lived overlapping generations framework, there are four possible hypotheses on family relationships, through which child, young parent, and old parent are connected each other: (i) self-interest, (ii) filial altruism, (iii) parental altruism, and (iv) reciprocal altruism. Each hypothesis results in a particular utility function, which in turn has distinctive implications on key household decisions. As discussed in our introduction, some authors assume one of these hypotheses based on objectives of their study. Other authors, who investigate which hypothesis is most applicable for a particular economy, derive predictions under more than one hypothesis and compare them. In this section, we group existing theoretical models from related studies into the four categories of family relationships and review them one by one.

(i) Households Connected via Self-interest

In the self-interest model, or the life cycle model, individuals are selfish and derive utility only from their own lifetime consumption. In its most basic form, family members behave independently and have no incentive to make transfers to the other members or to invest in each other’s human capital. In their life-cycle model section, Cigno and Rosati (1996) argue that individuals when they are children cannot

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[51] Family relationships might have moved from those with filial altruism, where people spend a significant amount of time to take care of their cohabiting old parents, to those with parental altruism, where young parents carefully raise their sole child in nuclear family setups. However, considering the current high old-age dependency rates in developed economies, family relationships have to move back to filial altruism. If we take our social security contributions into account, total financial supports for the elderly parents have not declined during the post war period (Ehrlich & Lui, 1991).

consume anything as their parents have no interest in expending resources on them. In turn, for young parents, private savings through the capital market is the only way to secure their old-age consumption because their grown-up children, if any, have no interest in making transfers to their elderly parents. In order to give these selfish individuals incentives to make voluntary intergenerational transfers of any directions, Cigno and Rosati institute a set of rules that all family members must follow. Their family rules dictate middle-age agents transfer resources to nurture their children and to support their old parents. Most importantly, these intergenerational transfers provide the agents with interests in having children, and fertility decisions become endogenous in the self-interest model with the family rules. As discussed in our introduction, the baseline self-interest model of Ehrlich and Lui (1991) introduces the family insurance system and shows that even selfish young parents invest in their children's education capital in order to enhance their old-age consumption.[52] According to Ehrlich and Lui, the incentives to educate successive generations are the motivating forces underlying economic growth. Our model in this study includes a variant of the family insurance system in order that even agents without parental altruism invest in their children's education capital, and those without filial altruism invest in their parents' health capital.

(ii) Households Connected via Filial (Backward) Altruism

The filial altruism model requires children who care about their parents' welfare. Individuals in this category derive utility not only from their own lifetime consumption but also from their parents' well-being. Nonetheless, as in the self-interest model, parents have no interest in their children's well-being and behave independent of them. Cigno and Rosati (1996), in their backward (filial) altruism section, suggest that grown-up children donate old-age supports to their elderly parents without expecting any returns. As a result, middle-age agents in this category demand children as assets. In their study of fertility decisions, Boldrin and Jones (2002) employ the hypothesis of filial altruism and criticize the more standard endogenous fertility models in Becker and Barro (1988) and Barro and Becker (1989), which assume the hypothesis of parental altruism. While the models of Becker and Barro predict constant fertility as individuals' income increases, this prediction goes against empirical data showing that fertility rates decrease with per capita income. Boldrin and Jones go further and point out that the parental altruism models fail to explain the process of demographic transition because these models predict rising fertility as infant mortality decreases. On the contrary, under the assumption of filial altruism children are investment goods therefore exogenous decreases in infant mortality lower fertility rates as the historical data suggest.[53] It has to be noted that because agents' mortality rates are exogenously given in studies reviewed in this section, a relationship between filial altruism and endogenous longevity has not been discussed despite its evident importance. As health investment decisions are

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[52] Ehrlich and Lui (1998) introduce Pay-As-You-Go social security system into the self-interest and altruism models of Ehrlich and Lui (1991).

[53] Boldrin, De Nardi, and Jones (2005), using two existing models of fertility decisions, Barro and Becker (1989) and Boldrin and Jones (2002), investigate which model is more consistent with data showing the strong negative correlation between public old-age pension benefits and fertility rates.

endogenized in this study, altruistic children will invest in their old parent's health capital because this directly increases their utility. Then, life expectancy of an economy with filial altruism should be higher than economies without it.

(iii) Households Connected via Parental (Forward, One-Sided) Altruism

The huge literature on fertility decisions, which is based on the hypothesis of parental altruism, proclaims that the motivation to have children is altruism on the part of parents. If members of households are connected via parental altruism, individuals get utility from their own lifetime consumption and their children's well-being (e.g. utility, consumption, quantity, or quality). As mentioned above, the two papers from Becker and Barro, which pioneer in endogenizing households' fertility decisions, are the most popular ones in this category. In addition to these two studies, Cigno and Rosati (1996) investigate the forward (parental) altruism model as well. Their findings in the respective section reveal that savings are motivated by individuals' wish to leave bequests for their children, and fertility must be positively related to their income and pensions. They show that the latter finding is counterfactual in their empirical analysis. In case of the endogenous growth model of Ehrlich and Lui (1991), altruistic parents invest more in children's education capital than selfish ones because the higher quality of children increases not only the amount of old-age supports from grown-up children but also their utility directly. Consequently, an economy under parental altruism grows faster than one populated by selfish agents.[54] Blackburn and Cipriani (2005) examine two hypotheses of household preferences. Their one-sided altruism flows from parents to children whereas their two-sided altruism corresponds to reciprocal altruism in this study. Under one-sided (parental) altruism, selfish children make a fixed amount of gifts to their parents, and the altruistic parents make an endogenously determined amount of bequests to their children.[55] The parental altruism model of Blackburn and Cipriani (2005) succeeds in replicating the process of demographic transition suggested by Caldwell (1978) and shows that the decline in fertility is explained by the inverse intra-family wealth flow from rich parents to their children at high levels of development. We will shortly discuss their reciprocal altruism model in the next sub-section.

(iv) Households Connected via Reciprocal (Two-Sided) Altruism

The last hypothesis of family relationships is reciprocal altruism, in which individuals are altruistic towards both their parents and children. They derive utility from well-being of all other family members in addition to their own lifetime utility. According to Horioka (2002), who is reporting the results of Japanese survey about "the willingness of respondents to give financial assistance to others" in 1998 and 2001, 92% of respondents are altruistic towards their own children, and similarly high 87% of them are altruistic towards their own parents. From these findings, Horioka

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[54] Although the relationship between filial altruism and endogenous longevity (health investments) has not studied yet, there is a large literature on the similarly important relationship between parental altruism and endogenous growth (education investments).

[55] The fixed amount of gifts shelled out by selfish children might be prescribed by their family rules (Cigno & Rosati, 1996).

suggests that the reciprocal (two-sided) altruism model would be highly applicable for Japanese economy. If we look into the theoretical literature, the two-sided (reciprocal) altruism model developed in Blackburn and Cipriani (2005) is the only example in this category. They argue that if members of households are connected via reciprocal altruism both gifts from children to parents and bequests from parents to children are endogenously determined. During the course of economic development, the direction of intergenerational transfers shifts from gifts to bequests, and according to Balckburn and Cipriani the process of demographic transition (e.g. the decline in fertility) is crucially linked to this shift. Compared to the predictions derived from their one-sided altruism model, those from the two-sided altruism model are qualitatively the same as their concluding remarks suggest that individuals eventually switch their type of behavior from either self-interest (if one-sided altruism) or filial altruism (if two-sided altruism) to parental altruism as the economy develops. It has to be noted that in their two-period-lived overlapping generations model the relatively old agents in the second period of life are more like middle-age agents, whose earning capacity is the highest among generations, rather than old-age agents, whose survival depends on supports from their children, in the three-period-lived models discussed in this review. Thus, a relationship between filial altruism, which is one side of reciprocal altruism, and endogenous provisions of old-age supports is not addressed in Blackburn and Cipriani (2005) either.

Through this literature review, we find that the parental altruism models of Barro and Becker (1989) and the filial altruism model of Boldrin and Jones (2002) have considerably different implications on individuals' fertility choices. Cigno and Rosati (1996) criticize both models of parental and filial altruism because their self-interest model is the only one that fits the historical data. In the next section, we set up a basic overlapping generations model in order to investigate implications of the four hypotheses of family relationships on households' education and health investments decisions.

4.3 The Model

The process of health capital accumulation is integrated into the endogenous growth model of Ehrlich and Lui (1991). While their main discussion is about the trade-off between the quality and quantity (fertility) of children, we are interested in an alternative trade-off between the quality of children and the quantity (longevity) of the elderly for investigating an economy under population aging. In order to address problems of the elderly, we introduce two rarely studied hypotheses of filial and reciprocal altruism in addition to the more popular hypotheses of self-interest and parental altruism seen in Ehrlich and Lui (1991). As in the highly developed economies fertility rates tend to be very close to the replacement level, and infant mortality rates are negligibly low, our simplifying assumptions are that a young parent always chooses to have one child, and the probability of survival through child to old equals to one. In this overlapping generations model, following a set of rules of the family insurance system, a young parent controls resources spent for educating her sole dependent child and for maintaining her old parent's health level and provides her old parent with intergenerational transfers in order to supplement her old-age consumption.

4.3.1 Preferences

The economy consists of generations of agents with perfect foresight. Each agent goes through three periods of lifetime: child, young parent, and old parent. A representative agent derives utility from her own young-age and old-age consumption and from old-age consumption of her parent and child. We assume that the agent's consumption when she is a dependent child has no effect on her life-time utility because in her childhood the agent makes no choice. For simplicity, we assume that there is no utility from retirement leisure therefore the agent works when she is old.[56] For an agent born in period $t-1$, her utility function is given by:[57]

$$U^{t-1} = \ln c_t^y + \beta \ln c_{t+1}^o + \alpha^F \ln c_t^o + \beta^2 \alpha^P \ln c_{t+2}^o, \quad 0 < \beta < 1, \quad 0 \le \alpha^F \le 1, \quad 0 \le \alpha^P \le 1, \tag{4-1}$$

where α^F and α^P denote the degrees of filial and parental altruism. The logarithmic utility function is used here in order to guarantee interior solutions. Utility from the agent's own old-age consumption, the second term, is discounted for the subjective rate of time preference ρ, where $\beta = \left(1/(1+\rho)\right)^{30}$. The third and fourth terms represent emotional benefits to the agent from old-age consumption of her parent and child, c_t^o and c_{t+2}^o, respectively. We have chosen old-age consumption as the measure of well-being of the family members because it includes both their education capital stocks E_{t-1}^y and E_{t+1}^y when young and their health capital stocks H_t^o and H_{t+2}^o when old as seen in Equations (4-10) and (4-11).[58] The fourth term is discounted by β^2 because the old consumption c_{t+2}^o of the child occurs two periods after the young-age consumption c_t^y of the agent born in $t-1$.

[56] An old parent's labor force participation can be exogenously or endogenously determined. If she is forced to retire, for example, due to a legal retirement age (exogenous retirement), an addition of utility from leisure to Equation (4-1) does not affect our main results shown in Equations (4-24) and (4-25). If she can freely choose whether she retires or not (endogenous retirement), the new problem requires a complex two-stage optimization procedure (Aisa et al., 2012; Matsuyama, 2008). We are currently working on an issue of endogenous retirement and longevity in a separate study under the simplest hypothesis of self-interest but assume away from it in this study.

[57] The first term in the right hand side of Equation (4-1) is the utility from consumption of a young parent (superscript y) at time t (subscript t), and, analogically, the second term is the utility from consumption of an old parent (superscript o) at time $t+1$ (subscript $t+1$).

[58] The measure of well-being is not necessarily agents' consumption level. In Ehrlich and Lui (1991), their companionship function has only two arguments, which are the number of surviving children and their human capital stock (the quantity and quality of children). In our study, the quality of children and the quantity of the elderly included in old-age consumption are their counterparts. Because an agent's own old-age consumption at time $t+1$ is already in place in Equation (4-1), the choice of old-age consumption as the measure of well-being significantly facilitates the derivation of first-order conditions as well.

4.3.2 Human Capital Accumulation

We introduce an endogenous health maintenance process in this study. Suppose that each agent is born with perfect health and can maintain the same health level H_t^y throughout her young parenthood at time t. The health level of the young parent H_t^y is normalized to one. The young parent born in $t-1$ devotes a fraction of her time v_t to take care of her old parent, who then maintain the health level of H_t^o. The health level of the old parent at time t is given by:

$$H_t^o = \psi v_t H_t^y = \psi v_t, \qquad \psi > 0, \tag{4-2}$$

where ψ is the constant parameter measuring the productivity of health capital investment. In order to guarantee that the young parent, who has supported her old parent at time t, will receive the same level of old-age supports from her child at time $t+1$, social norms of filial care for the elderly must exist. Assuming that the norms dictate young parents of two subsequent generations in Equation (4-1) to choose the same amount of time to take care of their elder parent, the health levels of the old parents at time $t+1$ and $t+2$ are given by:[59]

$$H_{t+2}^o = H_{t+1}^o = H_t^o = \psi v_t. \tag{4-3}$$

The young parent acts as an educator of her child. An intergenerational transfer of education capital is directed from the young parent, who possesses education stock E_t^y and spends her time u_t at time t, to her child. Then, the child maintains the education level E_{t+1}^y during her young parenthood at time $t+1$. The technology of imparting education capital to the child is given by:

$$E_{t+1}^y = \gamma u_t E_t^y, \qquad \gamma > 1, \tag{4-4}$$

where γ is the constant parameter measuring the productivity of education capital investment. The young parent's productivity depreciates as she gets old at an exogenous rate. An intertemporal transition of education capital from the young parent at time t to her old parenthood at time $t+1$ is given by:

$$E_{t+1}^o = \delta E_t^y, \qquad 0 < \delta < 1, \tag{4-5}$$

where δ is the exogenous fraction of education capital stock maintained at old-age.

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[59] While Ogawa and Retherford (1993) suggest that norms of filial care was stable during the post war period in Japan, the assumption that agents of three consecutive generations devote the exactly same fraction of time to health care services is rather of theoretical necessity and for simplicity. However, if we relax this assumption and change it to a more realistic one such that agents spend $(1-\sigma) \times 100\%$ less time for health investments than ones belong to a directly preceding generation, $\sigma^2 v_t = \sigma v_{t+1} = v_{t+2}, 0 < \sigma < 1$, this does not affect first-order conditions (4-12) and (4-13).

4.3.3 Time Allocation

The agent does not make any decision by herself during her childhood. The total amount of time endowed to the young parent at time t is H_t^y, which is her current health level and equals to one. As discussed in the previous sub-section, the young parent chooses fractions of time devoted to education investment of her child u_t, time devoted to health investment of her old parent v_t, and hence time spent for working, $1 - u_t - v_t$. The agent combines her earning capacity E_t^y with the time allocation for working and earns income.[60] In addition to time devoted to her old parent's health maintenance, the young parent makes a financial transfer to her old parent following implicit contracts of the family insurance system. Under this system, the old parent born in period $t - 1$ stands to receive from her grown-up child an amount of old-age supports, which is equal to a compensation rate $\bar{z}$ multiplied by the child's earning capacity E_{t+1}^y and by the old parent's own health level H_{t+1}^o. For simplicity, we assume that the compensation rate is fixed and exogenously given by the family contracts.[61] Accordingly, the cost to the young parent in time t is the financial transfer $\bar{z}E_t^y H_t^o$ to her old parent, and her consumption at time t is determined by:[62]

$$c_t^y = E_t^y(1 - u_t - v_t) - \bar{z}E_t^y H_t^o. \tag{4-6}$$

Using Equation (4-3), the young consumption at time t (4-6) can be restated as:

$$c_t^y = E_t^y[1 - u_t - v_t(1 + \bar{z}\psi)]. \tag{4-7}$$

Upon reaching her old parenthood, the same agent receives the transfer from her grown-up child $\bar{z}E_{t+1}^y H_{t+1}^o$ and also earns wage income from her old-age work $E_{t+1}^o H_{t+1}^o$. The consumption of the old parent at time $t + 1$ is given by:

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[60] The earning capacity of the young parent at time t is $wE_t^y H_t^y$, where w is the wage rate per effective labor, to be precise. As we normalized both w and H_t^y equal to one, the earning capacity is stated as E_t^y.

[61] If the compensation rate is endogenously determined as in Ehrlich and Lui (1991), the new problem requires a two-stage maximization procedure under the hypotheses without filial altruism. In the first stage, taking the compensation rate as given, a young agent chooses the optimal values of education and health investments, which are functions of the compensation rate. In the second stage, by maximizing her children's utility, the young agent chooses an optimal value of the compensation rate, which applies in all time period. However, even for the self-interest model we have no closed-form solution in the second stage. Though to endogenize the compensation rate is more straightforward under the hypotheses with filial altruism, we assume that the rate is exogenously given in order to derive consistent results from all the four hypotheses and compare them.

[62] A young parent may provide her child with a fixed fraction of time or a fixed amount of financial supports in order to raise the child (see Ehrlich and Lui (1991) for time supports and Cigno and Rosati (1996) for financial supports). However, because the young parent always has one child in this study, the fixed cost of raising a child does not affect first-order conditions (4-12) and (4-13). As discussed in note 57, the amount of financial supports has to be determined exogenously in order to compare results from the four hypotheses.

$$c^o_{t+1} = \bar{z}E^y_{t+1}H^o_{t+1} + E^o_{t+1}H^o_{t+1}. \tag{4-8}$$

Using Equations (4-3), (4-4), and (4-5), the old consumption at time $t+1$ (4-8) can be restated as:

$$c^o_{t+1} = E^y_t[\bar{z}\gamma u_t + \delta]\psi v_t. \tag{4-9}$$

By lagging and updating one period, we can derive the old parent's consumption at time t and $t+2$ as:

$$c^o_t = E^y_t[\bar{z}\gamma u_{t-1} + \delta]\frac{\psi v_t}{\gamma u_{t-1}}, \tag{4-10}$$

$$c^o_{t+2} = E^y_t[\bar{z}\gamma u_{t+1} + \delta]\gamma u_t \psi v_t. \tag{4-11}$$

4.3.4 The Optimization Problem

The agent born in period $t-1$ maximizes her utility function (4-1) with respect to fractions of time devoted to education and health investments, u_t and v_t, subject to budget constraints (4-7), (4-9), (4-10), and (4-11). The necessary optimal conditions are as follows:

$$u_t: \quad \frac{1}{1-u_t-v_t(1+\bar{z}\psi)} = \frac{\beta}{u_t+\frac{\delta}{\gamma\bar{z}}} + \frac{\beta^2\alpha^P}{u_t}, \tag{4-12}$$

$$v_t: \quad \frac{1+\bar{z}\psi}{1-u_t-v_t(1+\bar{z}\psi)} = \frac{1}{v_t}(\beta + \alpha^F + \beta^2\alpha^P). \tag{4-13}$$

Equation (4-12) says that the loss in utility from earning less and investing more for her child's education is compensated by utility obtained from the larger amount of old-age supports from the child and the utility gain from higher old-age consumption of her child via parental altruism. Equation (4-13) implies that utility foregone from reducing one more unit of the young parent's consumption for investing in health capital of her old parent is equal to utility obtained from higher old-age consumption of the young parent herself, of her parent via filial altruism, and of her child via parental altruism. The same agent's old-age consumption is higher because her better health level increases the old-age support coming from family insurance and her wage earning. In order to elucidate respective household decisions under the four hypotheses of family relationships, we set α^F and α^P, the parameters measuring degrees of filial and parental altruism, to either zero or one. Then, the four hypotheses are designated as follows: (i) self-interest if $\alpha^F = 0, \alpha^P = 0$, (ii) filial altruism if $\alpha^F = 1, \alpha^P = 0$, (iii) parental altruism if $\alpha^F = 0, \alpha^P = 1$, and (iv) reciprocal altruism if $\alpha^F = 1, \alpha^P = 1$.[63]

[63] These are the extreme cases for the clearer demonstration, but as discussed briefly in our conclusion the actual values of the two parameters should be somewhere between zero and one. For the more general cases of $0 < \alpha^F < 1$ and $0 < \alpha^P < 1$, the solutions for u_t and v_t are given

(i) Household Connected via Self-interest

By setting both α^F and α^P to zero in Equations (4-12) and (4-13), fractions of time spent on education and health by a selfish young parent at time t are given by:

$$u_t{}^S = \frac{\beta - (1+\beta)\frac{\delta}{\gamma\bar{z}}}{1+2\beta}, \qquad (4\text{-}14)$$

$$v_t{}^S = \frac{\beta\left(1+\frac{\delta}{\gamma\bar{z}}\right)}{(1+2\beta)(1+\bar{z}\psi)}, \qquad (4\text{-}15)$$

where S denotes the hypothesis of self-interest.

(ii) Household Connected via Filial Altruism

By setting α^F to one and α^P to zero in Equations (4-12) and (4-13), fractions of time spent on education and health by a young parent with filial altruism at time t are given by:

$$u_t{}^F = \frac{\beta - (2+\beta)\frac{\delta}{\gamma\bar{z}}}{2(1+\beta)}, \qquad (4\text{-}16)$$

$$v_t{}^F = \frac{1+\frac{\delta}{\gamma\bar{z}}}{2(1+\bar{z}\psi)}, \qquad (4\text{-}17)$$

where F denotes the hypothesis of filial altruism.

(iii) Household Connected via Parental Altruism

By setting α^F to zero and α^P to one in Equations (4-12) and (4-13), fractions of time spent on education and health by a young parent with parental altruism at time t are given by:

$$(1+2\beta+2\beta^2){u_t{}^P}^2 + \left[(1+\beta+2\beta^2)\frac{\delta}{\gamma\bar{z}} - (\beta+\beta^2)\right]u_t{}^P - \beta^2\frac{\delta}{\gamma\bar{z}} = 0, \qquad (4\text{-}18)$$

$$v_t{}^P = \frac{\beta+\beta^2}{(1+\beta+\beta^2)(1+\bar{z}\psi)}(1-u_t{}^P), \qquad (4\text{-}19)$$

where P denotes the hypothesis of parental altruism.

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by: $(1+2\beta+\alpha^F+2\beta^2\alpha^P)u_t{}^2 + \left[(1+\beta+\alpha^F+2\beta^2\alpha^P)\frac{\delta}{\gamma\bar{z}} - (\beta+\beta^2\alpha^P)\right]u_t - \beta^2\alpha^P\frac{\delta}{\gamma\bar{z}} = 0$ and $v_t = \frac{\beta+\alpha^F+\beta^2\alpha^P}{(1+\beta+\alpha^F+\beta^2\alpha^P)(1+\bar{z}\psi)}(1-u_t)$.

(iv) Household Connected via Reciprocal Altruism

By setting both α^F and α^P to one in Equations (4-12) and (4-13), fractions of time spent on education and health by a young parent with reciprocal altruism at time t are given by:

$$(2+2\beta+2\beta^2){u_t^R}^2+\left[(2+\beta+2\beta^2)\frac{\delta}{\gamma\bar{z}}-(\beta+\beta^2)\right]{u_t}^R-\beta^2\frac{\delta}{\gamma\bar{z}} = 0, \quad (4\text{-}20)$$

$${v_t}^R=\frac{1+\beta+\beta^2}{(2+\beta+\beta^2)(1+\bar{z}\psi)}(1-{u_t}^R), \quad (4\text{-}21)$$

where R denotes the hypothesis of reciprocal altruism.

Our analysis focuses on the balanced steady-state growth path and ignores transitional dynamics. On the balanced steady-growth path, both fractions of time devoted to education and health investments are constant, and hence the economy's growth rate and life expectancy are also constant. The steady-state balanced growth rate g^X and period life expectancy of old agents l^X are given by:

$$g^X=\gamma u^X-1, \quad (4\text{-}22)$$

$$l^X=30\psi v^X, \quad (4\text{-}23)$$

where X denotes each hypothesis of family relationships, S, F, P, or R. Equation (4-22) indicates that the growth rate depends linearly on time allocation of the young parent on education investment of her child. Similarly, Equation (4-23) shows that period life expectancy of old agents is linearly related to time allocation of the young parent on health investment of her old parent.[64] According to Equations (4-14) - (4-21) and also *Table 3* from our numerical analysis in the case of no government intervention ($\tau=0$), the predicted fractions of time devoted to education and health investments (or equivalently, the economy's growth rate and period life expectancy of old agents) among the four hypotheses of households preferences are ranked as:

$$u^F<u^S<u^R<u^P \qquad (g^F<g^S<g^R<g^P), \quad (4\text{-}24)$$

$$v^S<v^P<v^R<v^F \qquad (l^S<l^P<l^R<l^F). \quad (4\text{-}25)$$

Our results show that, depending on the assumption on family relationships, individuals who are identical other than their preferences spend their time in the considerably different manner. As shown in *Table 3*, whose first column ($\tau=0$) represents the case without government interventions discussed in this section, young parent with parental altruism devotes about 32% of her time to educate her child but spends less than half, 13%, for her old parent. Considering that economic growth rates and life expectancy tend to be negatively correlated, the model replicates the economy

[64] As the one period in this model spans 30 years, the annual growth rate is given by: $(1+g^X)^{\frac{1}{30}}-1$, and life expectancy at birth of the agents is given by: $30+30+l^X$.

at the low level of development, which attains an exceptionally high rate of growth whereas its life expectancy is predicted to be very low (Barro, 2013). On the contrary, a household connected by filial altruism spends 11% and 19% of its time for education and health investments, respectively. This leads to the longevity society with very slow economic growth that has reached the very high level of development. While the predicted fractions of time devoted to education and health investments from our highly stylized models are not meant to fit to empirical data well, these hypotheses of one-sided altruism would not be applicable for Japanese economy, to which the model is calibrated. This view is in accordance with the suggestion by Cigno and Rosati (1996), who reveal that their theoretical predictions derived from parental and filial altruism models are categorically rejected by empirical data. However, a young parent with self-interest or reciprocal altruism divides her time between education and health investments more evenly. The results also suggest that the selfish agent works more and invests less in human capital than the one with reciprocal altruism. Though both the hypotheses of self-interest and reciprocal altruism seems to be similarly applicable for real economies, we suggest a possibility that, in studies which do not take reciprocal altruism into account, the similar results from the two hypotheses may disguise households connected via reciprocal altruism as selfish ones. We will report aspects of social welfare for each hypothesis of family relationships in the next section.

4.4 The Government

4.4.1 The Setup with Government Interventions

The social security, which is added to our baseline model, is the Pay-As-You-Go (PAYG) system. At time t, a contribution to the social security is collected from a young parent at a contribution rate τ on her earning capacity E_t^y, and in the same period a social security benefit S_t is paid to her old parent as an intergenerational transfer. Consequently, the costs to the young parent are the financial transfer $\bar{z}E_t^y H_t^o$ to her old parent and the social security contribution τE_t^y to the government, and the budget constraint of the young parent at time t (4-7) is modified to:

$$c_t^y = E_t^y[1 - u_t - v_t(1 + \bar{z}\psi) - \tau]. \tag{4-26}$$

At time $t + 1$, bearing in mind that numbers of young and old agents are same, the benefit paid to the same agent in her old period equals a contribution collected from her child, $S_{t+1} = \tau E_{t+1}^y$.[65] Then, the budget constraints of the old parents belonging to three generations (4-9), (4-10), and (4-11) are modified to:

[65] To achieve the solution below, we assume that the social security benefit is a function of the contribution of the old agent's own child, and this motivates the young agent to invest in her child's education capital. In case of Japan, PAYG benefits are related to agents' previous earnings, that is, $S_{t+1} = \phi E_t^y$, where the parameter ϕ is called as the replacement rate ("Social Security Programs Throughout the World: Asia and the Pacific, 2012," 2013). If the benefit is recognized as a fraction of the old agent's own earning capacity when she was young, she has no additional incentive to invest in her child's education capital. As discussed in Ehrlich and Lui (1998), a

$$c_{t+1}^{o} = E_t^{y}\left[\left(1+\frac{\tau}{\bar{z}\psi v_t}\right)\bar{z}\gamma u_t + \delta\right]\psi v_t, \tag{4-27}$$

$$c_{t}^{o} = E_t^{y}\left[\left(1+\frac{\tau}{\bar{z}\psi v_t}\right)\bar{z}\gamma u_{t-1} + \delta\right]\frac{\psi v_t}{\gamma u_{t-1}}, \tag{4-28}$$

$$c_{t+2}^{o} = E_t^{y}\left[\left(1+\frac{\tau}{\bar{z}\psi v_t}\right)\bar{z}\gamma u_{t+1} + \delta\right]\gamma u_t \psi v_t. \tag{4-29}$$

The agent born in period $t-1$ maximizes her utility function (4-1) with respect to fractions of time devoted to education and health investments, u_t and v_t, subject to budget constraints (4-26), (4-27), (4-28), and (4-29). The necessary optimal conditions (4-12) and (4-13) change to:

$$u_t: \qquad \frac{1}{1-u_t-v_t(1+\bar{z}\psi)-\tau} = \frac{\beta\left(1+\frac{\tau}{\bar{z}\psi v_t}\right)}{u_t\left(1+\frac{\tau}{\bar{z}\psi v_t}\right)+\frac{\delta}{\gamma\bar{z}}} + \frac{\beta^2\alpha^P}{u_t}, \tag{4-30}$$

$$v_t: \qquad \frac{1+\bar{z}\psi}{1-u_t-v_t(1+\bar{z}\psi)-\tau} = \frac{1}{v_t}\left[\frac{\beta\left(u_t+\frac{\delta}{\gamma\bar{z}}\right)}{u_t\left(1+\frac{\tau}{\bar{z}\psi v_t}\right)+\frac{\delta}{\gamma\bar{z}}} + \frac{\alpha^F\left(u_{t-1}+\frac{\delta}{\gamma\bar{z}}\right)}{u_{t-1}\left(1+\frac{\tau}{\bar{z}\psi v_t}\right)+\frac{\delta}{\gamma\bar{z}}} + \frac{\beta^2\alpha^P\left(u_{t+1}+\frac{\delta}{\gamma\bar{z}}\right)}{u_{t+1}\left(1+\frac{\tau}{\bar{z}\psi v_t}\right)+\frac{\delta}{\gamma\bar{z}}}\right]. \tag{4-31}$$

However, we cannot obtain explicit solutions for this model with the government intervention. In order to examine net effects of raising the tax rate on the key household choice variables and social welfare, we will simulate a calibrated version of the extended model and report numerical results in the next sub-section.

4.4.2 Numerical Analyses

To quantify effects of the government intervention on household choice variables under the four hypotheses of family relationships, the parameters of our model are calibrated using actual Japanese data and consensus estimates from the existing literature. Following Ehrlich and Kim (2007), the time preference parameter ρ is set to be 0.01, so the discount factor is $\beta = (1/1.01)^{30}$ as the one period in this study spans thirty years.[66] As for the depreciation rate of education capital stock, Kotlikoff and Gokhale (1992) document that human capital declines to about a third of the peak level at the age of forty-five in following twenty years. Hence, we set the fraction of

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higher PAYG compensation rate lowers the return on education capital and slows economic growth.

[66] While Ehrlich and Kim (2007) calibrate their model's parameter values using actual data in the United States, the values of discount factor should be similar among the developed economies.

education capital maintained at old-age δ to 0.3 occurring over the period of thirty years.[67] From their numerical analysis in Ehrlich and Lui (1991), in which the compensation rate of the family insurance system is an endogenous variable, the rate relevant to this study without private savings is calculated to be 0.34.[68] The remaining productivity parameters γ and ψ are solved by combining the two first-order conditions with respect to u and v at the balanced steady-state growth and by using the survival probability of Japanese population from age 60 to 90, which is calculated to be $2/3$ from the last thirty years' average life expectancy of 80, and the long-run one period growth rate of $(1 + 0.011)^{30}$ from the average annual growth rate of the same period (World Development Indicators, 2013). For the selfish household hypothesis with no government intervention, the solution yields the following calibrated parameter values: $\gamma = 6.7200$ and $\psi = 5.9924$, and these values are used for the three other hypotheses of household preferences as well. We then perform comparative statics analyses numerically by changing the level of government intervention.

Table 3 reports the simulated effects of increases in the PAYG contribution rate τ, which is set to 0%, 4%, 8%, and 12%, on the key choice variables at the growth equilibrium steady state under each hypothesis. The increases in τ raise the education capital investments u hence the economy's growth rate, $g = \gamma u - 1$, under all the hypotheses because the more young agents invest in their children's education capital the higher the same agents receive PAYG benefits when they are old. As for the health investments v, the same increases in τ lower v hence the old agents' period life expectancy, $l = 30 \times \psi v$, under all the hypotheses. This can be explained by the trade-off between the two types of human capital investments. Households devote the larger fractions of time for their children's education as the contribution rate goes up therefore they substitute away from health investments.

Lastly, we report the respective changes in social welfare of the economy. Letting $\overline{U}^{t-1}$ denote the utility level of the agent born in $t - 1$ evaluated at the steady-state values of the choice variables, we can now express the social welfare function as:

$$\Omega^X = \frac{1}{1-\omega}\sum_{t=2}^{\infty} \omega^{t-1}\overline{U}^{t-1^X}, \qquad 0 < \omega < 1, \tag{4-32}$$

where ω is the generational discounting factor, and X denotes each type of family relationships, S, F, P, or R. Also, we set an initial level of young agents' education capital stock to be $E_1^y = 2.7182$ ($\ln E_1^y \cong 1$) for simplicity. Under the hypotheses of self-interest and parental altruism, the increases in the PAYG contribution rate, which

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[67] In this case, the annual depreciation rate is 0.04. The depreciation rate of education capital is also discussed in Sala-I-Martin (1996).

[68] In Ehrlich and Lui (1991), the compensation rate is 0.18 and the equilibrium value of savings rate is 0.16. Hence, the sum of these two means is available for the representative agent to consume in old-age period. In the absence of private savings, the compensation rate must be raised to a level close to the sum to provide the elder agent with sufficient old-age supports. We add up the above two rates as our compensation rate, which is exogenously given in this study without private savings.

motivate individuals to spend greater fractions of time for educating their children, promote social welfare as Ω^S and Ω^P increase. However, if family members are connected via filial altruism, social welfare of the economy Ω^F decreases with the same increases in the contribution rate. Under the hypothesis of reciprocal altruism, the diverging effects from individuals' parental and filial altruistic traits on social welfare nearly cancel each other, therefore the increases in the contribution rate only slightly improve social welfare Ω^R. This result suggests that the standard government interventions such as the increases in the PAYG social security contribution rate would have quite different impacts on social welfare of the economy depending on the assumption of family relationships. As a consequence, policy makers must possess knowledge about which hypothesis of family relationships is most appropriate for their society in order to accurately predict effects of new government policies.

Table 3: Simulation results: quantitative impacts of changes in the PAYG contribution rate

		$\tau = 0$	$\tau = 0.04$	$\tau = 0.08$	$\tau = 0.12$
Self-interest	u^S	0.2066	0.2214	0.2355	0.2491
	$(1 + g^S = \gamma u^S)$	(1.3885)	(1.4879)	(1.5828)	(1.6738)
	v^S	0.1113	0.0965	0.0815	0.0663
	$(l^S/30 = \psi v^S)$	(0.6667)	(0.5782)	(0.4884)	(0.3975)
	Ω^S	1.0035	1.0459	1.0858	1.1240
Filial altruism	u^S	0.1096	0.1176	0.1254	0.1330
	$(1 + g^S = \gamma u^S)$	(0.7367)	(0.7906)	(0.8430)	(0.8939)
	v^S	0.1862	0.1728	0.1592	0.1454
	$(l^S/30 = \psi v^S)$	(1.1159)	(1.0355)	(0.9539)	(0.8714)
	Ω^S	1.3697	1.3334	1.3030	1.2782
Parental altruism	u^S	0.3195	0.3257	0.3319	0.3381
	$(1 + g^S = \gamma u^S)$	(2.1470)	(2.1884)	(2.2301)	(2.2721)
	v^S	0.1263	0.1116	0.0969	0.0821
	$(l^S/30 = \psi v^S)$	(0.7569)	(0.6690)	(0.5806)	(0.4918)
	Ω^S	2.6837	2.7285	2.7749	2.8229
Reciprocal altruism	u^S	0.2381	0.2415	0.2450	0.2486
	$(1 + g^S = \gamma u^S)$	(1.5997)	(1.6230)	(1.6466)	(1.6706)
	v^S	0.1747	0.1608	0.1470	0.1330
	$(l^S/30 = \psi v^S)$	(1.0466)	(0.9638)	(0.8806)	(0.7972)
	Ω^S	2.8466	2.8578	2.8715	2.8876

Source: The authors' simulation.

Notes: Parameter values used are: $\beta = (1/1.01)^{30}$, $\delta = 0.30$, $\bar{z} = 0.34$, $\gamma = 6.7200$, $\psi = 5.9924$, $E_1^y = 2.7182$ ($\ln E_1^y \cong 1$), and $\omega = 0.95^{30}$.

4.5 Conclusion

By comparing the results from the four hypotheses of family relationships, we find that the respective households choose significantly different amounts of investments in education and health capital. The economy with parental altruism attains the highest growth rate though their life expectancy is the lowest among the four hypotheses. Individuals connected through filial altruism enjoy the longest lifespan but have to bear with the slowest economic growth. While we have investigated the only four extreme hypotheses of household preferences, intermediary shades of family relationships should exist in the real world. For example, in their numerical analysis, Blackburn and Cipriani (2005) estimate both parameters measuring the degrees of filial and parental altruism to be 0.60.[69] With these parameter values, households with reciprocal altruism would divide their time for working and investing in education and health capital in a well-balanced manner. This study also finds that the effects of government policies on the macroeconomy can be quite different depending on the hypotheses of family relationships. The increases in the PAYG social security contribution rate improve social welfare only in the economy under the hypothesis of self-interest or parental altruism. If individuals are altruistic only towards their parents, the same increases in the contribution rate lower social welfare. Under the hypothesis of reciprocal altruism, social welfare is only slightly improved by the expansion of social security. Then, policy makers have to know the exact hypothesis of household behavior applicable for their society before they institute government interventions which affect agents' human capital investments decisions.

In the recent past most of developed economies have undergone the demographic transition, and rapid population aging is accompanied by changes in intergenerational relationships within the family. Ogawa and Retherford (1993) report the sudden normative shift in Japan, where more and more people consider social norms of filial care for elderly parents as burdensome and inappropriate. In analogy, it should be reasonable to assume that individuals' altruistic traits might not be intact from the process of population aging. While we must wait future empirical studies to find which hypothesis of household behavior fits best to the real economy, our contribution to the existing literature with this study is to present the theoretical framework capable of replicating the various shades of family relationships.

[69] In our study, the relevant parameter values are given by: $\alpha^F = \beta^2\alpha^P = 0.6$.

Chapter 5: Conclusion

In the existing literature, the difference between education and health capital has been recognized for decades. As Ehrlich and Chuma (1990) suggest, the marginal benefit of education falls with agents' remaining work horizon, whereas that of health rises over the entire life span. However, the critical difference was masked by the existence of complementarity between education and health capital. In the past, the increases in agents' health capital stock were raising the rates of return on education investments by lengthening their work horizon. In other words, the inability of education capital to lengthen the work horizon was supplemented by increases in the stocks of health capital. However, as the economy grows, the elders' retirement decisions are not bounded by their health condition any more but by their remaining productivity. For that reason, the recent improvements in longevity solely lengthen the elders' retirement period but not extend their work horizon. In the second chapter of this dissertation, the theoretical model shows that, once the economy reaches the certain level of development, the higher chance of early retirement emerges as young agents invest more in their children's education. The numerical analyses also confirm that the more the economy develops the less the elders are inclined to stay in the labor market. On the whole, pension reforms that were raising the pension eligibility ages in the developed countries would reduce social welfare and increase unemployment at old-age.

In the absence of complementarity between health and education capital, their difference becomes pronounced. Firstly, as the age-efficiency profile discussed in the labor literature suggests, the inherently stagnant education capital of the elders is not capable of extending their work horizon by itself, and this leads to the retirement age rigidity. Secondly, individuals' investments in health capital are going to increase without any foreseeable limitation. The possible consequence of these trends is persistence of high old-age dependency until current low fertility reduces the size of future old population. The PAYG social security systems would not come to help this situation. While they are effective in terms of poverty relief and redistribution, the PAYG systems themselves are one of the major causes of the retirement age rigidity because they provide elder workers with incentives to retire at the designated ages. According to Ehrlich and Lui (1998), the PAYG systems reduce young parent's incentives to invest in their child's education capital due to the moral hazard effect. In addition to this, the existence of another moral hazard is shown in the third chapter. The same PAYG systems induce young parents to invest in their own health capital beyond the optimal level. Taking the two moral hazard effects into account, the current low economic growth and high old-age dependency in the developed economies should be explained as the endogenous outcomes of the PAYG social security systems.

As Japanese survey in Horioka (2002) suggests, people care not only their own lifetime utility but also their old parents' and children's welfare. In contrast to the selfish agent models of the previous two chapters, the model in the fourth chapter includes both filial and parental altruism. When filial altruism is in place, improvements in longevity are welfare improving, and young agents invest in their parents' health even if it hinders economic growth. If retirement or social security is added to the two-sided altruism model, where agents work until they die, health investments will increase further at the cost of education capital as demonstrated in the previous two chapters. Therefore, the possible policy recommendation in this study could be that the government speeded up the long-run growth by suppressing survival probability of the elderly. However, this argument contains more serious ethical problems than the one that concerns reduction of fertility. If the three chapters' findings are put together, under the PAYG social security systems, it would be difficult to lengthen the elders' work horizon corresponding to their improving longevity because the retirement age was bound by their remaining productivity. In order to resolve the retirement rigidity, the social security systems must be reformed so that the new schemes provide appropriate incentives for both education and health capital investments, and complementarity between the two components of human capital is recovered. In consequence, the age of retirement becomes the elders' choice variable spanning more than 20 years.

As for the future extensions to the current partial equilibrium models, the saving decision and physical capital accumulation process should be introduced in order to evolve the models into the general equilibrium ones. Then, the more general production function increasing with the agents' education capital stock and endogenous technological progress can be utilized for further examination of the problems of population aging.

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APPENDICES

A Characteristics of the Three Models

Chapter	Leisure	Retirement	Working in old period	PAYG social security	FF social security	Family insurance	Altruism
2	○	○	○	○			
3		○		○	○	○	
4			○	○		○	○

B Names and Definitions of Variables/Parameters in Chapter 2

U^{t-1}	Instantaneous utility of "quality of life" of agent born in time $t-1$
$c_t^y,\ c_{t+1}^o$	Consumption of young parent at time t and old parent at time $t+1$, respectively
ρ	The rate of time preference; $\beta = \left(1/(1+\rho)\right)^{30}$
l_{t+1}^o	The leisure time of old parent due to retirement at time $t+1$
η	The parameter measuring the intensity of utility from leisure
$H_t^o,\ H_t^y$	Stock of health capital of old and young parent at time t, respectively; $H_t^y = 1$
ψ	The parameter measuring the productivity of health capital investment
v_t	Time inputs used in the production of H_t^o
α	The scalar parameter to make the term $\alpha\psi v_t > 1$
l	Survival probability of the old parent; $l = \psi v$
$E_t^o,\ E_t^y$	Stock of education capital of old and young parent at time t, respectively
γ	The parameter measuring the productivity of education capital investment
u_t	Time inputs used in the production of E_{t+1}^y
g	The growth rate; $1+g = \gamma u$
$\bar{\delta}$	The exogenous of depreciation rate of education capital stock
$\mu,\ \phi$	The parameter measuring the effect of u_t and v_t on $\bar{\delta}$, respectively
S_t	The PAYG benefit
τ	The PAYG social security contribution rate
e_{t+1}	The indicator function; $e_{t+1} = 0$ if retired and $e_{t+1} = 1$ if working at time $t+1$
Ω	Social welfare
ω	The rate of generational discount; $\omega = \beta$

C Programing Code (Scilab) in Chapter 2

```
//calibration, t=0.18, retirement
clear;
```

```
r=0.023;
b=(1/(1+r))^30;
//b=0.5055114;
h=(80.4-59)/30;//0.7133333;
a=(1+0.0158)^30;//1.6004653;
t=0.18;

deff('[y]=f0(z)',[
'f_1=t*z(1)*(1+2*b+z(2))-b*(z(1)+1)',
'f_2=t*z(1)*z(2)-b*(1+h)',
'y=[f_1;f_2]'])
z0=[1,1]
zsol=fsolve(z0,f0)

p=2.5124008;
e=1.915188;

g=a/t
//g=8.8914737;


//retiring, t=0.18
clear;
r=0.023;
b=(1/(1+r))^30;
//b=0.5055114;
e=1.915188;
h=(80.4-59)/30;//0.7133333;
a=(1+0.0158)^30;//1.6004653;
p=2.5124008;
g=8.8914737;
//t=0.18;

u=b*(p+1)/(p*(1+2*b+e))
v=(e*p-1-2*b)/(p*(1+2*b+e))

//u=0.18;
//v=0.2839250;

u*g//1.6004653;
v*p//0.7133333;

t=u;

u^b*(1-u-v)^(1+b)*(1+p*v)^e//0.4610297;


//working, t=0.18, baseline
//m=-0, f=0
clear;
r=0.023;
b=(1/(1+r))^30;
//b=0.5055114;
e=1.915188;
```

```
h=(80.4-59)/30;//0.7133333;
a=(1+0.0158)^30;//1.6004653;
p=2.5124008;
g=8.8914737;
//t=0.18;
m=-0;
f=0;

d=a/p;//d=0.6370263;
alpha=a/(p*0.0953346);//6.6820048;

deff('[y]=f1(x)',[
'f_1=g*x(1)-d*(1+f)*(g*x(1))^m*(alpha*p*x(2))^f*p',
'f_2=(1+2*b)*(f+1)*x(1)-((1+b)*m-b*(f+1)-1)*x(2)-b*(f+1)',
'y=[f_1;f_2]'])
x0=[0.1,0.1]
xsol=fsolve(x0,f1)

u=0.18;
v=0.0953346;

u*g//1.6004653;
v*p//0.2395187;

t=u-m*v/(1+f)//t=0.18;

d*(g*u)^m*(alpha*p*v)^f//0.6370263;

g*u/(p*(1+f))//0.6370263;

u^b*(1-u-v+d*(g*u)^m*(alpha*p*v)^f*p*v/(g*u))^(1+b)//0.3117292;//retire


//m=-0, f=0.1
clear;
r=0.023;
b=(1/(1+r))^30;
//b=0.5055114;
e=1.915188;
h=(80.4-59)/30;//0.7133333;
a=(1+0.0158)^30;//1.6004653;
p=2.5124008;
g=8.8914737;
//t=0.18;
m=-0;
f=0.1;

d=a/p;//d=0.6370263;
alpha=a/(p*0.0953346);//6.6820048;

deff('[y]=f1(x)',[
'f_1=g*x(1)-d*(1+f)*(g*x(1))^m*(alpha*p*x(2))^f*p',
'f_2=(1+2*b)*(f+1)*x(1)-((1+b)*m-b*(f+1)-1)*x(2)-b*(f+1)',
'y=[f_1;f_2]'])
```

```
x0=[0.1,0.1]
xsol=fsolve(x0,f1)

u=0.2014842;
v=0.0709189;

u*g//1.7914915;
v*p//0.1781767;

t=u-m*v/(1+f)//t=0.2014842;

d*(g*u)^m*(alpha*p*v)^f//0.6482359;

g*u/(p*(1+f))//0.6482360;

u^b*(1-u-v+d*(g*u)^m*(alpha*p*v)^f*p*v/(g*u))^(1+b)//0.3132363;//retire


//m=-0.1, f=0
clear;
r=0.023;
b=(1/(1+r))^30;
//b=0.5055114;
e=1.915188;
h=(80.4-59)/30;//0.7133333;
a=(1+0.0158)^30;//1.6004653;
p=2.5124008;
g=8.8914737;
//t=0.18;
m=-0.1;
f=0;

d=a/p;//d=0.6370263;
alpha=a/(p*0.0953346);//6.6820048;

deff('[y]=f1(x)',[
'f_1=g*x(1)-d*(1+f)*(g*x(1))^m*(alpha*p*x(2))^f*p',
'f_2=(1+2*b)*(f+1)*x(1)-((1+b)*m-b*(f+1)-1)*x(2)-b*(f+1)',
'y=[f_1;f_2]'])
x0=[0.1,0.1]
xsol=fsolve(x0,f1)

u=0.1724665;
v=0.0958161;

u*g//1.5334813;
v*p//0.2407284;

t=u-m*v/(1+f)//t=0.1820481;

d*(g*u)^m*(alpha*p*v)^f//0.6103648;

g*u/(p*(1+f))//0.6103649;
```

```
u^b*(1-u-v+d*(g*u)^m*(alpha*p*v)^f*p*v/(g*u))^(1+b)//0.3092935;//retire
```

D Names and Definitions of Variables/Parameters in Chapter 3

H_t^o, H_t^y	Stock of health capital of old and young parent at time t, respectively
E_t^o, E_t^y	Stock of education capital of old and young parent at time t, respectively
ψ	The parameter measuring the productivity of health capital investment
v_t	Time inputs used in the production of H_{t+1}^o
γ	The parameter measuring the productivity of education capital investment
u_t	Time inputs used in the production of E_{t+1}^y
w_t	Market wage rate; $w_t = wE_t^y$; $w = 1$
r, ρ	Market rate of interest and rate of time preference; $\beta = \left(1/(1+\rho)\right)^{30}$
U^{t-1}	Instantaneous utility of "quality of life" of agent born in time $t-1$
c_t^y, c_{t+1}^o	Consumption of young parent at time t and old parent at time $t+1$, respectively
$\bar{z}$	The compensation rate of the family insurance system (exogenously given)
τ	The Pay-As-You-Go social security contribution rate
S_{t+1}	The earning-related social security benefit
$1/\sigma$	The intertemporal elasticity of substitution in consumption
R_u	The rate of return on education capital
R_v	The rate of return on health capital
ϕ_t	The replacement rate at time t
ς	The mandatory savings rate
Ω	Social welfare
ω	The rate of generational discount
$PYDC$	Pay As You Go social security - Decentralized
$PYSP$	Pay As You Go social security - Social planner
FF	Fully funded social security

E The Optimization Problems and the Laws of Motion of Education Capital Accumulation in Chapter 3

With (3-1), (3-2), (3-5), and (3-6), the lifetime utility of the agent born in $t-1$ (3-7) can be restated as:

$$U^{t-1} = \frac{1}{1-\sigma}\left[E_t^y(1-u_t-v_t)(1-\tau-\bar{z}\psi v_{t-1})\right]^{1-\sigma} + \beta\frac{\psi v_t}{1-\sigma}\left[\bar{z}\gamma u_t E_t^y(1-u_{t+1}-v_{t+1}) + \frac{S_{t+1}}{\psi v_t}\right]^{1-\sigma}. \tag{E1}$$

For the PAYG decentralized equilibrium solution, we set S_{t+1} to be $\phi_{t+1}E_t^y(1-u_t-v_t)\psi v_t$. The followings are the first-order conditions:

$$\frac{\partial U}{\partial u} = c_t^{y\,-\sigma}\left(-E_t^y(1-\tau-\bar{z}\psi v_{t-1})\right) + \beta\psi v_t c_{t+1}^{o\;-\sigma}\left(E_t^y(1-u_{t+1}-v_{t+1})\left(\bar{z}\gamma - \frac{\phi_{t+1}}{1-u_{t+1}-v_{t+1}}\right)\right) = 0, \tag{E2}$$

$$\frac{\partial U}{\partial v} = c_t^{y\,-\sigma}\left(-E_t^y(1-\tau-\bar{z}\psi v_{t-1})\right) + \beta\psi\frac{{c_{t+1}^o}^{1-\sigma}}{1-\sigma} + \beta\psi v_t {c_{t+1}^o}^{-\sigma}\left(E_t^y(1-u_{t+1}-v_{t+1})\left(-\frac{\phi_{t+1}}{1-u_{t+1}-v_{t+1}}\right)\right) = 0, \tag{E3}$$

which are rearranged to Equations (3-11) and (3-12), respectively. For the PAYG social planner solution, the term $\phi_{t+1}E_t^y(1-u_t-v_t)\psi v_t$ is replaced by $\tau\gamma u_t E_t^y(1-u_{t+1}-v_{t+1})$. The first-order conditions are given by:

$$\frac{\partial U}{\partial u} = c_t^{y\,-\sigma}\left(-E_t^y(1-\tau-\bar{z}\psi v_{t-1})\right) + \beta\psi v_t {c_{t+1}^o}^{-\sigma}\left(E_t^y(1-u_{t+1}-v_{t+1})\left(\bar{z}\gamma+\frac{\tau\gamma}{\psi v_t}\right)\right) = 0, \tag{E4}$$

$$\frac{\partial U}{\partial v} = c_t^{y\,-\sigma}\left(-E_t^y(1-\tau-\bar{z}\psi v_{t-1})\right) + \beta\psi\frac{{c_{t+1}^o}^{1-\sigma}}{1-\sigma} + \beta\psi v_t {c_{t+1}^o}^{-\sigma}\left(E_t^y(1-u_{t+1}-v_{t+1})\left(-\frac{\tau\gamma u_t}{\psi {v_t}^2}\right)\right) = 0, \tag{E5}$$

which are rearranged to Equations (3-13) and (3-14), respectively.

With (3-8) and (3-9), Equation (3-13) is rearranged to Equation (3-15), which is duplicated below for convenience:

$$E_{t+1}^y = (1-u_t-v_t)\psi v_t(\beta\gamma)^{\frac{1}{\sigma}}\left(\frac{(1-u_{t+1}-v_{t+1})(\bar{z}\psi v_t+\tau)}{1-\tau-\bar{z}\psi v_{t-1}}\right)^{\frac{1}{\sigma}-1} E_t^y, \tag{E6}$$

Together with (3-1) and setting $\sigma = 0.5$, (A6) gives the equation below:

$$\gamma u_t = (1-u_t-v_t)\psi v_t \beta^2\gamma^2\frac{(1-u_{t+1}-v_{t+1})(\bar{z}\psi v_t+\tau)}{1-\tau-\bar{z}\psi v_{t-1}}. \tag{E7}$$

In steady state, using (3-16) resulting from $R_u^{PYSP} = R_v^{PYSP}$, Equation (E7) is further rearranged to:

$$\begin{aligned} &1.5^2\beta^2\gamma\bar{z}^2\psi^3v^4 - 3(\bar{z}\psi-\tau)\beta^2\gamma\bar{z}\psi^2v^3 \\ &\quad - [1.5\tau\beta^2\gamma\bar{z}\psi - 0.5\bar{z}^2\psi - (\bar{z}\psi-\tau)^2\beta^2\gamma]\psi v^2 \\ &\quad - [0.5(1-1.5\tau)\bar{z}\psi - \tau(\bar{z}\psi-\tau)\beta^2\gamma\psi]v \\ &\quad - 0.5^2\tau(1-\tau-\tau\beta^2\gamma\psi) = 0. \end{aligned} \tag{E8}$$

Together with (3-16), the solution to Equation (E8) fully describes the law of motion of education capital (3-15) and (E6). The analogous processes fully describe the respective laws of motion of education capital (3-17) and (3-25) for each option of securing old-age consumption.

F Calibration of Parameters in Chapter 3

As mentioned in Section 3.4, $\sigma = 0.5$ and $\beta = (1/1.01)^{30} = 0.7419$, and the baseline contribution rate is set to $\tau = 0.08$ for the PAYG decentralized solution. At the growth equilibrium steady state, the analogical steps to derive (E8) give:

$$\begin{aligned}1.5^2\beta^2\gamma\bar{z}^3\psi^4v^5 - 3(\bar{z}\psi - 1.5\tau)\beta^2\gamma\bar{z}^2\psi^3v^4 \\ + \{[(\bar{z}\psi - 1.5\tau)^2 - 3\tau\bar{z}\psi]\beta^2\gamma + 0.5\bar{z}^2\psi\}\bar{z}\psi^2v^3 \\ - [0.5(1-3\tau)\bar{z} - 2\tau(\bar{z}\psi - 1.5\tau)\beta^2\gamma]\bar{z}\psi^2v^2 \\ - [0.5\tau(2-3\tau)\bar{z}\psi - \tau^2\beta^2\gamma\psi]\bar{z}\psi v - 0.5\tau^2(1-\tau) = 0.\end{aligned} \tag{F1}$$

Then, multiplying both sides by ψ gives:

$$\begin{aligned}1.5^2\beta^2\gamma\bar{z}^3h^5 - 3(\bar{z}\psi - 1.5\tau)\beta^2\gamma\bar{z}^2h^4 \\ + \{[(\bar{z}\psi - 1.5\tau)^2 - 3\tau\bar{z}\psi]\beta^2\gamma + 0.5\bar{z}^2\psi\}\bar{z}h^3 \\ - [0.5(1-3\tau)\bar{z} - 2\tau(\bar{z}\psi - 1.5\tau)\beta^2\gamma]\bar{z}\psi h^2 \\ - [0.5\tau(2-3\tau)\bar{z}\psi - \tau^2\beta^2\gamma\psi]\bar{z}\psi h - 0.5\tau^2(1-\tau)\psi = 0,\end{aligned} \tag{F2}$$

where, $h = \psi v = 0.6667$ is the economy's life expectancy at the steady state. Multiplying both side of Equation (3-18) with γ and ψ gives:

$$\psi a(\bar{z}h + \tau) = 0.5\gamma\bar{z}h^2, \tag{F3}$$

where $a = \gamma u = (1 + 0.019)^{30}$ is the economy's growth rate at the steady state. Then, solutions to Equations (F2) and (F3) yield the parameter values of $\gamma = 12.9566$ and $\psi = 1.7000$.

G Programing Code (Scilab) in Chapter 3

```
//calibration, payg decentralized, t=0.08
clear;
r=0.01;
b=(1/(1+r))^30;
//b=0.7419229;
b2=b^2;
h=2/3;
a=(1+0.019)^30;
z=0.27;
t=0.08;

deff('[y]=f1(x)', [
'f_1=(0.5*z*h*x(1)*h-(z*h+t)*x(2)*a)',
'f_2=(1.5^2*b2*x(1)*z^3*h^5-3*(z*x(2)-1.5*t)*b2*x(1)*z^2*h^4+(0.5*z^3*x(2)+((z*x(2)-1.5*t)^2-
3*t*z*x(2))*b2*x(1)*z)*h^3-(0.5*(1-3*t)*z^2*x(2)-2*t*(z*x(2)-1.5*t)*b2*x(1)*z*x(2))*h^2-(0.5*t*(2-
3*t)*z*x(2)-t^2*b2*x(1)*z*x(2)^2)*h-0.5*t^2*(1-t)*x(2))',
'y=[f_1; f_2]'])
x0=[10,10]
[xs, fxs, m]=fsolve(x0', f1)

g=12.956567;
p=1.6999742;


//payg decentralized, t=0.08
```

```
clear;
r=0.01;
b=(1/(1+r))^30;
//b=0.7419229;
b2=b^2;
h=2/3;
a=(1+0.019)^30;
z=0.27;
t=0.08;

g=12.956567;
p=1.6999742;

p1=poly([-0.5*t^2*(1-t) -(0.5*t*(2-3*t)*z*p-t^2*b2*g*z*p^2) -(0.5*(1-3*t)*z^2*p-2*t*(z*p-
1.5*t)*b2*g*z*p)*p (0.5*z^3*p+((z*p-1.5*t)^2-3*t*z*p)*b2*g*z)*p^2 -3*(z*p-1.5*t)*b2*g*z^2*p^3
1.5^2*b2*g*z^3*p^4], 'v', 'c')
roots(p1)

v=0.3921628;//2/5
u=v*0.5*z*p*v/(z*p*v+t)//u=0.1357487;

g*u//a=g*u=1.7588366;
p*v//h=p*v=0.6666666;

w=2*(((1-u-v)*(1-t-z*p*v))^0.5+b*p*v*((1-u-v)*(z*g*u+t*g*u/(p*v)))^0.5)

x=[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20];
y=sum(((0.95^30)^x).*((g*u)^x).^0.5)

w*y/(1-0.95^30)//3.1061396; welfare

t*g*u/(p*v)//0.2110604; replacement rate


//payg social planner, t=0.08
clear;
r=0.01;
b=(1/(1+r))^30;
//b=0.7419229;
b2=b^2;
h=2/3;
a=(1+0.019)^30;
z=0.27;
t=0.08;

g=12.956567;
p=1.6999742;

p2=poly([0.5^2*t*(t*b2*g*p-(1-t)) -(0.5*(1-1.5*t)*z*p-t*(z*p-t)*b2*g*p) (0.5*z^2*p+(z*p-t)^2*b2*g-
1.5*t*b2*g*z*p)*p -3*(z*p-t)*b2*g*z*p^2 1.5^2*b2*g*z^2*p^3], 'v', 'c')
roots(p2)

v=0.3957666;//2/4
u=v*0.5*(z*p*v+t)/(z*p*v+0.5*t)//u=0.2335936;
```

```
g*u//a=g*u=3.026571;
p*v//h=p*v=0.6727930;

w=2*(((1-u-v)*(1-t-z*p*v))^0.5+b*p*v*((1-u-v)*(z*g*u+t*g*u/(p*v)))^0.5)

x=[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20];
y=sum(((0.95^30)^x).*((g*u)^x).^0.5)

w*y/(1-0.95^30)//3.4660456; welfare

t*g*u/(p*v)//0.3598814; replacement rate


//fully funded, t=0.08, a>i
clear;
r=0.01;
b=(1/(1+r))^30;
//b=0.7419229;
b2=b^2;
h=2/3;
a=(1+0.019)^30;
z=0.27;
i=(1+0.012)^30;
t=0.08;

g=12.956567;
p=1.6999742;

p0=poly([-0.5*(1-t) (0.5+b2*g*p)*z*p -3*b2*g*z*p^2 1.5^2*b2*g*z*p^2], 'v', 'c')
roots(p0)

v=0.3921628;//2/3
u=0.5*v-t*i/(g*z*p*v)//u=0.1470197;

g*u//a=g*u=1.9048701;
p*v//h=p*v=0.6666666;

w=2*(((1-u-v)*(1-t-z*p*v))^0.5+b*p*v*((1-u-v)*(z*g*u+t*i/(p*v)))^0.5)

x=[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20];
y=sum(((0.95^30)^x).*((g*u)^x).^0.5)

w*y/(1-0.95^30)//3.1193397; welfare

t*g*u/(p*v)//0.2285844; replacement rate

t*i/(p*v)//0.1716314; replacement rate
```

H Names and Definitions of Variables/Parameters in Chapter 4

U^{t-1} Instantaneous utility of "quality of life" of agent born in time $t-1$

$c_t^y,\ c_{t+1}^o$ Consumption of young parent at time t and old parent at time $t+1$, respectively

ρ	The rate of time preference: $\beta = \left(1/(1+\rho)\right)^{30}$
α^F	The parameter measuring the intensity of utility from filial altruism
α^P	The parameter measuring the intensity of utility from parental altruism
$H_t^o,\ H_t^y$	Stock of health capital of old and young parent at time t, respectively: $H_t^y = 1$
ψ	The parameter measuring the productivity of health capital investment
v_t	Time inputs used in the production of H_t^o
l	Survival probability of the old parent: $l = \psi v$ in steady-state
$E_t^o,\ E_t^y$	Stock of education capital of old and young parent at time t, respectively
w	The wage rate per effective labor
γ	The parameter measuring the productivity of education capital investment
u_t	Time inputs used in the production of E_{t+1}^y
g	The growth rate: $1 + g = \gamma u$ in steady-state
δ	The depreciation rate of education capital stock
$\bar{z}$	The compensation rate of the family insurance system (exogenously given)
τ	The Pay-As-You-Go social security contribution rate
Ω	Social welfare
ω	The rate of generational discount
S	Self-interest
F	Filial altruism
P	Parental altruism
R	Reciprocal altruism

I Programing Code (Scilab) in Chapter 4

```
//calibration, z=0.34, d=0.7
clear;
r=0.01;
b=(1/(1+r))^30;
h=2/3;
a=(1+0.011)^30;
z=0.34;
d=0.7;

g=(a*(1+2*b)+(1+b)*(1-d)/z)/b//g=6.7199996;
p=((1+2*b)*h)/(b*(1+(1-d)/(g*z))-(1+2*b)*h*z) //p=5.9923713;


//self-interest, t=0
clear;
r=0.01;
b=(1/(1+r))^30;
g=6.7199996;
p=5.9923713;
z=0.34;
d=0.7;
a=(1+0.011)^30;
h=2/3;
t=0;
af=0;
ap=0;

m=(1-d)/(g*z)//m=0.1313025;
```

```
deff('[y]=f3(x)', ['f_1=x(2)*(1+z*p)/(1-x(1)-x(2)*(1+z*p)-t)-
(x(1)+m)*(b+af+b^2*ap)/((1+t/(z*p*x(2)))*x(1)+m)', 'f_2=x(1)/(1-x(1)-x(2)*(1+z*p)-t)-
(b*(1+t/(z*p*x(2)))*x(1)/((1+t/(z*p*x(2)))*x(1)+m)+b^2*ap)', 'y=[f_1; f_2]'])
x0=[0.2,0.2]
[xs, fxs, m]=fsolve(x0', f3)

usg=0.2066167;
vsg=0.1112526;

g*usg//g*usg=a=1.3884641;
p*vsg//p*vsg=h=0.6666669;


//filial altruism, t=0
clear;
r=0.01;
b=(1/(1+r))^30;
g=6.7199996;
p=5.9923713;
z=0.34;
d=0.7;
a=(1+0.011)^30;
h=2/3;
t=0;
af=1;
ap=0;

m=(1-d)/(g*z)//m=0.1313025;

deff('[y]=f3(x)', ['f_1=x(2)*(1+z*p)/(1-x(1)-x(2)*(1+z*p)-t)-
(x(1)+m)*(b+af+b^2*ap)/((1+t/(z*p*x(2)))*x(1)+m)', 'f_2=x(1)/(1-x(1)-x(2)*(1+z*p)-t)-
(b*(1+t/(z*p*x(2)))*x(1)/((1+t/(z*p*x(2)))*x(1)+m)+b^2*ap)', 'y=[f_1; f_2]'])
x0=[0.2,0.2]
[xs, fxs, m]=fsolve(x0', f3)

ufg=0.1096207;
vfg=0.1862284;

g*ufg//g*ufg=0.7366511;
p*vfg//p*vfg=1.1159497;


//parental altruism, t=0
clear;
r=0.01;
b=(1/(1+r))^30;
g=6.7199996;
p=5.9923713;
z=0.34;
d=0.7;
a=(1+0.011)^30;
h=2/3;
t=0;
```

```
af=0;
ap=1;

m=(1-d)/(g*z)//m=0.1313025;

deff('[y]=f3(x)', ['f_1=x(2)*(1+z*p)/(1-x(1)-x(2)*(1+z*p)-t)-
(x(1)+m)*(b+af+b^2*ap)/((1+t/(z*p*x(2)))*x(1)+m)', 'f_2=x(1)/(1-x(1)-x(2)*(1+z*p)-t)-
(b*(1+t/(z*p*x(2)))*x(1)/((1+t/(z*p*x(2)))*x(1)+m)+b^2*ap)', 'y=[f_1; f_2]'])
x0=[0.2,0.2]
[xs, fxs, m]=fsolve(x0', f3)

upg=0.3194979;
vpg=0.1263075;

g*upg//g*upg=2.1470258;
p*vpg//p*vpg=0.7568814;


//self-interest, t=0.08
clear;
r=0.01;
b=(1/(1+r))^30;
g=6.7199996;
p=5.9923713;
z=0.34;
d=0.7;
a=(1+0.011)^30;
h=2/3;
t=0.08;
af=0;
ap=0;

m=(1-d)/(g*z)//m=0.1313025;

deff('[y]=f3(x)', ['f_1=x(2)*(1+z*p)/(1-x(1)-x(2)*(1+z*p)-t)-
(x(1)+m)*(b+af+b^2*ap)/((1+t/(z*p*x(2)))*x(1)+m)', 'f_2=x(1)/(1-x(1)-x(2)*(1+z*p)-t)-
(b*(1+t/(z*p*x(2)))*x(1)/((1+t/(z*p*x(2)))*x(1)+m)+b^2*ap)', 'y=[f_1; f_2]'])
x0=[0.2,0.2]
[xs, fxs, m]=fsolve(x0', f3)

usg=0.2355320;
vsg=0.0815067;

g*usg//g*usg=1.5827749;
p*vsg//p*vsg=0.4884184;


//filial altruism, t=0.08
clear;
r=0.01;
b=(1/(1+r))^30;
g=6.7199996;
p=5.9923713;
z=0.34;
```

```
d=0.7;
a=(1+0.011)^30;
h=2/3;
t=0.08;
af=1;
ap=0;

m=(1-d)/(g*z)//m=0.1313025;

deff('[y]=f3(x)', ['f_1=x(2)*(1+z*p)/(1-x(1)-x(2)*(1+z*p)-t)-
(x(1)+m)*(b+af+b^2*ap)/((1+t/(z*p*x(2)))*x(1)+m)', 'f_2=x(1)/(1-x(1)-x(2)*(1+z*p)-t)-
(b*(1+t/(z*p*x(2)))*x(1)/((1+t/(z*p*x(2)))*x(1)+m)+b^2*ap)', 'y=[f_1; f_2]'])
x0=[0.2,0.2]
[xs, fxs, m]=fsolve(x0', f3)

ufg=0.1254397;
vfg=0.1591905;

g*ufg//g*ufg=0.8429547;
p*vfg//p*vfg=0.9539286;


//parental altruism, t=0.08
clear;
r=0.01;
b=(1/(1+r))^30;
g=6.7199996;
p=5.9923713;
z=0.34;
d=0.7;
a=(1+0.011)^30;
h=2/3;
t=0.08;
af=0;
ap=1;

m=(1-d)/(g*z)//m=0.1313025;

deff('[y]=f3(x)', ['f_1=x(2)*(1+z*p)/(1-x(1)-x(2)*(1+z*p)-t)-
(x(1)+m)*(b+af+b^2*ap)/((1+t/(z*p*x(2)))*x(1)+m)', 'f_2=x(1)/(1-x(1)-x(2)*(1+z*p)-t)-
(b*(1+t/(z*p*x(2)))*x(1)/((1+t/(z*p*x(2)))*x(1)+m)+b^2*ap)', 'y=[f_1; f_2]'])
x0=[0.2,0.2]
[xs, fxs, m]=fsolve(x0', f3)

upg=0.3318579;
vpg=0.0968917;

g*upg//g*upg=2.230085;
p*vpg//p*vpg=0.5806110;


//reciprocal altruism, t=0.08
clear;
r=0.01;
```

```
b=(1/(1+r))^30;
g=6.7199996;
p=5.9923713;
z=0.34;
d=0.7;
a=(1+0.011)^30;
h=2/3;
t=0.08;
af=1;
ap=1;

m=(1-d)/(g*z)//m=0.1313025;

deff('[y]=f3(x)', ['f_1=x(2)*(1+z*p)/(1-x(1)-x(2)*(1+z*p)-t)-
(x(1)+m)*(b+af+b^2*ap)/((1+t/(z*p*x(2)))*x(1)+m)', 'f_2=x(1)/(1-x(1)-x(2)*(1+z*p)-t)-
(b*(1+t/(z*p*x(2)))*x(1)/((1+t/(z*p*x(2)))*x(1)+m)+b^2*ap)', 'y=[f_1; f_2]'])
x0=[0.2,0.2]
[xs, fxs, m]=fsolve(x0', f3)

urg=0.2450268;
vrg=0.1469596;

g*urg//g*urg=1.64658;
p*vrg//p*vrg=0.8806365;
```

VITA

Yoshitaka Koda was born in Takamatsu, Japan in 1969. He received BFA in Architecture from Tokyo National University of Fine Arts and Music in 1995, BA in Art History from Waseda University in 1998, MUP in Urban Design from City College of New York, CUNY in 2004, MA in Exhibition Design from Fashion Institute of Technology, SUNY in 2008, MBA in Media Management from Metropolitan College of New York in 2010 and Ph.D. in Economics from Chulalongkorn University in 2016. His work experience includes the senior architect position at Norihiko Dan and Associates and the exhibition designer position at GES. His main areas of research interest are human capital accumulation and economic growth with special emphasis on the problems of population aging. His article, Altruism and Four Shades of Family Relationships, was published from Eurasian Economic Review in 2015.