CHAPTER 2

CHARACTERISTICS AND FLUCTUATION OF STATE-OF-POLARIZATION IN A SINGLE-MODE OPTICAL FIBER

2.1. Introduction

In this chapter, we look at characteristics and fluctuations of SOP of light transmitted through a single-mode fiber. We first present general expressions used to characterize SOP of a polarized light. Recent theoretical studies and measurements of polarization fluctuation in single-mode optical fiber are then reviewed.

2.2. Mathematical expressions of general state-of-polarization

2.2.1. Elliptical, linear and circular polarization of completely polarized light (monochromatic light)

A light is considered to be completely polarized if the end point of its electric (and also of magnetic) vector observed at a typical point in space (x,y,z-coordinates) moves periodically along a straight line, or a circle or an ellipse. In general, an ellipse is described and the light is said to be elliptically polarized. We shall now give the equation of the ellipse in the x-y-z system. Suppose the light propagates in z direction, the electric components can be written as

a) elliptical polarization:

$$E_x = a_x \exp[i(\omega t - kr + \delta_x)]$$
$$E_y = a_y \exp[i(\omega t - kr + \delta_y)]$$
$$E_z = 0 \qquad (1)$$

where r is a position vector of a point in x-y-z system, k is called wave number, ω is the light frequency in space, δ_x and δ_y are the lightwave phase in x and y coordinates. a is the amplitude of the lightwave.

By simple mathematical manipulation, the equation of an ellipse (conic) can be determined from eq.(1) as [32]

$$\left(\frac{E_x}{a_x}\right)^2 + \left(\frac{E_y}{a_y}\right)^2 - 2\,\frac{E_x E_y}{a_x a_y}\cos\delta = \sin^2\delta \qquad (2)$$

where $\delta = \delta_y - \delta_x$ is the phase difference.

b) linear polarization:

In special cases, the ellipse can degenerate into a straight line or a circle. According to eq.(1), the ellipse will reduce to a straight line, when

$$\delta = m\pi \quad (m = \text{integer})$$

$$\frac{E_y}{E_x} = (-1)^m \frac{a_y}{a_x} \tag{3}$$

In this case, only one component may remain along this line, e.g. E_x, and we say that electric field (E) is linearly polarized in the x direction.

c) circular polarization:

For circularly polarized wave, special conditions are also necessary, i.e. $a_x = a_y = a$, and $\delta = m\pi/2$ (m =odd integer). Then eq.(2) reduces to the equation of the circle

$$E_x^2 + E_y^2 = a^2 \tag{4}$$

For right-handed circular polarization :

$$\delta = m\pi/2$$

$$\frac{E_y}{E_x} = e^{-i\pi/2} = -i \tag{5}$$

For left-handed circular polarization :

$$\delta = -\pi/2$$

$$\frac{E_y}{E_x} = e^{i\pi/2} = i \tag{6}$$

Figure 2 illustrates how the polarization ellipse changes with varying δ. We shall see later how the above equation can be applied to express the property of a completely polarized light (coherency matrix).

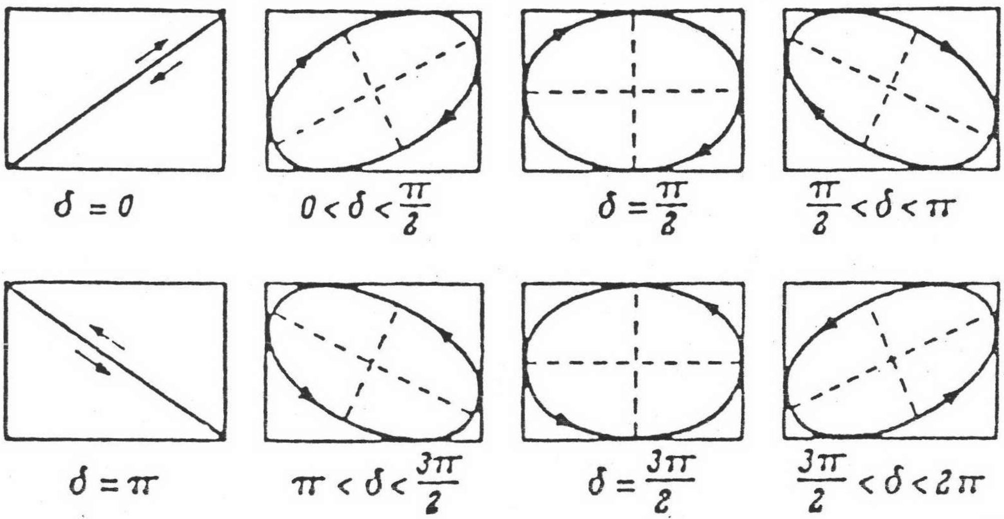


Fig. 2. Elliptical polarization with various values of the phase difference δ.

2.2.2. Expression of partially polarized light (quasi-monochrometric light)

In general, the variation of the field vectors is neither completely regular nor completely random and we may say that a light having properties between these two extremes is partially polarized.

Consider such a light of average angular frequency propagated in the z-direction. The electric field components in the x and y directions at right angles to the direction of propagation can be expressed as

$$E_x(t) = a_x(t) \exp[i(\omega t + \delta_x(t)]$$

$$E_y(t) = a_y(t) \exp[i(\omega t + \delta_y(t)] \tag{7}$$

where $a_i(t)$ and $\delta_i(t)$ denote amplitude and phase noises in the respective directions, respectively. If the light were completely polarized, the quantities would be constant see eq.(1). For a partially polarized light these quantities depend on the time, but they change only by small relative amounts in any time interval Δt that is small compared to the coherence time $1/\Delta\nu$, i.e. $\Delta t < 1/\Delta\nu$ for ν is the effective spectral width of the light.

Next, we consider the intensity of such a partially polarized light. Suppose that a phase delay

ε is given to the y-component using for example, a Babinet-Soleil compensator (BSC), and observe the intensity after the light passes through a linear polarizer oriented at θ with respect to the x-direction. Then the component of the electric vector in the θ direction can be expressed as

$$E(t;\theta,\varepsilon) = E_x\cos\theta + E_y e^{i\varepsilon}\sin\theta \tag{8}$$

Therefore, the intensity is given as

$$\begin{aligned} I(\theta,\varepsilon) &= \langle E(t;\theta,\varepsilon).E^*(t;\theta,\varepsilon) \rangle \\ &= J_{xx}\cos^2\theta + J_{yy}\sin^2\theta + J_{xy}e^{i\varepsilon}\cos\theta\sin\theta \\ &\quad + J_{yx}e^{-i\varepsilon}\sin\theta\cos\theta \end{aligned} \tag{9}$$

where * denotes complex conjugate, J_{xx}, J_{yy}, J_{xy} and J_{yx} are the elements of the following matrix

$$J = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} = \begin{bmatrix} \langle a_x^2 \rangle & \langle a_x a_y e^{i(\delta_x - \delta_y)} \rangle \\ \langle a_x a_y e^{-i(\delta_x - \delta_y)} \rangle & \langle a_y^2 \rangle \end{bmatrix} \tag{10}$$

Matrix J is called the coherency matrix. The diagonal elements J_{xx} and J_{yy} are real and represent the intensities in the x- and y- direction, respectively. The total intensity of the light is given by the sum of these elements i.e., the trace of the matrix,

$$Tr[J] = J_{xx} + J_{yy} = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle \tag{11}$$

The non-diagonal elements J_{xy} and J_{yx} are always complex-conjugate with each other.

The measurements method to determine the four elements of the coherency matrix will be discussed later in chapter 4. We shall now consider the forms of coherency matrix for completely unpolarized and completely polarized lights.

a) completely unpolarized light:

For a completely unpolarized light $I(\theta,\varepsilon)$ is independent of ε and θ. In other words

$$I(\theta,\varepsilon) = \text{constant}, \tag{12}$$

and it follows that the coherency matrix J_u of such a light is

$$J_u = \frac{1}{2} I_o \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{13}$$

where $I_o = J_{xx} + J_{yy}$ under the conditions of $J_{xy} = J_{yx} = 0$ and $J_{xx} = J_{yy}$.

b) completely polarized light:

As stated previously, for a completely polarized, the amplitudes a_x and a_y and the phase noises δ_x and δ_y in eq.(7) do not depend on the time, and coherency matrix has the form

$$J_c = \begin{bmatrix} a_x^2 & a_x a_y e^{i\delta} \\ a_x a_y e^{-i\delta} & a_y^2 \end{bmatrix} \tag{14}$$

where $\delta = \delta_x - \delta_y$.

In this case, the determinant of the coherency matrix is

$$\det[J_c] = J_{xx}J_{yy} - J_{xy}J_{yx} = 0 \tag{15}$$

We shall consider now for special cases when the light is linearly and circularly polarized.

According to eq.(3) the coherency matrices J_x and J_y represent linearly polarized light of intensity I, with the electric field vector in the x-direction and the y-direction respectively can be expressed as

$$J_x = I\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad J_y = I\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \tag{16}$$

and the coherency matrix $J_{45°}$ and $J_{135°}$ represents linearly polarized light of intensity I, with the electric vector tilted 45° and 135° with the x-direction, respectively are given by

$$J_{45°} = \frac{1}{2}I\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and } J_{135°} = \frac{1}{2}I\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{17}$$

where, in eq.(3), $a_x = a_y$, m = 0 and $a_x = a_y$, m = 1, respectively

For circularly-polarized light, the coherency matrix can be expressed as

$$J_{cir} = \frac{1}{2}I\begin{bmatrix} 1 & \pm i \\ \pm i & 1 \end{bmatrix} \tag{18}$$

where I is the intensity of the light, and $a_x = a_y$, $\delta = m\pi/2$ (m: odd integer) according to eq.(5) and (6). The upper or lower sign is for the respective right or left handed polarization.

2.2.3. Degree of polarization

Any partially polarized light can be regarded as the sum of a completely unpolarized and a completely polarized lights which are independent of each other. From this statement, any coherency matrix of a partially polarized light can be expressed uniquely as

$$J = J_u + J_c \tag{19}$$

where J_u and J_c denote the uncompletely polarized and completely polarized coherency matrices, in accordance with eq.(13) and (14), we write J_u and J_c as

$$J_u = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \quad J_c = \begin{bmatrix} B & D \\ D^* & C \end{bmatrix} \tag{20}$$

Hence, we have

$$J = \begin{bmatrix} A+B & D \\ D^* & A+C \end{bmatrix} \tag{21}$$

with $A \geq 0$, $B \geq 0$, $C \geq 0$ and $BC - DD^* = 0$. We can obtain immediately from eq.(19) and (20),

$$\begin{aligned} A + B &= J_{xx}, \quad D = J_{xy} \\ D^* &= J_{yx}, \quad A + C = J_{yy} \end{aligned} \tag{22}$$

where the J elements denote those of the given partially polarized light. Substituting from eq.(21) to $BC\text{-}DD^*=0$, we obtain the following equation for A :

$$(J_{xx} - A)(J_{yy} - A) - J_{xy}\, J_{yx} = 0 \tag{23}$$

The two roots of eq.(22) can be found as

$$A = \frac{1}{2}(J_{xx} + J_{yy}) \pm \frac{1}{2}\sqrt{(J_{xx} + J_{yy})^2 - 4\det|J|} \qquad (24)$$

From eq.(20) it follows that B and C must be positive. This condition is satisfied when we take the negative sign in front of the square root in eq.(22).

Hence, we have

$$A = \frac{1}{2}(J_{xx} + J_{yy}) - \frac{1}{2}\sqrt{(J_{xx} + J_{yy})^2 - 4\det|J|} \qquad (25)$$

$$B = \frac{1}{2}(J_{xx} - J_{yy}) - \frac{1}{2}\sqrt{(J_{xx} + J_{yy})^2 - 4\det|J|} \qquad (26)$$

$$C = \frac{1}{2}(J_{yy} - J_{xx}) - \frac{1}{2}\sqrt{(J_{xx} + J_{yy})^2 - 4\det|J|} \qquad (27)$$

$$D = J_{xy} \text{ and } D^* = J_{yx} \qquad (28)$$

These are solutions for elements of the coherency matrices in eq.(20).

According to e.q.(11) the trace of the matrix of e.q.(21) is the total intensity of a partially polarized light and it is given by

$$I_{total} = Tr[J] = 2A + B + C \qquad (29)$$

and the total intensity of its completely polarized part is

$$I_{pol} = Tr[J_c] = B + C \qquad (30)$$

The ratio of polarized component intensity to total intensity is called the degree of polarization (hereafter. DOP) P, and is given by

$$P = \frac{I_{pol}}{I_{total}} = \frac{B + C}{2A + B + C} \tag{31}$$

Hence from eq.(25) to eq.(28), P is

$$P = \sqrt{1 - \frac{4\det|J|}{(J_{xx} + J_{yy})^2}} \tag{32}$$

From eq.(32) and (25), it follows that $0 \leq P \leq 1$ the property of a light can be characterized by the degree of polarization.

(1) For a completely polarized light, i.e. there is no unpolarized component, P = 1 and hence, $\det|J| = 0$.

(2) For a completely unpolarized light, i.e. when the polarized component is absent, P = 0 and hence $(J_{xx} + J_{yy})^2 = 4\det|J|$

(3) For a partially polarized light, $0 < P < 1$. Thus, for eq.(31) the condition

$$\det|J| \leq J_{xx}J_{yy} \leq \frac{1}{4}(J_{xx} + J_{yy})^2 \tag{33}$$

always hold.

2.3. Graphical representation of the SOP of a completely polarized light

There are two methods for geometrical representation of all the different SOP's: the Poincare and the planar chart. The former method is well-known and widely used for expressing any SOP of a completletely polarized light. This will be discussed first.

2.3.1. Poincaré sphere

The geometrical representation of different SOP's by points on a sphere is proposed by Poincaré in 1982 [33], and is now well-known as the Poincare sphere. Figure 3 shows the Poincare sphere representation of polarization. Any general elliptical polarization state which is characterized by its inclination angle α and ellipticity $\psi = \pm \arctan b_s/a_s$ can be represented by a unique point C of longitude 2ψ and latitude 2α on the surface of a unit sphere. Hence, all linear polarization states lie on the equator and the poles represent the left L and right R circular polarization states. X and Y are the horizontal and vertical linear polarizations. P and Q represent the linear polarization at ±45° to the polarization mode. The remainder of the surface of the sphere represents all possible elliptical polarization states of which the state C is one.

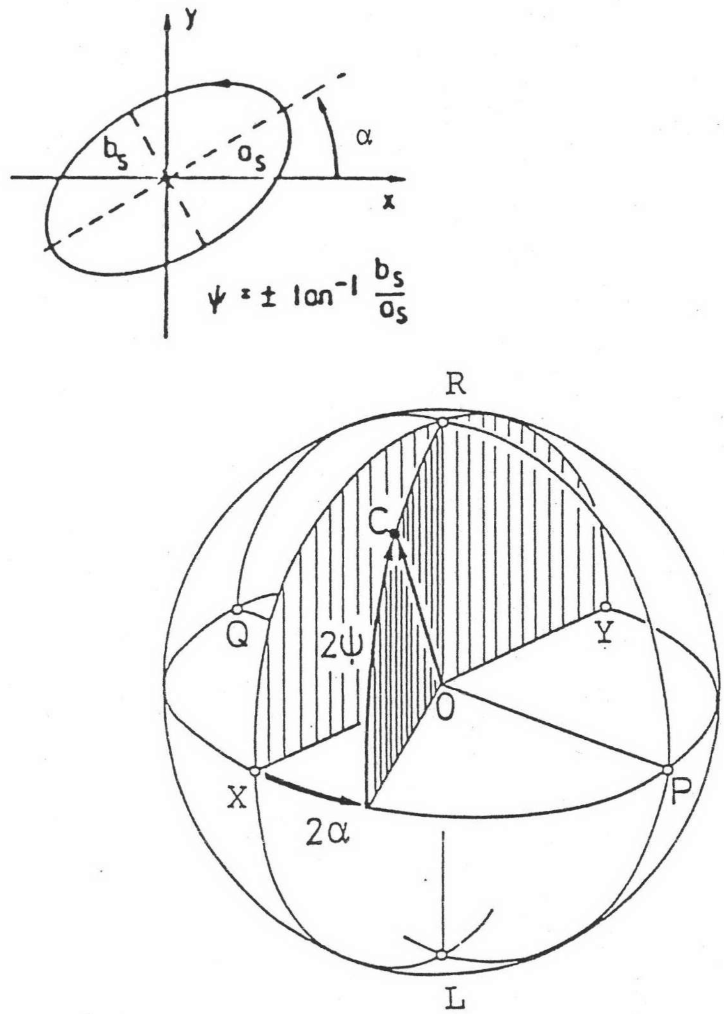


Fig. 3. The Poincaré sphere (after Ulrich and Simon [33]).

2.3.2. Planar chart

The Poincaré sphere has widely been used to express the SOP of a light. However, it has a drawback in that it can not be drawn or printed on a paper because it is curve.

A solution to this problem is to project the Poincare sphere onto a planar chart. Recently, Okoshi (1986,[34]) proposed one method using the stereoscopic projection of the Poincare sphere from a fictitious light source at the North Pole R onto a phase touching the sphere at the South Pole L; for details, see [34].

Figure 4 shows this chart drawn by a computer. In this figure, four groups of characteristic curves exist : equi R-circles, equi-ϕ circles, the concentric equi-ψ circles and radial straight lines giving equi-α contours. The application of the chart to the understanding of SOP-control device proposed in this thesis will be shown latter.

2.4. Review of theoretical analysis of degree of polarization and measurements of state-of polarization (SOP) fluctuation in single-mode optical fiber.

2.4.1. Degradation of degree of polarization in single-mode fiber

The degree of polarization (DOP) defined in eq.(31) of Subsection 2.2.3 is an important quantity

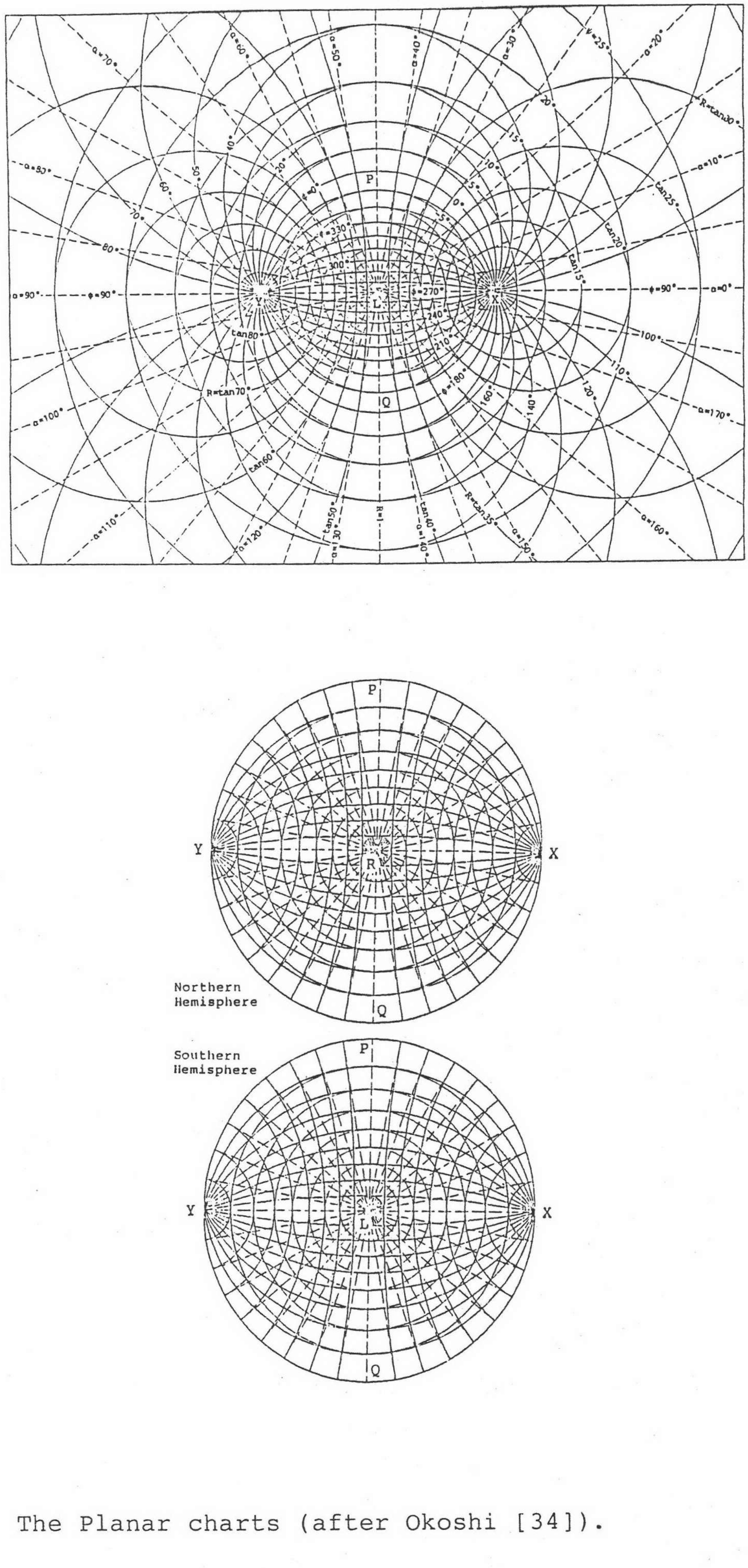


Fig. 4. The Planar charts (after Okoshi [34]).

in optical fiber application that involve interference, such as coherent optical fiber communication systems and fiber optic interferometric sensors. These systems rely on some way on the SOP of light both along the fiber and at its output. It is well-known that polarization mode dispersion known as depolarization contributes to the deterioration of DOP along the fiber [35]. Preservation of a high DOP is required in coherent optical fiber transmission line, while depolarization is desirable in certain sensors such as fiber optic gyroscope, for reducing noise in the signal [36,37]. This section reviews the theoretical analysis and measurement of polarization fluctuation in single-mode fiber that have been reported in the past years.

Monerie and Jeunhomme (1980, [38]) have shown that for uniform mode coupling, the DOP depends on the fiber birefringence, the incident condition, and is oscillating with the fiber length. Deterioration in the DOP after a long distance transmission has been shown to be recovered by a phase-compensation technique at the fiber output [39].

Sakai et al. (1982, [40]) have presented a general expression for the DOP in anisotropic single-mode fiber without mode coupling effect, as a function of the degree of coherence, associated with the fiber parameters, light source spectrum , and the input conditions. It has been pointed that any incident light at the fiber input can be splitted into two eigenpolarization modes which propagate at different group velocities with each other along the fiber. The DOP depends on the mutual correlation function of the two eigenpolarization modes, and DOP = 1 can be obtained (i.e. for completely polarized light at the fiber exit) when only one of these modes is excited at the fiber input. If two eigenpolarization modes are excited identically, the DOP will reduce to zero with increasing fiber length. However, this analysis cannot be applied to practical fibers where mode couplings between the polarization modes take place due to irregular imperfections and external disturbances along the fiber [38].

Burns et al. (1983, [41]) have shown that the DOP of broad-band light at the fiber output depends on the position of the coupling center and the coupled power for the fiber model with one discrete mode-coupling. Experimental results on the measurements of DOP as a function of fiber length for 1-km-long fiber with different incident conditions were given. The results indicated that DOP did not reduce to zero with increasing fiber length. The existence of nonzero DOP in long lengths of fiber is shown to be due to mode coupling at particular positions along the fiber.

In the same year, results on polarization tests for long fibers reported by British Telecom [42] and KDD [43] researches showed that there is still a high DOP even after a 30-50 km-long fiber transmission.

In 1984, Sakai [44] has presented another theoretical analysis for the DOP considering the mode coupling effect. He concluded that the DOP depended on the light source spectrum, the fiber polarization dispersion, the incident condition, and would approach zero value with increasing fiber length. However, from a mathematical point of view, the convergence of the successive interations is not likely to hold when the fiber is sufficiently long.

Grundinin and Sulimov [45] have also derived an expression for the DOP which is found to decrease with the decreasing source coherence time and depends on the fiber characteristics and the incident condition.

The more recent theoretical treatment has been presented by Shangyunan et al. (1986, [46]). The DOP is analized with discrete mode-coupling centers having random coupling coefficients at regular intervals. Their results differ from the results of the iteration method used by Sakai [44] in some aspects such as the DOP with random mode coupling does not reduce to zero with increasing fiber length but approaches a nonzero value the magnitude of which depends on the coupling intensity, the light source spectrum, the fiber birefringence, and is independent of incident condition.

Tian et al. (1987, [47]) has presented the first theoretical expression for the polarization in single-mode fibers with random coupling between the two eigenpolarization modes. The calculation formulae for the polarization fluctuation are given, and can be used to calculate the fluctuation noise and estimate the effect of polarization on coherent optical fiber communication systems. They concluded that the effect of polarization fluctuation on long distance coherent transmission system could be minimized, if fibers with shorter correlation lengths and small disturbances variances were chosen.

So far, the measurement of DOP as functions of the fiber properties, the fiber length, and source spectrum linewidth has not yet been reported. Such measurement will be worthwide at least to evaluate the validity of the theory.

2.4.2. Measurements of polarization fluctuation in single-mode fiber

Measurement of polarization fluctuations is important for the study of the propagation characteristics of the polarization state of light in single-mode optical fibers.

Since mechanical vibration, pressure changes, and temperature fluctuations all affect the residual birefringence, the polarization state of the light along the fiber and at its output is not temporally constant. Measurements of the polarization stability of a conventional single-mode fiber over a period of 96 hours reported by Smith et al. (1983, [48]) are shown in Fig. 5 for fiber wound on a drum and a cable fiber in underground ducts. For these measurements, it is clear that polarization changes do occur, but only over periods of minutes or hours. They have also been reported that the polarization characteristics in the optical fiber submarine under various stress conditions were stable [49-51]. Recent polarization fluctuation characteristics measured during and after the submarine cable installation [52] is shown in Fig. 6.

Since measurements of the polarization on long cable fibers have shown that the polarization state drifts slow it suggests that polarization compensation on conventional single-mode optical fiber can be performed using various SOP-conversion devices.

2.5. Summary

The mathematical expression of degree of polarization (DOP) defined as the ratio of the power of the polarized component to the total power is given. Theoretically, the deterioration in DOP depends on the light source spectrum, fiber properties, and fiber length. Mode coupling caused by various kinds of

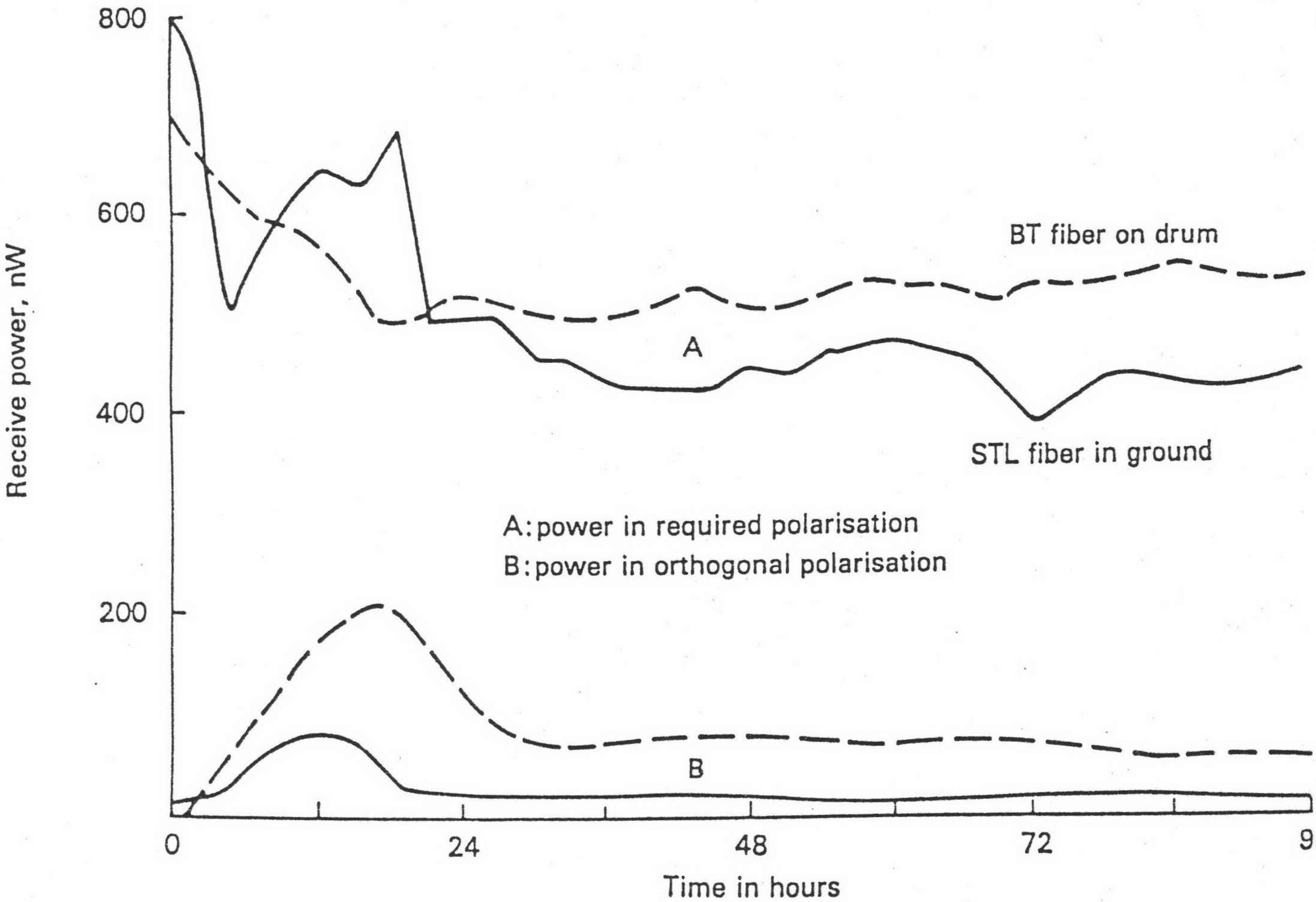
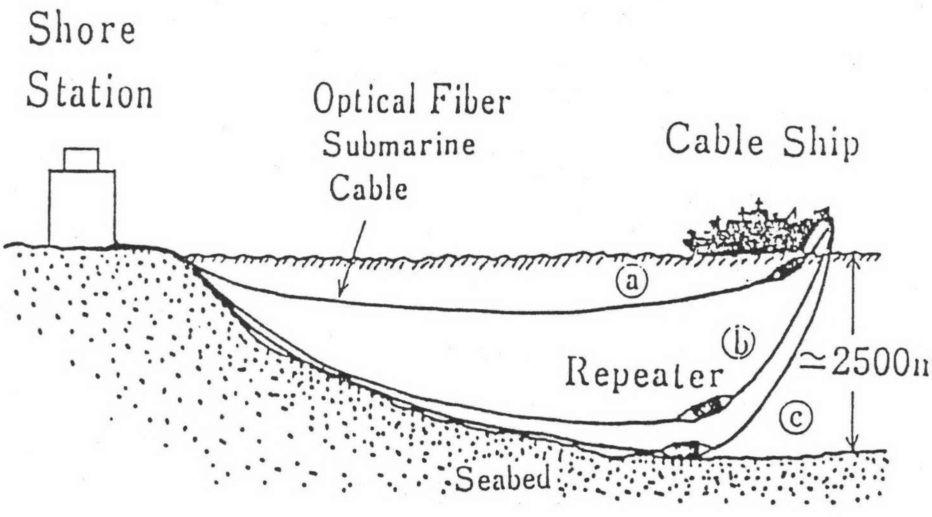
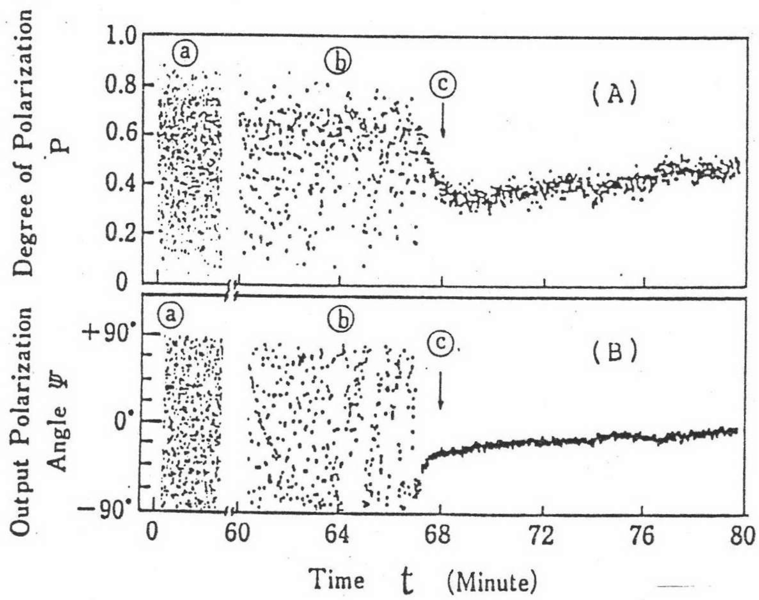


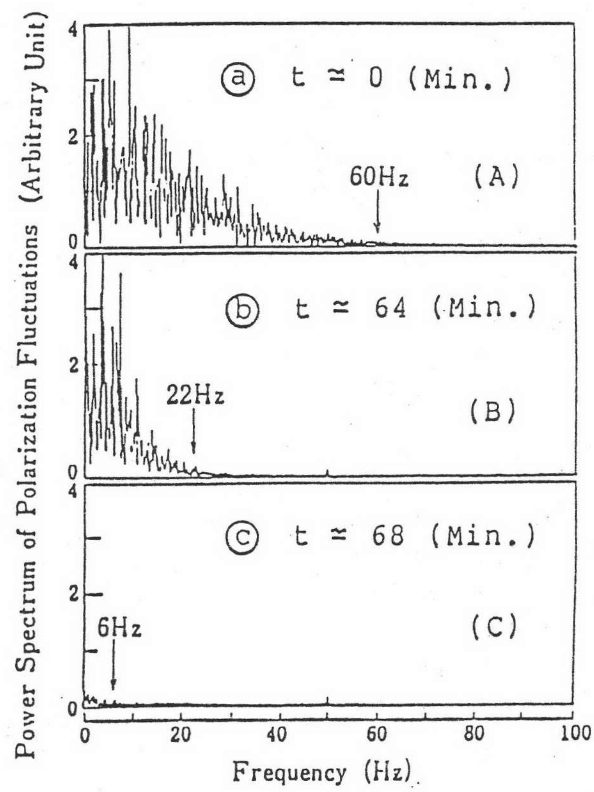
Fig. 5. Polarization stability for cabled single-mode fibers (after Smith et al.[48]).



General view of the optical fiber submarine cable installation in sea trial.



Polarization characteristics in optical fiber submarine cable installation.
(A): Degree of polarization P .
(B): Output polarization angle Ψ.



Power spectrum during and after the cable installation at states ⓐ (4-A), ⓑ (4-B), and ⓒ (4-C), respectively.

Fig. 6. Polarization fluctuation characteristics of optical fiber submarine cable during and after installation (after Namihira et al.[52]).

imperfection in the fiber and external disturbances along the fiber contributes to the polarization fluctuations. It has been shown experimentally that there is still a high DOP even after along fiber (>30 km) transmission, and polarization-state drifts slowly. In addition, polarization fluctuations in installed fiber cables have also been measured and found to be quite stable under the static stress conditions.