AN APPLICATION OF LINE OF BALANCE AND BUILDING INFORMATION MODELING FOR OPTIMAL RESOURCE AND SCHEDULE: A CASE STUDY OF AN ELEVATED HIGHWAY CONSTRUCTION

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การประยุกต์ใช้ LINE OF BALANCE และแบบจำลองสารสนเทศอาคาร (BIM) เพื่อแผนงานและทรัพยากรที่เหมาะสม: กรณีศึกษาการก่อสร้างทางยกระดับ

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การวางแผนงานคือ กระบวนการสำคัญในการก่อสร้างเพื่อให้โครงการสำเร็จลุล่วงภายใต้เวลา และต้นทุนที่มีอยู่อย่างจำกัด สำหรับโครงการประเภทโครงสร้างพื้นฐานถนนและทางยกระดับซึ่งลักษณะเป็นโครงการที่มีขนาดใหญ่และมีความซับซ้อนในการบริหารจัดการ กระบวนการวางแผนในปัจจุบันนั้นยังอาศัยความสามารถของผู้วางแผนงานเป็นหลักซึ่งอาจส่งผลให้เกิดความล่าช้าและความคลาดเคลื่อน อีกทั้งวิธีการนำเสนอแผนงานการดำเนินงานก่อสร้างที่มีอยู่ยังไม่สามารถแสดงภาพรวมของโครงการให้ครอบคลุมในมิติต่างๆ งานวิจัยนี้จึงนำเสนอ การประยุกต์ใช้ Line of Balance และแบบจำลองสารสนเทศอาคาร (BIM) เพื่อความเหมาะสมของแผนงานและทรัพยากร ซึ่งนำไปสู่การพัฒนาระบบวางแผนก่อสร้าง Line of Balance จากแบบจำลองสารสนเทศอาคาร ที่เรียกว่า BIM-LOB-SS (BIM-based Line of Balance Scheduling System) โดยเริ่มจากการคิดค้นตัวแบบการหาค่าเหมาะสมที่สุด (Optimization model) เพื่อหาทรัพยากรที่เหมาะสม สำหรับโครงการที่มีการก่อสร้างซ้ำกันของโครงสร้างที่คล้ายกันหลายประเภทในตำแหน่งที่แตกต่างกัน (Multi-identical types of units) ที่พบได้ในการก่อสร้างทางยกระดับนั้น ตัวแบบการหาค่าเหมาะสมที่สุดมีฟังก์ชันวัตถุประสงค์คือ การหาต้นทุนของทรัพยากรที่ต่ำที่สุดซึ่งสามารถทำให้โครงการแล้วเสร็จภายในระยะเวลาที่กำหนดและทรัพยากรถูกใช้งานอย่างต่อเนื่อง ในการพิสูจน์กรอบแนวคิดของตัวแบบการหาค่าเหมาะสมและตัวสร้างแผนงานนั้น งานวิจัยได้ตรวจสอบความถูกต้องกับตัวอย่างซึ่งมีคำตอบที่ชัดเจน 3 ตัวอย่าง ซึ่งสามารถหาคำตอบที่เหมาะสมได้อย่างถูกต้องรวดเร็ว ในลำดับต่อมางานวิจัยได้ประยุกต์ใช้แบบจำลองสารสนเทศอาคาร (BIM) เพื่อลดการป้อนข้อมูลโดยผู้วางแผนซึ่งช่วยให้การป้อนข้อมูลนั้นมีความถูกต้องและรวดเร็ว แบบจำลองยังถูกใช้เพื่อนำเสนอภาพรวมของโครงการด้วยแบบจำลองเวลาการดำเนินงานของโครงการ (4D construction simulation) อีกด้วย ในส่วนสุดท้ายงานวิจัยได้ประเมินประสิทธิภาพของระบบวางแผนก่อสร้างกับข้อมูลตัวอย่างของโครงการก่อสร้างทางยกระดับ พบว่าระบบนั้นสามารถเป็นเครื่องมือบริหารจัดการโครงการ ซึ่งผู้วางแผนสามารถใช้เพื่อประกอบการตัดสินใจในการดำเนินงาน

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Project scheduling is an essential tool to support construction operations completing the project under limited time and cost. The linear infrastructure project such as elevated highway construction involves a large scale of working area and complexity of management. Recently, schedules are planned manually based on planners' experience and intuition which may consume time and lead to human-errors. Moreover, the presentations of schedule by using the existing methods dose not cover the overview of projects in various aspects. The objective of this research is to establish an application of Line of Balance (LOB) and Building Information Modeling (BIM) for optimal resource and schedule, which is called BIM-LOB-SS (BIM-based Line of Balance Scheduling System). The development of the system begins with the creation of an optimization model for the construction of several types of repetitive structures located in different locations (Multi-identical types of units). For the verification, three example projects with known solutions are employed. The model can compute the optimal solution correctly with a short time. The utilization of the BIM model is presented in order to reduce massive input by using the BIM information and to improve visualization of the project operation with 4D construction simulation. Finally, the proposed scheduling system is demonstrated with the example information from a section of elevated highway construction in Thailand. As a result, this research proposes a new management tool that can support the decision making of the manager in project operations.

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Chapter 1
Introduction

1.1 Problem background

The linear infrastructure projects such as water pipelines, highway, railway, and elevated way primarily comprise repetitive activities in sections or units. The same resources perform the typical tasks in various units (locations, sections) by moving from one unit to the next unit. Because of the characteristics, proper management of the resources is necessary to achieve the projects efficiently.

The construction schedule is an essential communication tool in construction projects. It illustrates work associated with delivering the project on time, what work needs to be performed, and which resources of the organization operate the tasks. Without a complete schedule, the project manager cannot efficiently manage the work in the project. Generally, experienced planners use scheduling software such as Microsoft Project (MP), Primavera P6, and TILOS depending on the project's characteristics to create a construction schedule. The software has been developed based on several scheduling techniques such as network scheduling technique, Linear Scheduling Method (LSM), and Line of Balance (LOB) to facilitate the planners in the schedule creation. In the construction industry, network-based methods such as the Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) have been proven to be generic scheduling and progress control tools. However, they are not suitable for projects of a repetitive nature because the techniques focus on minimizing project duration rather than dealing with time/space conflicts and work continuity. Repetitive activities in linear infrastructure projects generally consist of different production rates. This phenomenon of production rate for scheduling repetitive projects with network-based methods has negative impacts to project performance by causing work interruption, inefficient utilization of resources, and complicating display with large network diagram (Arditi and Albulak, 1986; Arditi et al., 2002; Chrzanowski and Johnston, 1986; Yamin and Harmelink, 2001).

On the other hand, specific scheduling methods, such as LSM and LOB, have shown their potential in linear repetitive projects. Both methods emphasize the continuity of resource utilization rather than minimizing project duration and provide production rate and duration information in the form of an easily interpreted graphical format. Maintaining resource utilization continuity leads to minimizing the idle time of the resource and maximizing learning curve effect. LSM and LOB allow a better grasp of a project composed of repetitive activities compared to any other scheduling techniques because activities' rates of production can be adjusted to the smooth and efficient flow of resources (Arditi et al., 2002). However, the limitations of LSM and LOB still exist. These techniques are time-consuming for the user, rely heavily on the user's intuition and massive trial-and-error to acquire an optimal schedule. Moreover, if the trial-and-error process is not too exhaustive, the solution may be far from optimum (Srisuwanrat et al., 2008);(Leu and Hwang, 2001); (Lutz, 1993).

Thus, to deal with the limitations of LOB/LSM and complicated linear repetitive scheduling problems involving resource utilization continuity and resource optimization, optimization models with computer techniques for repetitive projects have been developed. For example, Hegazy and Wassef (2001) proposed a generalized CPM/LOB method to determine the minimum total project cost of non-serial linear construction projects by using the Genetic Algorithm (GA). Hyari and El-Rayes (2006) utilized the Genetic Algorithm (GA) for repetitive project scheduling by presenting a multi-objective optimization model to minimize project duration and maximize resource utilization continuity concurrently. S.-S. Liu and Wang (2012) presented an optimization model using Constraint Programming (CP) to minimize the project duration and considered the concept of optimizing single-skilled crew and multi-skilled crew in different tasks. Damci et al. (2013b) proposed an algorithm for resource-leveling by applying Line of Balance and genetic algorithm. The algorithm considered a single resource type performing in all activities. The previous studies tried to minimize the sum of the absolute deviations of daily resource requirement from average resource usage that implied the resource optimization in the schedule.

However, the studies hold an assumption that units in the project are only one identical type while problems of projects that contain multi-identical types of units have not been covered. For example, when a project has many repetitive units which they are classified into several types based on resource requirements of each type. To illustrate, in an elevated highway construction project, the project is comprised of many piers (units). The piers in the project may be designed in different three types including type P1, type P2, and type P3. Each type of pier is designed for different heights suitable for the terrain of the elevated highway at different locations in the project to serve the elevation and alignment. Normally, one type of pier contains structural elements such as foundation, column, and viaduct segment. To construct each structural element, specific resources are required. Specific resources of the structural elements are generally provided to serve specific structural element such as a fabricated formwork for unique geometry. The formwork is specifically designed for certain types of structure which may not be used for other types in the project. Moreover, the viaduct of the elevated highway requires the span-by-span method which performs the viaduct erection for every pier of the project. The span-by-span method erects the viaduct on the completely built piers with one direction from the first pier to the last pier. With the condition of the type of pier, location of pier, resource requirement of each type, the individual cost per unit of resource, and the direction of erection, it causes the complex scheduling problem to contractors that how many formworks for each type they should prepare in order to complete the project on time with the lowest total cost of specific resources. This problem is different from previous research. The previous studies mainly consider that resources can perform similar tasks for all units of the project and only quantity of work are different such as backhoes for excavation or workers for concrete work. The influence of locations for the project operation is not in consideration. Moreover, the models by previous studies require massive input from users, which may cause human errors.

In order to reduce massive input assignment, Building Information Modeling (BIM) is described as a digital representation of the physical and functional characteristics of a facility. BIM has been regarded as a potential solution to the challenges within the Architecture, Engineering, and Construction (AEC) industry. BIM facilitates information exchanges and interoperability between software applications during the project life cycle. BIM model stores all the information of the buildings or infrastructures. The information lays the foundation by which the BIM tools perform a variety of analyses, such as structural analysis, cost analysis, and schedule planning analysis (NIBS, 2015). BIM project management software including Autodesk Navisworks and VISCO provide the integrated platforms linking between BIM applications and project visualization. The software can be used to perform clash detections, quantity take-off, 4D simulation, and other useful features. However, scheduling features in Autodesk Navisworks and VICO still rely on the manual operation (Kim et al., 2013).

A few studies have attempted to use information stored in either 3D CAD models or BIM for processes related to schedule systems. For example, Vries and Harink (2007) proposed a unique algorithm to generate construction schedules at the building component level from a 3D CAD model. This algorithm creates construction orders by topology/geometry of building components. Kim et al. (2013) established a prototype for the generation of construction schedules using open BIM technology. Their work primarily involved in automating data extraction from a BIM file stored in an industry foundation classes (IFC) format and parsing building information as the inputs for scheduling. As a result of the studies conducting and exploring the process of BIM-based schedule generation by automatic approach, the nature of building projects has been significantly considered in several works whereas the conditions of linear infrastructure projects are rarely investigated.

1.2 Problem statement

An efficient schedule is an important communication tool reflecting all the tasks required to deliver the project on time, achieve high profit, and fulfill the specifications. In order to create an efficient construction schedule, the planner has to consider several factors in the construction project such as activity duration, cost, resources, machines, workers, and etc. These factors have direct and indirect interrelationships and adjustment of them can affect the project duration and cost. For linear infrastructure projects, repetitive scheduling techniques such as Line of Balance and Linear Scheduling Method have been proven by many scholars that they are suitable for the characteristic of linear repetitive projects. However, the techniques require massive manual processes to accomplish an acceptable schedule and do not guarantee the exact optimal solution.

On the other hand, the optimization models for repetitive projects proposed by the previous studies were capable of solving optimization problems related to project scheduling. Nevertheless, the conditions of multi-identical types of units existing in elevated highway/railway construction projects were not covered. Moreover, the optimization models needed substantial manual input which may cause human-error affecting the optimal solution.

To eliminate the manual process, schedule systems by utilizing 3D/BIM models to generate construction schedules were developed. Information stored in 3D/BIM models was utilized as the input. The systems could reduce the manual process and provided more convenient approaches to create efficient schedules. Nevertheless, the previous studies mainly implemented the system for building construction while linear infrastructure project was rarely investigated.

This study proposes a BIM-based Line of Balance Scheduling System (BIM-LOB-SS) for elevated highway/railway construction projects. A conceptual framework of the application of Line of Balance is first developed to invent an optimizing and scheduling process that provides the optimal solution and the generated schedule for the problem of the multi-identical types of units. An application of Building Information Modeling is subsequently developed to utilize the BIM model in both as the input for the system and for the creation of 4D construction simulation. The proposed scheduling system aims to provide planners retrieving an information management tool with preliminary solutions for planning the construction project. The outcomes of the system include Line of Balance diagram, Bar chart, 4D construction simulation, and optimal solution.

1.3 Research objective

1) To propose an application of Line of Balance scheduling and Building Information Modeling for optimal resource and schedule.

2) To develop an optimization model by an application of Line of Balance scheduling technique for the condition of the multi-identical types of units.

3) To invent a BIM-based Line of Balance Scheduling System (BIM-LOB-SS) for elevated highway/railway construction projects by a combination of an optimization model and an application of Building Information Modeling.

1.4 Scope of research

This study has selected scheduling problems of an elevated highway construction project as a case study. The elevated highway project contains large numbers of repetitive activities and resource utilization.

The scopes of the study are listed as the following issues:

1) This study considers the conditions of an elevated highway project that employs a span by span method for viaduct spans construction.
2) This study focuses on the activities in the project at the structural element level. The Line of Balance concept is used to schedule the activities of these elements such as footing, column, and segment erection. The last activity to be scheduled is the segment erection where the launching gantry installs the segment on the first span and move to the next span continuously.
3) This study examines the fixed cost per unit of resource. The resource is a one-time investment and is used repeatedly throughout its life cycle. Thus, the resources that cost depend on times such as workers or rental equipment are not considered.
4) This study focuses on the optimization model and the scheduling system.
5) This study develops the scheduling system by the incorporation of several programs. The following programs are used to create the proposed system
    - Autodesk Revit 2018
    - Matlab 2018
    - Jupyter in Anaconda Navigator
    - Microsoft project
    - Autodesk Naviswork 2018

1.5 Research methodology

1) Study the use of BIM information and related BIM software in term of schedule creation for the development of BIM model that which information of the project should be attached to the BIM model in order to develop the proposed system.

2) Investigate the elevated highway construction project to understand scheduling problems, conditions, and practical operations of the project,
   - Interview with the project manager about the scheduling problems
   - Study the behavior of resource utilization
   - Study drawings and construction schedules of the project
   - Gather relevant information for the development of BIM model

3) Analyze the scheduling problems and identify the conditions of the case study to develop the proposed automatic schedule system

4) Develop the proposed scheduling system,
   - Develop an optimization model for multi-identical types of units by applying a repetitive scheduling technique
   - Verify the optimization model to guarantee its capability
   - Develop a schedule generator for generating the start and finish times of the optimal solution
   - Verify the schedule generator to assure its performance
   - Develop BIM model based on the information from the case study
   - Invent BIM information transformer which transforms the BIM information and parse to the input of the optimization model

5) Validate the proposed scheduling system with the case study

6) Summarize and present the application of LOB and BIM

1.6 Expected benefits of research

1) The application presents an alternative method of project duration calculation for linear repetitive projects

2) The optimization model facilitates the planner to deal with resource optimization for linear scheduling problems.

3) The preliminary schedule and the optimal solution assist managers in decision making.

4) The application of LOB and BIM addresses a prototype of a BIM-based scheduling system called BIM-LOB-SS for the elevated highway/railway construction projects.

Chapter 2
Literature review

This chapter focuses on review of relevant literature and textbooks including 1) Elevated highway/railway construction projects, 2) Scheduling of the linear repetitive projects, 3) Optimization models, 4) Building Information modeling (BIM), 5) Automatic schedule system, and 6) Summary.

2.1 Elevated highway/railway construction projects

2.1.1 Introduction of elevated highway construction projects

Highway/railway construction projects involve a complex combination of at grade, bridges, tunnels, elevated ways. An elevated highway/railway construction project is a controlled-access highway/railway that is raised above grade for its entire length. In highway/railway transportation systems, the term “elevated way” is a structure that crosses over a body of water, traffic, or other obstruction, permitting the smooth and safe passage of vehicles. The construction of elevated highway/railway has a strong positive impact on the economic and social development of the areas to be served. The reliability of the connection is the major factor affecting the decision to invest in elevated highway/railway. A major elevated highway/railway represents a significant investment, and considerable social-political involvement is necessary for deciding on the construction of an elevated highway/railway. An elevated highway/railway is more expensive to build than at-grade, and are usually only used where there is some combination of the following issues on the desired route (Hart, 2007).

Figure 2.1 An elevated highway construction project

1) Difficulty of controlling access at-grade, for example, where it would be very disruptive or expensive to eliminate existing crossings at grade,

2) Unable to reach optimal traffic flow due to hilly terrain or existing crossings road,

3) Hills that are costly to level or crave a path through,

4) A safety issue at grade, for example, where there are many pedestrians or wildlife is concerned. Budget or time to eliminate impeding structures is high due to acquisition costs, demolition costs, or environmental factors,

5) Right of way through an urban area, where private property would have to be purchased or condemned and might have to be litigated,

Normally, elevated highway/railway structure is a combination of two components: substructure and superstructure. For bridges with bearings, all the elements which transfer the loads from bearing to the ground are called substructures. The substructure consists of piles, foundation, columns, and crossbeam. The superstructure consists of decks, girders, and viaducts (Ostenfeld et al., 2000).

2.1.2 Construction of viaduct with span by span method

Recently, the construction of viaducts with span by span method is the most economical and rapid method of construction available for long bridges and viaducts with individual spans. There are several systems that are recognized as the span-by-span method in the construction industry.

1) Span-by-span casting with Movable Scaffolding Systems (MSS)

Movable Scaffold System (MSS) has been developed for multi-span bridges over difficult terrain or water where scaffolding would be expensive or simply not feasible. A launching girder moves forward on the bridge piers, span-by-span to allow the placing of the cast-in-situ concrete. The traveling gantry system is most suited for spans of 30 m. to 60 m. (Zoli, 2012).

Figure 2.2 Movable Scaffold System (MSS)

2) Span-by-span erection with launching gantry (LG)

Launching gantry system is the recognized technology for the construction of modern bridges with a tight horizontal radius and has become one of the most widely used for the construction of precast segmental bridges. Pre-cast segment erection with launching gantry offers a very high speed of construction. It is most often used in conjunction with an erection truss overhead erection gantry to guide the precast elements into position. The most common use of launching gantry system is to build long viaducts with spans of similar length. The method has been used most often for spans ranging from 25 m. to 45 m. (Rosignoli, 2016).

Figure 2.3 Pre-cast segment erection with launching gantry (LG)

3) Span-by-span full span precast system

Alternatively, full-span precast beams can be delivered from the precast beam production to the erection front by a launching gantry. This system allows for a fast rate of erection. The full span pre-cast system of erection is suited for specific structures comprising of multiple spans of similar lengths and minimum curvature. With this method, the entire bridge spans of a viaduct are produced in a costing yard located at the beginning of the viaduct. The pre-cast spans are then transported along the pre-made part of the bridge structure to its destination without disruption the existing road traffic (Rosignoli, 2016).

Figure 2.4 Full Span Precast Method (FSPM)

These methods are significantly well-established construction methods that offer many benefits on suitable projects. The advantages include minor site disruption and easy maintenance of highway and railway traffic at the erection site. However, these systems are still restricted. First, the systems must perform span by span straightly from the current unit to the next unit. Second, assemblies of the systems consume significant time and contribute to high cost due to their massive sizes. Hence, skipping operation is rarely assigned in practical execution. These conditions limit the production rate of segment erection to be improved by adding more equipment. Thus, the delivery rate of the system is controlled by a single launching gantry performance. The preceding substructure must be complete in advance to allow the smoothest construction process of the viaduct.

Figure 2.5 Straight-line performance of launching gantry

2.1.3 Construction of pier and substructure

An elevated highway/railway commonly comprises a massive number of piers and spans along the horizon alignment. Pier is used to present any substructure that supports the spans of the viaduct and transfers the loads from the viaducts to the ground. Pier's elements generally consist of piles, foundation, column, and crossbeam, depending on the design of the project. The piers are usually designed to be several types. A type represents characters of piers. Each type of piers has a specific design with covering height, specific geometric structure, and unique appearance. The designers technically consider several factors such as elevation from the ground, alignment accuracy, constructability on terrain, dead load, live load, facilities, and utilization, etc. Thus, elevated highway/railway projects containing serval types of units are then called as **multi-identical types of units**.

Figure 2.6 Piers in an elevated railway construction project

2.2 Scheduling of the linear repetitive projects

Linear construction projects such as water pipelines and high-rise buildings primarily comprise repetitive activities in sections or units, and the same operations are repeated within each unit. Due to the characteristics of linear projects, the same resource generally executes each similar activity from one end of the unit to the other. Therefore, it creates a critical need for a construction schedule that facilitates the uninterrupted flow of resource (i.e., work crews)(Harris and Ioannou, 1998; S.-S. Liu and Wang, 2012).

Project scheduling is a mechanism to communicate what tasks need to be done and which resources will be allocated to complete those tasks in what timeframe. A project schedule is a document collecting all the work required to deliver the project on time. In addition to assigning dates to project activities, project scheduling is intended to match the resources of equipment, materials, and labor with project work tasks over time. Traditional scheduling methods were developed by many scholars to deal with several characteristics of construction projects (Daniel W. et al., 2017; Yamin and Harmelink, 2001).

Table 2.1 Recommended scheduling methods for different types of projects (Yamin and Harmelink, 2001)

Type of Project	Scheduling Method	Main Characteristice
Linear Project (Pipe lines, Railways tunnels, highways)	Linear Scheduling Method (LSM)	Few activities Excuted along a linear path/space Hard sequece logic Work continuity crucial for effective performance
Multi-unit repetitive project (housing, building)	Line of Balance (LOB)	Final product a group of similar units Same activities during all projects Balance between different activties achieved to reach objective production
Hugh-rise buildings	Line of Balance (LOB) Verical Production Method (VPM)	Repetitive activties Hard logic for some activities, soft for others Large amount of activties Every floor considered a production unit
Refineries and other very Complex projects	Program Evaluation and Review Technique (PERT) Critical Path Method (CPM)	Extremely large number of activities Complex design Activities discrete in nature Crucial to keep project in critical path
Simple projecrs (of any kind)	Bar chart	Indicates only time dimension (when tostart and end activties) Reively few activties

2.2.1 Network scheduling: Critical Path Method (CPM)

Several scholars, researchers, and authors have proven and listed the incapability of CPM dealing with linear project, repetitive project, and linear repetitive project by following these aspects:

1) Difficulties in the visualization of a large network of repetitive activities

2) Difficulties in using multiple crews in a large network of repetitive activities

3) The focus on minimizing project duration rather than dealing with time/space conflicts and resource constraints

4) Not clearly showing activities' rates of progress to the units to be produced

5) Ambiguity in the continuity of repetitive activities that may create idle times for crews

2.2.2 Linear Scheduling Method (LSM)

Linear scheduling method is a graphical scheduling method that focuses on continuous resource utilization in repetitive activities. It was first applied in a highway construction project (Chrzanowski and Johnston, 1986) (Johnston, 1981). LSM is used mainly in the construction industry to schedule resources in repetitive activities commonly found in highway, pipeline, high-rise building, and rail construction projects. These projects are called repetitive or linear projects. The main advantages of LSM over Critical Path Method (CPM) is its underlying idea of keeping resources continuously at work. In other words, it schedules activities in the following ways: (1) resource utilization is maximized, (2) interruption in the on-going process is minimized, including hiring-and-firing, and (3) the effect of the learning curve phenomenon is maximized. The basic format for the presentation of the linear scheduling method is illustrated in the below figure. One axis of scheduling diagram represents time while the perpendicular axis represents location or station along the length of the project.

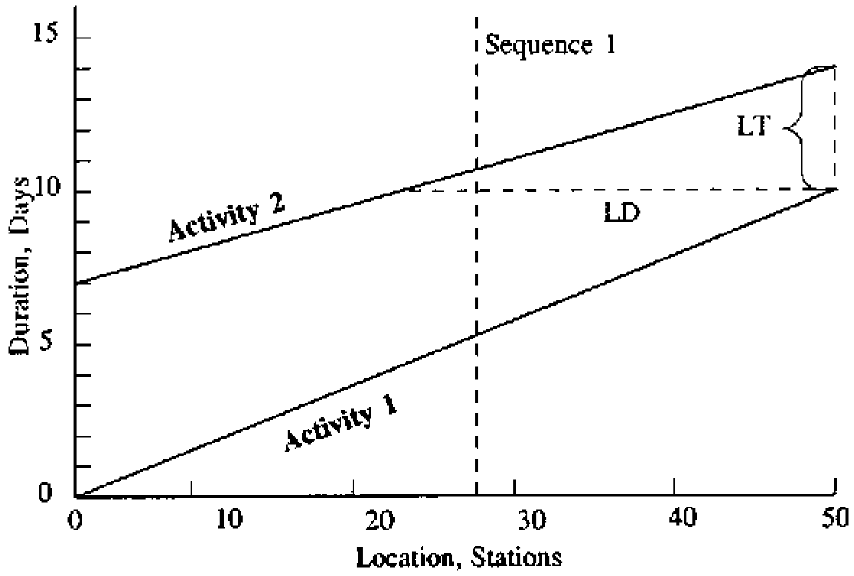


Figure 2.7 The basic format of the linear scheduling method (Chrzanowski and Johnston, 1986)

2.2.3 Line of Balance (LOB)

The origin of the Line of Balance method came from manufacturing and was improved by the U.S Navy Department in 1942 for scheduling and controlling of repetitive projects. Then, the method was strengthened to deal with repetitive housing project by the National Building Agency (in the UK). The method was officially published as the Line of Balance method by Lumsden in 1968. LOB is oriented toward the required delivery of completed units and is based on knowledge of how many units must be completed on any day so that the programmed delivery of units can be achieved. Once a target rate of delivery has been established for the project, the rate of production of each activity is expected not to be less than this target rate of delivery (Lumsden, 1968),(Arditi and Albulak, 1986).

The objectives of LOB are to ensure that:

1) A programmed rate of completed units is met.
2) A constant rate of repetitive work is maintained.
3) Labor and plant move through the project in a continuous manner such that a balance labor force is maintained and kept fully employed.
4) The cost benefits of repetitive working are achieved.

To meet these objectives, a network diagram for one of the many units to be produced is prepared as a 1st step. Then, the man-hours necessary, as well as the optimum crew sizes are estimated for each activity. The optimum rate of output that a crew of optimum size will be able to produce is called the ''**natural rhythm**'' of the activity. Any rate of production that differs from a multiple of the natural rhythm is bound to yield some idle time for labor and equipment. The target rate of delivery in a project is expressed in terms of the number of units to be completed per each time period (e.g., units/day, units/week, units/month, and so on). The target rate of delivery is the slope of the Line of Balance joining the start times of the repetitive activity in each unit and is calculated as shown in Eq. 1 (Arditi and Albulak, 1986).

$$m = \frac{(Q_b - Q_a)}{(t_b - t_a)} \; where \quad a < b \ldots\ldots\ldots Eq.1$$

m = Rate of delivery

Q_a, Q_b = Quantity of complete units at a^{th} and b^{th} unit

t_a, t_b = Finish time at a^{th} and b^{th} unit

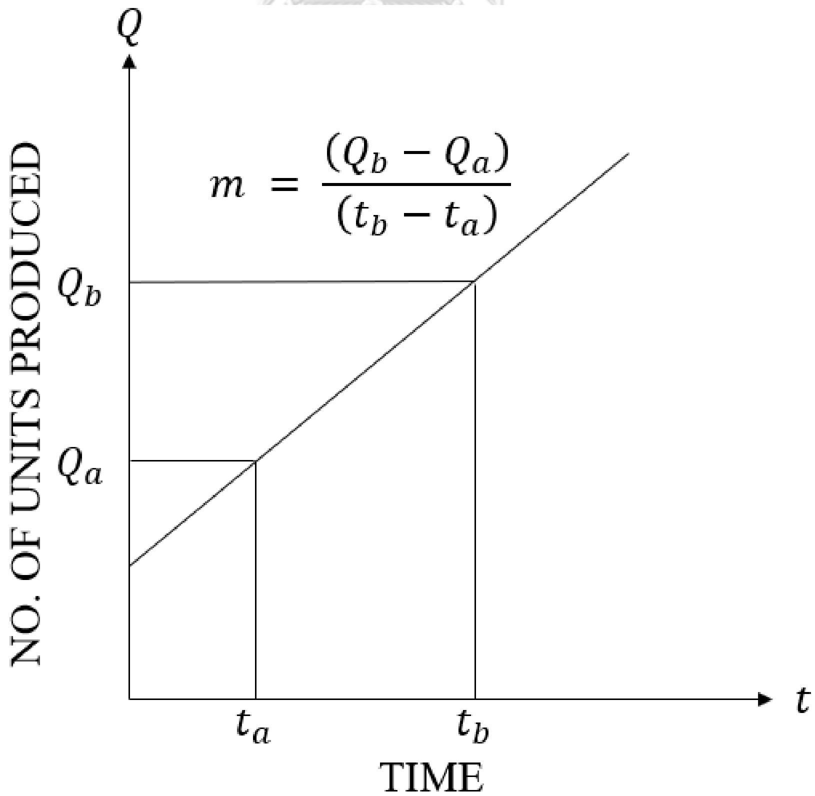


Figure 2.8 Relationship between LOB quantities and time (Damci et al., 2013a)

The slope of the line of balance joining the finish times of the repetitive activity in each unit is denoted as m. If the duration of the activity is known and if the actual rate of output is limited to a multiple of the natural rhythm, then the Eq. 1 is effectively reduced to

$$m = \frac{R}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.2$$

m = Rate of delivery

R = Number of resources used in the activity (i.g. crews, equipment)

D = Duration of the activity

The principle of natural rhythm is an essential part of the LOB technique (Lumsden, 1968). The principle of "natural rhythm" allows shifting of the start times of repetitive activities forward or backward with at different units by changing the number of crews of the activity. This procedure can be implemented only if it does not violate the precedence relationships between activities.

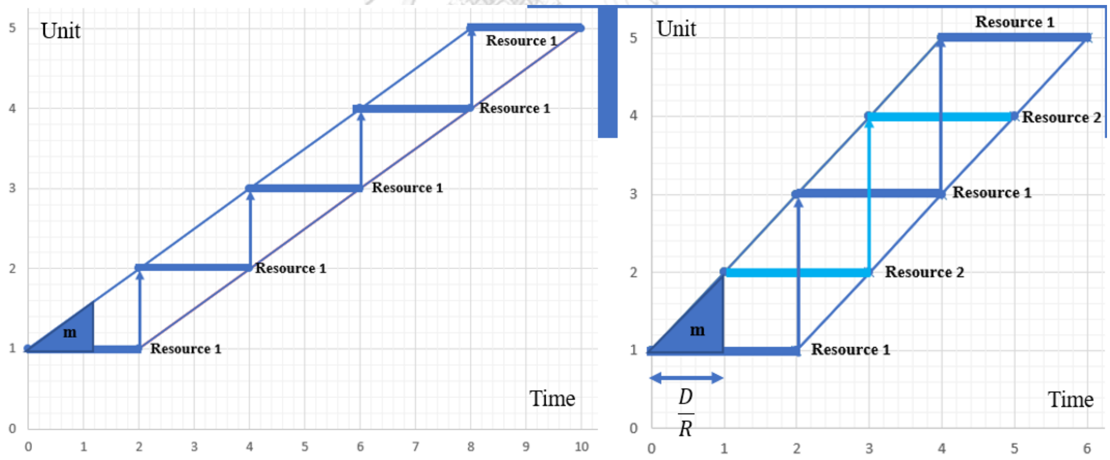


Figure 2.9 LOB diagram for activity with different numbers of resources

LOB diagram is drawn in a system of coordinates where the x-axis shows time and the y-axis shows the number of units to be produced. Every activity is represented by two oblique and parallel lines, whose slope is calculated according to Eq.1 or Eq.2. These two oblique and parallel lines denote the start and finish times, respectively, of activity at each unit. In this study, the slope of these two oblique and parallel lines is called a start-to-start (or finish-to-finish) delivery rate (Arditi et al., 2002).

LOB diagram is created by following these steps:

1) Create a sequence logic of one unit.

2) Provide the number of resources for activity to calculate the delivery rate.

3) Start plotting two oblique and parallel lines of the first activity, and their slope is equal to the delivery rate output.

4) Draw the successive activity by considering control points at top and bottom. If the delivery rate of the successive activity is higher, the control point is located at the top of the preceding line as the 2nd activity in Fig 2.10 Else the control point is located at the bottom as 3rd, 4th activity in Fig 2.10

5) repeat the 4th process until all activities are scheduled. Project duration is illustrated on the finish date of the last activity of the last unit is done as the 4th activity.

These are the general procedures to create LOB diagram.

To find optimum resources in the LOB procedures, trial-and-error by changing the number of resources is the general approach. The delivery rate of all activities gets adjusted one by one until the overall result reaches a satisfying solution. The delivery rates are basically changed by raising or reducing the number of resources (crew). This approach consumes the human labor force and time to complete. It may cause human-error, and the optimum solution is not generally guaranteed.

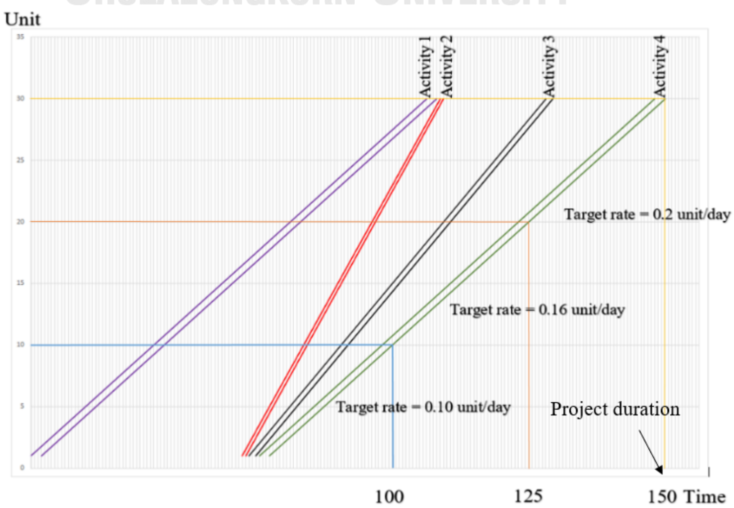


Figure 2.10 LOB diagram (Arditi et al., 2002)

A manual approach is proposed to overcome this problem, where all activities start out using one crew and therefore operate with a rate of production equal to their natural rhythm. The project duration obtained after performing the LOB analysis then compared with the contract duration. If the LOB duration for the project is equal to or less than the contract duration, there is no problem. But if the LOB duration for the project turns out to be greater than the contract duration, which is the most likely outcome, then the rates of production of certain activities are increased in a given order of priority, based on resource availability and utility costs (Arditi et al., 2002).

2.2.4 Scheduling software

During the years, scheduling software has been developed to facilitate users in the AEC industry. There are several capable scheduling applications and excellent management software which commercially exist.

2.2.4.1 Network scheduling-based software

Network scheduling-based software is widely used around the world. Scheduling techniques such as CPM, Bar chart, and PERT are used to develop commercial management software like Microsoft Project and Primavera P6. They are designed to assist a project manager in developing a plan, assigning resources to tasks, tracking progress, managing the budget, and analyzing workloads. Although, the software contains luminous features that aid users to accomplish the whole project plan conveniently even though the applications are originated based on network scheduling theories. However, the software is still limited and lacks optimization features.

2.2.4.2 Linear scheduling-based software

The linear project management software (ex. TILOS) is powerful for road, pipeline, transmission line, railway, tunnel, and other linear infrastructure projects. The linear scheduling-based software presents a graphical link between the location where the work is performed (the distance axis) and the time when it is executed (the time axis). Time-distance diagrams clarify the scope by showing the project details and the schedule in one view. The software shows that for linear and repetitive projects, TILOS strongly predominates MS project & Primavera. Nevertheless, to generate the project schedule in TILOS, it still requires manual creation by users. And in terms of searching optimal resource schedules, the software needs the trial-and-error process by adjusting production rates to reach optimal solutions (G V and Shankar, 2015).

2.3 Optimization model

Mathematical optimization is the branch of computational science that seeks to answer the question `What is best?' for problems in which the quality of any solution can be expressed as a numerical value. Such problems arise in all areas of business, physical, chemical and biological sciences, engineering, architecture, economics, and management. Optimization models are used extensively in almost all areas of decision-making such as engineering design, and financial portfolio selection. If the mathematical model is a valid representation of the performance of the system, as shown by applying the appropriate analytical techniques, then the solution obtained from the model should also be the solution to the system problem. The effectiveness of the results of the application of optimization technique is largely a function of the degree to which the model represents the system studied. A mathematical optimization model consists of an objective function and a set of constraints expressed in the form of a system of equations or inequalities (Adeli, 2001).

2.3.1 Objective function of optimization model

To define those conditions that will lead to the solution of problems, the analyst must first identify a criterion by which the performance of the system may be measured. This criterion is often referred to as the measure of the system performance or the measure of effectiveness. The mathematical (i.e., analytical) model that describes the behavior of the measure of effectiveness is called the objective function. An objective function attempts to maximize profits or minimize losses based on a set of constraints and the relationship between one or more decision variables. If the objective function is to describe the behavior of the measure of effectiveness, it must capture the relationship between that measure and those variables that cause it to change (Adeli and Karim, 2001).

2.3.2 Constraints of optimization model

Constraints serve to bound a parameter or variable with upper and lower limits. Variable constraints may be expressed as absolute numbers or functions of parameters or variable initial conditions. A variable constraint is included in the variable declarations section along with the initial condition. The constraints could refer to capacity, availability, resources, technology, and etc. and reflect the limitations of the environment in which the operates. Each combination of values that apply to decision variables forms the solution of the problem. When these values satisfy the constraints of the problem, the solution is the feasible solution. System variables can be categorized as decision variables and parameters. A decision variable is a variable, the decision-maker can directly control that. There are also some parameters whose values might be uncertain for the decision-maker. The below figure shows the objective function which is to minimize function f(x) while constraints are $g_i(x)$ and $h_j(x)$.

$$
\begin{array}{rlll}
\min & f(\mathbf{x}) & & \\
\text{subject to} & g_i(\mathbf{x}) = c_i & \text{for } i = 1, \ldots, n & \text{Equality constraints} \\
 & h_j(\mathbf{x}) \geqq d_j & \text{for } j = 1, \ldots, m & \text{Inequality constraints}
\end{array}
$$

Figure 2.11 Objective function and constraints (Adeli and Karim, 2001)

2.3.3 Continuous function

In mathematics, a continuous function is a function for which sufficiently small changes in the input result in arbitrarily small changes in the output. A continuous function allows the x-values to be ANY points in the interval, including fractions, decimals, and irrational values. A set of data is said to be continuous if the values belonging to the set can take on any value within a finite or infinite interval. These functions may be evaluated at any point along the number line where the function is defined. For example, The height of a horse (could be any value within the range of horse heights), time to complete a task (which could be measured to fractions of seconds), the outdoor temperature at noon (any value within possible temperatures ranges), and the speed of a car on Route 3 (assuming legal speed limits).

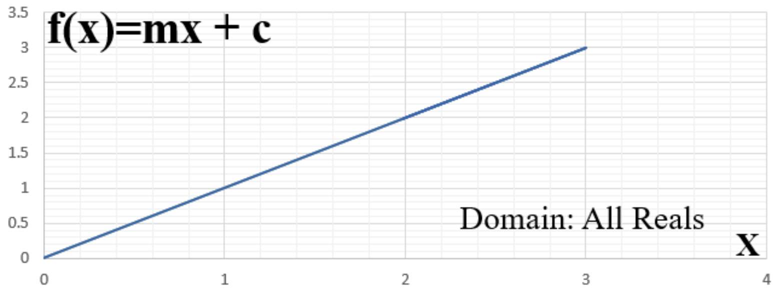


Figure 2.12 A continuous function (Adeli and Karim, 2001)

2.3.4 Discrete function

A discrete function is a function with distinct and separate values. This means that the values of the functions are not connected with each other. A discrete function allows the x-values to be only certain points in the interval, usually only integers or whole numbers. A set of data is said to be discrete if the values belonging to the set are distinct and separated (unconnected values). For example, the number of workers in a project (no fractional parts of a person) or the number of TV sets in a home (no fractional parts of a TV set).

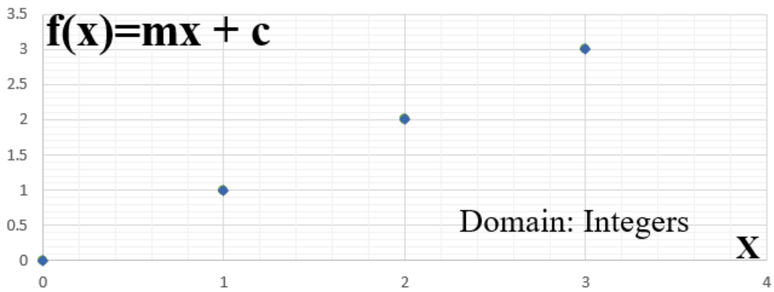


Figure 2.13 A discrete function (Adeli and Karim, 2001)

2.3.5 Verification of optimization model

The verification of the model could be carried out by comparison between experimental data and numerical simulations. If the problem is new and there is no experience data, the verification by trial-and-error approach is acceptable. However, the solutions from trial-and-error must have principles to support in order to guarantee that the solution is exact optimum. For example, the generation of a significant quantity of random numbers, corresponding to the decision variables values and the verification that none of them is better than the optimal one by the optimization criteria selected (Adeli and Karim, 2001).

2.3.6 Optimization model for repetitive projects

In the construction industry, several developments in optimization models were proposed to handle resource optimization problems in repetitive projects. The problems are too complicated and exhausting to be carried out by the traditional scheduling techniques. (El-Rayes and Moselhi, 2001; Hegazy and Wassef, 2001; Hyari and El-Rayes, 2006; S.-S. Liu and Wang, 2012; Long and Ohsato, 2009).

2.3.6.1 Optimization of crew formations

1) El-Rayes and Moselhi (2001) proposed a topic "optimizing resource utilization for repetitive construction project". They developed an optimization model based on a Dynamic Programming formulation (DP) in order to find the optimum crew formations that provided minimum project duration. The model incorporated a scheduling algorithm and an interruption algorithm. The scheduling algorithm followed sequence logic, crew availability, and crew work continuity constraints. The interruption algorithm was used to generate a set of available interruption for each crew formation. With the similar crews, their work considered that allowing work interruption could reduce minimum project duration shorter that maintaining work continuity. The process of the model started with retrieving the input; it then selected crew formation of each activity and began searching start dates and finish dates of all activities. The algorithm could identify an optimal crew formation and interruption option for each activity that led to minimum project duration. A bridge project which contains five successive activities as showing below was an example to illustrate the algorithm's capability.

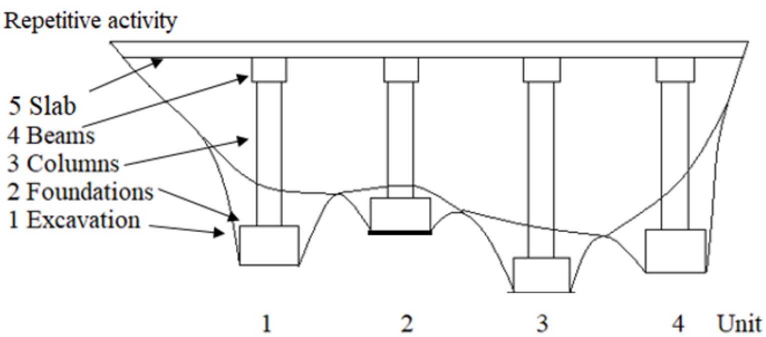


Figure 2.14 The example bridge shown in El-Rayes and Moselhi (2001)

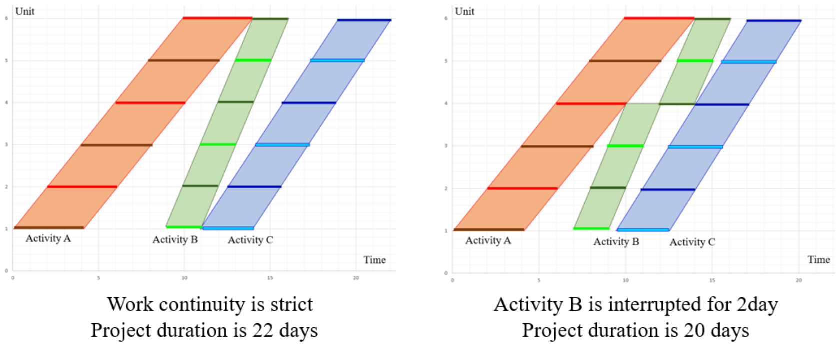


Figure 2.15 Project duration with work interruption (El-Rayes and Moselhi, 2001)

2) Hyari and El-Rayes (2006) presented a multi-objective optimization model for repetitive projects which applied a genetic algorithm as a searcher. The model concurrently minimized project duration and maximized crew work continuity. The processes of their model consisted of three main modules: scheduling, optimization, and ranking modules. First, the schedule module was designed to consider the availability of typical and nontypical repetitive activities; it was also assigned to compute start and finish dates of all activities. Second, the optimization module was developed to search and identify a set of optimal construction plans. The optimization module computed optimal solutions by considering trading off between project duration and crew work continuity. Third, the ranking model used a linear utility function to rank all the optimal solutions from the second module that which solution was fit for the project's desirability.

3) S.-S. Liu and Wang (2007) used Constraint Programming (CP) to develop a flexible scheduling model that optimized either total cost or project duration for linear construction projects. They investigated the concept of outsourcing resources, which optionally accelerated the production rate of repetitive activities. Additionally, discrete activities were also involved in the model. The model initially selected a crew formation of each activity and searched outsourcing, which provided either minimum project duration or minimum total cost under an assigned duration.

4) Long and Ohsato (2009) invented a GA-based method for repetitive project scheduling problems by considering the different attributes (interruption availability) of activities and relationships between direct costs and activity duration. The method

could deal with several objectives such as minimum project duration, minimum project cost, or both. Optimization process began with selecting crew formation and searching for the available start and finish dates. The start and finish dates were then moved in available time distance depending on the interruption availability of each activity. The moves affected the total project cost and project duration. Thus, the optimum crew and optimum distances providing the minimum objective function was the optimum solution.

5) S.-S. Liu and Wang (2012) developed an optimization model using constraint programming (CP) to minimize the project duration and studied the concept of multi-skilling for raising productivity. They enhanced the model in 2007, which considered outsourcing resources as the multi-skilled crew. The model searched for the optimal mixing between single skill crew and multi-skilled crew performing in an activity that provided the minimum project.

According to the reviews of previous studies on crew formation optimization, Their approaches try to search the number of workers in the crew that fulfills the optimum criteria. Only one crew is assigned to performs the repetitive activities for every unit, so the effect of multiple-crew in repetitive activities is not covered as showing below.

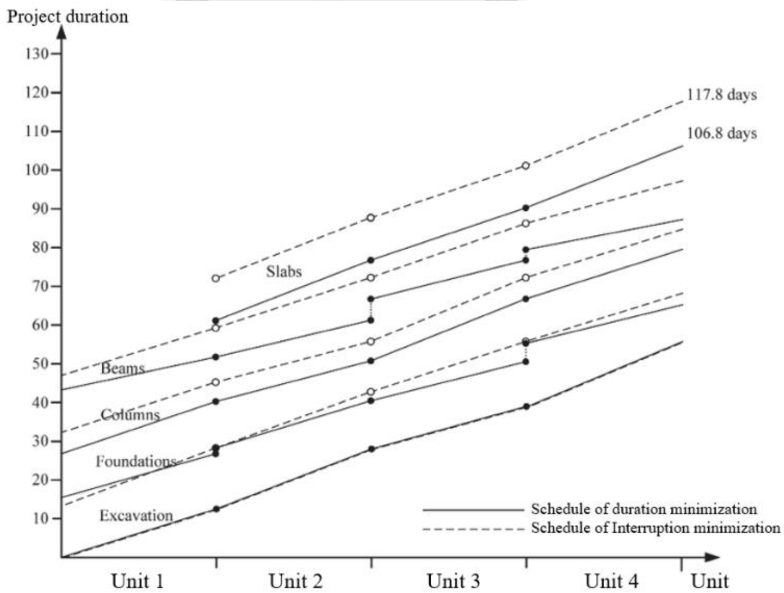


Figure 2.16 LSM diagram of the example bridge (S.-S. Liu and Wang, 2007)

2.3.6.2 Optimization of multi-crews' performance

1) Hegazy and Wassef (2001) proposed a generalized CPM/LOB method to determine the minimum total project cost of non-serial linear construction projects by using the genetic algorithm (GA). The model utilized GA to identify a combination of construction methods, number of crews, and work interruptions for each activity. The project deadline was met at the minimum total cost. Their work considered indirect cost and delayed cost during the time-cost tradeoff in the optimization process. The optimization process was developed based on the condition that the repetitive activity duration was constant, and all units are identical.

2) Kang et al. (2001) established a construction scheduling model using a conceptual approach to improve the efficiency of construction resources for a multiple, repetitive construction process (MRCP). The study involved construction projects consisting of both horizontal (n) and vertical repetitive processes (A) among several multi-story structures (r). They developed the model by considering basic repetitive activities such as wall steel and slab formwork, which exist in both vertical and horizontal meaning that every unit in the project must have the same type of structure. The model could analyze MRCP with all identical units by a conceptual approach including the optimized project duration and defined repetitive work zone. Rotation technique of crew group improved work continuity. The objective of the model was to find the minimum loss in construction cost by work interruption.

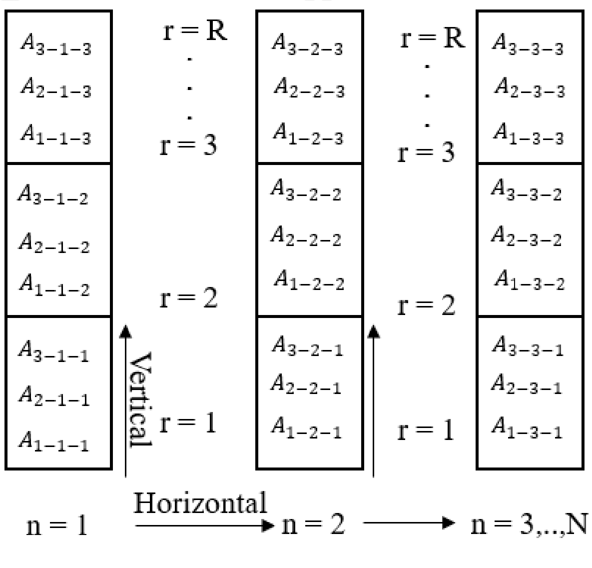


Figure 2.17 Horizontal and vertical work areas for MRCP

3) Georgy (2008) adopted an integrated CAD/GA approach for the resource leveling of linear schedules and cleaned up the shortcomings of mathematical solutions for a resource-leveling problem. The method considers the productivity rate of the crew (resource) as a decision variable by aiming to minimize either the day-to-day fluctuations in resource usage or the daily deviation from the average resource utilization during the project duration. One primary limitation of the model is that it cannot handle occasional activities or unstable resource usage for individual activities. This method only considered the performance of a single common resource.

4) Damci et al. (2013b) presented Resource leveling in Line-of-Balance scheduling and allow Multi-resource leveling. Both models were named as Excel-based LOB model, which were developed in MS Excel. The model used the genetic algorithm (GA) to search the solutions which considered minimizing the sum of the absolute deviations of the daily resource usage from the average resource usage as Georgy (2008). They regarded as a single resource (crew) performing the whole project which related to Georgy (2008). The multi-resource was considered in the second study. The model assumed that the activities were performed by two different resources which have individual production rates, the production rate of the activity was controlled by the lower which was slightly different from the first model in term of resource emphasizing factor.

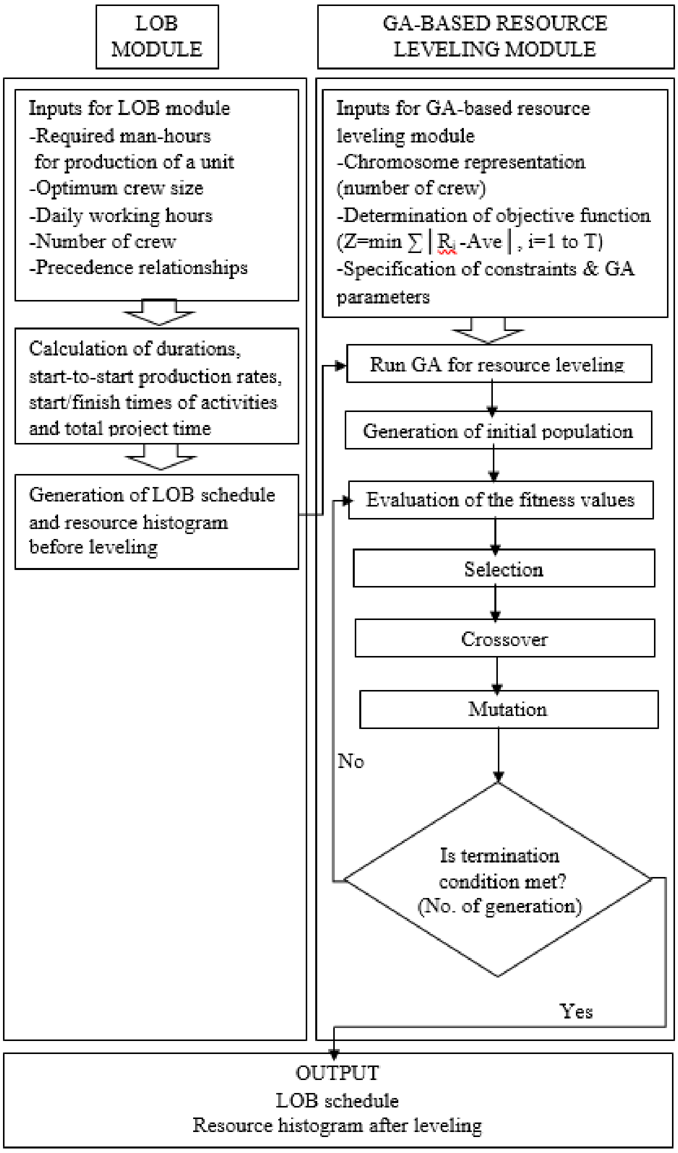


Figure 2.18 Flowchart of the development process by Damci et al. (2013a)

2.3.3 Summary of optimization models

Models and algorithms in the previous studies demonstrated their potential dealing with optimization problems in linear and repetitive projects. Many conditions have been considered to improve the scheduling process as more advance. However, the studies had a similar assumption that the project only had a single identical type of units. They mainly emphasized on similar resources performing the repetitive activities in every unit. Thus, problems of projects that contain multi-identical types of units have not been covered. In summary, the table below is a comparison of the conditions and assumptions of the previous studies.

Table 2.2 Comparison of the optimization models

Author	Searching algorithm	One-identical type of unit	Typical actvity	Resource continuity	Interruption avialability	Multi-objectives	Multiple resource assignment	Multi-identical types of unit
El-Rayes & Moselhi (2001)	DP	Yes	Yes	No	Yes	No	No	Not Cover
El-Rayes & Hyari (2006)	GA	Yes	Yes	No	Yes	Yes	No	Not Cover
Liu & Wang (2007)	CP	Yes	Yes	No	Yes	Yes	Yes	Not Cover
Long & Ohsato (2009)	GA	Yes	Yes	No	Yes	Yes	No	Not Cover
Liu & Wang (2012)	CP	Yes	Yes	No	Yes	No	Yes	Not Cover
Hegazy & Wassef (2001)	GA	Yes	Yes	No	Yes	No	No	Not Cover
Kang et al. (2001)	-	Yes	Yes	Yes	No	No	No	Not Cover
Georgy (2008)	GA	Yes	Yes	Yes	No	No	No	Not Cover
Damci et al. (2013a)	GA	Yes	Yes	Yes	No	No	No	Not Cover
Damci et al. (2013b)	GA	Yes	Yes	Yes	No	No	Yes	Not Cover

2.4 Building Information Modeling (BIM)

Building Information Modeling (BIM) is an intelligent 3D model-based process that provides architecture, engineering, and construction (AEC) professionals the insight and tools to more efficiently plan, design, construct and manage construction projects. BIM is used to design and document building and infrastructure designs. Every detail of a building is modeled in BIM. The model can be used for analysis to explore design options and to create visualizations that help stakeholders understand what the building will look like before it is built. The goal of using a BIM solution is to create a 3D model that users can operate the project over its life-cycle. A 3D model enables users to understand relationships between spaces, materials, and various systems within a physical structure. BIM software can be used for every step of the process, from planning to design to construction. Every step of the process is vital to the BIM users building a structure in the real world (NIBS, 2015).

4D Simulation is a tool that combines the 3D model with the project schedule during Create, Realise and Enhance phases. It allows stakeholders to visualize the construction phase in a virtual environment to realize opportunities to enhance construction sequencing. 4D Simulation reduces the time required to review and challenge the schedule by providing dynamic visualization of any spatial or sequence constraints in the construction schedule, thus enhancing communication and coordination between all disciplines to reduce missing activities (Dang and Tarar, 2012).

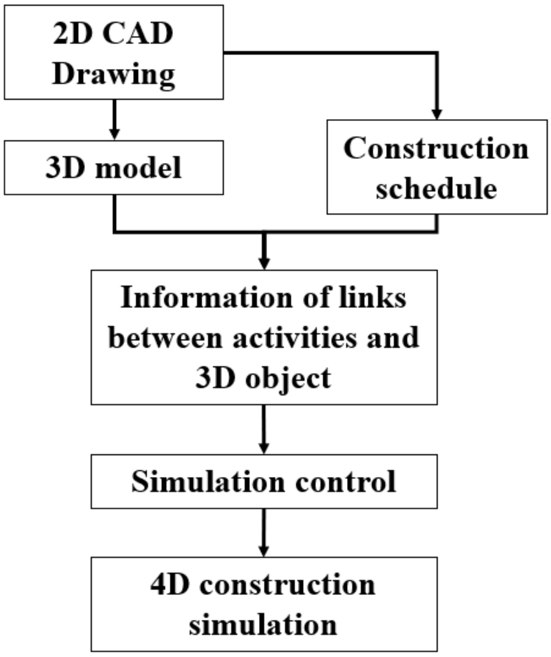


Figure 2.19 The implementation procedure of 4D construction simulation

Excellent BIM/4D tools such as Navisworks and VICO office are the integrated platforms where construction estimating, scheduling, and design management all come together. BIM models could be imported into the software and then automatically perform clash detections, quantity take-off, 4D simulation, etc. Navisworks is developed for the purposes of project visualization, and it does not have scheduling feature for project management, whereas the VICO office contains scheduling features and automated work-time calculation from the imported BIM objects. However, optimization process still requires the trial-and-error by adjusting production rates as TILOS (H. Liu et al., 2014).

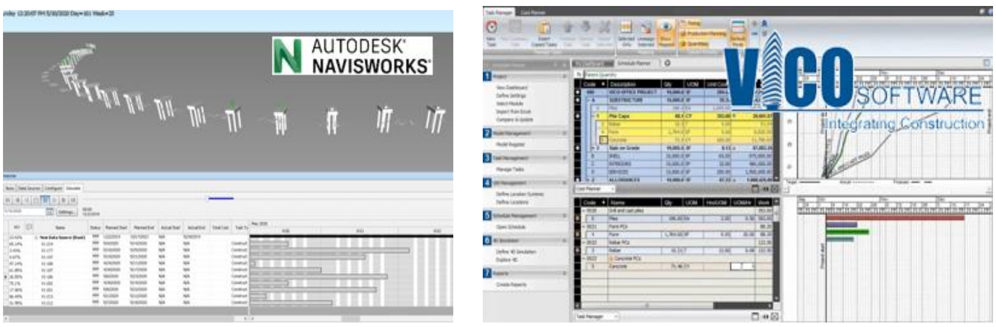


Figure 2.20 Naviswork and VICO interfaces.

2.5 Scheduling system

1. Kataoka (2008) presented a system by which to generate construction schedules from simple 3D building geometries and predefined construction method. The system was specified to provide schedules for structures processes in the very early stage of projects.

2. Wu et al. (2010) introduced a methodology for creating input data for a constraint-based discrete-event simulation of construction processes on software “Preparator” for a bridge construction schedule. A 3D model of a bridge was used as the input data for an optimization model in discrete-event simulation. The system could deal with uncertain productivity and complexity of the project through sequence reasoning in discrete-event simulation. However, the system required massive manual involvement during the implementation phase. Furthermore, the 3D CAD model was used only to provide the quantity of work but the process of sequence logic did not exist.

3. Chen et al. (2013) developed a framework which involved a manual process for obviously illustrating a complete activity network and assigning quantity take-offs from a house 3D CAD to activities. In their studies, 3D CAD only provided the quantity of work for the process simulation model. The near-optimal schedule was acquired by simply picking the best solution in multiple runs.

4. Kim et al. (2013) proposed a prototype for automating the generation of construction schedules using open BIM technology by parsing an Industry Foundation Classes (IFC) of the BIM model. The study listed sequencing rules by dividing work zone of two buildings in order to determine the precedence relationships among pre-defined activities. The prototype could create a construction schedule of two separate building with BIM information, and calculate activity duration by using productivity rates from a database and applying sequencing rules.

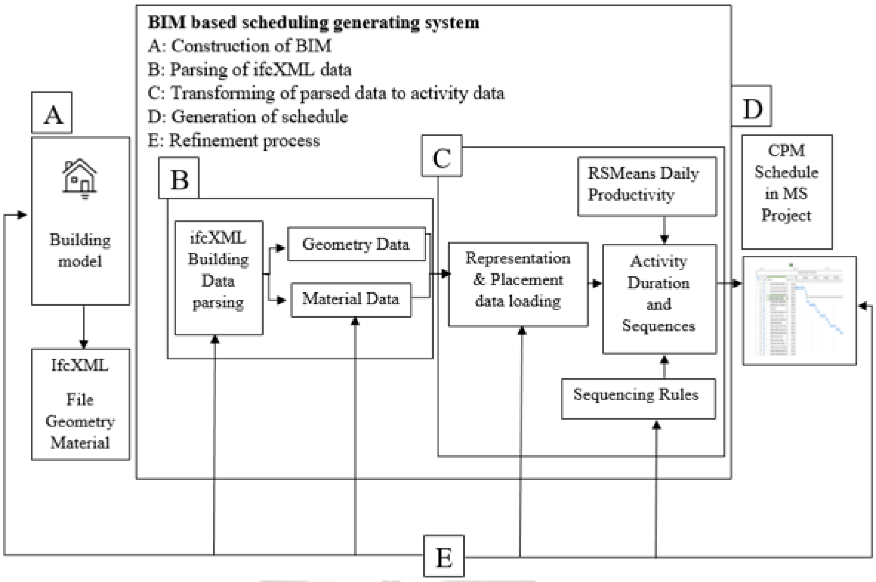


Figure 2.21 Methodology flowchart by Kim et al. (2013)

5. H. Liu et al. (2014) devised a BIM-based approach to automatically generate on-site schedules for panelized construction which employed the Light Gauge Steel method. The approach used MS Access as a resource database that provided production rates. The BIM model in Revit has attached sequences by applying information from the joints of the Light Gauge Steel method. The schedule was generated from the information to be displayed in the MS project for better communication. H. Liu et al. (2015) enhanced their previous system more efficient in activity level and resource constraint issues. The process simulation model was added to search the optimized resource schedules.

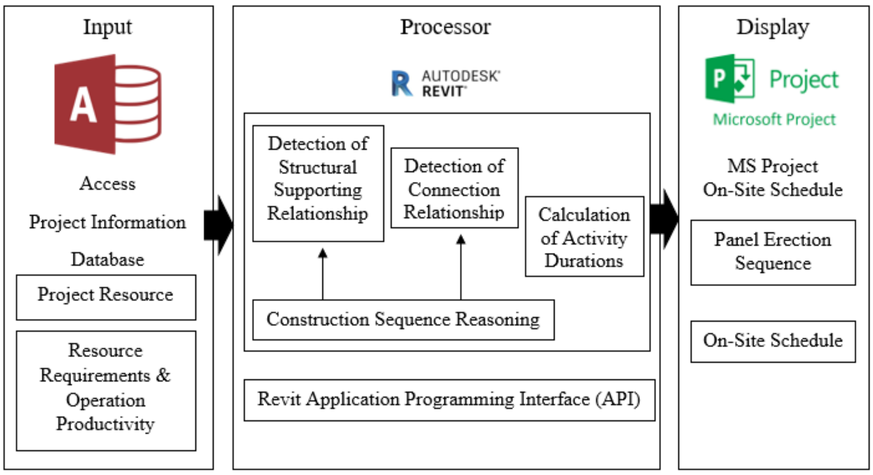


Figure 2.22 Architecture of automatic schedule system by H. Liu et al. (2014)

According to the previous studies, they have investigated the feasibility of scheduling systems that could utilize enriched information in 3D/BIM model. The systems used 3D models either for the quantity of work or for sequence logic. The systems were capable to generate construction schedules that facilitated the planners by reducing manual process and human error.

2.6 Summary

From the literature review related to this study, it illustrates that an excellent construction schedule can lead a project to success in term of time, cost, and quality. To create a construction schedule, the planner has to concern many complex factors which have both direct and indirect interrelationships in term of time and cost. For linear infrastructure projects, Line of Balance and Linear Scheduling Method have been proven that they are capable of dealing with the characteristic of linear repetitive projects. However, they require manual processes and their solution may not be optimum.

The optimization models developed by previous studies provided alternatives to create effective construction schedules. However, the models were developed for specific conditions where some conditions were not covered. For example, the conditions of multi-identical types of units exist in elevated highway/railway construction projects where the repetitive activities of different types of piers are performed independently. Furthermore, the optimization models require substantial manual input which may cause human-error and incorrect solutions.

The utilization of BIM information proposes an approach to eliminate the substantial manual input. The previous studies developed systems with the use of BIM information to automatically generate construction schedules. Most of them were implemented for the information of the building construction while there is only one work by Wu at el. (2010) related to the implementation of a scheduling system for linear infrastructure construction.

Chapter 3
Research methodology

The purpose of this chapter is to explain the research methodology for developing BIM-based Line of Balance Scheduling System (BIM-LOB-SS). The research methodology includes research characteristic, research design, and research method. Research characteristic presents the research type and research description. Research design expresses the main phases of this research. Research method explains the procedures to solve the research problems.

3.1 Research characteristic

This research is quantitative and applied research. The research explores a case study of the construction of an elevated highway to find a scheduling problem of multi-identical types of units. An application of Line of Balance is proposed to solve the scheduling problem. The application is then verified to determine capability and limitation. This study also presents the application of BIM to facilitate the project operation by a proposed BIM-based Line of Balance Scheduling System (BIM-LOB-SS). Finally, the proposed system is validated with the case study to define the system's contribution in the real-world project.

3.2 Research design

The main procedures of this study are divided into five main phases including (A) Research problem identification, (B) Development of conceptual framework, (C) Verification of conceptual framework, (D) Development of the proposed system, and (E) Validation of the proposed system. The research methodology is shown in Figure 3.1.

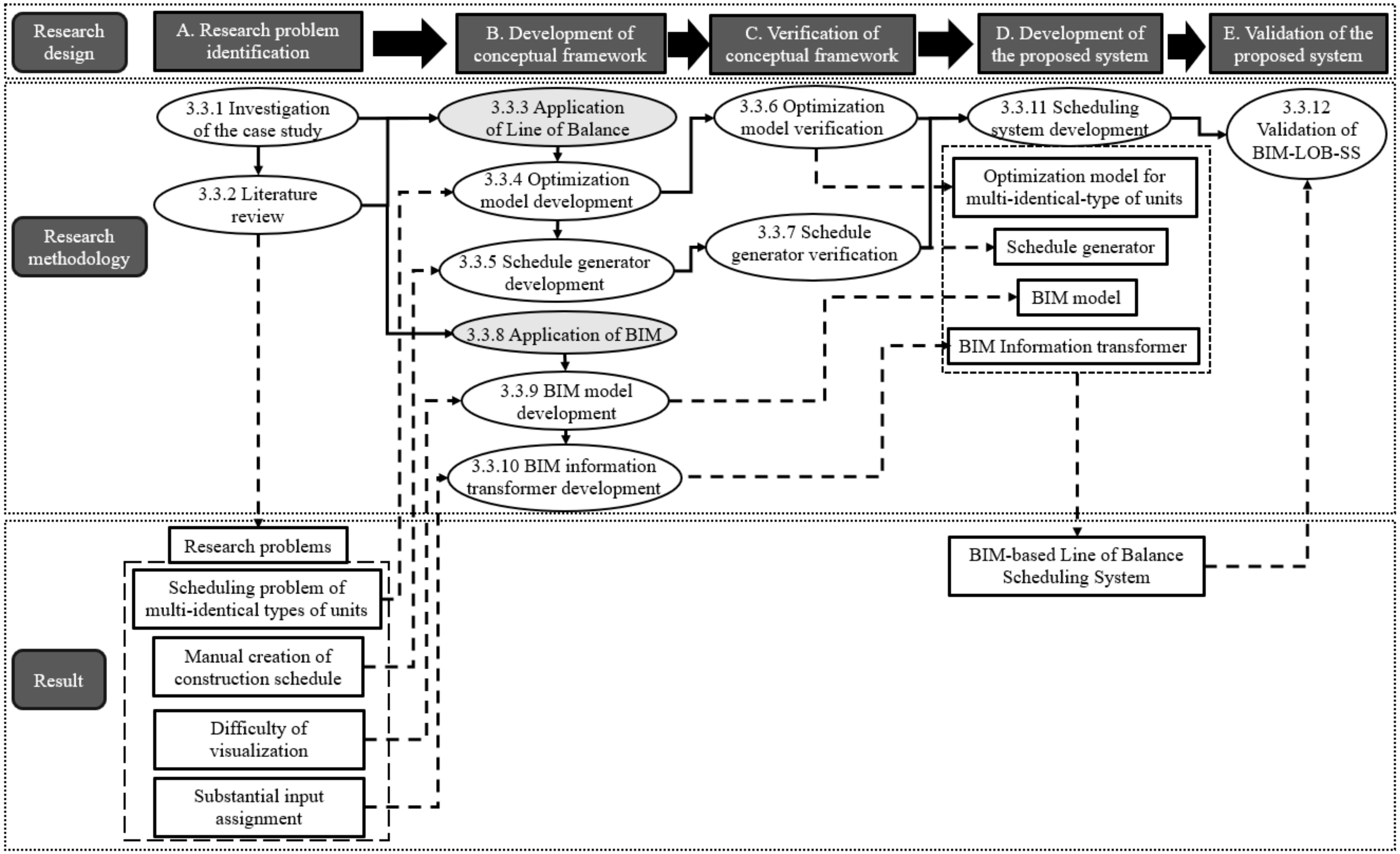


Figure 3.1 Research design and research methodology

3.3 Research methods

3.3.1 Investigation of the case study

To understand the problem of the elevated highway construction project, this study investigates a 3.3. km elevated highway construction project which is a section of the Bang Pa-In – Nakhon Ratchasima intercity motorway. The investigation includes an interview with the project manager, a study of construction methods of elevated highway, and data collection (drawing and other relevant documents related to the scheduling problem). The interview with the project manager expresses the scheduling objectives of the contractor and scheduling problems of the project. The study of construction methods provides a better understanding of the project's performance, the restrictions of resource utilization, and the conditions of the project. The data collection provides information to develop the BIM model fo the project.

Figure 3.2 Case study: A section of Bang Pa-In – Nakhon Ratchasima Motorway

3.3.1.1 Multi-identical types of units

The case study is a section of an elevated highway at Lam Takhong. The 12.30m width carriageway of the elevated highway is designed for 2-lanes roadway. The length is 3.3 km where the piers of 69 stations are provided to support the elevation of the highway. The station codes of 69 piers start from station V1-155 to station V1-223 successively as shown in the following figure.

Figure 3.3 Piers and station codes

To support the carriageway of the elevated highway along the length 3.3km, the substructure piers are classified into three types (P11, P12, and P13) to preserve different elevations and alignment of the carriageway. A type of pier is designed to a station individually. For example, pier type P11 belongs to station V1-219. The following detail information is for three types of piers in this project

1) Pier type P11 is the two-column pier in one row. The maximum height from the top of the footing to the top of the deck slab is limited to 13.50m. The structural elements of P11 consist of piles, footing, bottom column, and top column.

Figure 3.4 Pier type P11

2) Pier type P12 is the Y-shape pier with a single column. The maximum height from the top of the footing to the top of the deck slab is limited to 16.50m. The structural elements of P12 consist of piles, footing, bottom column, Y-shape column, and crossbeam.

Figure 3.5 Pier type P12

3) Pier type P13 is the Y-shape pier with a single column. The maximum height from the top of the footing to the top of the deck slab is limited to 26.50m. The structural elements of P13 consist of piles, footing, bottom column, Y-shape column, and crossbeam.

Figure 3.6 Pier type P13

These three types of piers locate in different stations depending on the terrain, elevation, and alignment. The combination of these three types of piers along the alignment is described as the multi-identical types of units.

3.3.1.2 Construction of the pier in the elevated highway

To construct the piers, the piers are built by the cast in situ method with metal formworks and fresh concrete. Each pier element of each type requires a specific resource such as drilling crane for piles or fabricated formwork for the column. The specific resource is designed for a certain structure and it can not be used for the other structures in the project, especially the fabricated formwork. The fabricated formwork is specific for a certain structure of a type such as fabricated formwork for top column P11, fabricated formwork for Y-shape column P12, and fabricated formwork for Y-shape column P13 as shown in Figure 3.7, 3.8, and 3.9. These formworks are built as fabricated modules with metal frames. A fabricated formwork can be repetitively used to construct the same structures of each type in different locations.

Figure 3.7 Fabricated formwork for top column P11

Figure 3.8 Fabricated formwork for Y-shape column P12

Figure 3.9 Fabricated formwork for Y-shape column P13

These formworks have to be prepared and fabricated in the early stage of the project. They are costly and specially designed for their project. The constructor confronts with the issue of how many of these formworks should be used to complete the project on time and with maximum profit. To find an optimal set of the formworks, an efficient construction schedule is necessary.

3.3.1.3 Construction of carriageway of the elevated highway

The carriageway of the elevated highway is constructed by the viaduct precast segment method on launching gantry. The launching gantry straightly performs segment erection for every station starting from the station V1-223 to the station V1-155. In this project, the pier at station V1-155 is the destination of the segment erection.

Figure 3.10 Launching gantry for the viaduct precast segment erection

The launching gantry method is restricted to straightly erect the viaduct segment from a pier to the next pier. In Figure 3.10, the launching gantry erects the viaduct segment for P11 at the station V-214. Before that, the launching gantry has erected the viaduct segment for P12 at the previous station, V1-215.

3.3.1.4 Scheduling problem of multi-identical types of units

The project comprises many piers located along the project's alignment. The piers in the project are in different types including type P11, type P12, and P13. A type of pier contains structural elements such as foundation, column, and crossbeam. These structural elements individually require specific resources to construct in which each specific resource generates cost for its activity. Specific resources of the structural elements are generally provided to serve specific elements. The example LOB diagram of multi-identical types of units with single launching gantry is shown below.

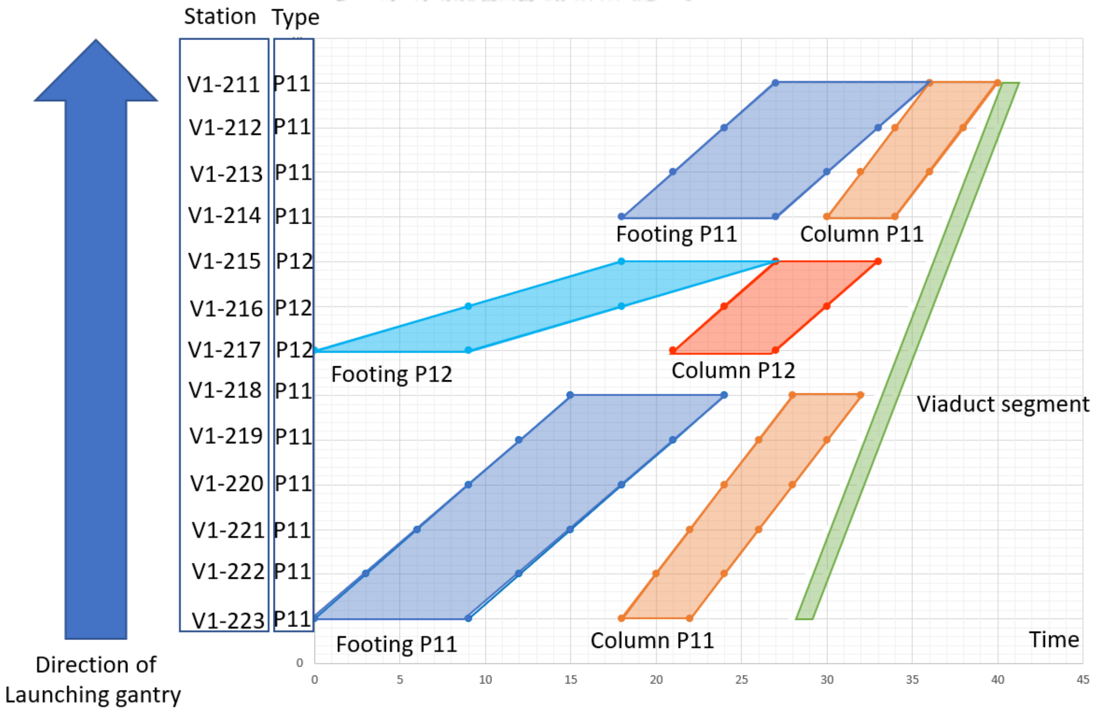


Figure 3.11 LOB diagram of multi-identical types of units with a launching gantry

To maintain the continuity of the segment erection, the piers have to be completely constructed before the launching gantry reached. If the piers are close to the starting point of segment erection, the contractor has to complete the piers earlier in order to avoid the interruption of worker and maintaining the continuity of the segment erection. Such demand directly affects the numbers of the specific resources which control the production rate of the piers' elements. From Figure 3.11, the LOB diagram shows the continuity of operation of launching gantry to install the viaduct segments. The resources are fully utilized with no interruption. The production rate of the launching gantry controls the activity duration. However, for column construction, the scheduling becomes more complex since there are 2 types of columns (P11 and P12), and their locations of each type are not continuity. For example, column P11 is built at station V1-233 to V1-218 and is built again at V1-214 to V1-221. There are 3 columns to be built with P12 in the piers between V-217 and V1-215. The constraints are that (1) the resources of each type must be used with no interruption in order to reduce the cost of idle time and (2) the numbers of resources must be enough to finish construction as fast as possible but must also be at the lowest possible cost. Since the production rate of the column affects both its duration and the launching gantry for segment erection, the optimal set of resources and schedule to operate at a certain pier is essential to consider the project duration. In addition, when this relationship occurs with other pairs of consecutive activities such as footing and column, planning or operating of the succeeding activity becomes more complicated. This character exists in the real linear infrastructure projects where there are several types of identical structure to be built at different locations. As a result, this study calls "the multi-identical types of units". Other factors including activity duration, location of the pier, type of pier, number of resources, resource cost, and work sequence have the complex interrelationships affecting the project duration and total cost of specific resources. Thus, the constructor confronts the complexity of a scheduling problem of multi-identical types of units. For example, how many of each specific resources should be used to complete the project on time with the lowest total cost of specific resources.

The objectives of the contractor are listed as the following:

1) Maintain continuity of resource utilization

2) Complete the project with the minimum total cost of specific resources

3) Complete the project within a desirable project duration

With the objectives and the complexity, scheduling problem can be summarized as the following:

1) The project contains multi-identical types of units.
2) The different piers are at different stations.
3) Only a single launching gantry performs segment erection. This is the last activity to be scheduled in this case where this activity performs natural rhythm in the concept of LOB.
4) The contractor aims to utilize the same specific resources continuously to reach a high learning curve.
5) The project must be completed with the minimum total cost of specific resources.
6) The project must be completed within the desirable project duration.

3.3.2 Literature review

The literature review is presented in Chapter 2 to study the previous research, to explore the state of the art and uncovered issues, and to provide knowledge to support the development of BIM-LOB-SS. This study finds that the existing approaches to create construction schedules mainly rely on manual creation. The optimization models proposed by the previous studies did not cover the condition of multi0identical types of units. The previous models require massive manual input which may cause human-error. Moreover, their presentation of schedules still not cover the overview of the project.

3.3.3 Application of Line of Balance

The application of Line of Balance aims to accomplish a concept of scheduling for multi-identical types of units. The developing concept is designed to consider the repetitive scheduling in mathematical terms of representative equations and variables. The concept of application of LOB is used to create an optimization model for computing the optimal solution, and a schedule generator for generating the start times and finish times of activities. Matlab 2018 is selected to develop the model and the generator.

3.3.4 Optimization model development

From the scheduling problem of the case study project, this research proposes the optimization model for multi-identical types of units with the objective of minimizing the total cost of specific resources and continuity of resource utilization under the desired duration of a project. A code of computer language is designed by using Matlab 2018 to create the optimization model.

The following conditions are considered to develop the optimization model.

1) This study considers the sequence of construction of piers to timely support the erection of segments by one-direction of single launching gantry.
2) The piers along the alignment are the combination of more than one type of identical structures (multi-identical types of units).
3) Activities of piers start from substructure to superstructures such as piling, foundation, column, and the last activity is segment erection.
4) All resources are continuously utilized without interruption.
5) The number of specific resources mainly controls the rate of delivery.
6) A specific resource is required to build a structural element.

The optimization model designed by this study provides the optimal solution including an optimal set of specific resources, an optimal total cost of specific resources, and an optimal duration of the project.

3.3.5 Schedule generator development

To solve the problem of manual creation of the construction schedule, a schedule generator is invented to automatically create the construction schedule by computing start and finish times of all activities from the optimal solution. In this study, the schedule generator is designed to retrieve inputs from the optimization model which are activity duration, the sequence of activities, number of piers, and the optimal set of specific resources.

3.3.6 Optimization model verification

Before validating the optimization model with the case study, this study examines the optimization model with three small examples. The case study provides the uncommon conditions of linear repetitive scheduling problems. From the literature review, if there is no previous example which can be used to verify the optimization model, the trial-and-error is an alternative to prove the optimization model's capability. A comparison between the optimal solutions solved by trial-and-error and the optimum solutions provided by the optimization model can carry out the verification of the optimization model.

3.3.7 Schedule generator verification

This study utilizes a scheduling software called Asta Powerproject which has Line of Balance feature to verify the schedule generator by comparing the result from the software and those from the schedule generator. The software automatically displays the start and finish times when the workflow (number of resources) of any activity is changed. The comparison analyzes the start and finish dates of all activities with a similar set of resources. With a similar set of resources, activity duration, and sequential logic, if the results from software and the result the schedule generator are exactly the same, the capability of the generator is verified.

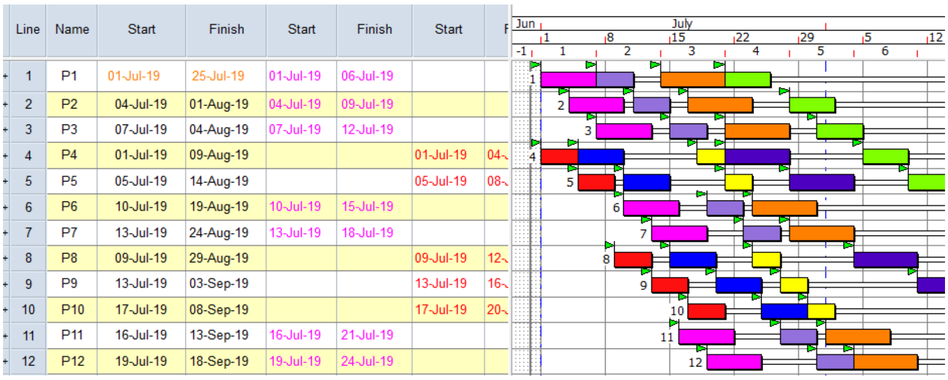


Figure 3.12 Interface of Powerproject

3.3.8 Application of Building Information Modeling

The application of building information modeling consists of the development of the BIM model and the BIM information transformer. The application aims to utilize the BIM model and the BIM information to enhance the scheduling system in terms of visualization and input assignment.

3.3.9 Development of BIM model

Development of BIM model aims to create a database of the case study and to provide better visualization. This study employs Autodesk Revit 2018 to develop the BIM model of the case study. The BIM model is developed by obtaining the construction drawing and relevant documents.

3.3.10 Development of BIM information transformation

Development of BIM information transformation has the objectives of reducing massive manual input. The transformation consists of two components which are an information extractor for extracting the selected information from the BIM model and inventing an information transformer for parsing the extracted information. The BIM model is considered as a 3D database that contains data of the project such as geometry, location, and name of the element. The BIM information can provide the input of the optimizing and scheduling process by matching the data required for input and BIM information. Because the BIM information may not be utilized directly by the proposed system. Thus, a BIM data transformation is essential to make the BIM information suitable for the optimization model. In other words, this study develops an information transformer to automatically transform the selected BIM information to be the input of the optimization model.

3.3.11 Scheduling system development

Finally, this study develops the BIM-based Line of Balance Scheduling system (BIM-LOB-SS) by the combination several programs. The development presents the link and flow of data between each programs from the sources of input to the output management tools. The proposed system comprises the following four components as shown in Figure 3.13: (1) Source of input (database and BIM model of the project), where relevant information associated project scheduling is stored; (2) BIM information transformation, where the selected BIM information is transformed to be the input of the optimizing and scheduling process; (3) Optimizing and scheduling process, where the optimal solution and schedule are computed; (4) Output management tools, where various project presentations are presented. The BIM-LOB-SS is shown in Figure 3.13.

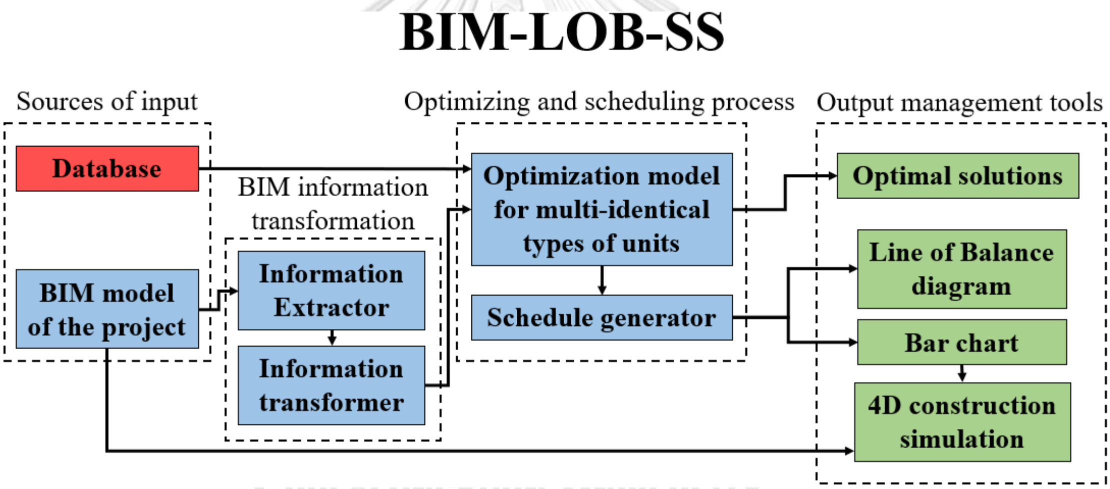


Figure 3.13Framework of BIM-based Line of Balance Scheduling System

3.3.12 Validation of BIM-LOB-SS

This part aims to validate BIM-LOB-SS with the case study. The system is demonstrated to the practitioners in the case study to obtain feedback and suggestion. The results of validation express the capability and limitations of the proposed system dealing with the practical problem.

3.4 Conclusion

This chapter explains the procedures to accomplish the research. This study begins with site investigation and literature review to gather knowledge, and to determine the problem statement of research. Then, BIM-LOB-SS is proposed to solve the problems of optimization and scheduling. To develop the system, an application of Line of Balance and BIM is established. The application of Line of Balance aims to invent the optimizing and scheduling process for the scheduling problem of multi-identical types of units. Three small examples are utilized to verify the application of Line of Balance. The application of BIM intends to utilize the BIM information for the optimizing and scheduling process and to improve the project planning and scheduling with 4D construction simulation. Finally, this study demonstrates BIM-LOB-SS with the case study to determine system capability and limitations. In the next sections, the procedures of this study are explained.

Chapter 4
Application of Line of Balance

This chapter presents an application of Line of Balance to develop the optimizing and scheduling process. The development consists of optimization problem, application of Line of Balance, optimization model, optimization model verification, schedule generator, and conclusion.

4.1 Optimization problem

The scheduling problem is transformed into an optimization problem. The factors which relate to the scheduling problem consist of activity duration, location of unit, type of units, number of specific resources, specific resource cost, and work sequence. These factors have interrelationships affecting the total cost of specific resources and project duration, so they must be identified in terms of components of an optimization model to form a representation of the optimization problem. They are described as the components of an optimization problem in the following section.

4.1.1 Objective function

One objective is to complete the project with the lowest total cost of the specific resources. This study has a condition that the last activity is the viaduct segment erection, where only a launching gantry operates. Hence, the cost of launching gantry is separately calculated and is not included in the objective function. The specific resources for the other activities are provided to be purchased resources with a fixed individual cost per unit. Therefore, the objective function of the model can be defined as Eq. 3.

$$C_{TR} = (R_{(1)})(C_{(1)}) + (R_{(2)})(C_{(2)}) + (R_{(3)})(C_{(3)}) + \ldots + (R_{(i)})(C_{(i)}) \text{........} Eq.3$$

Where i is the number of total repetitive activities (not include the last activity: the segment erection)

C_{TR} = The total cost of specific resources

$R_{(i)}$ = Number of resources for repetitive activity i

$C_{(i)}$ = Cost per unit of specific resource for repetitive activity i

The objective function is to minimize the value C_{TR} which varies upon variable R and variable C. The function is a discrete function with the domain of integer variable R where variable C is constant. Thus, the variable R is the decision variable of the optimization model.

4.1.2 Constraint

One of the goals is to complete the project within a desirable duration. This study defines the goal as a constraint of the optimization model. R is the decision variable of the objective function. The function of the project duration must follow the objective function by using the decision variable R to calculate the project duration. Thus, the discrete function of project duration is written as Eq. 4.

$$P\left(R_{(1)}, R_{(2)}, R_{(3)}, \ldots, R_{(i)}\right) \leq \text{desirable duration} \ldots\ldots\ldots Eq.4$$

Where i is the number of total repetitive activities (not include the last activity: the segment erection)

$P\left(R_{(1)}, R_{(2)}, R_{(3)}, \ldots, R_{(i)}\right)$ is the function of project duration

$R_{(i)}$ = Number of resources for repetitive activity i

To acquire to project duration, the function of project duration has to consider the decision variable R with the constant variables including activity duration, number of units, and sequence logic. These variables have the interrelationship that causes the function of the project duration calculation too complex to be conducted by the general linear equation. Therefore, this study proposes an application of LOB to create the function of project duration as the following.

4.2 Application of Line of Balance

The application of Line of Balance aims to develop a concept of scheduling for multi-identical types of units which leads to an indirect method of project duration calculation for the linear repetitive projects.

4.2.1 Method of project duration calculation

The method of project duration presented in this study is based on the principle of natural rhythm. The principle has the assumption that productivity can be achieved the highest when an optimum size crew performs. The assumption controls multiple crews of optimum size being used to increase the delivery rate. Thus, if multiple crews of optimum size are used, no idle time occurs when crews move from unit to unit. The slope of the delivery rate becomes steeper following the increased number of crews. Thus, this study considers <u>the number of multiple crews of optimum size as the number of specific resources</u> of the function of project duration.

From the principle of natural rhythm by considering on work continuity and resource synchronization, a relationship that determines the optimum crew size to reach the highest delivery rate, without work interruption. This relationship can be achieved by examining resource synchronization in Figure 4.1. In this figure, three resources are used to perform a repetitive activity that is repeated at six units. Only one resource has a duty to work in a single unit, then instantly moves to the next unit without idle time. Dividing the activity duration (D) among the number of specific resources (R) implies that each resource must start after a time (D/R) relative to its preceding unit. This allows shifting the start times of a repetitive activity forward or backward at the different units by changing the number of resources. Thus, the slope of the repetitive activity is equal to m in Eq.2 (Hegazy and Wassef, 2001).

$$m = \frac{R}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.2$$

m = Rate of delivery

R = Number of specific resources

D = Activity duration

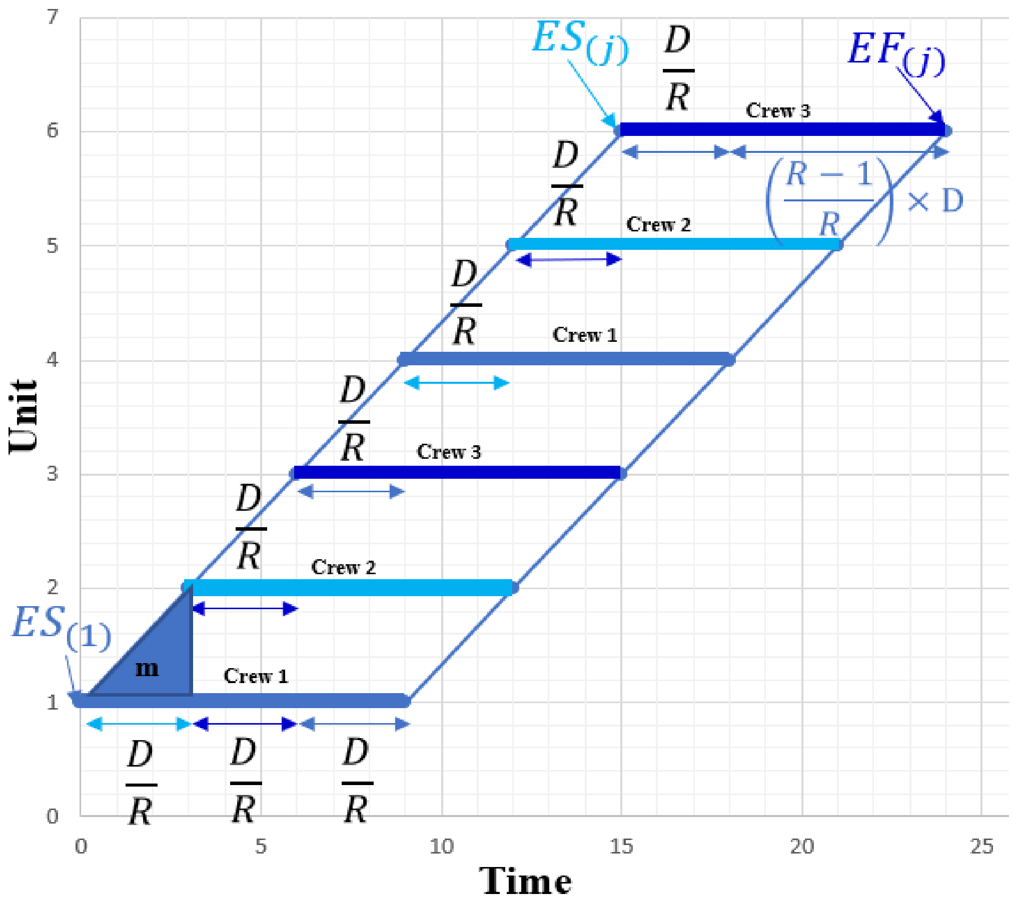


Figure 4.1 LOB diagram with resource synchronization

With consideration on work continuity and resource synchronization, Early Start time of activity at j[th] unit can be derived by the linear equation as below. The Eq. 5 is an essential equation to find the early start time of activity at any unit j. Where j is the unit in consideration.

(1) Basic linear equation; $y = mx + c$..(1)

(2) At unit 1, y = 1 and x = $ES_{(1)}$; $1 = m(ES_{(1)}) + c$..................................(2)

(3) At unit N, y = N and x = $ES_{(j)}$; $N = m(ES_{(j)}) + c$.................................(3)

(4) = (3) - (1); $N - 1 = m(ES_{(j)} - ES_{(1)})$ ....(4)

$$ES_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N - 1) \ldots\ldots Eq.5$$

$ES_{(j)}$ = Early Start time of activity at j[th] unit

$ES_{(1)}$ = Early Start time of activity at 1[st] unit

m = Rate of delivery, N = Number of unit

Early Finish time of activity can be derived as the following:

(5) $EF_{(j)}$ from $ES_{(j)}$;

$$EF_{(j)} = ES_{(j)} + D \text{..} (5)$$

(6) From Eq. 5 and (5);

$$EF_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N-1) + D \text{..........} (6)$$

(7) Multiple term $\frac{1}{m}$;

$$EF_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N) - \frac{1}{m} + D \text{.........} (7)$$

(8) $m = \frac{R}{D}$;

$$EF_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N) - \frac{D}{R} + D \text{.........} (8)$$

(9) $D = \frac{RD}{R}$;

$$EF_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N) - \frac{D}{R} + \frac{RD}{R} \text{.....} (9)$$

$$EF_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N) + \left(\frac{R-1}{R}\right) \times D \text{..} Eq.6$$

$EF_{(j)}$ = Early Finish time of activity at jth unit

$ES_{(j)}$ = Early Start time of activity at jth unit

$ES_{(1)}$ = Early Start time of activity at 1st unit, m = Rate of delivery

N = Number of unit

R = Number of specific resources

D = Activity duration

4.2.2 Equation of two consecutive activities of the identical type of units

From the previous section, Eq.5 and Eq.6 only provide Early Start time *($ES_{(j)}$)* and Early Finish time *($EF_{(j)}$)* of each activity individually. This section presents the consideration of two consecutive activities in one representative equation. The representative equation aims to provide an undefined variable that can be used to calculate project duration and verify sequence logic indirectly. In Figure 4.2, three teams of specific resources are utilized to complete the preceding activity (repetitive activity i - 1) that is repeated for 6 units while two teams of specific resources are utilized to complete the succeeding activity (repetitive activity i) that is repeated for 6 units. Where i is the repetitive activity and j is the unit in consideration. In this section, the number of repetitive activities must be at least two activities to make the representative equation usable.

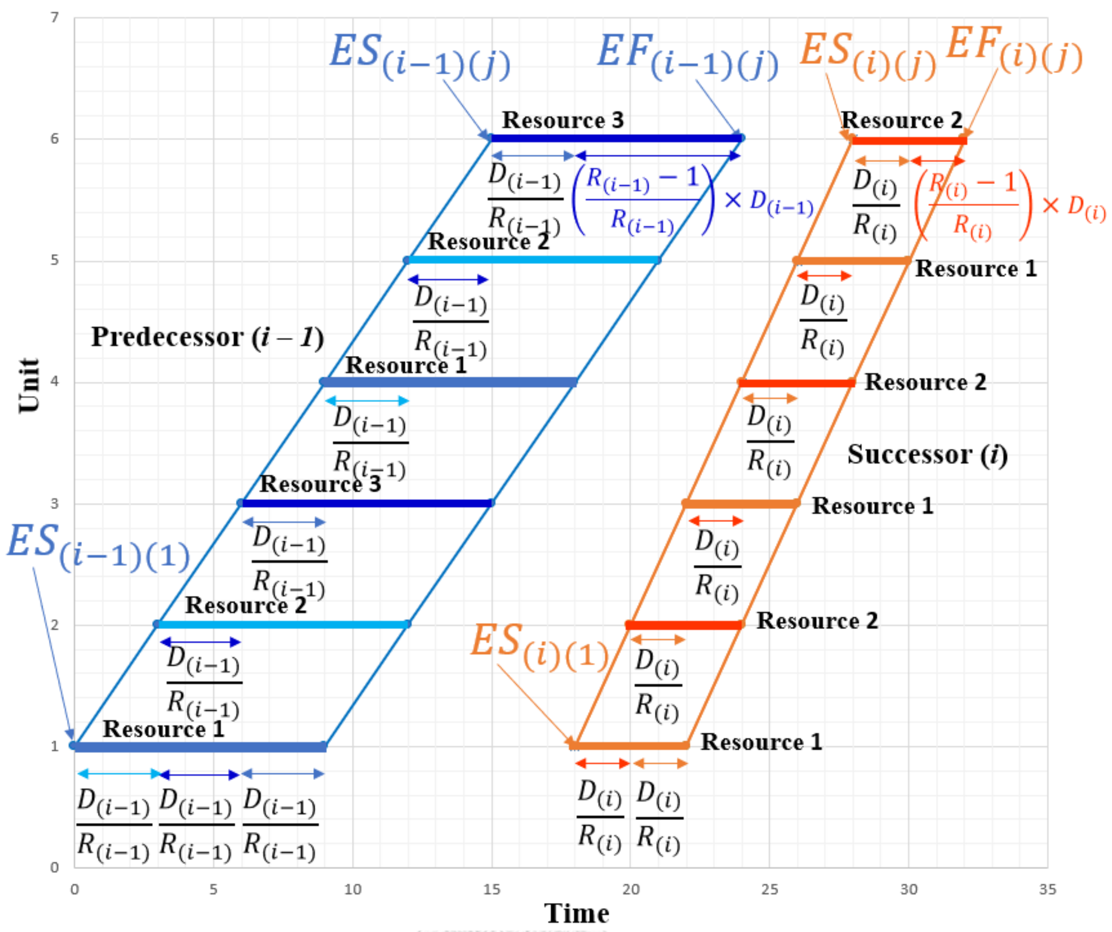


Figure 4.2 LOB diagram of predecessor (activity i – 1) and successor (activity i)

With Eq.5 and Eq.6, Early Start time and Early Finish time of the two successive activities are shown below.

The Early Start time of the predecessor at jth unit from Eq. 5

$$ES_{(i-1)(j)} = ES_{(i-1)(1)} + \left(\frac{1}{m_{(i-1)}}\right) \times (N-1) \quad \text{...} (10)$$

The Early Finish time of the predecessor at jth unit from Eq. 6

$$EF_{(i-1)(j)} = ES_{(i-1)(1)} + \left(\frac{1}{m_{(i-1)}}\right) \times (N) + \left(\frac{R_{(i-1)}-1}{R_{(i-1)}}\right) \times D_{(i-1)} \quad \text{...........} (11)$$

The Early Start time of the successor at jth unit from Eq. 5

$$ES_{(i)(j)} = ES_{(i)(1)} + \left(\frac{1}{m_{(i)}}\right) \times (N-1) \quad \text{..} (12)$$

The Early Finish time of the successor at jth unit from Eq. 6

$$EF_{(i)(j)} = ES_{(i)(1)} + \left(\frac{1}{m_{(i)}}\right) \times (N) + \left(\frac{R_{(i)} - 1}{R_{(i)}}\right) \times D_{(i)} \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$ES_{(i-1)(j)}$ = Early Start time of predecessor at the jth unit

$ES_{(i-1)(1)}$ = Early Start time of predecessor at the 1st unit

$EF_{(i-1)(j)}$ =Early Finish time of predecessor at the jth unit

$m_{(i-1)}$ = Rate of delivery of predecessor

N = Number of unit

$R_{(i-1)}$ = Number of specific resources of predecessor

$D_{(i-1)}$ = Activity duration of predecessor

$ES_{(i)(j)}$ = Early Start time of successor at the jth unit

$ES_{(i)(1)}$ = Early Start time of successor at the 1st unit

$EF_{(i)(j)}$ =Early Finish time of successor at the jth unit

$m_{(i)}$ = Rate of delivery of successor

$R_{(i)}$ = Number of specific resources of successor

$D_{(i)}$ = Activity duration of successor

To consider predecessor and successor in one representative equation, this study introduces two new variables $DSS_{(1)}$ and $DFS_{(j)}$ as representatives duration between early start times and duration between early finish times of two consecutive activities.

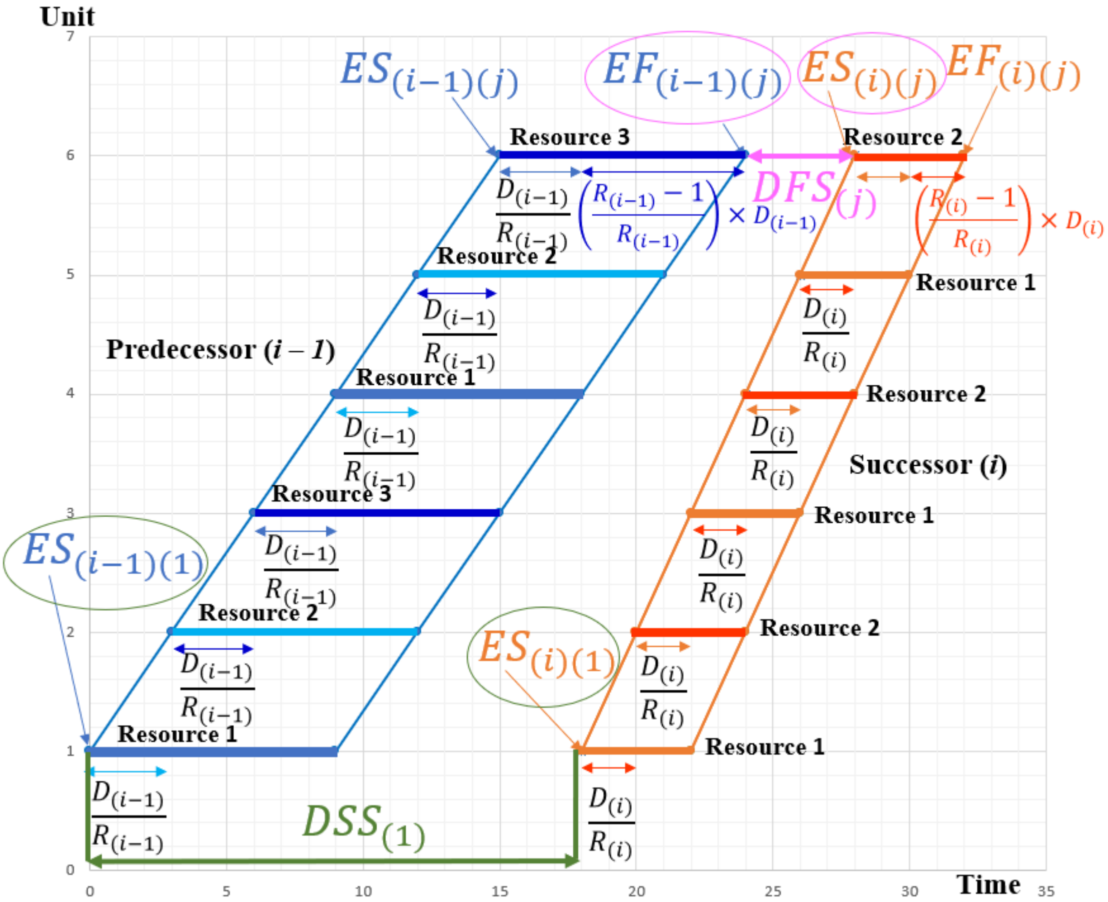


Figure 4.3 $DSS_{(1)}$ and $DFS_{(j)}$ of LOB diagram

The first variable is $DSS_{(1)}$, the difference time between the early start time of successor at the 1st unit and the early start time of predecessor at the 1st unit, is shown in Eq. 7. The second variable is $DFS_{(j)}$, the difference time between the early start time of successor at jth unit and the early finish time of predecessor at jth unit, is shown in Eq. 8.

$$DSS_{(1)} = ES_{(i)(1)} - ES_{(i-1)(1)} \text{.........} Eq.7$$

$DSS_{(1)}$ = The difference time between the early start time of successor at the 1st unit and the early start time of predecessor at the 1st unit

$ES_{(i-1)(1)}$ = Early Start time of predecessor at the 1st unit

$ES_{(i)(1)}$ = Early Start time of successor at the 1st unit

$$DFS_{(j)} = ES_{(i)(j)} - EF_{(i-1)(j)} \text{.........} Eq.8$$

$DFS_{(j)}$ = The difference time between the early start time of successor at j[th] unit and the early finish time of predecessor at j[th] unit

$ES_{(i)(j)}$ = Early Start time of successor at the j[th] unit

$EF_{(i-1)(j)}$ = Early Finish time of predecessor at the j[th] unit

With variables $DSS_{(1)}$ and $DFS_{(i)}$, the representative equation of predecessor and successor is derived as the following:

Repeat from (12);

$$ES_{(i)(j)} = ES_{(i)(1)} + \left(\frac{1}{m_{(i)}}\right) \times (N-1) \qquad (12)$$

Substitute $ES_{(i)(j)}$ in Eq. 8 with $ES_{(i)(j)}$ from (12);

$$DFS_{(j)} = ES_{(i)(1)} + \left(\frac{1}{m_{(i)}}\right) \times (N-1) - EF_{(i-1)(j)} \qquad (14)$$

Repeat from (11);

$$EF_{(i-1)(j)} = ES_{(i-1)(1)} + \left(\frac{1}{m_{(i-1)}}\right) \times (N) + \left(\frac{R_{(i-1)}-1}{R_{(i-1)}}\right) \times D_{(i-1)} \qquad (11)$$

Substitute $EF_{(i-1)(j)}$ in (14) with $EF_{(i-1)(j)}\ from$ (11);

$$DFS_{(j)} = ES_{(i)(1)} + \left(\frac{1}{m_{(i)}}\right) \times (N-1) - (ES_{(i-1)(1)} + \left(\frac{1}{m_{(i-1)}}\right) \times (N) + \left(\frac{R_{(i-1)}-1}{R_{(i-1)}}\right) \times D_{(i-1)}) \qquad (15)$$

Rearrange (15) in the form of $ES_{(i)(1)} - ES_{(i-1)(1)}$;

$$DFS_{(j)} = ES_{(i)(1)} - ES_{(i-1)(1)} + \left(\frac{1}{m_{(i)}}\right) \times (N-1) - \left(\frac{1}{m_{(i-1)}}\right) \times (N) - \left(\frac{R_{(i-1)}-1}{R_{(i-1)}}\right) \times D_{(i-1)}) \qquad (16)$$

Substitute $ES_{(i)(1)} - ES_{(i-1)(1)}$with $DSS_{(1)}$ from Eq.7;

$$DFS_{(j)} = DSS_{(1)} + \left(\frac{1}{m_{(i)}}\right) \times (N-1) - \left(\frac{1}{m_{(i-1)}}\right) \times (N) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)}) \text{.......} Eq.9$$

Eq.9 is the representative equation for two consecutive activities (predecessor and successor) by examining $DSS_{(1)}$ and $DFS_{(j)}$ instead of $ES_{(i)(j)}$ and $ES_{(i-1)(j)}$.

4.2.3 Definition of $DSS_{(1)}$ value and $DFS_{(j)}$ value

In the LOB scheduling, the slope line of activity is an essential factor to create the LOB diagram. This study considers the slope line and proposes the utilization of $DSS_{(1)}$ and $DFS_{(j)}$ as follows. For the case of convergent lines in Figure 4.4, the slope line of the predecessor ($m_{(i-1)}$) is lower than the slope line of the successor ($m_{(i)}$). The line of predecessor and the line of successor converge to each other when the numbers of units increase. Thus, the critical point of sequence logic locates at the last units. Thus, $DSS_{(1)}$ and $DFS_{(j)}$ can be used not only to provides the key for project duration but also control the sequence logic of two consecutive activities.

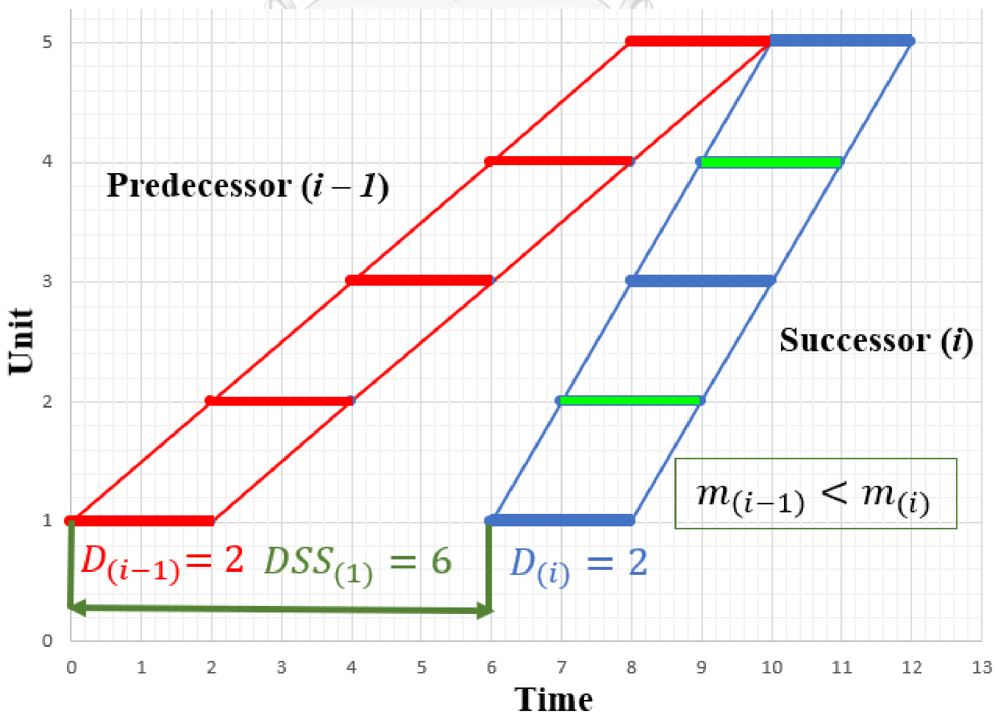


Figure 4.4 Case 1 convergent lines of slopes of two consecutive activities

From Figure 4.4 and Eq.8, the value of $DFS_{(j)}$ presents conditions that can be used to verify sequence logic for the convergent lines for any unit j in consideration as follows:

If $DFS_{(j)} < 0$ (negative value) then ($EF_{(i-1)(j)} > ES_{(i)(j)}$)

- Successor starts before predecessor finished and the logical sequence is violated.

If $DFS_{(j)} > 0$ (positive value) then ($EF_{(i-1)(j)} < ES_{(i)(j)}$)

- Successor starts after predecessor finished with some space-time. This fulfills sequence and logical sequence is acceptable.

If $DFS_{(j)} = 0$ then ($EF_{(i-1)(j)} = ES_{(i)(j)}$)

- Successor instantly starts when predecessor has just finished. This condition fulfills sequence logic and $DSS_{(1)}$ is minimum.

In the case of the convergent lines, Eq. 9 can provide minimum $DSS_{(1)}$ by assigning $DFS_{(J)}$ as 0 where J is the number of total units in consideration. A demonstration of the application of Eq.9 is presented below.

From Figure 4.4, $D_{(i-1)}$= 2 days, $D_{(i)}$= 2 days, N = 5 units, $R_{(i-1)}$= 1, $R_{(i)}$= 2

$$DFS_{(J)} = DSS_{(1)} + \left(\frac{2}{2}\right) \times (5-1) - \left(\frac{2}{1}\right) \times (5) - \left(\frac{1-1}{1}\right) \times 2 \text{..................} (17)$$

$$DFS_{(J)} = 0, DSS_{(1)} = 6$$

For the case of divergent lines in Figure 4.5, the slope line of predecessor ($m_{(i-1)}$) is higher than or equal to the slope line of successor ($m_{(i)}$). The line of predecessor and the line of successor diverge from each other when the numbers of units increase. Thus, the critical point of sequence logic always locates at the first unit.

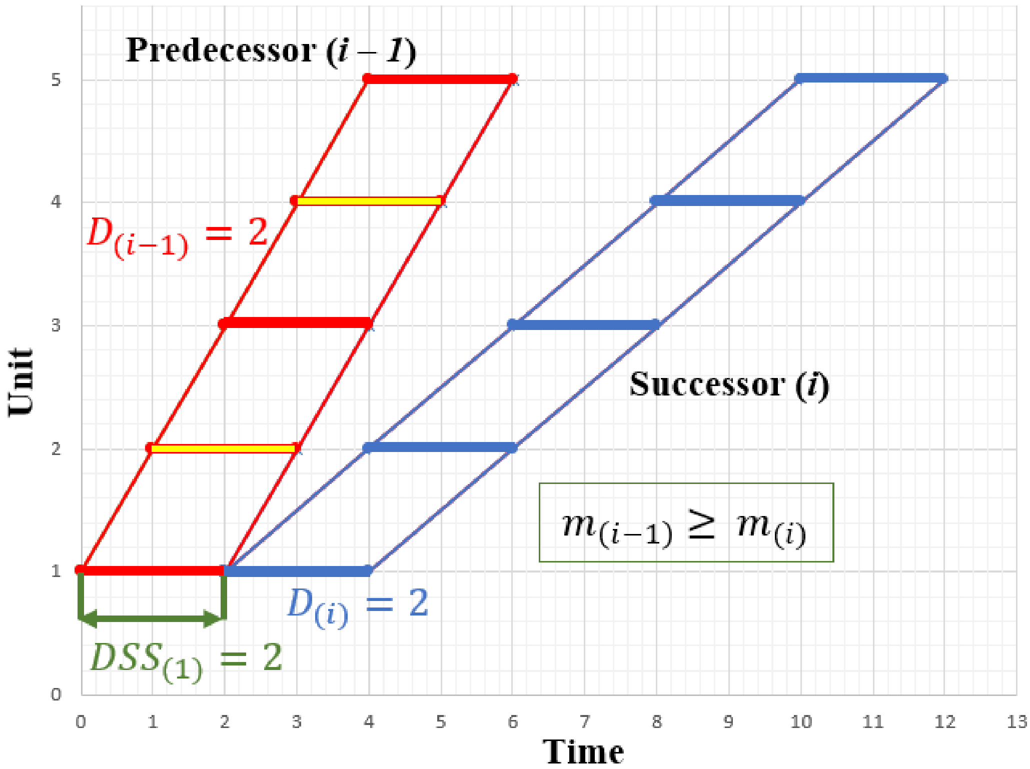


Figure 4.5 Case 2 divergent lines of slopes of two consecutive activities

From Figure 4.5 and Eq.7, the value of $DSS_{(1)}$ presents conditions that can be used to verify sequence logic for the divergent lines like the following:

If $DSS_{(1)} < D_{(i-1)}$ then $(EF_{(i-1)(1)} > ES_{(i)(1)})$

- Successor starts when predecessor has not finished, meaning that sequence logic is violated.

If $DSS_{(1)} > D_{(i-1)}$ then $(EF_{(i-1)(1)} < ES_{(i)(1)})$

- Successor starts after predecessor finished with some space-time and the sequence logic of two activity is fulfilled.

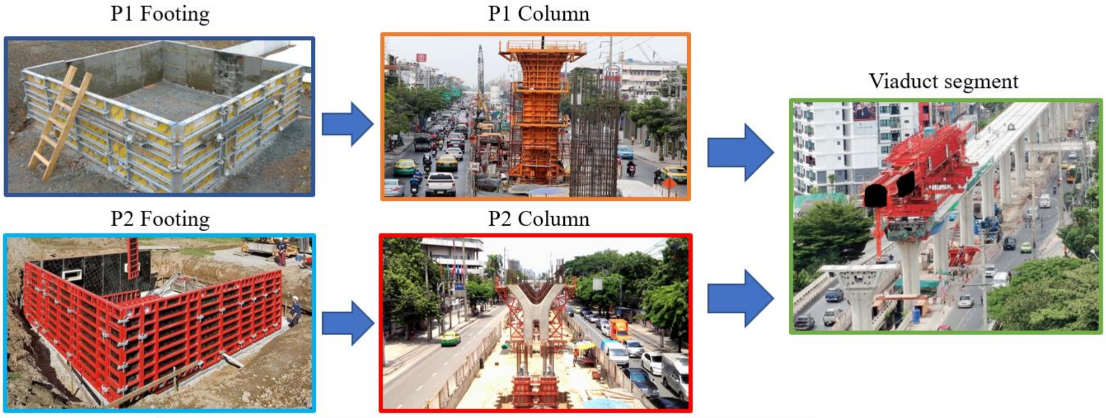
If $DSS_{(1)} = D_{(i-1)}$ then $(EF_{(i-1)(j)} = ES_{(i)(j)})$

- Successor instantly starts when predecessor just finished. The sequence is fulfilled logic and $DSS_{(1)}$ is minimum.

This section presents the representative equation where predecessor and successor are considered together in one equation. The representative equation can be used to verify the sequence logical at the last unit for two consecutive activities or, in other words, for an identical type of unit. However, in the case that units of an identical type are separated to be sets of units along the alignment and between the sets are the units of other types. So when the resources of an identical type complete the task at the current set, they have to move to the next set by focusing only on its own type. Thus, the operation of the specific resource for an identical type is then independent of the other type. Therefore, this study defines these various types of units, many sets of units, and the operation of resources containing in a linear project as the multi-identical type of units. The difference between an identical type of unit and the multi-identical type of units is the consideration of the sequence logic of the convergent lines ($m_{(i-1)} < m_{(i)}$). The multi-identical type of units may cause the critical point of sequence logic not located at the last unit of the first set because the presented representative equation (Eq.9) is developed by examining only the first set of the type of units, so it may provide the incorrect result when the critical point does not locate at the first set. Thus, the better representative equations for multi-identical types of units are essentially required to cover the critical point at any set of units. The next section explains the advance modification of the representative equations.

4.2.4 Equations for multi-identical types of units

This section uses the scheduling problem of an example project to explain the advance modification of representative equations for multi-identical types of units. The example project contains 13 piers which are designed into two types of piers; P1 and P2. Each type consists of two structural elements which are Footing and Column. The location and type of pier are shown in Figure 4.6. For example, at the station (unit) 1-6 and station (unit) 11-13 structure type P1 is used, whereas structure type P2 is located at station 7-9. The sequence of work starts from Footing, Column, and the viaduct segment erection. The viaduct segment installed by using a launching gantry is the last activity and starts from station 1 to station 13. Hence, the LOB diagram of the example project can be created as showing in Figure 4.7. P1 Footing, P2 Footing, P1 Column, and P2 Column require specific formworks for their unique geometries.



Station	1	2	3	4	5	6	7	8	9	10	11	12	13
Type	P1	P1	P1	P1	P1	P1	P2	P2	P2	P1	P1	P1	P1
Footing													
Column													
Segment													

Figure 4.6 Example project for multi-identical types of units

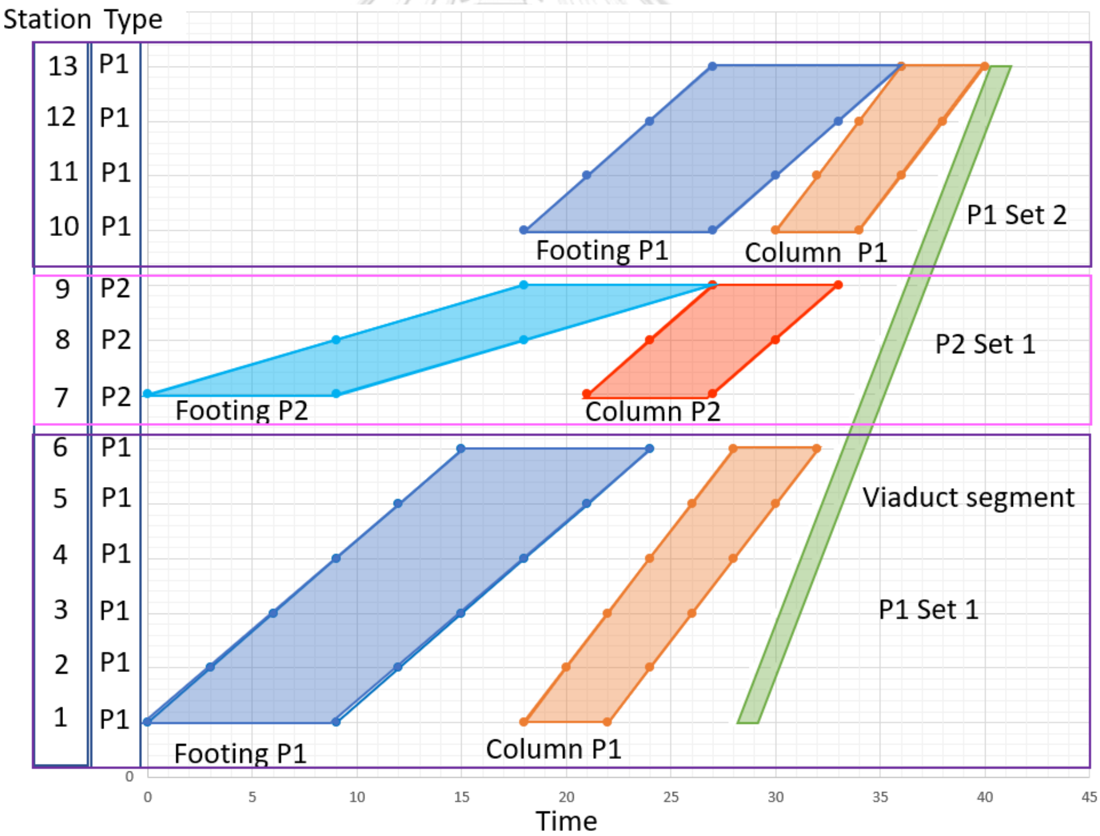


Figure 4.7 LOB diagram of the example project

In this example, Footing P1 has two sets of units where the first identical set is for station 1-6 and the second identical set is for station 10-13. Footing P2 has one set of identical units and the set includes station 7-9. This combination of various identical types of units and many sets of units is what this study describes as the multi-identical types of units.

In addition, specific resources such as formworks or workers must be used at a certain location at a suitable time, so that all the structural element can be finished before the launching gantry installed the segments. The resources must be used continuity in order to reduce the cost and the optimal size of the resource must be allocated in all multi-identical types of units to make the overall operation complete under the desired project duration.

4.2.4.1 Viaduct segment erection and its predecessor

From Figure 4.7, the P1 column and segment erection are selected to demonstrate the deriving of representative equations for the activity of launching gantry for the viaduct segment erection and its predecessor as shown in Figure 4.8.

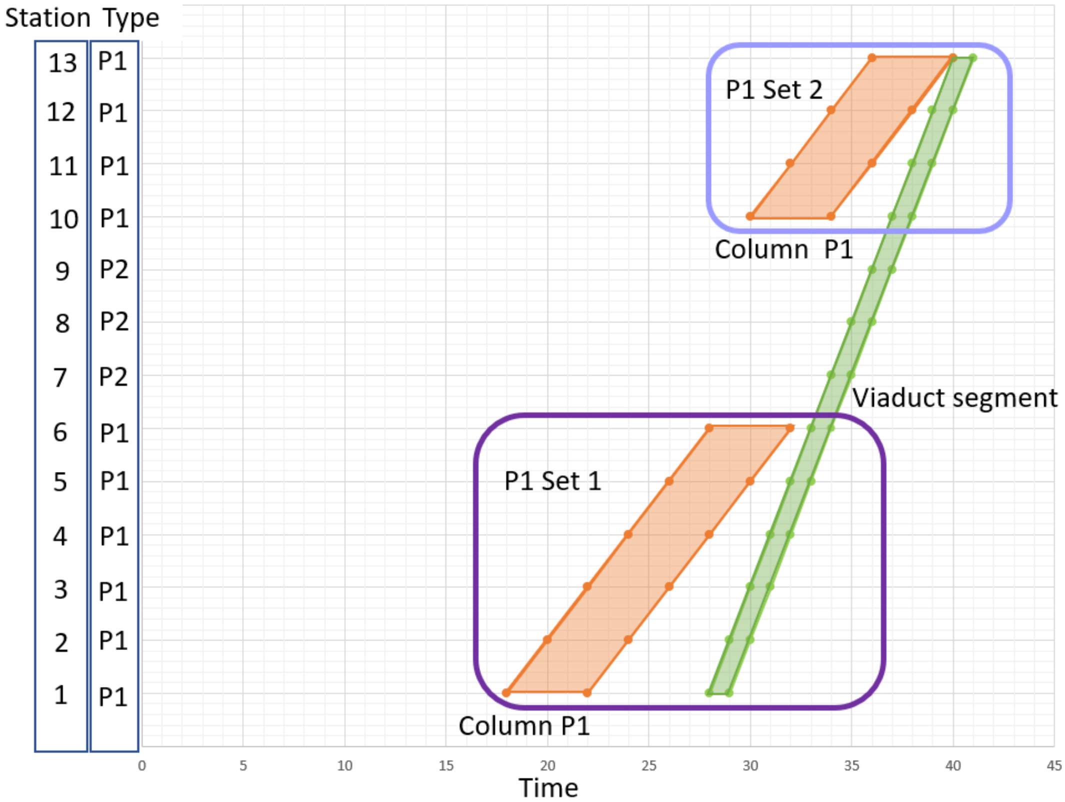


Figure 4.8 A pair of viaduct segment erection and its predecessor

Figure 4.8 is showing that units of column P1 are split into two sets. The specific resources for column P1 build the column continuously by started from station 1 to station 6 of set 1, then move to build at station 10 to station 13 of set 2. Concurrently, the launching gantry continuously erects the viaduct segments from station 1 to station 13. The Eq. 9 for each set of units is presented as the following.

Equation of set 1 from Eq.9. Where J is the number of total units of the set.

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \quad (18)$$

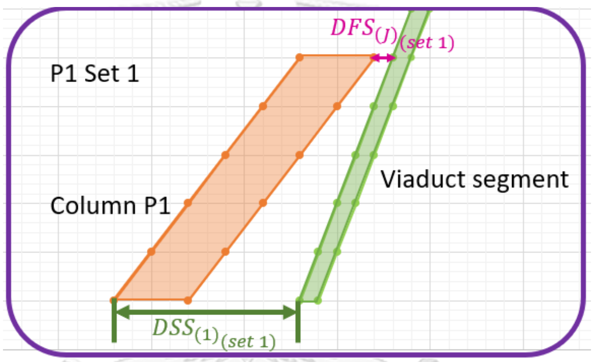


Figure 4.9 $DFS_{(J)}$ and $DSS_{(1)}$ of P1 set 1 for column P1 and segment erection

Equation of set 2 from Eq.9. Where J is the number of total units of the set.

$$DFS_{(J)(set\ 2)} = DSS_{(1)(set\ 2)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 2)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 2)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \quad (19)$$

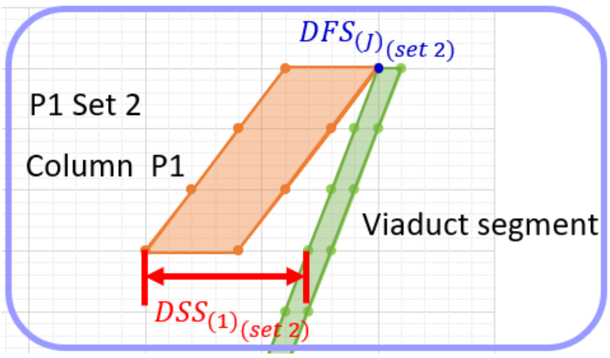


Figure 4.10 $DFS_{(J)}$ and $DSS_{(1)}$ of P1 set 2 for column P1 and segment erection

The equation of each set contains $DSS_{(1)}$ which is an unknown variable. To reduce the unknown variable, this study tries to change $DSS_{(1)}$ set 2 to a variable of set 1 with the following procedures. The first reason is that $DSS_{(1)}$ of set 2 is the difference time between the start time of successor at the 1st unit of set 2 and start time of predecessor at the 1st unit of set 2 and the second reason is that the conditions that all resources are utilized continuously. The consideration of work continuity presents that each resource starts after a time (D/R) relatively to its start time of the earlier preceding unit. Thus, the first activity at the first unit of set 2 technically starts after the last unit of set 1 begins for time D/R days as shown in Figure 4.11.

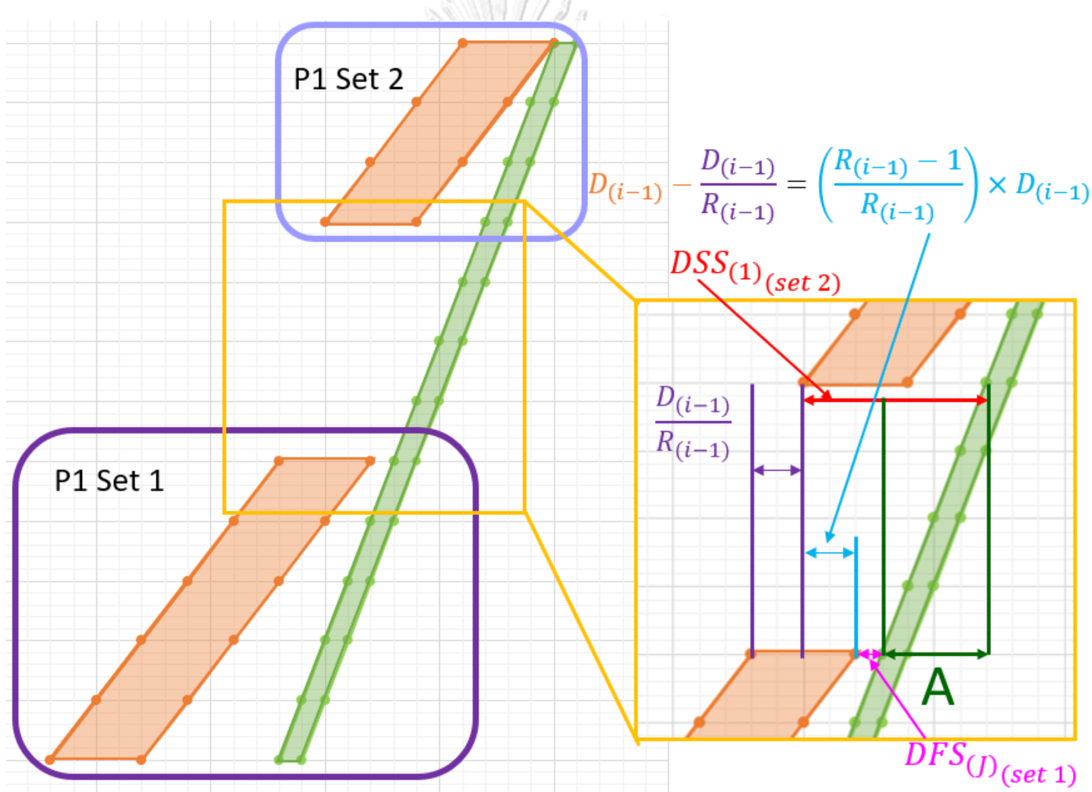


Figure 4.11 Analysis of variables between 2 consecutive sets

To connect the equation of set 1 (18) and the equation of set 2 (19), this study examines to transform variable $DSS_{(1)}$ of set 2 in terms of variable $DFS_{(J)}$ of set 1. From Figure 4.11, $DSS_{(1)}$ of set 2 is equal to the summation of the term of $R_{(i-1)}$, $DFS_{(J)}$ of set 1, and the A. A is the difference time between the early start time of successor at 1st unit of set 2 and the early start time of successor at Jth unit of set 1.

$$DSS_{(1)(set\ 2)} = \left(\frac{R_{(i-1)}-1}{R_{(i-1)}}\right) \times D_{(i-1)} + DFS_{(J)(set\ 1)} + A \text{........................} (20)$$

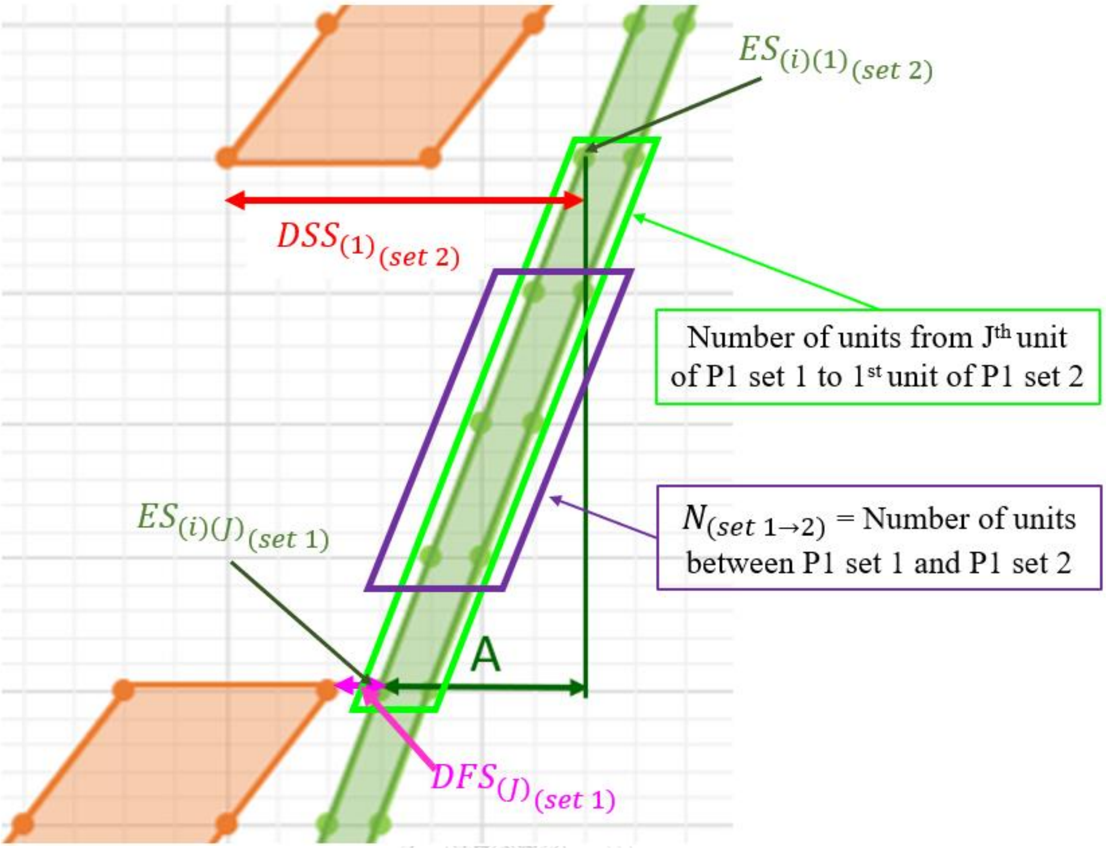


Figure 4.12 Consideration of the number of units between two sets

The value of the A is the difference between the start time of successor at jth unit of set 1 ($ES_{(i)(J)_{(set\ 1)}}$) and the start time of successor at 1st unit of set 2 ($ES_{(i)(1)_{(st\ 2)}}$) as showing in Figure 4.12. From Eq. 5, Early Start time at any jth unit can be retrieved from Early Start time at any afore unit. Thus, the successor can provide the value of the A with Eq. 5 as the following.

From Eq. 5;

$$ES_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N-1) \qquad Eq. 5$$

Substitute $ES_{(i)(J)_{(set\ 1)}}$, $ES_{(i)(1)_{(set\ 2)}}$ and $m_{(i)}$ into Eq. 5;

$$ES_{(i)(1)_{(set\ 2)}} = ES_{(i)(J)_{(set\ 1)}} + \left(\frac{1}{m_{(i)}}\right) \times (N-1) \qquad (21)$$

From Eq. 5, N is defined as the number of units from the first unit to any jth unit. Technically, N is the number of units from any afore unit of jth unit to jth unit. Therefore, N of this case can be obtained from the number of units from Jth unit of P1 set 1 to 1st unit of P1 set 2 as shown in Figure 4.12. N is equal to the summation of the number of units between P1 set1 and P1 set 2 ($N_{(set\ 1\rightarrow 2)}$) with one unit of Jth unit of set 1 (1 unit) and one unit of 1st unit of set 2 (1 unit).

$$N = N_{(set\ 1\rightarrow 2)} + 1 + 1 = N_{(set\ 1\rightarrow 2)} + 2 \quad (22)$$

Substitute N in (21) with N from (22);

$$ES_{(i)(1)_{(set\ 2)}} = ES_{(i)(J)_{(set\ 1)}} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1\rightarrow 2)} + 2 - 1\right) \quad (23)$$

Value of A from the definition as showing in Figure 4.12;

$$A = ES_{(i)(1)_{(set\ 2)}} - ES_{(i)(J)_{(set\ 1)}} \quad (24)$$

Rearrange the equation (23);

$$ES_{(i)(1)_{(set\ 2)}} - ES_{(i)(J)_{(set\ 1)}} = \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1\rightarrow 2)} + 1\right) \quad (25)$$

Substitute $ES_{(i)(1)_{(set\ 2)}} - ES_{(i)(J)_{(set\ 1)}}$ in (25) with A from (24);

$$A = \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1\rightarrow 2)} + 1\right) \quad (26)$$

Repeat from equation (20);

$$DSS_{(1)_{(set\ 2)}} = \left(\frac{R_{(i-1)}-1}{R_{(i-1)}}\right) \times D_{(i-1)} + DFS_{(J)_{(set\ 1)}} + A \quad (20)$$

Substitute A in (20) with A from (26);

$$DSS_{(1)_{(set\ 2)}} = \left(\frac{R_{(i-1)}-1}{R_{(i-1)}}\right) \times D_{(i-1)} + DFS_{(J)_{(set\ 1)}} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1\rightarrow 2)} + 1\right) \quad (27)$$

Repeat from equation (19);

$$DFS_{(J)(set\ 2)} = DSS_{(1)(set\ 2)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 2)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 2)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \text{.................................}(19)$$

Substitute $DSS_{(1)(set\ 2)}$ in (19) with $DSS_{(1)(set\ 2)}$ from (27);

$$DFS_{(J)(set\ 2)} = \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} + DFS_{(J)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1 \to 2)} + 1\right) + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 2)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 2)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \text{.......................................}(28)$$

Rearrange the equation (28);

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 2)} + N_{(set\ 1 \to 2)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 2)}\right) \text{....................}(29)$$

Equation (29) is the representative equation for P1 set 2 which contains the variable ($DFS_{(J)(set\ 1)}$) from the equation P1 set 1 (18). In these equations, the unknown two variables are then reduced to one. The $DFS_{(J)(set\ 1)}$ is not only used to verify the sequence logic but also connects the equations of two consecutive sets.

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \text{..............................}(18)$$

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 2)} + N_{(set\ 1 \to 2)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 2)}\right) \text{..................}(29)$$

From the Eq. 9 and the critical point locating at the last unit (J^{th}) of the sets, the equation for set 1 can be rewritten as Eq. 10, where i is the repetitive activity and J is the number of total units of the set in consideration.

$$DFS_{(J)}{}_{(set\ 1)} = DSS_{(1)}{}_{(set\ 1)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \dots\dots Eq. 10$$

$DFS_{(J)}{}_{(set\ 1)}$ = The difference time between the early start time of successor at J^{th} unit of set 1 and the early finish time of predecessor at J^{th} unit of set 1

$DSS_{(1)}{}_{(set\ 1)}$= The difference time between the early start time of successor at the 1^{st} unit of set 1 and the early start time of predecessor at the 1^{st} unit of set 1

$m_{(i)}$ = Rate of delivery of successor, $m_{(i-1)}$ = Rate of delivery of predecessor

$N_{(set\ 1)}$ = Number of units J of set 1

$R_{(i-1)}$ = Number of specific resources of predecessor

$D_{(i-1)}$ = Activity duration of predecessor

Furthermore, when the sets of units of an identical type are more than two sets, the similar modifications as the previous section can provide representative equations of set 3, set 4,…, and any set v while containing $DFS_{(J)}$ of the previous set ($DFS_{(J)}{}_{(set\ v-1)}$). Hence, from equation (29), the representative equation of set v where v > 1 is described into a general term as Eq. 10, where i is the activity repetitive, J is the number of total units of the set, and v is the set in consideration.

$$DFS_{(J)}{}_{(set\ v)} = DFS_{(J)}{}_{(set\ v-1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ v)} + N_{(set\ (v-1)\rightarrow v)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ v)}\right) \dots\dots Eq. 11$$

$DFS_{(J)}{}_{(set\ v)}$ = The difference time between the early start time of successor at J^{th} unit of set v and the early finish time of predecessor at J^{th} unit of set v

$DFS_{(J)(set\ v-1)}$ = The difference time between the early start time of successor at J^th^ unit of set (v - 1) and the early finish time of predecessor at J^th^ unit of set (v – 1)

$m_{(i)}$ = Rate of delivery of successor

$m_{(i-1)}$ = Rate of delivery of predecessor

$N_{(set\ v)}$ = Number of units J of set v

$N_{(set\ (v-1)\rightarrow v)}$ = Number of units between set v and set (v-1)

For the convergent lines of two consecutive activities with many sets of units, the Eq.10 and Eq.11 are used to verify the sequence logic when the successor is the repetitive activity for all types of units and the predecessor is the repetitive activity for an identical type of units. The number of Eq. 11 depends on the number of sets from set 2 to any set v. Therefore, the sequence logic of two consecutive activities is verified by examining the $DFS_{(J)}$ from every set. From the definition of $DFS_{(J)}$, the sequence logic is fulfilled whenever $DFS_{(J)}$ equal to or higher than 0. Thus, it can be summarized that the minimum $DSS_{(1)}$ of set 1 is found when at least $DFS_{(J)}$ of one set equal to 0 and $DFS_{(J)}$ of other sets equal to or higher than 0. To verify the sequence logic, set 1 always uses Eq.10 and set v, when v > 1, uses Eq. 11. The following equations are the examples of using Eq. 10 and Eq. 11 when there are 4 sets of units (V = 4). Then, v is the set in consideration and V is the number of total sets.

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)}$$

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 2)} + N_{(set\ 1\rightarrow 2)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 2)}\right)$$

$$DFS_{(J)(set\ 3)} = DFS_{(J)(set\ 2)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 3)} + N_{(set\ 2\rightarrow 3)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 3)}\right)$$

$$DFS_{(J)(set\ 4)} = DFS_{(J)(set\ 3)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 4)} + N_{(set\ 3\rightarrow 4)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 4)}\right)$$

The example of using Eq. 10 and Eq. 11 illustrates that every equation has $DFS_{(J)}$ which can be used to verify the sequence logic of its set. $DFS_{(J)}$ also connects between two equations of consecutive sets that can reduce many unknown $DFS_{(j)}$ to one unknown $DFS_{(J)}$. From the example, the unknown $DFS_{(J)}$ is the $DFS_{(J)}{}_{(set\ 4)}$ which is the last set of units in the example. Thus, the calculation of $DFS_{(J)}$ by Eq.10 and Eq. 11 starts with a trial of first $DSS_{(1)}{}_{(set\ 1)}$ to find $DFS_{(J)}{}_{(set\ 1)}$, then transfers $DFS_{(J)}{}_{(set\ 1)}$ to the equation of set 2. This process is repeated until acquiring the $DFS_{(J)}$ of the last set ($DFS_{(J)}{}_{(set\ V)}$). After that, the $DFS_{(J)}$ of every set is used to verify the sequence logic. If the sequence logic is violated, the whole process must be repeated by trial more value of $DSS_{(1)}{}_{(set\ 1)}$ until the sequence logic is fulfilled.

The following section is the demonstration of Eq. 10 and Eq. 11. The example project of multi-identical types of units is utilized. From Figure 4.13, the considering pair of consecutive activities is the pair of the column P1 and viaduct segment erection. The viaduct segment is erected continuously from station 1 to station 13. The relevant information for the calculation is shown in Figure 4.13.

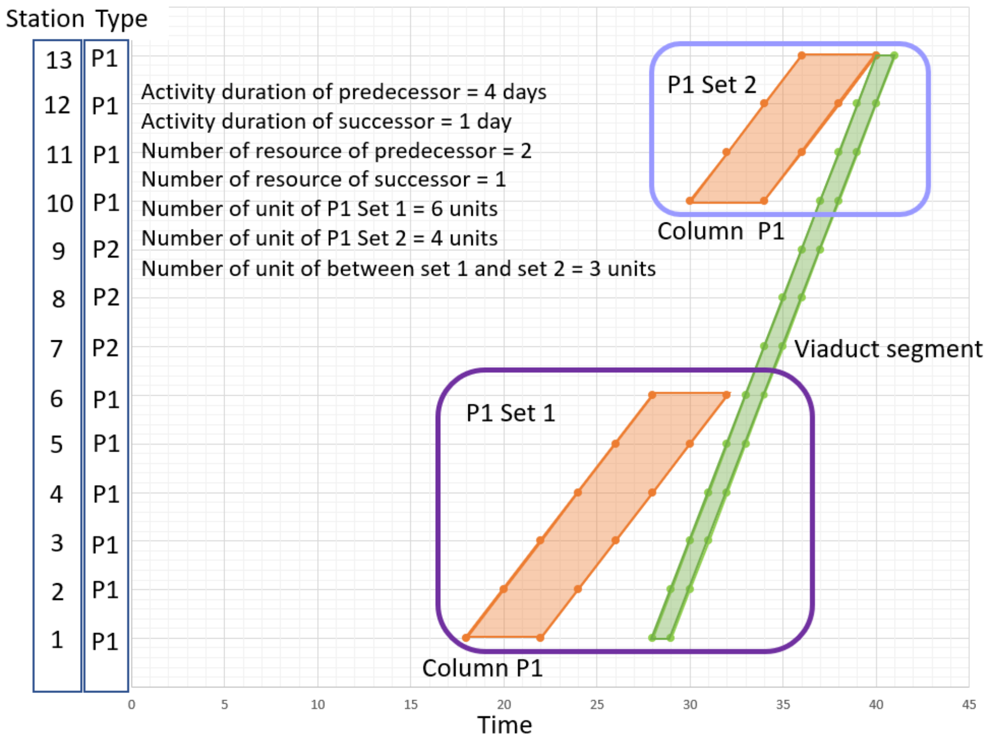


Figure 4.13 Information of the pair of the column P1 and viaduct segment erection

There are two sets of column P1, so Eq.10 and one Eq.11 are required.

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)}$$

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 2)} + N_{(set\ 1 \to 2)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 2)}\right)$$

Substitute the known variables into the equations;

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} + \left(\frac{1}{(1)}\right) \times (6 - 1) - \left(\frac{4}{2}\right) \times (6) - \left(\frac{2-1}{2}\right) \times 4$$

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} - 9 \text{ ....}(30)$$

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} + \left(\frac{1}{1}\right)(4 + 3) - \left(\frac{4}{2}\right)(4)$$

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} - 1 \text{ ....}(31)$$

$$DSS_{(1)(set\ 1)} = D_{(i-1)} = 4,\ DFS_{(J)(set\ 1)} = -5,\ DFS_{(J)(set\ 2)} = -6 \text{.....................}(32)$$

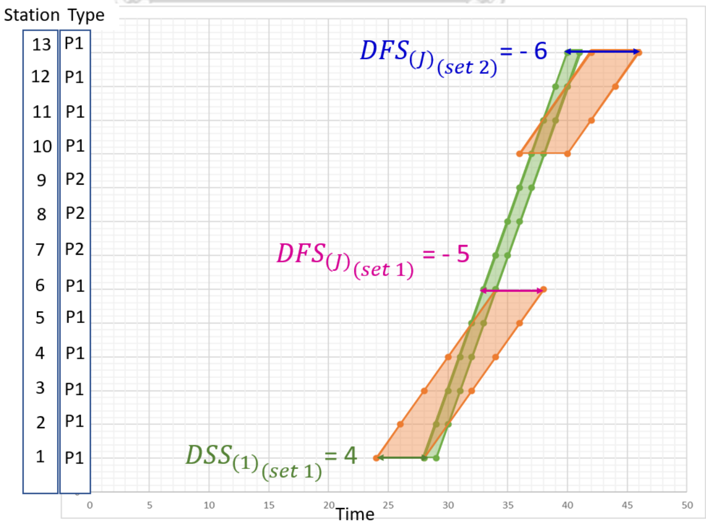


Figure 4.14 LOB diagram of $DSS_{(1)}$ is equal to the duration of the preceding activity

The result of $DFS_{(j)}$ of 2 sets shows that the sequence logic of the two sets is violated for 4 days of $DSS_{(1)}$.

$$DSS_{(1)(set\ 1)} = 9,\ DFS_{(J)(set\ 1)} = 0,\ DFS_{(J)(set\ 2)} = -1 \text{..............................}(33)$$

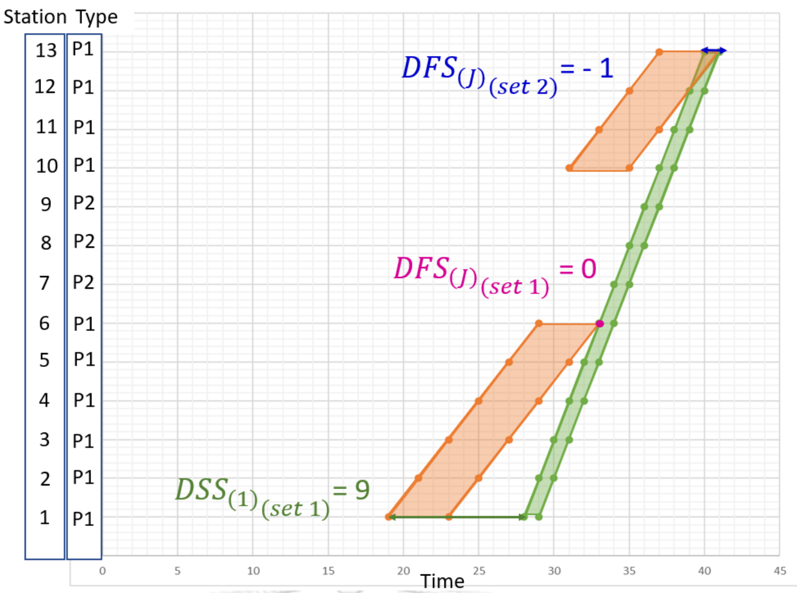


Figure 4.15 LOB diagram of $DSS_{(1)}$ is equal to 9 days

The result of $DFS_{(j)}$ of 2 sets shows that the sequence logic of set 1 is fulfilled but the sequence logic of set 2 is still violated for 9 days of $DSS_{(1)}$.

$$DSS_{(1)}{}_{(set\ 1)} = 10,\ DFS_{(j)}{}_{(set\ 1)} = 1,\ DFS_{(j)}{}_{(set\ 2)} = 0 \qquad (34)$$

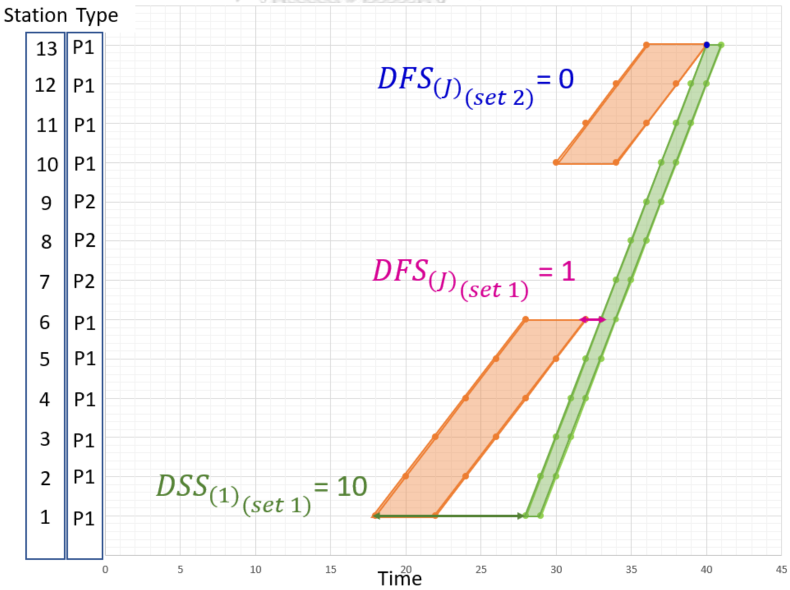


Figure 4.16 LOB diagram of $DSS_{(1)}$ is equal to 10 days

The result of $DFS_{(j)}$ of 2 sets shows that the sequence logic of the two sets is fulfilled for 9 days of $DSS_{(1)}$, so the minimum $DSS_{(1)}{}_{(set\ 1)}$ is equal to 10 days.

The previous example has shown the use of Eq.10 and Eq.11 for two sets of units., the next example with three sets of units is utilized to demonstrate the use of the representative equations. The preceding activity (i-1) is column construction with 6 days duration and two teams of resources are assigned. The succeeding activity (i) is segment erection which has 2 days duration and one team of a launching machine is assigned. As showing in Figure 4.17, the numbers of units in each set are 6, 3, and 4, respectively. The number of units between set 1st and set 2nd is 3 units and between set 2nd and set 3rd is 3 units. There are three sets in the example, so V is equal to 3. Eq.10 and two Eq.11 are required to verify the sequence logic.

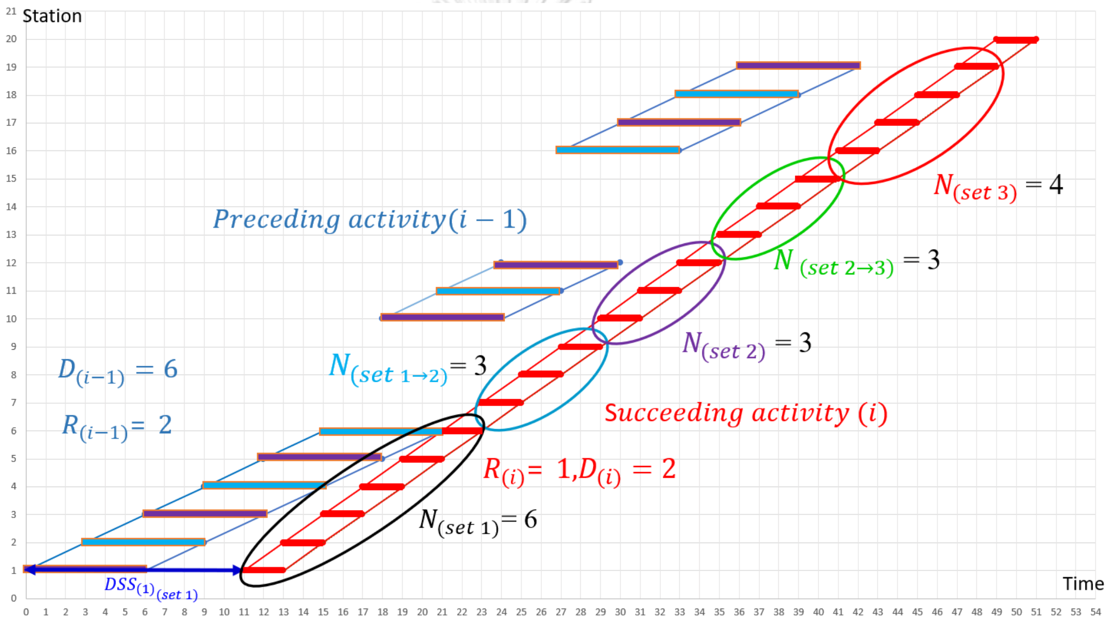


Figure 4.17 Example of the representative equation for multi-identical types of units

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)}$$

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 2)} + N_{(set\ 1\rightarrow 2)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 2)}\right)$$

$$DFS_{(J)(set\ 3)} = DFS_{(J)(set\ 2)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 3)} + N_{(set\ 2\rightarrow 3)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 3)}\right)$$

Substitute the known variables into the equations;

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} + \left(\frac{2}{1}\right) \times (6-1) - \left(\frac{6}{2}\right) \times (6) - \left(\frac{2-1}{2}\right) \times 6$$

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} - 11 \quad \text{......(35)}$$

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} + \left(\frac{2}{1}\right)(3+3) - \left(\frac{6}{2}\right)(3)$$

$$DFS_{(J)(set\ 2)} = DFS_{(J)(set\ 1)} + 3 \quad \text{......(36)}$$

$$DFS_{(J)(se\ 3)} = DFS_{(J)(set\ 2)} + \left(\frac{2}{1}\right)(3+4) - \left(\frac{6}{2}\right)(4)$$

$$DFS_{(J)(set\ 3)} = DFS_{(J)(set\ 2)} + 2 \quad \text{......(37)}$$

Trial first $DSS_{(1)(set\ 1)}$ with $DSS_{(1)(set\ 1)} = D_{(i-1)} = 6$

$$DSS_{(1)(set\ 1)} = 6,\ DFS_{(J)(set\ 1)} = -5,\ DFS_{(J)(set\ 2)} = -2,\ DFS_{(J)(set\ 3)} = 0 \quad \text{......(38)}$$

The result of $DFS_{(j)}$ of set 3 shows that the set 3 does not violate sequence logic, but $DFS_{(j)}$ of set 1 and set 2 still make violation of logical sequence.

Trial $DSS_{(1)(set\ 1)}$ with $DSS_{(1)(set\ 1)} = 8$

$$DSS_{(1)(set\ 1)} = 8,\ DFS_{(J)(set\ 1)} = -3,\ DFS_{(J)(set\ 2)} = 0,\ DFS_{(J)(set\ 3)} = 2 \quad \text{......(39)}$$

The results of $DFS_{(j)}$ of set 2 and set 3 show that the sequences are valid but DFS of set 1 still makes violation of logical sequence.

Trial $DSS_{(1)(set\ 1)}$ with $DSS_{(1)(set\ 1)} = 11$

$$DSS_{(1)(set\ 1)} = 11,\ DFS_{(J)(set\ 1)} = 0,\ DFS_{(J)(set\ 2)} = 3,\ DFS_{(J)(set\ 3)} = 5 \quad \text{......(40)}$$

The results of $DFS_{(J)}$ of all sets show that the sequences are fulfilled and $DFS_{(J)}$ of set 1 is equal to 0. Hence, the minimum $DSS_{(1)}$ of set 1 which achieves the sequence logic of all set is equal to 11 days.

For convergent lines, these two examples have shown that the critical point can be located at any set of units depending on the slope of the two lines, so the verification of sequence logic for every set of units is then necessary.

4.2.4.2 Two consecutive activities with the same type

From Figure 4.7, the P1 footing and the P1 column are chosen to illustrate the deriving of representative equations for the case that the successor and the predecessor are the same types as shown in Figure 4.18.

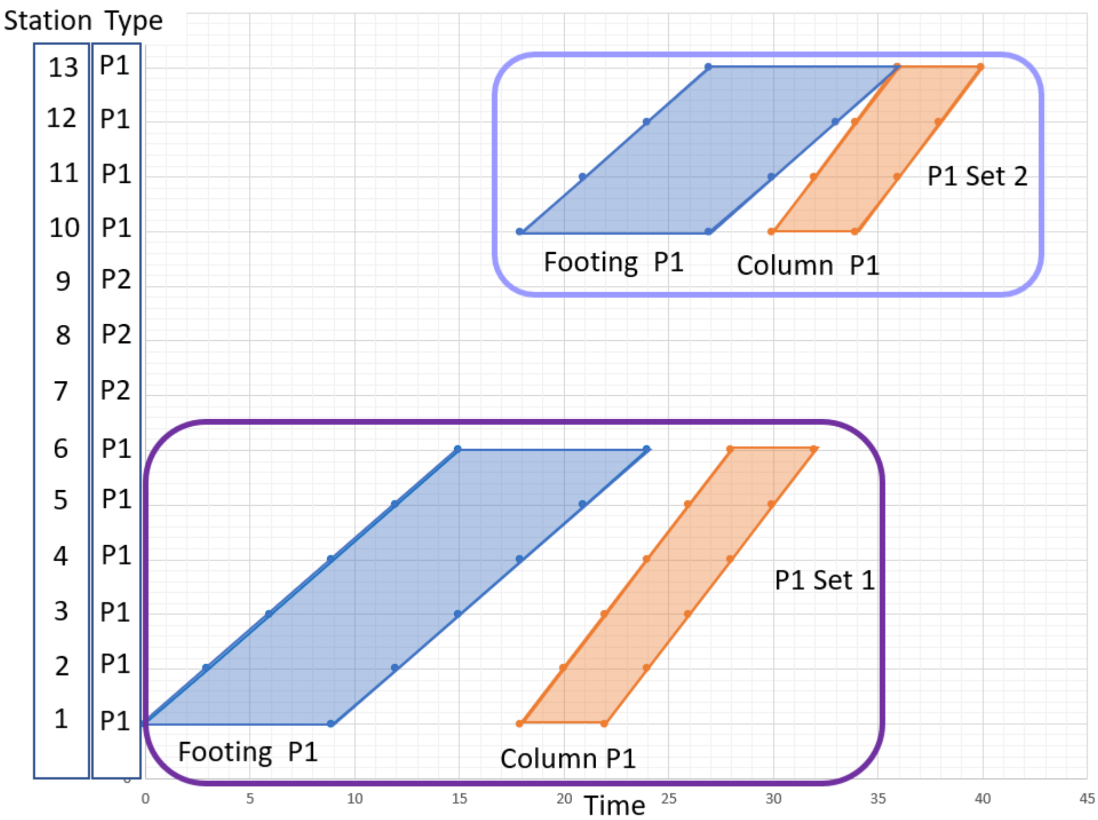


Figure 4.18 A pair of successor and predecessor of the same type

Figure 4.18 shows that units of Footing P1 and Column P1 are split into two sets. The specific resources for Footing P1 and column P1 are utilized continuously by starting from station 1 to station 6 of set 1, then moving to perform at station 10 to station 13 of set 2. The Eq. 10 for set 1 and Eq. 11 for set 2 are written as the following.

From Eq.10, the equation for set 1 is written as (41);

$$DFS_{(J)(set\ 1)} = DSS_{(1)(set\ 1)} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \quad (41)$$

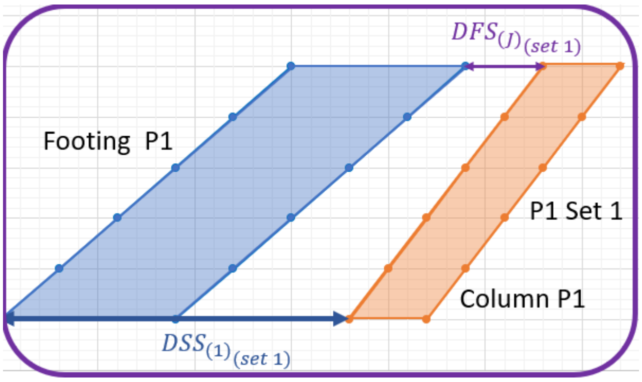


Figure 4.19 $DFS_{(J)}$ and $DSS_{(1)}$ of P1 set 1 for Footing P1 and Column P1

From Eq.11, the equation for set 2 is written as (42);

$$DFS_{(J)}{}_{(set\ 2)} = DFS_{(J)}{}_{(set\ 1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 2)} + N_{(set\ 1\to 2)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 2)}\right) \quad (42)$$

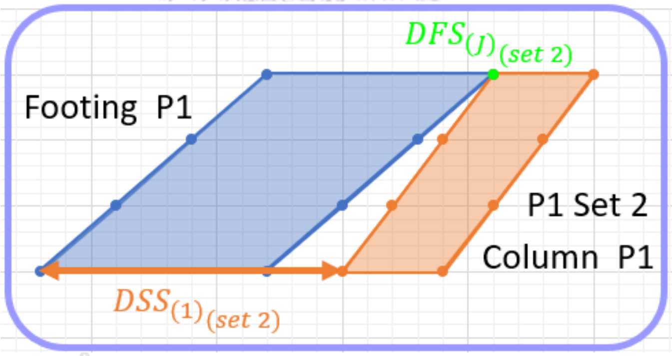


Figure 4.20 $DFS_{(J)}$ and $DSS_{(1)}$ of P1 set 2 for Footing P1 and Column P1

From (42), variable $N_{(set\ 1\to 2)}$ is the number of units between P1 set 1 and P2 set 2. Unlike the segment erection, the units in each set of column P1 (successor) in this case are equal to the number of units of footing P1 (predecessor) in each set as well. Thus, the variable $N_{(set\ 1\to 2)}$ is then zero. (42) can be rewritten as below.

$$DFS_{(J)}{}_{(set\ 2)} = DFS_{(J)}{}_{(set\ 1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 2)} + 0\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 2)}\right) \quad (43)$$

$$DFS_{(J)}{}_{(set\ 2)} = DFS_{(J)}{}_{(set\ 1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ 2)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ 2)}\right) \quad (44)$$

According to the condition of continuity of resource utilization, the resources of a type are independently utilized from the other types in the project. The numbers of units in each set of the two consecutive activities are equal. Therefore, for the convergent lines of this case, the critical point of the sequence logic is certainly located at the last units of the last set (J^{th} unit of set V). Thus, examining only $DFS_{(J)_{(set\,V)}}$ is sufficient to verify the sequence logic. In order to calculate $DFS_{(J)_{(set\,V)}}$ directly, the equations for the sets of units as Eq.10 and Eq.11 can be reduced the form in only one representative equation. The representative equation is derived as the following equation.

Substitute $DFS_{(J)_{(set\,1)}}$ in (44) with $DFS_{(J)_{(set\,1)}}$ from (41);

$$DFS_{(J)_{(set\,2)}} = DSS_{(1)_{(set\,1)}} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\,1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\,1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\,2)}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\,2)}\right) \quad \ldots\ldots (45)$$

Rearrange (45);

$$DFS_{(J)_{(set\,2)}} = DSS_{(1)_{(set\,1)}} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\,1)} + N_{(set\,2)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\,1)} + N_{(set\,2)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \quad \ldots\ldots (46)$$

From (46), the equation (46) has a similar form as Eq. 9;

$$DFS_{(i)} = DSS_{(1)} + \left(\frac{1}{m_{(i)}}\right) \times (N - 1) - \left(\frac{1}{m_{(i-1)}}\right) \times (N) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)}) \quad \ldots\ldots Eq.\,9$$

The difference is the number of units that Eq.9 considers only set 1. Hence, the summation of units of all sets as (46) is then defined as a new variable Q where Q is obtained from Eq. 12 as the following equation.

$$Q = \sum_{v=1}^{v=V} N_{(set\ v)} \qquad \text{Eq.12}$$

Where v is the set in consideration and V is the number of total sets

Q = Quantity of units from the summation of all $N_{(set\ v)}$

$N_{(set\ v)}$= Number of units of set v

With the variable Q, (46) is rewritten as below;

$$DFS_{(J)_{(set\ 2)}} = DSS_{(1)_{(set\ 1)}} + \left(\frac{1}{m_{(i)}}\right) \times (Q-1) - \left(\frac{1}{m_{(i-1)}}\right) \times (Q) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \qquad (47)$$

For any set v, the general form of the representative equation of two consecutive activities with the same type is the Eq 13, where i is the activity in consideration, J is the number of total units of the set, and v is the set in consideration.

$$DFS_{(J)_{(set\ V)}} = DSS_{(1)_{(set\ 1)}} + \left(\frac{1}{m_{(i)}}\right) \times (Q-1) - \left(\frac{1}{m_{(i-1)}}\right) \times (Q) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \qquad \text{Eq.13}$$

$DFS_{(J)_{(set\ V)}}$= Difference time between the start time of successor at Jth unit of set v and finish time of predecessor at Jth unit of set V

$DSS_{(1)_{(set\ 1)}}$= Difference time between the start time of successor at 1st unit of set 1 and the start time of predecessor at 1st unit of set 1

$m_{(i)}$ = Delivery rate of successor, $m_{(i-1)}$ = Delivery rate of predecessor

Q = Quantity of units from the summation of all $N_{(set\ v)}$

$R_{(i-1)}$ = Number of resources of predecessor, $D_{(i-1)}$ = Duration of predecessor

From the example project of the multi-identical type of units in Figure 4.18, Footing P1 has 9 days of activity duration and three teams of specific resources are assigned. Column P1 has 4 days of activity duration and two teams of specific resources are assigned. The numbers of units in set 1 and set 2 are 6 and 4, respectively. Hence, Q in Eq. 12 is then equal to 10 units.

$$Q = \sum_{v=1}^{v=2} N_{(set\ v)} = N_{(set\ 1)} + N_{(set\ 2)} = 6 + 4 = 10 \quad (48)$$

Substitute all known variables in Eq. 13;

$$DFS_{(J)_{(set\ 2)}} = DSS_{(1)_{(set\ 1)}} + \left(\frac{4}{2}\right) \times (10 - 1) - \left(\frac{9}{3}\right) \times (10) - \left(\frac{3-1}{3}\right) \times 9 \quad (49)$$

To determine the minimum $DSS_{(1)_{(set\ 1)}}$, $DFS_{(J)_{(set\ 2)}}$ is then equal to zero due to the critical point.

$$0 = DSS_{(1)_{(set\ 1)}} + \left(\frac{4}{2}\right) \times (10 - 1) - \left(\frac{9}{3}\right) \times (10) - \left(\frac{3-1}{3}\right) \times 9 \quad (50)$$

Rearrange the equation and determine $DSS_{(1)_{(set\ 1)}}$;

$DSS_{(1)_{(set\ 1)}}$ = 18 days as in Figure 4.18 ....(51)

For convergent lines, the example has shown that the critical point always locates at last set v of units, so it is sufficient to verify sequence logic only at Jth unit of set V.

In this section, this study has explained the advance modification of the representative equations for the convergent lines. Two cases of consecutive activities were utilized to demonstrate the modification. For the convergent lines of multi-identical types of units, examining the case of the pair of consecutive activities is essential. This section has illustrated how to utilize representative equations to determine $DSS_{(1)_{(set\ 1)}}$ with the valid sequence logic.

4.2.4.3 Representative equations for the multi-identical types of units

From the previous section, the equations for all possible cases of converging lines have been derived. The variables in the equations only consider one type of units and one pair of consecutive activities, so this section then rewrites the variables in the modified equations by adding the term of type k and the term of pair u, where i is the repetitive activity, j is the unit in the set, J is the number of total units of the set, k is the type of units, u is pair of two consecutive activities, v is the set of units, and V is the number of total sets in consideration.

The representative equations for the viaduct segment and its predecessor

From Eq. 10 (Equation for set 1);

$$DFS_{(J)_{(set\ 1)}} = DSS_{(1)_{(set\ 1)}} + \left(\frac{1}{m_{(i)}}\right) \times \left(N_{(set\ 1)} - 1\right) - \left(\frac{1}{m_{(i-1)}}\right) \times \left(N_{(set\ 1)}\right) - \left(\frac{R_{(i-1)} - 1}{R_{(i-1)}}\right) \times D_{(i-1)} \ldots\ldots Eq.10$$

Rewrite Eq. 10 to Eq. 14 by adding the term type k and term pair u;

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)} - 1\right) - \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)}\right) - \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \ldots\ldots Eq.14$$

$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$ = The difference time between the early start time of successor at Jth unit of set 1 and the early finish time of predecessor at Jth unit of set 1 of pair u with type k

$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$= The difference time between the early start time of successor at the 1st unit of set 1 and the early start time of predecessor at the 1st unit of set 1 of pair u with type k

$m_{(i)}{}^{(typek)}$ = Rate of delivery of successor of pair u with type k

$m_{(i-1)}{}^{(type\ k)}$ = Rate of delivery of predecessor of pair u with type k

$N_{(set\ 1)}{}^{(type\ k)}$ = Number of units j of set 1 of pair u with type k

$R_{(i-1)}{}^{(type\ k)}$ = Number of specific resources of predecessor of pair u with type k

$D_{(i-1)}{}^{(type\ k)}$ = Activity duration of predecessor of pair u with type k

From Eq. 11 (Equation for set v where v is the number of sets and v > 1);

$$DFS_{(J)}{}_{(set\ v)} = DFS_{(J)}{}_{(set\ v-1)} + \left(\frac{1}{m_{(i)}}\right)\left(N_{(set\ \mathrm{v})} + N_{(set\ (\mathrm{v}-1)\rightarrow \mathrm{v})}\right) - \left(\frac{1}{m_{(i-1)}}\right)\left(N_{(set\ \mathrm{v})}\right) \dots\dots\dots Eq. 11$$

Rewrite Eq. 11 to Eq. 15 by adding the term type k and term pair u;

$$DFS_{(J)}{}_{(set\ v)}{}^{(Pair\ u)(type\ k)} = DFS_{(J)}{}_{(set\ v-1)}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right)\left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ (v-1)\rightarrow v)}{}^{(type\ k)}\right) - \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right)\left(N_{(set\ v)}{}^{(type\ k)}\right) \dots\dots\dots Eq. 15$$

$DFS_{(J)}{}_{(set\ v)}{}^{(Pair\ u)(type\ k)}$ = The difference time between the early start time of successor at J^th unit of set v and the early finish time of predecessor at J^th unit of set v of pair u, type k

$DFS_{(J)}{}_{(set\ v-1)}{}^{(Pair\ u)(type\ k)}$ = The difference time between the early start time of successor at J^th unit of set (v - 1) and the early finish time of predecessor at J^th unit of set (v – 1) of pair u, type k

$m_{(i)}{}^{(type\ k)}$ = Rate of delivery of successor of pair u, type k

$m_{(i-1)}{}^{(type\ k)}$ = Rate of delivery of predecessor of pair u, type k

$N_{(set\ v)}{}^{(type\ k)}$ = Number of units j of set v of pair u, type k

$N_{(set\ (v-1)\rightarrow v)}{}^{(type\ k)}$ = Number of units between set v and set (v-1) of pair u, type k

The representative equations for two consecutive activities with the same type

From Eq. 12;

$$Q = \sum_{v=1}^{v=V} N_{(set\,v)} \text{ .... } Eq.12$$

Rewrite Eq. 12 to Eq. 16 for type k;

$$Q^{(type\,k)} = \sum_{v=1}^{v=V} {N_{(set\,v)}}^{(type\,k)} \text{ .... } Eq.16$$

$Q^{(type\,k)}$ = Quantity of units from the summation of all $N_{(set\,v)}$ of type k

${N_{(set\,v)}}^{(type\,k)}$= Number of units of set v of type k

From Eq. 13;

$$DFS_{(J)_{(set\,V)}} = DSS_{(1)_{(set\,1)}} + \left(\frac{1}{m_{(i)}}\right) \times (Q-1) - \left(\frac{1}{m_{(i-1)}}\right) \times (Q) - \left(\frac{R_{(i-1)}-1}{R_{(i-1)}}\right) \times D_{(i-1)} \text{ .... } Eq.13$$

Rewrite Eq. 13 to Eq. 17 by adding the term type k and term pair u;

$$\begin{aligned} {DFS_{(J)_{(set\,V)}}}^{(Pair\,u)(type\,k)} &= {DSS_{(1)_{(set\,1)}}}^{(Pair\,u)(type\,k)} + \left(\frac{1}{{m_{(i)}}^{(type\,k)}}\right) \times \left(Q^{(type\,k)}-1\right) \\ &- \left(\frac{1}{{m_{(i-1)}}^{(type\,k)}}\right) \times \left(Q^{(type\,k)}\right) \\ &- \left(\frac{{R_{(i-1)}}^{(type\,k)}-1}{{R_{(i-1)}}^{(type\,k)}}\right) \times {D_{(i-1)}}^{(type\,k)} \text{ .... } Eq.17 \end{aligned}$$

${DFS_{(J)_{(set\,V)}}}^{(Pair\,u)(type\,k)}$= Difference time between the start time of successor at Jth unit of set V and finish time of predecessor at Jth unit of set V of pair u, type k

${DSS_{(1)_{(set\,1)}}}^{(Pair\,u)(type\,k)}$= Difference time between the start time of successor at 1st unit of set 1 and the start time of predecessor at 1st unit of set 1 of pair u, type k

${m_{(i)}}^{(type\,k)}$ = Delivery rate of Successor of pair u, type k

$m_{(i-1)}{}^{(type\ k)}$ = Delivery rate of Predecessor of pair u, type k

$Q^{(type\ k)}$ = Quantity of units from the summation of all $N_{(set\ v)}$ of type k

$R_{(i-1)}{}^{(type\ k)}$ = Number of resources of Predecessor of pair u, type k

$D_{(i-1)}{}^{(type\ k)}$ = Duration of Predecessor of pair u, type k

The variables in the Eq.10, Eq.11, Eq.12, and Eq.13 have been rewritten in terms of type k and pair u as Eq. 14, Eq.15, Eq.16, and Eq.17, respectively. These terms aim to clearly identify which activity, unit, type, pair, and set are being considered in the calculation process of the convergent lines.

For the divergent lines, the slope lines cause the lines of two consecutive activities diverging from each other for any increasing units. Therefore, the critical point must belong to the first unit of set 1. In other word, $DFS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$ is then equal to zero. Thus, the minimum $DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$ is always equal to the duration of Predecessor of pair u type k ($D_{(i-1)}{}^{(type\ k)}$) for diverging lines of any cases as showing in Figure 4.5 in section 4.2.3. Hence, the representative equation for the diverging lines for type k and pair u can be described as Eq. 18.

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = D_{(i-1)}{}^{(type\ k)} \quad \text{...} Eq.18$$

$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$= Difference time between the start time of successor at 1st unit of set 1 and the start time of predecessor at 1st unit of set 1 of pair u, type k

$D_{(i-1)}{}^{(type\ k)}$ = Duration of Predecessor of pair u, type k

In the calculation of $DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$, selection of the case and the representative equation is the most important part. The utilization of this method must consider the slope lines (converging lines or diverging line) and the case of the pair of consecutive activities (pair of viaduct segment erection and its predecessor, or pair of two consecutive activities with the same type) to select the procedure that provides the valid $DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$ in a consideration.

4.2.5 Project duration calculation

This section explains the calculation of the project duration after $DSS_{(1)}{}_{(set\ 1)}$ of every pair u of two consecutive activities of considering type k is acquired, where u is the number of pairs and k is the type of units in consideration. The project duration is retrieved from the summation of the minimum $DSS_{(1)}{}_{(set\ 1)}$ plus with the duration of the last activity from the first unit to the last unit. The summation of all $DSS_{(1)}{}_{(set\ 1)}$ then provides the minimum duration from the start time of the 1st activity (i = 1) at the 1st unit to the start time of the last activity (i = I) at the first unit as showing in Figure 4.20, where i is the repetitive activity and I is the number of total repetitive activities including the segment erection.

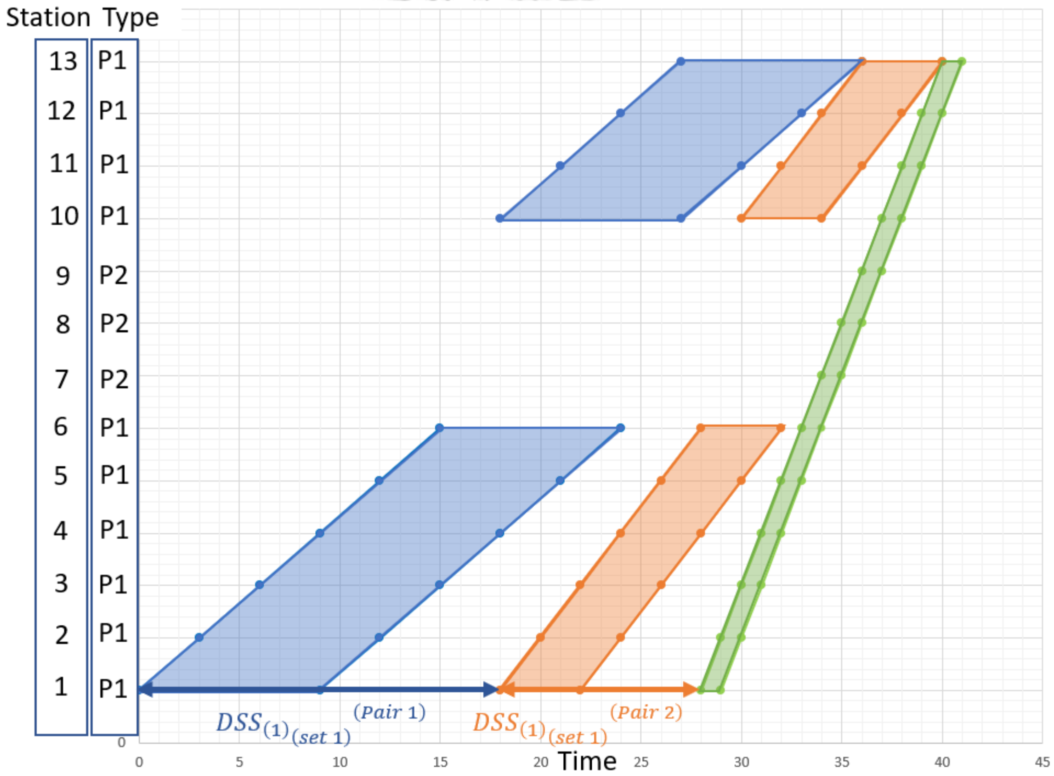


Figure 4.21 The summation of all $DSS_{(1)}$ of set 1

The duration of the last activity I from the first unit to the last unit can be determined by the duration of activity I plus the multiple of the slope of activity I with the quantity of units from the 1st unit of the considering type to the last unit of the project (Q_A) minus one as showing in Figure 4.21. The equation of project duration calculation can be written as Eq.14

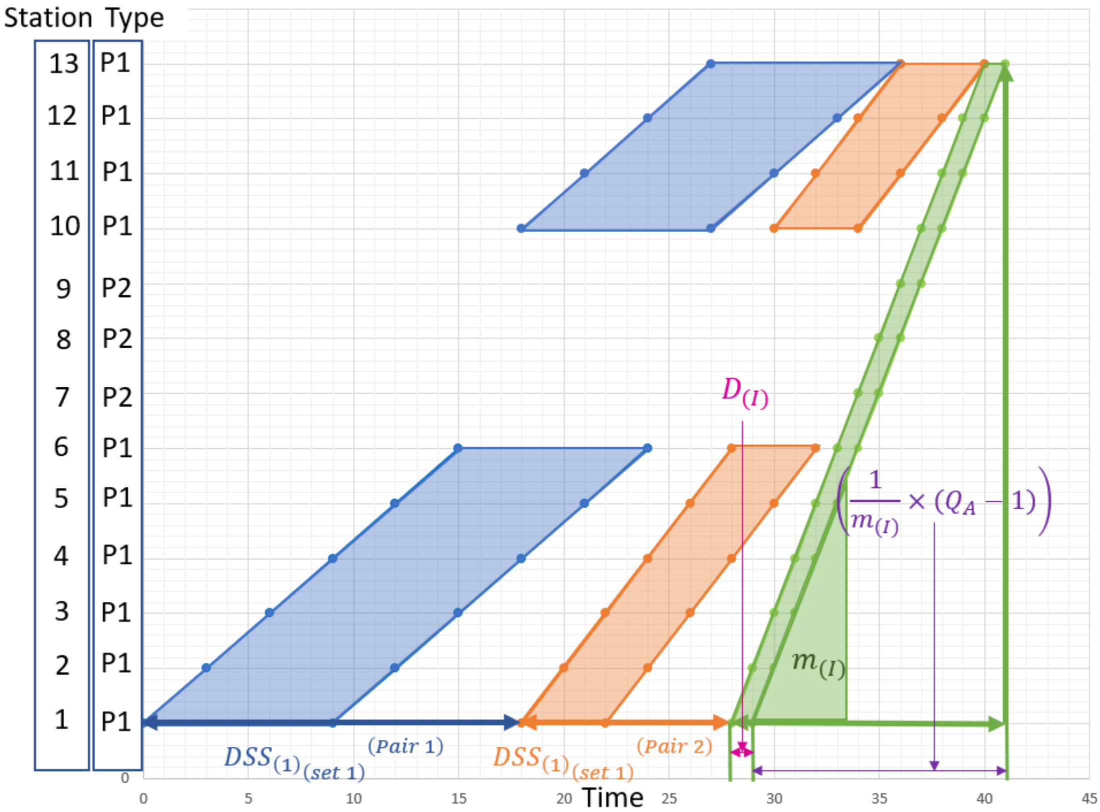


Figure 4.22 Calculation of project duration of a considering type of units

$$P = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}^{(Pair\ u)} + \left(D_{(I)} + \left(\frac{1}{m_{(I)}} \times (Q_A - 1)\right)\right) \text{..................................} Eq.19$$

Where I is the number of total repetitive activities including the segment erection, u is the pair of two consecutive activities and U is the number of total pairs of two consecutive activities in consideration of type.

P = Project duration, $D_{(I)}$ = Duration of activity I, $m_{(I)}$ = Rate of delivery of activity I

$DSS_{(1)_{(set\ 1)}}^{(Pair\ u)}$ = Difference time between the start time of successor at 1st unit of set 1 and the start time of predecessor at 1st unit of set 1 of pair u

Q_A = Quantity of units from the 1st unit of the considering type to the last unit of the project

The equation of project duration can be rewritten for any type k as Eq.20.

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} + \left(\frac{1}{m_{(I)}{}^{(type\,k)}} \times (Q_A{}^{(type\,k)} - 1) \right) \right) \quad \text{.....................} Eq.20$$

Where I is the number of total repetitive activities, u is the pair of two consecutive activities, U is the number of total pairs of two consecutive activities, and k is the type of units in consideration.

$P^{(type\,k)}$ = Project duration of type k

$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)}$ = Difference time between the start time of successor at 1st unit of set 1 and the start time of predecessor at 1st unit of set 1 of pair u type k

$D_{(I)}{}^{(type\,k)}$ = Duration of activity i type k

$m_{(I)}{}^{(type\,k)}$ = Rate of delivery of activity i type k

$Q_A{}^{(type\,k)}$ = Quantity of units from the 1st unit of the considering type k to the last unit of the project

$$m = \frac{R}{D} \quad \text{...} Eq.2$$

m = Rate of delivery, R = Number of specific resources, and D = Activity duration

From Eq. 2, the slope m depends on the variable R and D. Any number of resources can be assigned to the last activity for Eq. 19 and Eq.20. However, this study has the condition that the last activity is segment erection where a single launching gantry performs segment erection continuously. Therefore, the number of resources R in Eq. 2 is equal to one. The Eq.20 can be derived for a single launching gantry as below.

Substitute m from Eq.2 into m Eq. 20;

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} + \left(\frac{D_{(I)}{}^{(type\,k)}}{R_{(I)}{}^{(type\,k)}} \times (Q_A{}^{(type\,k)} - 1) \right) \right) \text{....................}(52)$$

Substitute 1 into R in Eq. 20;

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} + \left(\frac{D_{(I)}{}^{(type\,k)}}{1} \times (Q_A{}^{(type\,k)} - 1) \right) \right) \text{....................}(53)$$

Rearrange equation (53);

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} \times Q_A{}^{(type\,k)} \right) \text{..............} \textit{Eq. 21}$$

$P^{(type\,k)}$ = Project duration of type k

$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)}$ = Difference time between the start time of successor at 1st unit of set 1 and the start time of predecessor at 1st unit of set 1 of pair u, type k

$D_{(I)}{}^{(type\,k)}$ = Duration of activity I type k

$Q_A{}^{(type\,k)}$ = Quantity of units from the 1st unit of the considering type k to the last unit of the project

4.2.5.1 Project duration of the example provided by type P1

Eq.21 is used when one resource is assigned to the last activity. To demonstrate the calculation of project duration, the example from Figure 4.21 is utilized. From section 4.2.4.1 and 4.2.4.2, every $DSS_{(1)_{(set\,1)}}$ of unit type P1 was determined in equation (34) and (51). Pair 1 is Footing P1 and Column P1. Pair 2 is Column P1 and segment erection.

$DSS_{(1)_{(set\ 1)}}{}^{(Pai\ 1)(type\ P1)} = 10$.....................(34)

$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P1)} = 18$....................(51)

The activity duration of the segment erection is one day ($D_{(I)}{}^{(type\ k)}$= 1) and the quantity of units ($Q_A{}^{(type\ k)}$) is 13 units. Therefore, the calculation of project duration for type P1 with Eq. 21 is performed as follows.

Repeat from Eq.21, where U = 2;

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(D_{(I)}{}^{(type\ k)} \times Q_A{}^{(type\ k)}\right) \text{..............} Eq.\ 21$$

Substitute known variable into m Eq. 21;

$P^{(type\ P1)} = (10 + 18) + (1 \times 13)$..(54)

$P^{(type\ P1)} = 41$..(55)

The project duration for type P1 is 41 days as shown in Figure 23.

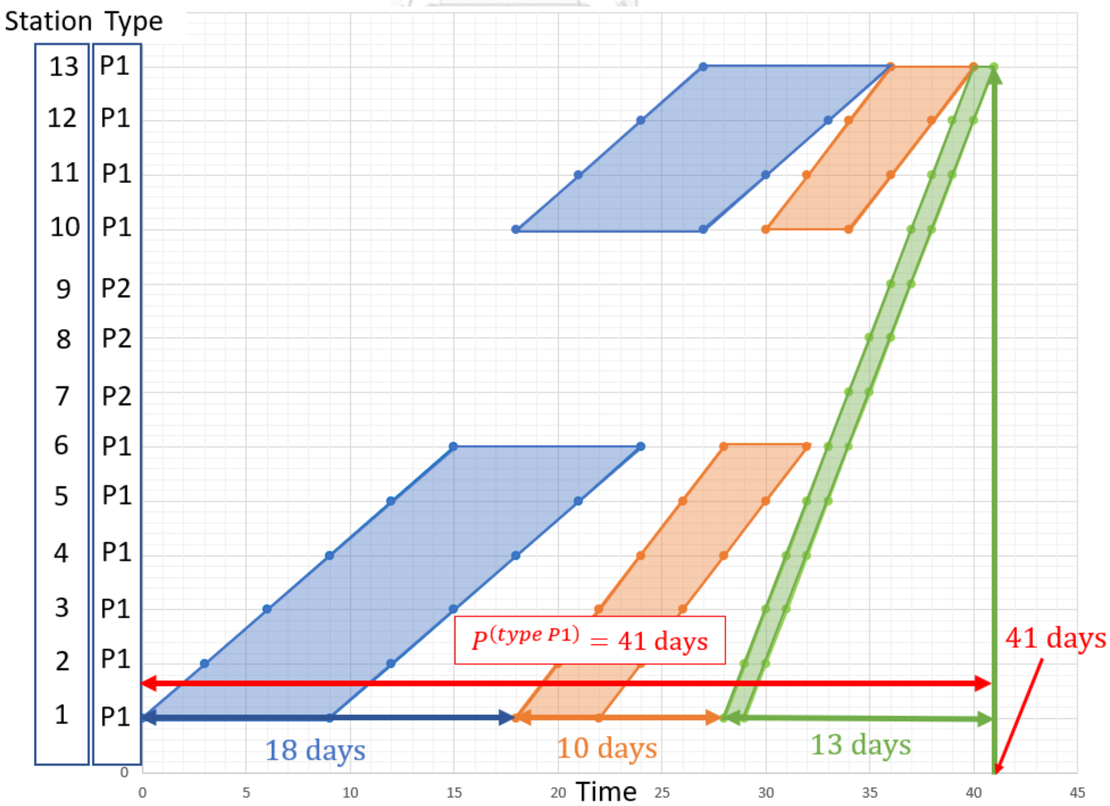


Figure 4.23 The project duration for type P1 from the example project

4.2.5.2 Project duration of the example provided by type P2

From figure 4.7, type P2 contains three repetitive activities, which are Footing P2, column P2, and segment erection. The activities are repeated for 3 units from station 7 to station 9. Footing P2 has 9 days of activity duration and one team of the specific resource is assigned. Column P2 has 6 days of activity duration and two teams of the specific resources are assigned. Segment erection has 1 day of activity duration and one launching gantry is provided. The LOB diagram of type P2 is shown in Figure 4.24. The calculation of $DSS_{(1)}$ of set 1 for each pair is illustrated as follows.

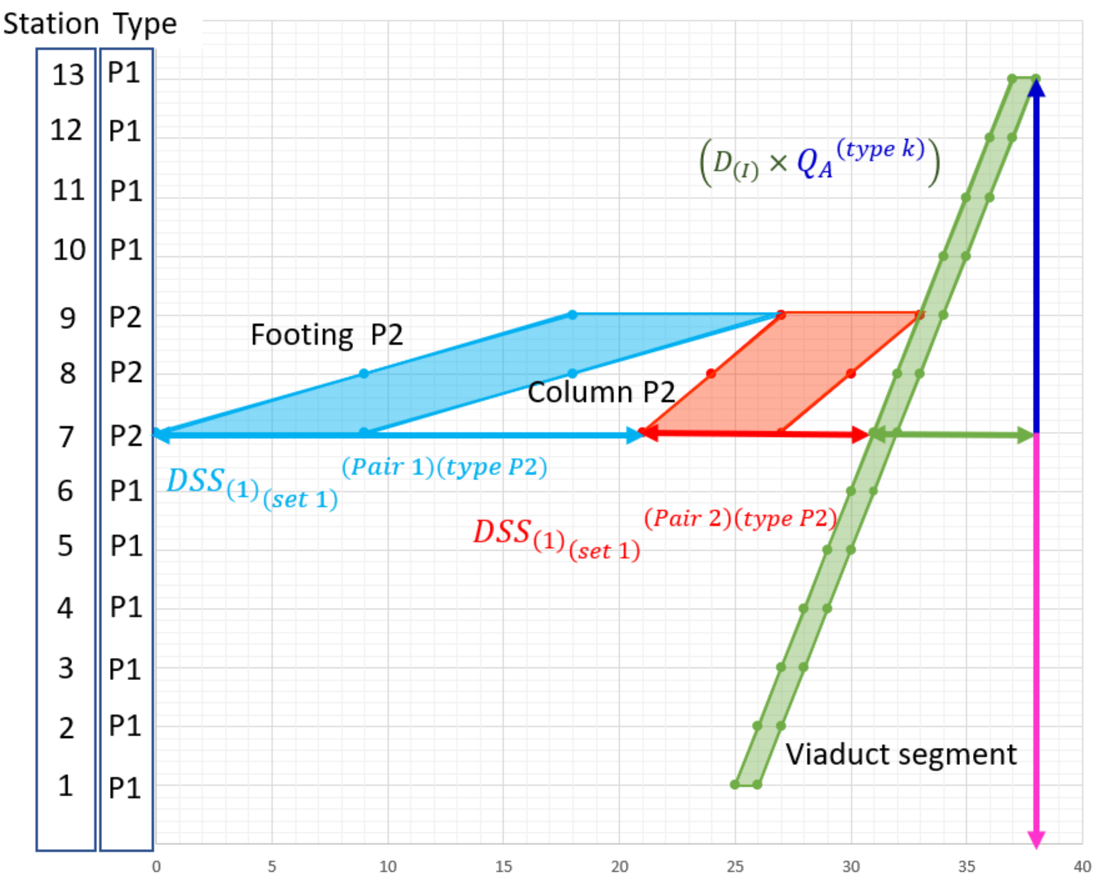


Figure 4.24 LOB diagram of type P2 from the project example

For the pair of Footing P2 and Column P2, there is one set of units. Thus, $DSS_{(1)}$ of set 1 can be determined by Eq. 17, where u is the pair consecutive activities in consideration and k is the type in consideration.

Repeat from Eq.17 to determine *DSS(1)* of set 1, where u = 1, k = P2, V = 1;

$$DFS_{(J)_{(set\,V)}}{}^{(Pair\,u)(type\,k)}$$

$$= DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(\frac{1}{m_{(i)}{}^{(type\,k)}}\right) \times \left(Q^{(type\,k)} - 1\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\,k)}}\right) \times \left(Q^{(type\,k)}\right)$$

$$- \left(\frac{R_{(i-1)}{}^{(type\,k)} - 1}{R_{(i-1)}{}^{(type\,k)}}\right) \times D_{(i-1)}{}^{(type\,k)} \quad Eq.17$$

Substitute known variable into Eq. 17, where $Q^{(type\,k)}$ is equal to N of set 1;

$$DFS_{(J)_{(set\,1)}}{}^{(Pair\,1)(type\,P2)}$$

$$= DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P2)} + \left(\frac{6}{2}\right) \times (3-1) - \left(\frac{9}{1}\right) \times (3)$$

$$- \left(\frac{1-1}{1}\right) \times 9 \quad (56)$$

DFS(J) of set 1 is zero due to the location of the critical point;

$$0 = DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P2)} + \left(\frac{6}{2}\right) \times (3-1) - \left(\frac{9}{1}\right) \times (3) - \left(\frac{1-1}{1}\right) \times 9 \quad (57)$$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P2)} = 21 \quad (58)$$

From Eq.17 to determine *DSS(1)* of set 1, where u = 2, k = P2, V = 1;

$$DFS_{(J)_{(set\,1)}}{}^{(Pair\,2)(type\,P2)}$$

$$= DSS_{(1)_{(set\,1)}}{}^{(Pair\,2)(type\,P2)} + \left(\frac{1}{1}\right) \times (3-1) - \left(\frac{6}{2}\right) \times (3)$$

$$- \left(\frac{2-1}{1}\right) \times 6 \quad (59)$$

DFS(J) of set 1 is zero due to the location of the critical point;

$$0 = DSS_{(1)_{(set\,1)}}{}^{(Pair\,2)(type\,P2)} + \left(\frac{1}{1}\right) \times (3-1) - \left(\frac{6}{2}\right) \times (3) - \left(\frac{2-1}{2}\right) \times 6 \quad (60)$$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,2)(type\,P2)} = 10 \quad (61)$$

Repeat from Eq. 21 to calculate the project duration provided by type P2;

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} \times Q_A{}^{(type\,k)}\right) \text{..............} \textit{Eq. 21}$$

Substitute known variable into Eq. 21 with $DFS_{(J)}$ of set 1 from (58) and (61);

$$P^{(type\,P2)} = 21 + 10 + \left(1 \times Q_A{}^{(type\,P2)}\right) \text{..........} (62)$$

From Eq. 21, $Q_A{}^{(type\,k)}$ is the quantity of units from the 1st unit of the considering type k to the last unit of the project. In the case of type P2, it is the number of units from station 7 to station 13 as shown in Figure 4.24. $Q_A{}^{(type\,k)}$ can be retrieved from the summation of all $N_{(set\,v)}{}^{(type\,k)}$ plus with the summation of all $N_{(set\,(v-1)\rightarrow v)}{}^{(type\,k)}$. However, for the case as the type P2, there are units after the last set V. To cover that unit, this study creates $N_{(set\,v+1)}{}^{(type\,k)}$ and $N_{(set\,v\rightarrow(v+1))}{}^{(type\,k)}$, where $N_{(set\,v+1)}{}^{(type\,k)}$ is always zero. If the last unit of type k is the last unit of the project, $N_{(set\,v\rightarrow(v+1))}{}^{(type\,k)}$ is equal to zero. On another hand, if the last unit of type k is not the last unit of the project, $N_{(set\,v\rightarrow(v+1))}{}^{(type\,k)}$ is the number of units between set V to set V+1. Thus, $Q_A{}^{(type\,k)}$ can be obtained from Eq. 22, where V is the number of total sets of type k.

$$Q_A{}^{(type\,k)} = \sum_{v=1}^{v=V} \left(N_{(set\,v)}{}^{(type\,k)} + N_{(set\,v\rightarrow(v+1))}{}^{(type\,k)}\right) \text{..........} \textit{Eq.22}$$

$Q_A{}^{(type\,k)}$ = the quantity of units from the 1st unit of the considering type k to the last unit of the project

$N_{(set\,v)}{}^{(type\,k)}$ = Number of units of set v type k

$N_{(set\,v\rightarrow(v+1))}{}^{(type\,k)}$= Number of units between set v and set v +1 of type k

From Eq. 22, $Q_A{}^{(type\,k)}$ can be obtained as the following;

$$Q_A{}^{(type\,P2)} = N_{(set\,1)}{}^{(type\,P2)} + N_{(set\,1\rightarrow 2)}{}^{(type\,P2)}) \text{..........} (63)$$

$$Q_A{}^{(type\,P2)} = 3 + 4 \text{..........} (64)$$

From (62) and (63), the project duration of type P2 is calculated as below.

$P^{(type\,P2)} = 21 + 10 + (1 \times 7)$ ....(65)

$P^{(type\,P2)} = 38$ ....(66)

The project duration provided by type P2 is 38 days as shown in Figure 4.25.

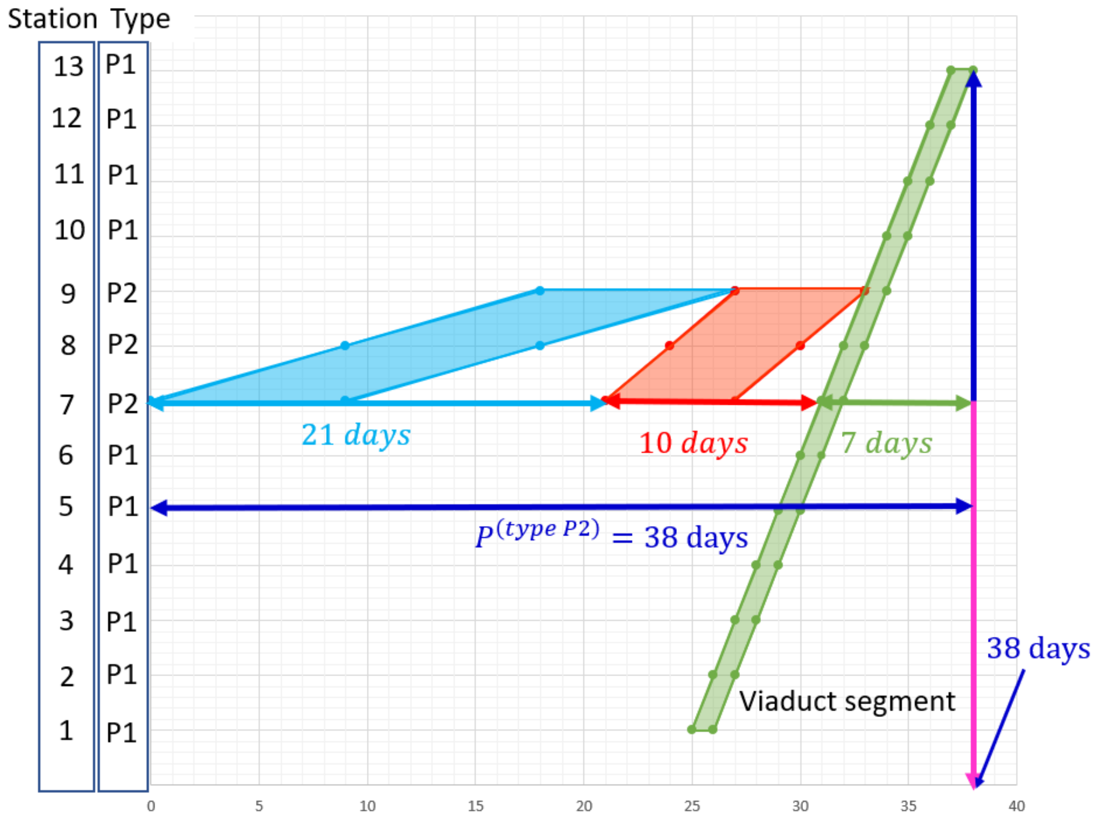


Figure 4.25 The project duration for type P2 from the example project

4.2.5.3 The control type of the project duration

In the previous section, the project duration of type P1 and type P2 have been determined. Type P1 makes the project finished in 41 days with the assigned resources while type P2 make the project finished in 38 days with the provided resources. Certainly, type P1 has the longest project duration. Thus, the project duration is 41 days. This study calls the type that contains the longest project duration as the control type of the project duration. In the example, type P1 is the control type where the segment erection performs on its schedule. This causes type P2 having 3 days of free-float (41-38) that can be adjusted for any uncover conditions. Thus, the function of the project duration for multi-identical types of units can be written as Eq. 23.

$$P = Max\left(P^{(type\,1)}, P^{(type\,2)}, P^{(type\,3)}, \dots, P^{(type\,K)}\right) \quad \text{.................................} Eq.\,23$$

Where k is the type of units and K is the number of total types in the project. Max is the function to determine the highest value among the considering variables

P = The project duration for multi-identical types of units

$P^{(type\,k)}$ = The project duration provided by type k

4.2.6 Procedure of the method of project duration calculation

In conclusion, this section explains the procedure of the method of project duration in detail from the very start to the end of the method.

The procedure of the method of project duration calculation is as follows:

1. Estimate all activity duration (D)
2. Create a sequence logic of one unit for each type with the segment erection is the last (define a number of i and k to D, ${D_{(i)}}^{(type\,k)}$)
3. Determine the units in sets and the units between sets of type k from v = 1 to v =V (${N_{(set\,v)}}^{(type\,k)}$, ${N_{(set\,\mathrm{v}\to(\mathrm{v}+1))}}^{(type\,k)}$).
4. Select a type k of units to determine project duration beginning with k = 1
5. Trial a set of specific resources (${R_{(1)}}^{(type\,k)}, {R_{(2)}}^{(type\,k)}, \dots, {R_{(I-1)}}^{(type\,k)}$)
6. Start determining $DSS_{(1)}$ of set 1 from the first pair (u=1)
7. Compare slope $m_{(i-1)}$ and slope $m_{(i)}$ of pair u type k to define the case of two lines. where i is the repetitive activity in consideration.
   7.1 If the diverging lines ($m_{(i-1)} \geq m_{(i)}$), uses Eq.18 to determine $DSS_{(1)}$ of set 1
   7.2. If the converging lines ($m_{(i-1)} < m_{(i)}$) and u < U, uses Eq.16 for $Q^{(type\,k)}$ and Eq.17 to determine $DSS_{(1)}$ of set 1.
   7.3. If the converging lines ($m_{(i-1)} < m_{(i)}$) and u = U, uses Eq.14 and Eq.15 to determine $DSS_{(1)}$ of set 1. The number of Eq.15 depends on the number of sets but not include the first set (V-1).

8. Keep the $DSS_{(1)}$ of set 1 of pair u type k and move to consider the next pair u by repeating step 7 until the $DSS_{(1)}$ of set 1 of last pair (u =U) is acquired.

9. Calculate $Q_A{}^{(type\ k)}$ from Eq. 22 and the project duration of type k ($P^{(type\ k)}$) from Eq. 21 with all $DSS_{(1)}$ of set 1 from step 8.

10. Keep the $P^{(type\ k)}$ and move to consider the next type k by repeating steps 3 - 9 until the $P^{(type\ k)}$ of the last type K (k = K) is acquired.

11. Determine the project duration among the types of units from Eq. 23.

This section has expressed the manual procedure of the method of project duration calculation. In the next section, the method will be utilized to create the proposed optimization model for computing the optimal set of specific resources and the project duration.

4.3 The proposed optimization model

This section explains the development of the proposed optimization model. In this study, Matlab 2018, programming software is selected to develop the model. From the previous section, the relationship between all variables associated with the total cost of the specific resources and the project duration are described in terms of the objective function and the constraint. The objective function of the proposed model is to minimize the total cost of the specific resources as Eq. 3.

$$C_{TR} = (R_1)(C_1) + (R_2)(C_2) + (R_3)(C_3) + \ldots + (R_{(i)})(C_{(i)}) \ldots\ldots\ldots\ldots Eq.3$$

Where i is the number of total repetitive activities (except segment erection).

C_{TC} = The total cost of specific resources

$R_{(i)}$ = Number of resources for repetitive activity i

$C_{(i)}$ = Cost per unit of specific resource for repetitive activity i

In this study, Eq. 3 is created to consider the total cost of the specific resources only for one identical type of units. Based on the condition that the specific resources of one identical type of units are independently utilized from each other, the total cost of the specific resources can be individually calculated as well. Thus, the total cost of the specific resources of the project is then determined by Eq.25.

$$C_{TR}{}^{(type\ k)} = \left(R_{(1)}{}^{(type\ k)}\right)\left(C_{(1)}{}^{(type\ k)}\right) + \left(R_{(2)}{}^{(type\ k)}\right)\left(C_{(2)}{}^{(type\ k)}\right)$$
$$+ \left(R_{(3)}{}^{(type\ k)}\right)\left(C_{(3)}{}^{(type\ k)}\right) + \ldots + \left(R_{(i)}{}^{(type\ k)}\right)\left(C_{(i)}{}^{(type\ k)}\right) \ldots\ldots\ldots\ldots\ldots Eq.24$$

Where i is the number of total repetitive activities (except segment erection) and k is the type.

$C_{TR}{}^{(type\ k)}$ = The total cost of specific resources of type k

$R_{(i)}{}^{(type\ k)}$ = Number of resources for repetitive activity i of type k

$C_{(i)}{}^{(type\ k)}$ = Cost per unit of resource for repetitive activity i of type k

$$C_{TP} = \sum_{k=1}^{k=K} \left(C_{TR}{}^{(type\ k)}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.25$$

Where k is the type and K is the number of total types in the project.

C_{TP} = The total cost of specific resources of the project

$C_{TR}{}^{(type\ k)}$ = The total cost of specific resources of type k

For the constraint, Eq. 4 from section 4.1.2 was created to provide the project duration only for one identical type of units Hence, the function of project duration for any type k of units is then defined as Eq. 26 and the project duration for multi-identical types of units can be retrieved from Eq. 23.

$$P\left(R_{(1)}, R_{(2)}, R_{(3)}, \ldots, R_{(i)}\right) \leq \text{desirable duration} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.4$$

Where i is the number of total repetitive activities (except segment erection)

$P\left(R_{(1)}, R_{(2)}, R_{(3)}, \ldots, R_{(i)}\right)$ is Function of project duration

$R_{(i)}$ = Number of resources for repetitive activity i

$$P^{(type\ k)}\left(R_{(1)}{}^{(type\ k)}, R_{(2)}{}^{(type\ k)}, \ldots, R_{(i)}{}^{(type\ k)}\right) \leq \text{desirable duration} \ldots\ldots\ldots Eq.26$$

Where i is the number of total repetitive activities (except segment erection) and Where k is the type of units in consideration.

$P^{(type\ k)}\left(R_{(1)}{}^{(type\ k)}, R_{(2)}{}^{(type\ k)}, \ldots, R_{(i)}{}^{(type\ k)}\right)$ is the function of project duration of type k

$R_{(i)}{}^{(type\ k)}$ = Number of resources for repetitive activity i of type k

$$P = Max\left(P^{(type\ 1)}, P^{(type\ 2)}, P^{(type\ 3)}, \ldots, P^{(type\ K)}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.23$$

Where k is the type of units and K is the number of total types in the project. Max is the function to determine the highest value among the considering variables.

P = The project duration for multi-identical types of units

$P^{(type\ k)}$ = The project duration provided by type k

4.3.1 Input of the proposed optimization model

From the variables in the modified equations, the input of the optimization model is defined as the following. Input 1 to 5 are retrieved from Excel. Input 6 and 7 are assigned directly to the model.

1) Sequences of activity of one unit for every type
2) Activity duration for every type (all $D_{(i)}^{(type\ k)}$)
3) Number of units in sets for every type (all $N_{(set\ v)}^{(type\ k)}$)
4) Number of units between sets for every type (all $N_{(set\ v \rightarrow (v+1))}^{(type\ k)}$)
5) Cost per unit of each specific resource for every type (all $C_{(i)}^{(type\ k)}$)
6) Number of maximum available resources for every type (all $M^{(type\ k)}$)
7) Desired project duration

Activtiy duration & Sequecnes			
Activity	Duration	Activity	Duration
Name	P1	Name	P2
Segment	1	Segment	1
Column	4	Column	6
Footing	9	Footing	9

Number of units of in set		
Type/Set	Set 1	Set 2
P1	6	4
P2	3	0

Cost per unit of specific resource			
Activity	Cost/Unit	Activity	Cost/Unit
Name	Milion	Name	Milion
	P1		P2
Column	2.00	Column	1.00
Footing	1.00	Footing	2.00

Number of units between sets		
Type/Set	Set 1-2	Set 2-3
P1	3	0
P2	4	0

Figure 4.26 The input from the example project in Figure 4.6

4.3.2 Search space of decision variables

In an optimization model, search space is an essential component. It is the domain of the objective function to determine the optimal solution. From the condition of the independently utilizing resources, the optimal solution for each type of units is individually computed type by type. Hence, the search space of type k is then provided to be a finite search space for each type of unit. The domain in search space is the sets of specific resources or it can be called as the sets ($Set_{(s)}{}^{(type\,k)}$) of decision variables (($R_{(1)}{}^{(type\,k)}, \ldots, R_{(i)}{}^{(type\,k)}$)). The size of the search space of each type ($S^{(type\,k)}$) is upon to the number of total activities i of type k and the maximum available resources of type k ($M^{(type\,k)}$) where the domain starts from 1 to $M^{(type\,k)}$, so $S^{(type\,k)}$ is equal to $(i)^{(M^{(type\,k)})}$. The example of the search space of a type of units is in Figure 4.27 where i = 4 and $M^{(type\,k)} = 5$.

		$R_{(i)}{}^{(type\,k)}$ i = 1	i = 2	i = 3	i = 4
		1	2	3	4
s = 1	1	1	1	1	1
s = 2	2	1	1	1	2
s = 3	3	1	1	1	3
s = 4	4	1	1	1	4
.	5	1	1	1	5
.	6	1	1	2	1
.	7	1	1	2	2
$Set_{(s)}{}^{(type\,k)}$.	8	1	1	2	3
.	9	1	1	2	4
.	10	1	1	2	5
.	11	1	1	3	1
.	12	1	1	3	2
.	13	1	1	3	3
s = 14	14	1	1	3	4
s = 15	15	1	1	3	5
s = 16	16	1	1	4	1
s = 17	17	1	1	4	2

Figure 4.27 Example of a search space of type k where i = 4 and $M^{(type\,k)} = 5$

4.3.3 Flow of the proposed optimization model

The computation flow showing in Figure 4.28 begins with retrieving the input of all types of units. The input of type k is taken to determine the optimal solution of type k. The number of total activity i of type k and Maximum available resources of type k ($M^{(type\,k)}$) are used to create a search space of type k. Then, a set ($Set_{(s)}{}^{(type\,k)}$) of decision variables from the search space and input of type k are assigned to the function of project duration calculation. Each set of decision variables from the search space will be assigned to the Eq.26 to determine the project duration from the first set (s = 1) of decision variables (set of all $R_{(i)}{}^{(type\,k)}$ =1) to the last set (s = $S^{(type\,k)}$) of decision variables (set of all $R_{(i)}{}^{(type\,k)}$ = $M^{(type\,k)}$). The process of the function of project duration calculation is explained in section 4.3.4. After that, the sets are classified by the desired duration to find the possible sets for the constraint. Then, each possible set get assigned to Eq.24 to calculate the total cost of specific resources of type k ($C_{TR}{}^{(type\,k)}$). Next, a function of determining the lowest value selects the set that provides the minimum total cost of type k ($min\ (C_{TR}{}^{(type\,k)}$)). Then, the optimal set of specific resources (optimal set of the decision variable), the minimum total cost of type k ($min\ (C_{TR}{}^{(type\,k)}$)), and the project duration ($P^{(type\,k)}$) are stored and move to considers the next type. When all types acquired their optimal solutions, the project duration for multi-identical types of units (P) is acquired from Eq. 23 and the total cost of specific resources of the project (C_{TP}) is retrieved from Eq. 25.

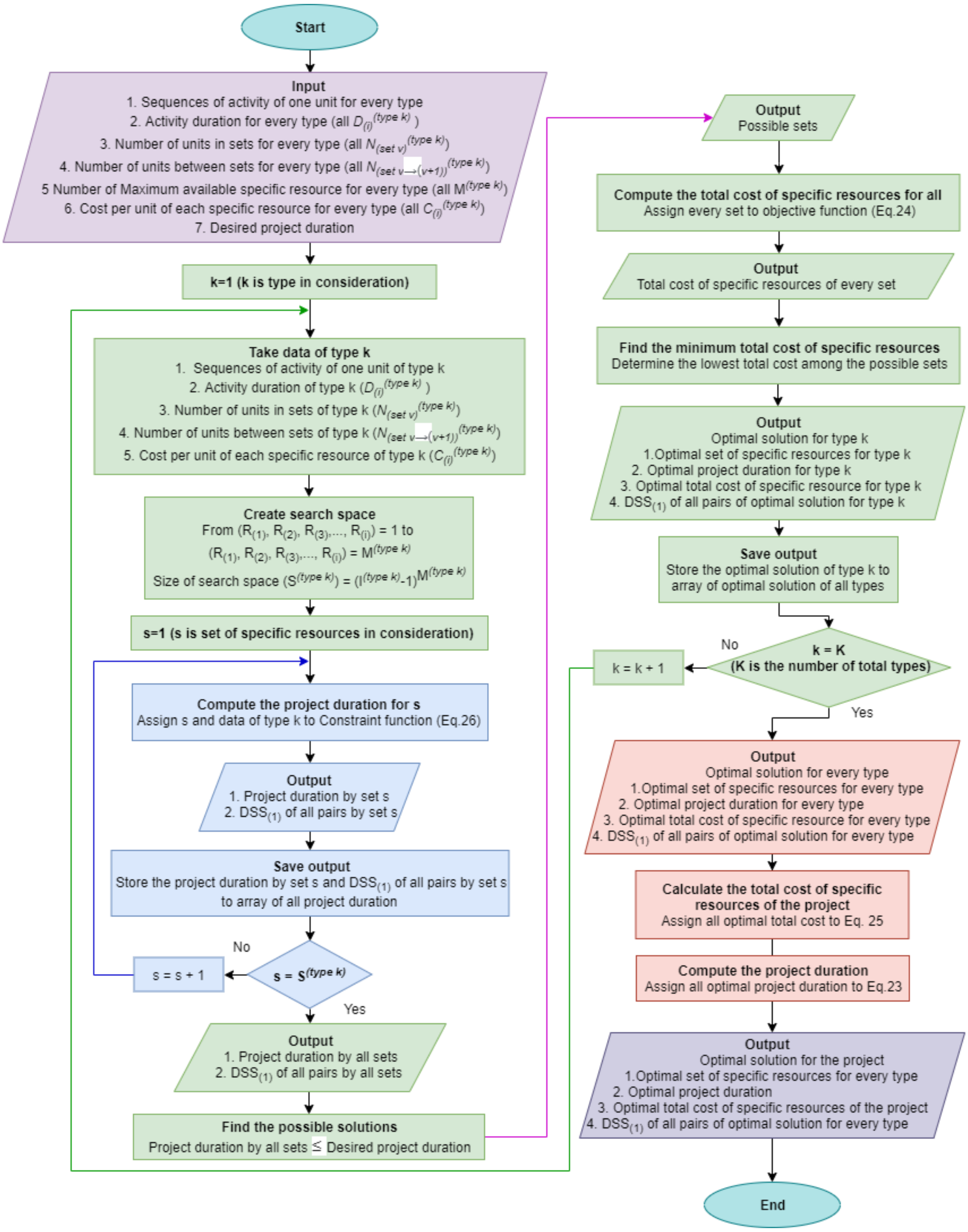


Figure 4.28 Flow of the proposed optimization model

4.3.4 Flow of the function of project duration calculation

Inside the flow of the proposed optimization model, there is the flow of project duration calculation (constraint) for determining the project duration by any set of decision variables. In Figure 4.29, the flow starts with getting the input of type k and set of decision variables (set of specific resources, $Set_{(s)}^{(type\ k)}$) from the optimization model. Then, the first pair of consecutive activities (u = 1) is considered to determine its DSS. The activity duration and the number of specific resources for predecessor and successor of pair u are taken into the Eq.2 to calculate and identify the case of slope lines (converging or diverging). If the pair u is the diverging lines, the DSS is equal to the duration of the predecessor (Eq.18) as the orange flow in Figure 4.29. For the converging lines, if u is lower than U (U is the number of total pairs), the DSS is acquired from Eq.16 and Eq.17 as the yellow flow in Figure 4.29. For the case that u is equal to U (u = U), it is the case of segment erection and its predecessor. The computation follows the red line to the trial of the first DSS where DSS is equal to the duration of the predecessor. DFS of set 1 (v = 1) is firstly calculated. Then, if v is still lower V (V is the number of total sets), the process moves to determine DFS of the next set (v = v +1) by using the DFS from the previous of current set v as the procedure of Eq. 15. The process is repeated until v is equal to V (v = V) meaning all sets have acquired their DFS as the blue flow in Figure 4.29. Next, the process verifies the sequence logic that if all DFS is higher or equal to 0, the sequence logic is not violated. For the case of violated sequence logic, new DSS is assigned by increased duration by one and the whole process of determining DFS of all sets is repeated until all DFS fulfill the sequence logic. After the determination of DSS of pair u completed, the DSS of pair u is stored in an array of all DSS. Then, the flow moves to consider the next pair (u = u+1) and repeats the process of determination of DSS until the pair U is considered (u =U). Finally, the process computes and provides the project duration from all DSS and Eq. 21 and Eq.22.

The input of the function of project duration calculation

1) Sequences of activity of one unit for type k
2) Activity duration for type k ($D_{(i)}^{(type\ k)}$)
3) Number of units in sets for type k ($N_{(set\ v)}^{(type\ k)}$)
4) Number of units between sets for type k ($N_{(set\ v \rightarrow (v+1))}^{(type\ k)}$)
5) Set of specific resources s of type k ($Set_{(s)}^{(type\ k)}$)

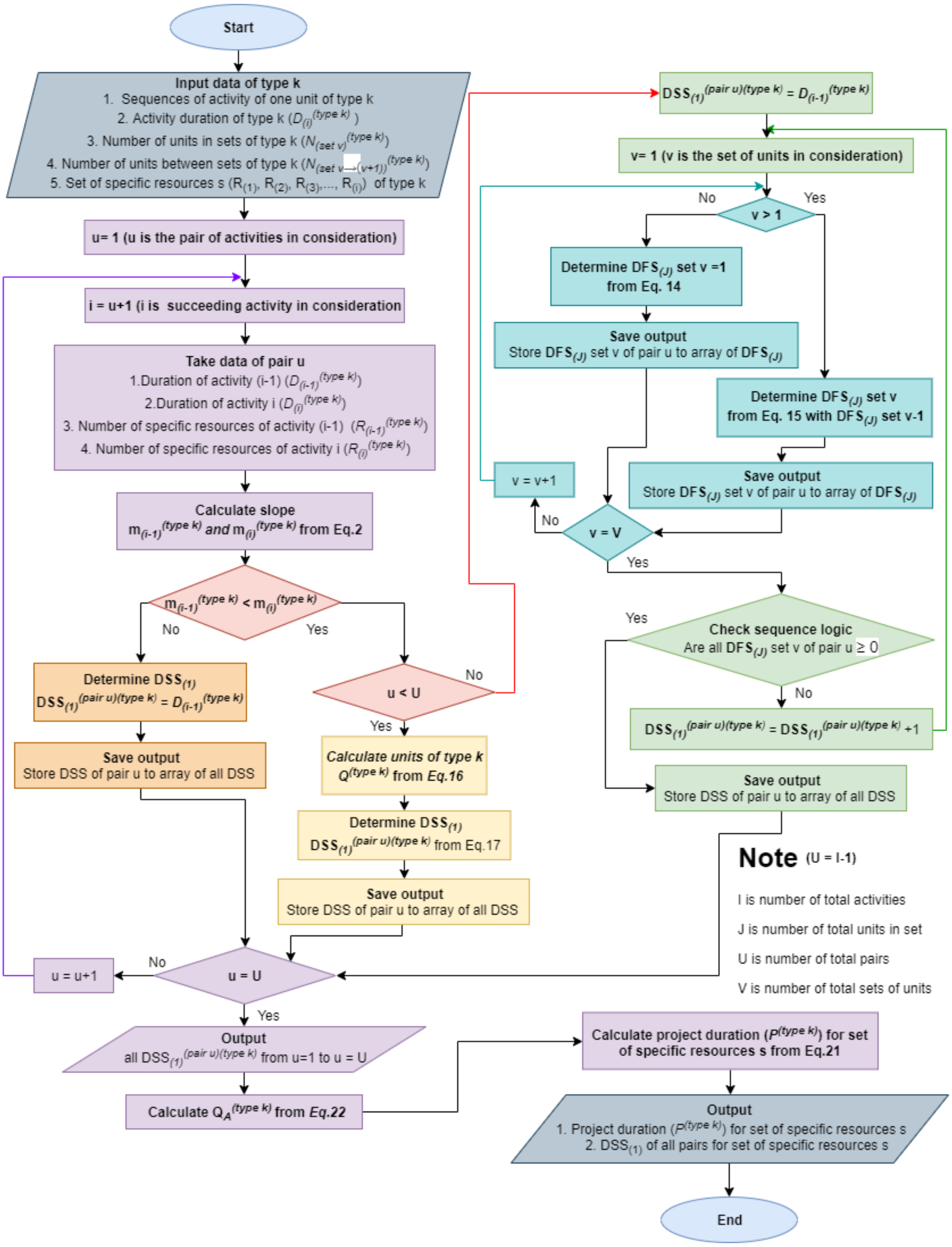


Figure 4.29 Flow of function of project duration calculation

The output of the function of project duration calculation

1) The project duration by type k ($P^{(type\ k)}$) for a set of specific resources ($Set_{(s)}^{(type\ k)}$)

2) $DSS_{(1)}$ of all pairs for a set of specific resources

4.3.5 Output of the proposed optimization model

From the objective function and the constraint, the output of the optimization model is as follows.

1) The optimal set of specific resources for every type ($R_{(i)}{}^{(type\ k)}$).

2) The optimal project duration (P)

3) The optimal total cost of specific resources of the project (C_{TP})

4) $DSS_{(1)}$ of all pairs by the optimal set of specific resources

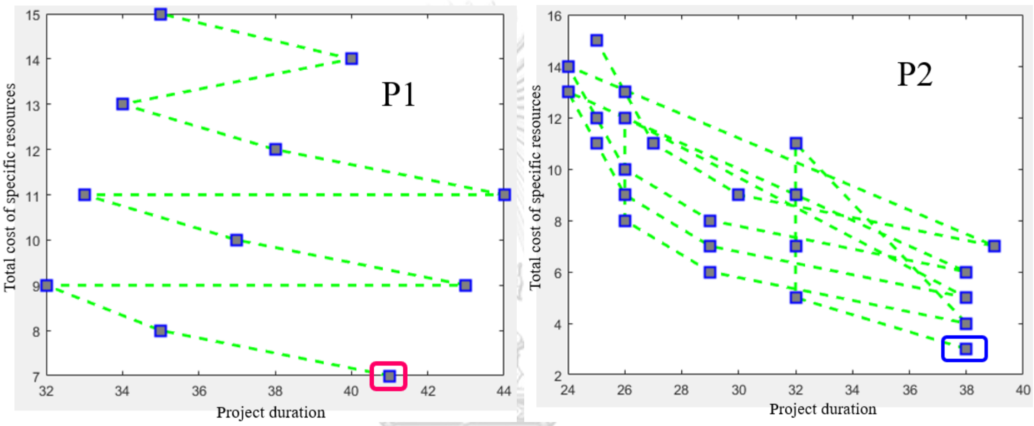


Figure 4.30 Searching paths in the optimization model for the example project

From the example project in Figure 4.6, Figure 4.30 illustrates that the searching path of P1 has the possible solutions from 25 domains which complete under the desired project duration of 45 days. There are 11 possible solutions of P1 but the optimal solution is 41 days of project duration with 7 million baht total cost of specific resources. For the type P2, there are 24 possible solutions and the optimal solution is 38 days of project duration with 3 million baht total cost of specific resources. Thus, the actual project duration is 41 days from control type P1 and the total cost of specific resources is 10 million baht.

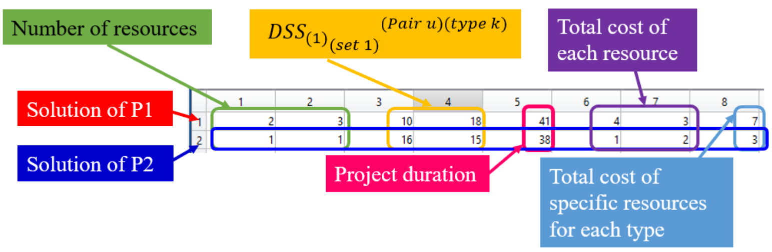


Figure 4.31 Optimal solution from the optimization model in Matlab 2018

In Figure 4.31, the output from the model is displayed in a table. The first two-column 1 and 2 are the number of specific resources. The number of columns depends on the number of decision variable $R_{(i)}$ of type k. In this example, there are two specific resources because there are two categories (i =1 and i = 2) of specific resources searched for two activities (Footing and Column). Next, column 3 and 4 are DSS of pair 1 and pair 2, respectively. This number of columns also relies on the number of decision variable R of type k. Column 5 is the project duration by the optimal set of specific resources of type k. Column 6 and 7 are the total cost of each category of specific resource. The last column 8 is the total cost from all categories of the type k. To display the output clearly, Table 4.1 is the example of the optimal solution displayed on Excel.

Table 4.1 The example of the optimal solution displayed on Excel

Type /Activtiy	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resource (m baht)
Type P1 (k = 1)				
Column	Formwork for Column P1	2.0	2.0	4.0
Footing	Formwork for Footing P1	1.0	3.0	3.0
Type P1	Project duration by P1	41 days	Total cost of resources	7.0
Type P2 (k = 2)				
Column	Formwork for Column P2	1.0	1.0	1.0
Footing	Formwork for Footing P2	2.0	1.0	2.0
Type P2	Project duration by P2	38 days	Total cost of resources	3.0
Project duration	41 days	Total cost of specific resource of the project		10.0

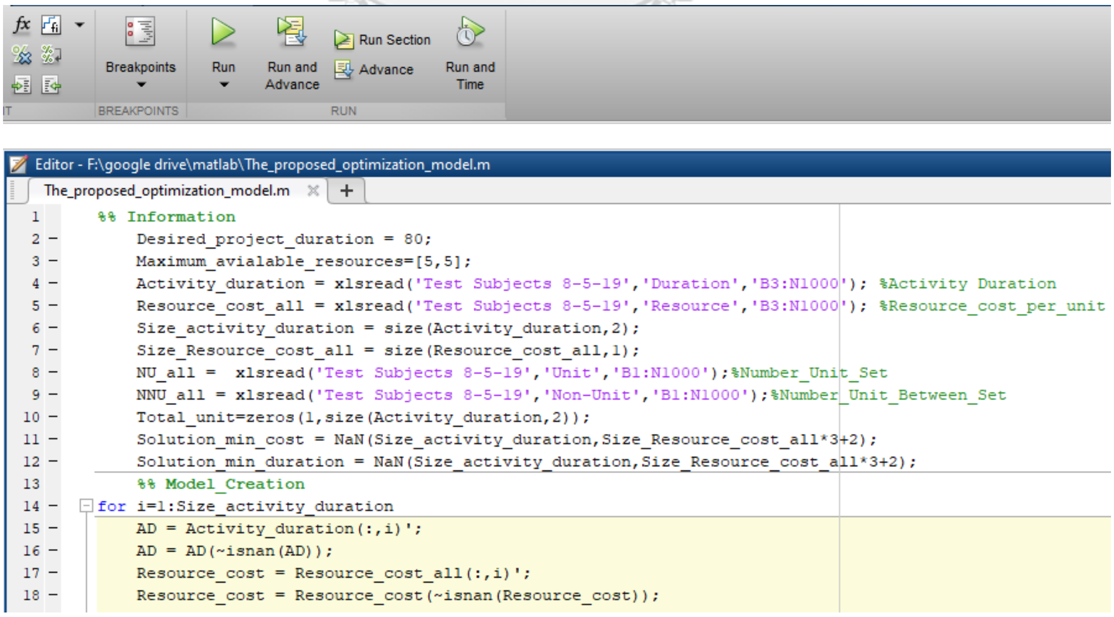


Figure 4.32 Interface of proposed optimization model in Matlab 2018

4.4 Verification of optimization model

This section presents the verification of the proposed optimization model. Comparing the optimal solution generated by the model with the optimal solution solved by trial-and-error is employed to prove the optimization model's capability. The trial-and-error process considers that the best solution from the massive random solutions relating to optimal criteria is the optimal solution. According to the objective function and constraint, trial-and-error probably considers only some domains and selects the optimal solution with the following criteria. When trying the solutions by changing the sets of specific resources until the minimum project duration or desired project duration is reached and changing the number of specific resources can not reduce the total cost. The optimal solutions can exist in the tried solution, so the solution that provides the lowest cost is the optimal solution. For the minimum project duration, it occurs when all pairs of consecutive activities are the divergent lines. The critical point of sequence logic of all pair is then located at the first unit of the type k in consideration, so the minimum $DSS_{(1)}$ of each pair is then equal to the duration of the preceding activity. With the all minimum $DSS_{(1)}$, the minimum project duration by type k ($min\ (P^{(type\ k)})$) is retrieved from Eq. 21. In this section, the verification compares the number of resources, project duration, total cost, start dates, and finish dates. This study uses three examples to verify the proposed concept of the optimization model. If the optimal solutions from the proposed optimization model are the same or better than trial-and-error, the capability of the optimization model can be guaranteed.

4.4.1 First example

The first example is a small project with five identical units (type P1) where four typical activities are repeated including pile, footing, column, and segment. The desired project duration is 50 days and the maximum available resources are 5 teams for each category $R_{(i)}$. The purpose of the first example is to verify the optimization model's capability dealing with general cases of repetitive projects (one identical type of units). Activity, sequence, duration, specific resources, cost per unit in million baht per unit, and pier station are illustrated in Table 4.2 and Figure 4.33.

Table 4.2 Information for the first example project

Activtiy	Sequence	Duration (days)	Sepcific resource	Cost per unit (m baht/unit)
Segment	i = 4	5		
Column	i = 3	7	Formwork for Column	2
Footing	i = 2	4	Formwork for Footing	1
Pile	i = 1	6	Casing Ø 1.5 m	1

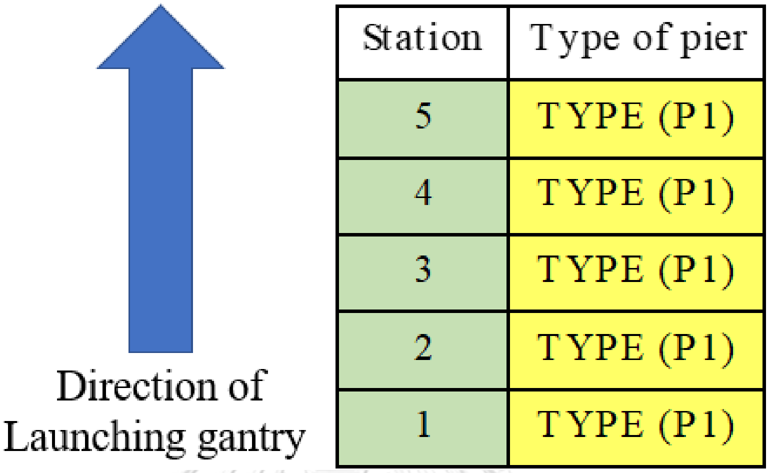


Figure 4.33 Direction of launching gantry, station, and type of pier for the first example

To find the minimum project duration, the $DSS_{(1)}$ of each pair of predecessor and successor must be the minimum value. The minimum $DSS_{(1)}$ is the case that all pairs are divergent lines as shown in Figure 4.34. Thus, $DSS_{(1)}$ of each pair is equal to the duration of its predecessor. The project duration retrieves from Eq. 21.

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)(set\,1)}^{(Pair\,u)(type\,k)} + \left(D_{(I)}^{(type\,k)} \times Q_A^{(type\,k)}\right) \text{.............} \textit{Eq. 21}$$

Minimum $P^{(type\,P1)} = D_{(1)} + D_{(2)} + D_{(3)} + (D_{(I)}^{(type\,P1)} \text{ x } Q_A^{(type\,P1)})$

Minimum $P^{(type\,P1)} = 6 + 4 + 7 + (5 \times 5) = 42$ days

For the first example, the trial-and-error is stopped when the project duration reached 42 days or the desired project duration (50 days) and the increasing number of resources does not reduce the total cost.

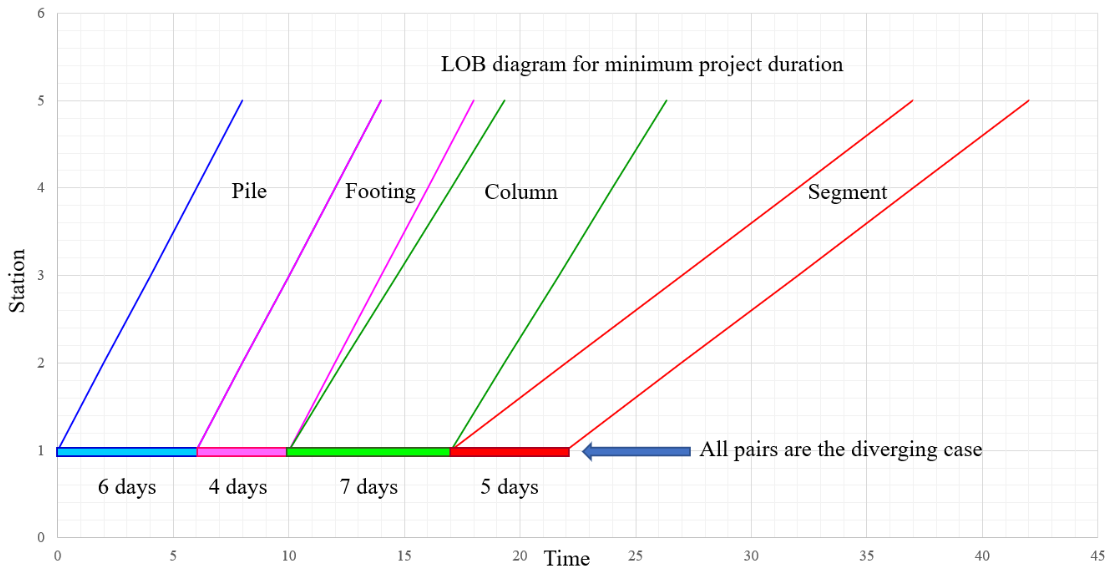


Figure 4.34 LOB diagram for the minimum project duration of the first example

To find the optimal solution with the trial-and-error, a search space of the sets $(Set_{(s)}{}^{(type\ k)})$ of decision variables is created and the first trial solution is s = 1 where all $R_{(i)}{}^{(type\ k)}$ is equal to 1. The number of total pairs (U) is three and there is only one set (V=1), so the first pair (u=1) is the pair of the pile and footing.

From Eq.2, the slope of the pile and slope of the footing are calculated as the following. $D_{(1)} = 6, R_{(1)} = 1, D_{(2)} = 4, and\ R_{(2)} = 1$

$$m_{(1)} = \frac{1}{6}\ and\ m_{(2)} = \frac{1}{4} \quad \text{...} (67)$$

From (67), the slopes of lines are the converging case, u = 1 < U, and V = 1, so $DSS_{(1)}$ is retrieved from Eq.17.

$$Q^{(type\ k)} = 5,\ m_{(i)}{}^{(type\ k)} = m_{(2)},\ m_{(i-1)}{}^{(type\ k)} = m_{(1)}, D_{(i-1)}{}^{(type\ k)} = D_{(1)},$$

$$R_{(i-1)}{}^{(type\ k)} = R_{(1)}, and\ DFS_{(J)}{}_{(set\ V)}{}^{(Pair\ u)(type\ k)} = 0$$

$$DFS_{(J)_{(set\ V)}}{}^{(Pair\ u)(type\ k)}$$

$$= DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(Q^{(type\ k)} - 1\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(Q^{(type\ k)}\right)$$

$$- \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \text{......................................} \textit{Eq.17}$$

$$0 = DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 1)(type\ P1)} + \left(\frac{4}{1}\right) \times (5-1) - \left(\frac{6}{1}\right) \times (5) - \left(\frac{1-1}{1}\right) \times 6 \text{.....}(68)$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 1)(type\ P1)} = 14 \text{......................................}(69)$$

Next, the second pair (u=2) in consideration is the pair of footing and column. From Eq.2, the slope of the footing and slope of the column are calculated as the following. $D_{(2)} = 4, R_{(2)} = 1, D_{(3)} = 7,\ R_{(3)} = 1$

$$m_{(2)} = \frac{1}{4}\ \ and\ m_{(3)} = \frac{1}{7} \text{......................................}(70)$$

From (70), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\ k)} = D_{(2)}$;

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = D_{(i-1)}{}^{(type\ k)} \text{......................................} \textit{Eq.18}$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P1)} = 4 \text{......................................}(71)$$

The third pair (u=3) in consideration is the pair of column and segment. From Eq.2, the slope of the column and slope of the segment are calculated as the following.$D_{(3)} = 7, R_{(3)} = 1, D_{(4)} = 5, and\ R_{(4)} = 1$

$$m_{(3)} = \frac{1}{7}\ \ and\ m_{(4)} = \frac{1}{5} \text{......................................}(72)$$

From (72), the slopes of lines are the converging case and u = 3 = U and v = 1 = V, so $DSS_{(1)}$ is retrieved from Eq.14.

$N_{(set\ 1)}{}^{(type\ k)} = 5,\ m_{(i)}{}^{(type\ k)} = m_{(4)},\ m_{(i-1)}{}^{(type\ k)} = m_{(3)}, D_{(i-1)}{}^{(type\ k)} = D_{(3)}$, and $R_{(i-1)}{}^{(type\ k)} = R_{(3)}$. First trial $DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P1)} = D_{(3)}$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$$

$$= DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)} - 1\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)}\right)$$

$$- \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \text{...} Eq.\ 14$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P1)} = 7 + \left(\frac{5}{1}\right) \times (5-1) - \left(\frac{7}{1}\right) \times (5) - \left(\frac{1-1}{1}\right) \text{..............(73)}$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P1)} = -8 \text{..(74)}$$

Second trial $DSS_{(1)}$ of pair 3 $= D_{(3)} + 8 = 15$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P1)} = 15 + \left(\frac{5}{1}\right) \times (5-1) - \left(\frac{7}{1}\right) \times (5) - \left(\frac{1-1}{1}\right) \text{...........(75)}$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P1)} = 0 \text{..(76)}$$

From (76), $DFS_{(J)}$ is 0, so the minimum $DSS_{(1)}$ of pair 3 type P1 is obtained.

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P1)} = 15 \text{..(77)}$$

From (69), (71), and (77), the project duration is obtained by Eq.21 and Eq.22.

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(D_{(I)}{}^{(type\ k)} \times Q_A{}^{(type\ k)}\right) \text{..............} Eq.\ 21$$

$$Q_A{}^{(type\ k)} = \sum_{v=1}^{v=V} \left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ v\to(v+1))}{}^{(type\ k)}\right) \text{.....................................} Eq.22$$

$$Q_A{}^{(type\ P1)} = 5 \text{..(78)}$$

$$P^{(type\ P1)} = 14 + 4 + 15 + (5 \times 5) \text{..(79)}$$

$$P^{(type\ P1)} = 58 \text{..(80)}$$

To proves the method of project duration calculation, Figure 4.35 is the LOB diagram of the first example for solution s = 1. The diagram is created by the general approach of the LOB technique. From (80) and Figure 4.35, the project duration for solution s = 1 is the same, so the method can be guaranteed its capability dealing with one identical type of units.

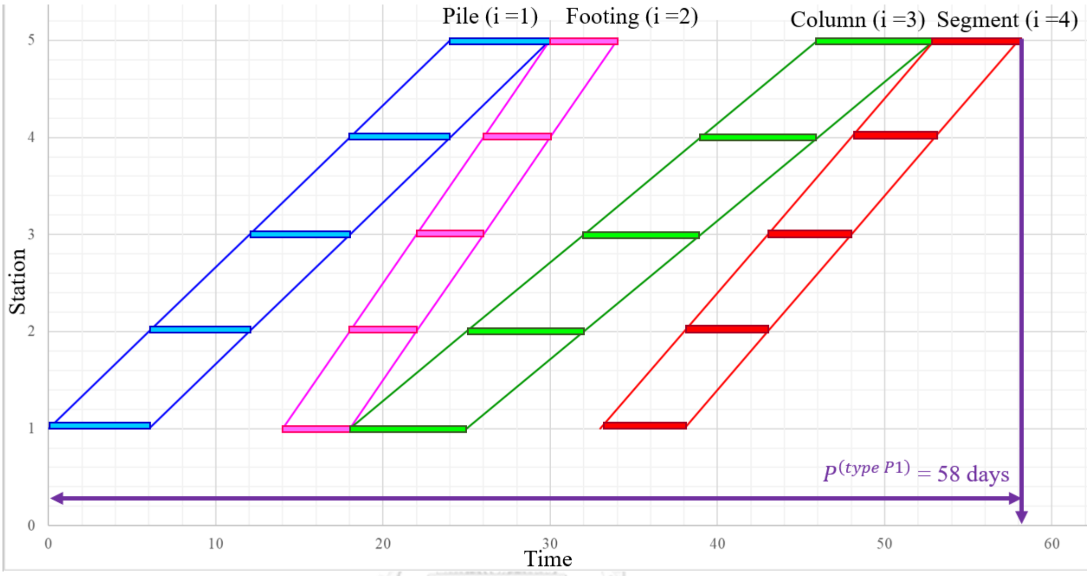


Figure 4.35 LOB diagram of the first example by the first trial solution

For the solution s = 1, the project duration is exceeded longer than the desired project duration that is required to complete within 50 days. Thus, the next trial solution is needed to find possible solutions. In this manual approach, trial all solutions would heavily take time and labor force. To facilitate the trial, a trick to select the next trial solution can consider the value of $DSS_{(1)}$ as an assistant. The value of $DSS_{(1)}$ reflects the duration of the first unit between two consecutive activities. If the value of $DSS_{(1)}$ is very high compared with the duration of the preceding activity of the pair, the number of resources for the preceding activity should be increased in the other to reduce the $DSS_{(1)}$ which directly affects the project duration. Moreover, if there are many pairs provided high different $DSS_{(1)}$, the pair that the preceding activity has the lowest cost per unit should be firstly concerned.

In this case, the pair 1 and pair 3 has the most different value of $DSS_{(1)}$ and the preceding activity of pair 1 (pile) has the lowest cost per unit. Thus, the next trial solution should be increased the number of resources for pile.

From the trick to select the trial solution, the next solution is the set of decision variables that $R_{(1)}^{(type\ P1)}, R_{(2)}^{(type\ P1)}, R_{(3)}^{(type\ P1)}$ is 2, 1, 1, respectively.

From Eq.2, the slope of the pile and slope of the footing are calculated as follows. $D_{(1)} = 6, R_{(1)} = 2, D_{(2)} = 4, and\ R_{(2)} = 1$

$$m_{(1)} = \frac{2}{6} = \frac{1}{3}\ and\ m_{(2)} = \frac{1}{4} \text{ ....} (81)$$

From (81), the slopes of lines are the diverging case, so DSS(1) is retrieved from Eq.18. $D_{(i-1)}^{(type\ k)} = D_{(1)}$

$$DSS_{(1)_{(set\ 1)}}^{(Pair\ u)(type\ k)} = D_{(i-1)}^{(type\ k)} \text{ ....} Eq.18$$

$$DSS_{(1)_{(set\ 1)}}^{(Pair\ 1)(type\ P1)} = 6 \text{ ....} (82)$$

From solution s = 1 in the first trial, only $R_{(1)}^{(type\ P1)}$ is increased from 1 to 2. The other decision variables remain as 1 as the first trial. Thus, the $DSS_{(1)}$ for pair 2 and pair 3 is the same. The project duration is then calculated from (71), (77), (78), Eq. 21, and Eq.22.

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}^{(Pair\ u)(type\ k)} + \left(D_{(I)}^{(type\ k)} \times Q_A^{(type\ k)}\right) \text{ ....} Eq.\ 21$$

$$Q_A^{(type\ P1)} = 5 \text{ ....} (78)$$

$$P^{(type\ P1)} = 6 + 4 + 15 + (5 \times 5) \text{ ....} (83)$$

$$P^{(type\ P1)} = 50 \text{ ....} (84)$$

For the second trial solution, the project is completed on day 50 which is within 50 of the desired project duration. So, the second trial solution is the possible solution which may be the optimal solution.

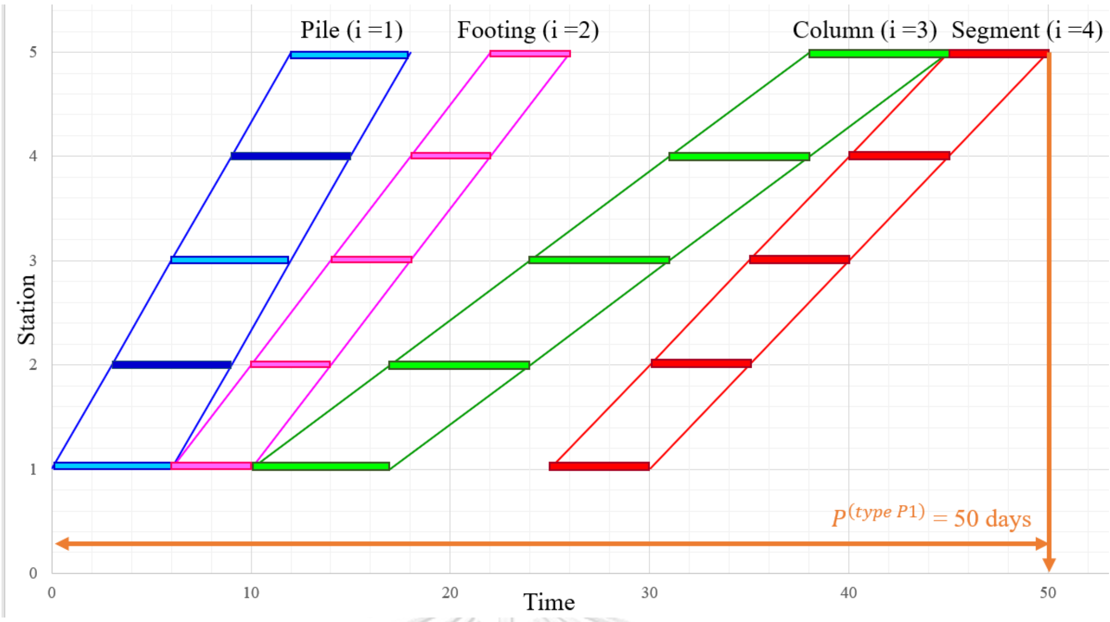


Figure 4.36 LOB diagram of the first example by the second trial solution

To ensure the optimal solution, this trial moves to consider one more solution which $R_{(1)}{}^{(type\ P1)}, R_{(2)}{}^{(type\ P1)}, R_{(3)}{}^{(type\ P1)}$ is 1, 1, 2, respectively. This third trial solution has an increased number of specific resources for the column. From (69), the first pair u of the third trial solution has the same number of specific resources (R=1). Hence, only pair 2 and pair 3 are demonstrated.

The second pair (u=2) in consideration is the pair of footing and column. From Eq.2, the slope of the footing and slope of the column are calculated as the following.$D_{(2)} = 4, R_{(2)} = 1, D_{(3)} = 7, and\ R_{(3)} = 2$

$$m_{(2)} = \frac{1}{4}\ and\ m_{(3)} = \frac{2}{7} \quad \text{.......(85)}$$

From (85), the slopes of lines are the converging case and u < U, so $DSS_{(1)}$ is retrieved from Eq.17.

$Q^{(type\ k)} = 5,\ m_{(i)}{}^{(type\ k)} = m_{(3)},\ m_{(i-1)}{}^{(type\ k)} = m_{(2)}, D_{(i-1)}{}^{(type\ k)} = D_{(2)},$

$R_{(i-1)}{}^{(type\ k)} = R_{(2)}$, and $DFS_{(J)}{}_{(set\ V)}^{(Pair\ u)(type\ k)} = 0$

$$DFS_{(J)_{(set\ V)}}{}^{(Pair\ u)(type\ k)}$$
$$= DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(Q^{(type\ k)} - 1\right)$$
$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(Q^{(type\ k)}\right)$$
$$- \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \ldots\ldots\ldots\ldots Eq.17$$

$$0 = DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P1)} + \left(\frac{7}{2}\right) \times (5-1) - \left(\frac{4}{1}\right) \times (5) - \left(\frac{1-1}{1}\right) \times 4 \ldots\ldots(86)$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P1)} = 6 \ldots\ldots\ldots\ldots(87)$$

The third pair (u=3) in consideration is the pair of the column and the segment. From Eq.2, the slope of the column and slope of the segment are calculated as the following. $D_{(3)} = 7, R_{(3)} = 2, D_{(4)} = 5, and\ R_{(4)} = 1$

$$m_{(3)} = \frac{2}{7}\ and\ m_{(4)} = \frac{1}{5} \ldots\ldots\ldots\ldots(88)$$

From (88), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\ k)} = D_{(3)}$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = D_{(i-1)}{}^{(type\ k)} \ldots\ldots\ldots\ldots Eq.18$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P1)} = 7 \ldots\ldots\ldots\ldots(89)$$

From (69), (87), and (89), the project duration for the third trial solution can be obtained by using Eq.21 and Eq.22.

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(D_{(I)}{}^{(type\ k)} \times Q_A{}^{(type\ k)}\right) \ldots\ldots Eq.\ 21$$

$$Q_A{}^{(type\ P1)} = 5 \ldots\ldots\ldots\ldots(78)$$

$$P^{(type\ P1)} = 14 + 6 + 7 + (5 \times 5) \ldots\ldots\ldots\ldots(90)$$

$$P^{(type\ P1)} = 52 \ldots\ldots\ldots\ldots(91)$$

The project duration is extended to be 52 days by the third trial. It means that the third trial solution can not achieve the constraint of the optimization problem.

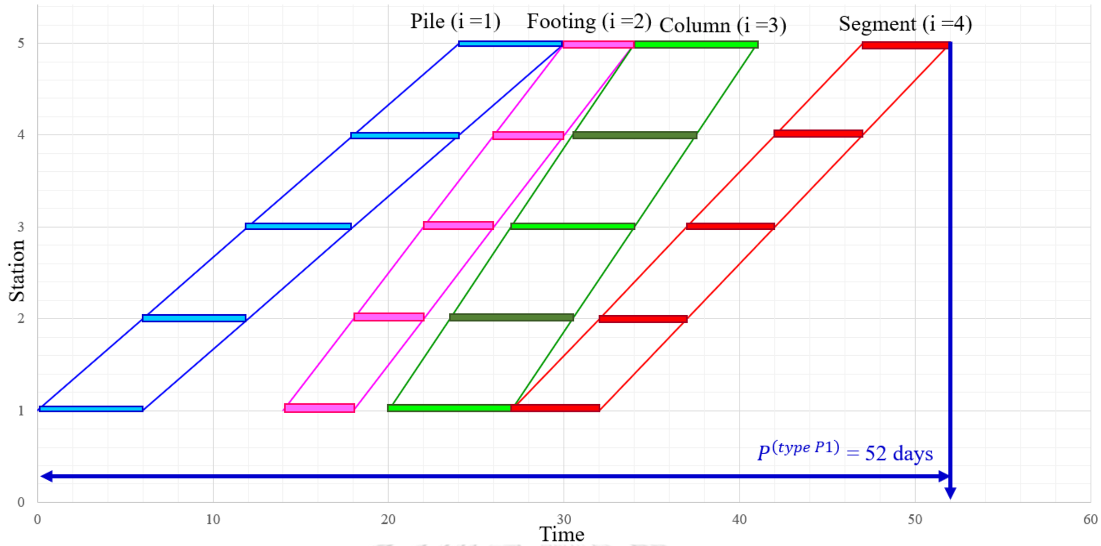


Figure 4.37 LOB diagram of the first example by using the third trial solution

From the three trial solutions, only the second solution achieves the constraint of the desired project duration as shown in Table 4.3. Even though more solutions can be tried to find more possible solutions, the optimal solution can be decided with these 3 solutions. The objective function is to minimize the total cost of specific resources. The solution that provides the lowest total cost without considering the constraint of the desired project duration is always the solution that all resources are only one unit. In this example, the first trial solution provides the project duration 58 days and the total cost is 4 million baht (calculated by Eq. 24). The second trial solution proposes 50 days of project duration and 5 million baht of the total cost (calculated by Eq. 24). From the information of the first example, the lowest cost per unit of specific resource is 1 million baht (the pile and the footing). To reduce the project duration with minimum cost, the resource of the pile or the resource of footing should be added. The second trial solution shows that adding one more the resource of the pile shortens the project duration from 58 days to 50 days which fulfills the constraint. So, the other solution with higher numbers of resources may provide shorter project durations but the total cost is also higher according to the number of resources. This can decide that the optimal solution is the second trial solution.

Table 4.3 Solution of the first example by using trial-and-error

Activtiy	Sequence	Sepcific resource	Number of specific resources		
			First trial	Second trial	Thrid trial
Column	i = 3	Formwork for Column	1	1	2
Footing	i = 2	Formwork for Footing	1	1	1
Pile	i = 1	Casing Ø 1.5 m	1	2	1
		Project duration	58	50	52

Table 4.4 Optimal solution of the first example by using trial-and-error

Activtiy	Sepcific resource	Cost per unit (m Baht/unit)	Number of resource	Cost of resource (m baht)
Segment	-	-	-	-
Column	Formwork for Column	2	1	2
Footing	Formwork for Footing	1	1	1
Pile	Casing Ø 1.5 m	1	2	2
Project duration	50 days	Total cost of resources	5 million baht	

For the optimization model, the information of the first example from Table 4.2 is assigned to the optimization model. The searching path for the first example is shown in Figure 4.38. The result from the model shows that the optimal solution is exact as trial-and-error as shown in Table 4.5.

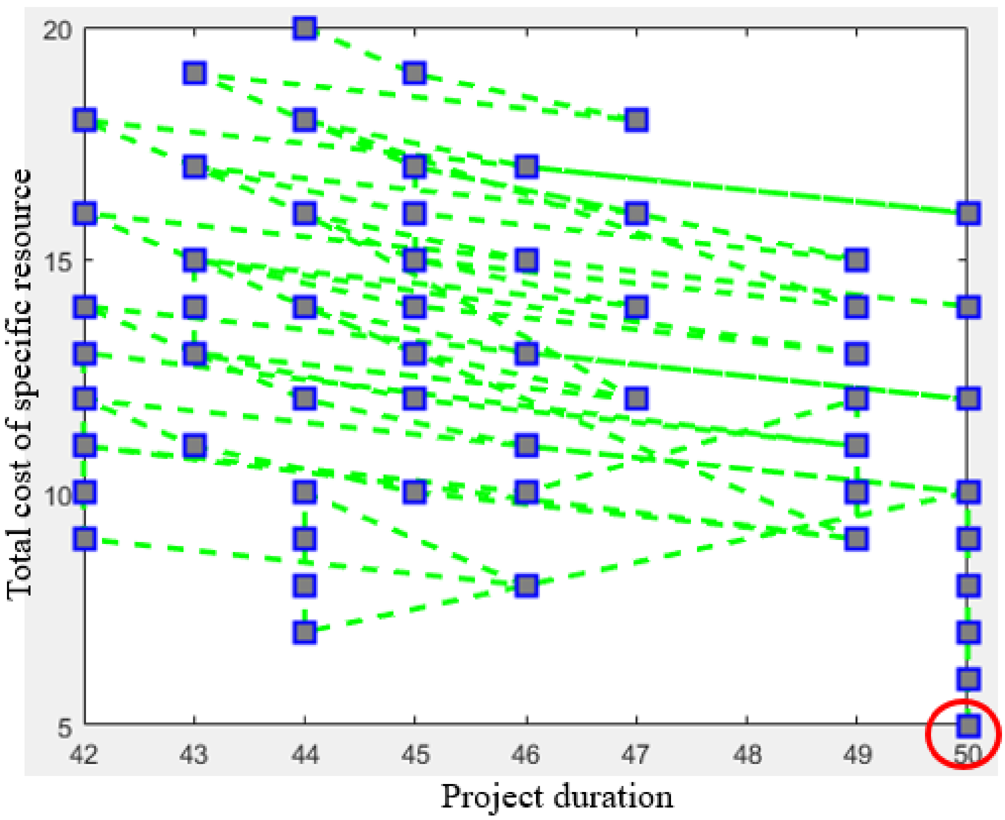


Figure 4.38 Searching path for the first example

Table 4.5 Optimal solution of the first example by using the optimization model

Activtiy	Sepcific resource	Cost per unit (m Baht/unit)	Number of resource	Cost of resource (m baht)
Segment	-	-	-	-
Column	Formwork for Column	2	1	2
Footing	Formwork for Footing	1	1	1
Pile	Casing Ø 1.5 m	1	2	2
Project duration	50 days	Total cost of resources	5 million baht	

The trial-and-error process can find the optimal solution with 50 days and 5 million baht of the total cost. With 5 maximum available resources, the optimization model searched for 125 solutions and retrieved the optimal solution with 50 days and 5 million baht of the total cost as the trial-and-error. The verification with the first example has shown that the optimization can solve the optimal solution compared with the solution from trial-and-error. This result could summarize that the optimization model is capable of dealing with the project that all units are identical.

4.4.2 Second example

The second example is a project with 15 units. The units are classified into three types. Each type has three short-duration activities. The desired project duration is 18 days. The second example aims to test the optimization model with the problem of multi-identical types of units. Activity, sequence, duration, specific resources, cost per unit in million baht per unit, and pier station are illustrated in Table 4.6 and Figure 4.39.

Table 4.6 Information of the second example project

Type /Activtiy	Sequence	Duration (days)	Sepcific resource	Cost per unit (m baht/unit)
Type P1 (k = 1)				
Segment	i = 3	1	-	-
Column	i = 2	1	Formwork for Column P1	1
Footing	i = 1	2	Formwork for Footing P1	1
Type P2 (k = 2)				
Segment	i = 3	1	-	-
Column	i = 2	2	Formwork for Column P2	1
Footing	i = 1	1	Formwork for Footing P2	1
Type P3 (k = 3)				
Segment	i = 3	1	-	-
Column	i = 2	3	Formwork for Column P3	1
Footing	i = 1	3	Formwork for Footing P3	1

Station	Type of Pier
15	TYPE (P2)
14	TYPE (P3)
13	TYPE (P3)
12	TYPE (P1)
11	TYPE (P1)
10	TYPE (P1)
9	TYPE (P2)
8	TYPE (P2)
7	TYPE (P2)
6	TYPE (P2)
5	TYPE (P2)
4	TYPE (P2)
3	TYPE (P1)
2	TYPE (P1)
1	TYPE (P1)

Figure 4.39 Direction of launching gantry, station, and type of pier for the second example

The minimum project duration for multi-identical types of units is obtained by calculating minimum project duration by each type with the case of diverging lines as the one identical type of unit in the first example. Then, the minimum duration by all types in the project is compared and the type that provides the longest duration is selected to be the control type of minimum project duration. From Figure 4.39, the variable $N_{(set\ v)}{}^{(type\ k)}$ and the variable $N_{(set\ v \to (v+1)}{}^{(type\ k)}$ are as the following.

$$N_{(set\ 1)}{}^{(type\ P1)} = 3, N_{(set\ 2)}{}^{(type\ P1)} = 3$$

$$N_{(set\ 1\to 2)}{}^{(type\ P1)} = 6, N_{(set\ 2\to 3)}{}^{(type\ P1)} = 3$$

$$N_{(set\ 1)}{}^{(type\ P2)} = 6, N_{(set\ 2)}{}^{(type\ P2)} = 1$$

$$N_{(set\ 1\to 2)}{}^{(type\ P2)} = 5$$

$$N_{(set\ 1)}{}^{(type\ P3)} = 2, N_{(set\ 1\to 2)}{}^{(type\ P3)} = 1$$

$$Q_A{}^{(type\ k)} = \sum_{v=1}^{v=V} \left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ v\to(v+1))}{}^{(type\ k)}\right) \text{......................................} Eq.22$$

The project duration is obtained by Eq. 21 and $DSS_{(1)}$ in the previous section.

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} \times Q_A{}^{(type\,k)}\right) \text{..............} \textit{Eq. 21}$$

Minimum $P^{(type\,P1)} = D_{(1)}{}^{(type\,P1)} + D_{(2)}{}^{(type\,P1)} + \left(D_{(3)}{}^{(type\,P1)} \times Q_A{}^{(type\,P1)}\right)$

Minimum $P^{(type\,P1)} = 2 + 1 + (1 \times 15) = 18$ days

Minimum $P^{(type\,P2)} = D_{(1)}{}^{(type\,P2)} + D_{(2)}{}^{(type\,P2)} + \left(D_{(3)}{}^{(type\,P2)} \times Q_A{}^{(type\,P2)}\right)$

Minimum $P^{(type\,P2)} = 1 + 2 + (1 \times 12) = 15$ days

Minimum $P^{(type\,P3)} = D_{(1)}{}^{(type\,P3)} + D_{(2)}{}^{(type\,P3)} + \left(D_{(3)}{}^{(type\,P3)} \times Q_A{}^{(type\,P3)}\right)$

Minimum $P^{(type\,P3)} = 3 + 3 + (1 \times 3) = 9$ days

Type P1 contains the longest of the minimum project duration. This means the project duration by type P2 and P3 that is shorter than 18 days having no effect on project duration. So the control type of minimum project duration is type P1 with 18 days.

For the second example, the trial-and-error is stopped when the project duration reached 18 days and the increasing number of resources does not reduce the total cost. For multi-identical types of units, the optimization process considers each type independently. Thus, the process runs three times following the number of types.

The first type to find the optimal solution is type P1 (k=1). To find the optimal solution with the trial-and-error, a search space of the sets ($Set_{(s)}{}^{(type\,k)}$) of decision variables of type P1 is created and the first trial solution is s = 1 where all $R_{(i)}{}^{(type\,P1)}$ is equal to 1. The number of total pairs (U = 2) is two and there are two sets (V=2), so the first pair (u=1) is the pair of the footing P1 and column P1. The variable $N_{(set\,\mathrm{v})}{}^{(type\,P1)}$ and the variable $N_{(set\,v\rightarrow(v+1))}{}^{(type\,P1)}$ are as the following.

$N_{(set\,1)}{}^{(type\,P1)} = 3, N_{(set\,2)}{}^{(type\,\mathrm{P1})} = 3$

$N_{(set\,1\rightarrow 2)}{}^{(type\,P1)} = 6, N_{(set\,2\rightarrow 3)}{}^{(type\,P1)} = 3$

From Eq.2, the slope of the footing P1 and slope of the column P1 are calculated as the following.

$$D_{(1)}{}^{(type\,P1)} = 2, R_{(1)}{}^{(type\,P1)} = 1, D_{(2)}{}^{(type\,P1)} = 1,\; R_{(2)}{}^{(type\,P1)} = 1$$

$$m_{(1)}{}^{(type\,P1)} = \frac{1}{2}\; and\; m_{(2)}{}^{(type\,P1)} = \frac{1}{1} \quad \text{......}(92)$$

From (92), the slopes of lines are the converging case and u = 1 < U, so $DSS_{(1)}$ is retrieved from Eq.17.

$$Q^{(type\,k)} = 6,\; m_{(i)}{}^{(type\,k)} = m_{(2)}{}^{(type\,P1)},\; m_{(i-1)}{}^{(type\,k)} = m_{(1)}{}^{(type\,P1)},$$

$$D_{(i-1)}{}^{(type\,k)} = D_{(1)}{}^{(type\,P1)}, R_{(i-1)}{}^{(type\,k)} = R_{(1)}{}^{(type\,P1)},$$

$$and\; DFS_{(J)_{(set\,V)}}{}^{(Pair\,u)(type\,k)} = 0$$

$$DFS_{(J)_{(set\,V)}}{}^{(Pair\,u)(type\,k)}$$

$$= DSS_{(1)_{(et\,1)}}{}^{(Pair\,u)(type\,k)} + \left(\frac{1}{m_{(i)}{}^{(type\,k)}}\right) \times \left(Q^{(type\,k)} - 1\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\,k)}}\right) \times \left(Q^{(type\,k)}\right)$$

$$- \left(\frac{R_{(i-1)}{}^{(type\,k)} - 1}{R_{(i-1)}{}^{(type\,k)}}\right) \times D_{(i-1)}{}^{(type\,k)} \quad \text{......} Eq.17$$

$$0 = DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P1)} + \left(\frac{1}{1}\right) \times (6-1) - \left(\frac{2}{1}\right) \times (6) - \left(\frac{1-1}{1}\right) \times 2 \text{.....}(93)$$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P1)} = 7 \text{......}(94)$$

The second pair (u=2) is the pair of column P1 and segment. From Eq.2, the slope of column P1 and slope of the segment are calculated as the following.

$$D_{(2)}{}^{(type\,P1)} = 1, R_{(2)}{}^{(type\,P1)} = 1, D_{(3)}{}^{(type\,P1)} = 1, and\; R_{(3)}{}^{(type\,P1)} = 1$$

$$m_{(2)}{}^{(type\,P1)} = \frac{1}{1}\; and\; m_{(3)}{}^{(type\,P1)} = \frac{1}{1} \quad \text{......}(95)$$

From (95), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\,k)} = D_{(2)}{}^{(type\,P1)}$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} = D_{(i-1)}{}^{(type\,k)} \quad \text{......} Eq.18$$

$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P1)} = 1$ ....(96)

From (94) and (96), the project duration is obtained by Eq.21 an Eq.22.

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(D_{(I)}{}^{(type\ k)} \times Q_A{}^{(type\ k)}\right) \text{.............} Eq.\ 21$$

$$Q_A{}^{(type\ k)} = \sum\nolimits_{v=1}^{v=V} (N_{(set\ v)}{}^{(type\ k)} + N_{(set\ v\rightarrow(v+1))}{}^{(type\ k)}) \text{....................} Eq.22$$

$Q_A{}^{(type\ P1)} = 3 + 3 + 6 + 3$ ....(97)

$P^{(type\ P1)} = 7 + 1 + (1 \times 15)$ ....(98)

$P^{(type\ P1)} = 23$ ....(99)

For the first trial solution of type P1, the project duration is 23 days which exceeds longer than 18 days of the desired project duration. Hence, the second trial solution should examine to shorten the project duration. From (94), $DSS_{(1)}$ has the longest different duration with 7 days. Thus, increasing the number of resources for footing type P1 would reduce the project duration significantly.

From the trick to select the trial solution, the next solution is the set of decision variables that $R_{(1)}{}^{(type\ P1)}, R_{(2)}{}^{(type\ P1)}$ is 2, 1, respectively.

From Eq.2, the slope of the footing P1 and slope of the column P1 are calculated as the following. $D_{(1)} = 2, R_{(1)} = 2, D_{(2)} = 1, and\ R_{(2)} = 1$

$$m_{(1)}{}^{(type\ P1)} = \frac{2}{2}\ and\ m_{(2)}{}^{(type\ P1)} = \frac{1}{1} \text{....................} (100)$$

From (100), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\ k)} = D_{(1)}{}^{(type\ P2)}$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = D_{(i-1)}{}^{(type\ k)} \text{....................} Eq.18$$

$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 1)(type\ P1)} = 2$ ....(101)

For pair 2, the numbers of resources are not changed, so the $DSS_{(1)}$ from (96) can be used. From (96) and (101), the project duration is obtained by Eq.21 an Eq.22.

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} \times Q_A{}^{(type\,k)}\right) \quad\text{.............}Eq.\ 21$$

$$Q_A{}^{(type\,P1)} = 15 \quad\text{..........}(97)$$

$$P^{(type\,P1)} = 2 + 1 + (1 \times 15) \quad\text{..........}(102)$$

$$P^{(type\,P1)} = 18 \quad\text{..........}(103)$$

For the second trial solution, the project is completed on day 18 which is within 18 of the desired project duration. The second trial solution not only provides a possible solution but also with all pairs are the case of diverging lines, so the optimal solution for type P1 can be decided that there is no better solution that could beat the second trial solution of P1 considering in term of cost and project duration.

Next is the determination of the optimal solution for type P2 (k=2). The first trial solution is s = 1 where all $R_{(i)}{}^{(type\,1)}$ is equal to 1. The number of total pairs (U = 2) is two and there are two sets (V=2), so the first pair (u=1) is the pair of the footing P2 and column P2.

The variable $N_{(set\,v)}{}^{(type\,P2)}$ and the variable $N_{(set\,v\to(v+1)}{}^{(type\,P2)}$ from set 1 to set 2 must be determined. From Figure 4.38, the values of these variables are as the following.

$$N_{(set\,1)}{}^{(type\,P2)} = 6, N_{(set\,2)}{}^{(type\,\mathrm{P2})} = 1$$

$$N_{(set\,1\to2)}{}^{(type\,P2)} = 5$$

From Eq.2, the slope of the footing P2 and slope of the column P2 are calculated as the following.

$$D_{(1)}{}^{(type\,P2)} = 1, R_{(1)}{}^{(type\,P2)} = 1, D_{(2)}{}^{(type\,P2)} = 2,\ R_{(2)}{}^{(type\,P2)} = 1$$

$$m_{(1)}{}^{(type\,P2)} = \frac{1}{1}\ and\ m_{(2)}{}^{(type\,P2)} = \frac{1}{2} \quad\text{..........}(104)$$

From (104), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\,k)} = D_{(1)}{}^{(type\,P2)}$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} = D_{(i-1)}{}^{(type\,k)} \quad\text{..........}Eq.18$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 1)(type\ P2)} = 1 \text{......} (105)$$

The second pair (u=2) is the pair of column P2 and segment. From Eq.2, the slope of column P2 and slope of the segment are calculated as the following.

$$D_{(2)}{}^{(type\ P2)} = 2, R_{(2)}{}^{(type\ P2)} = 1, D_{(3)}{}^{(type\ P2)} = 1, and\ R_{(3)}{}^{(type\ P2)} = 1$$

$$m_{(2)}{}^{(type\ P2)} = \frac{1}{2}\ and\ m_{(3)}{}^{(type\ P2)} = \frac{1}{1} \text{......} (106)$$

From (106), the slopes of lines are the converging case, u = U = 2, and V = 2, so $DSS_{(1)}$ is retrieved from Eq.14 and Eq.15. First trial $DSS_{(1)}$ of set 1= $D_{(2)}{}^{(type\ P2)}$= 2

$$N_{(set\ 1)}{}^{(type\ P2)} = 6,\ N_{(set\ 2)}{}^{(type\ P2)} = 1, N_{(set\ 1\rightarrow 2)}{}^{(type\ P2)} = 5$$

$$m_{(3)}{}^{(type\ P2)} = 1, m_{(2)}{}^{(type\ P2)} = \frac{1}{2}, D_{(2)}{}^{(type\ P2)} = 2, R_{(2)}{}^{(type\ P2)} = 1$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$$

$$= DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)} - 1\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)}\right)$$

$$- \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \text{......} Eq. 14$$

$$DFS_{(J)_{(set\ v)}}{}^{(Pair\ u)(type\ k)}$$

$$= DFS_{(J)_{(set\ v-1)}}{}^{(Pair\ u)(type\ k)}$$

$$+ \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ (v-1)\rightarrow v)}{}^{(type\ k)}\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \left(N_{(set\ v)}{}^{(type\ k)}\right) \text{......} Eq. 15$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P2)} = 2 + \left(\frac{1}{1}\right) \times (6-1) - \left(\frac{2}{1}\right) \times (6) - \left(\frac{1-1}{1}\right) \times 2 \text{....} (107)$$

$$DFS_{(J)_{(set\ 2)}}{}^{(Pair\ 2)(type\ P2)}$$

$$= DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P2)} + \left(\frac{1}{1}\right)(1+5) - \left(\frac{2}{1}\right)(1) \text{......} (108)$$

$DFS_{(J)_{(set\ 1)}}^{(Pair\ 2)(type\ P2)} = -5$ ....(109)

$DFS_{(J)_{(set\ 2)}}^{(Pair\ 2)(type\ P2)} = -1$ ....(110)

From (109) and (110), $DFS_{(J)}$ still violates the sequence logic, so trial new $DSS_{(1)}$ of set 1 with more increased value.

$DSS_{(1)_{(set\ 1)}}^{(Pair\ 2)(type\ P2)} = 7$ ....(111)

$DFS_{(J)_{(set\ 1)}}^{(Pair\ 2)(type\ P2)} = 7 + \left(\frac{1}{1}\right) \times (6-1) - \left(\frac{2}{1}\right) \times (6) - \left(\frac{1-1}{1}\right) \times 2$ ....(112)

$DFS_{(J)_{(set\ 1)}}^{(Pair\ 2)(type\ P2)} = 0$ ....(113)

$DFS_{(J)_{(set\ 2)}}^{(Pair\ 2)(type\ P2)} = 4$ ....(114)

From (113) and (114), $DSS_{(1)}$ of set 1 = 7 is the minimum $DSS_{(1)}$ of set 1 which fulfills the sequence logic for set 1 and set 2 of type P2. To calculate the project duration by type P2, $DSS_{(1)}$ of set 1 from (94) and (96) are used to determine project duration by Eq.21 and Eq.22.

$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}^{(Pair\ u)(type\ k)} + \left(D_{(I)}^{(type\ k)} \times Q_A^{(type\ k)}\right)$ ....*Eq. 21*

$Q_A^{(type\ k)} = \sum_{v=1}^{v=V} (N_{(set\ v)}^{(type\ k)} + N_{(set\ v \rightarrow (v+1))}^{(type\ k)})$ *Eq.22*

$Q_A^{(type\ P2)} = 6 + 1 + 5$ ....(115)

$P^{(type\ P2)} = 1 + 7 + (1 \times 12)$ ....(116)

$P^{(type\ P2)} = 20$ ....(117)

For the first trial solution of type P2, the project duration is 20 days which can not achieve 18 days of the desired project duration. From (106), the slope lines are converging case. Increasing the number of resources for column type P2 would shorten the project duration. Thus, the second trial solution of type P2 is the set of decision variables that $R_{(1)}^{(type\ P2)}$, $R_{(2)}^{(type\ P2)}$ is 1, 2, respectively.

For pair u =1, From Eq.2, the slope of the footing P2 and slope of the column P2 are calculated as the following.

$D_{(1)}{}^{(type\,P2)} = 1, R_{(1)}{}^{(type\,P2)} = 1, D_{(2)}{}^{(type\,P2)} = 2,\ R_{(2)}{}^{(type\,P2)} = 2$

$$m_{(1)}{}^{(type\,P2)} = \frac{1}{1}\ and\ m_{(2)}{}^{(type\,P2)} = \frac{2}{2} \quad \text{..........}(118)$$

From (118), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\,k)} = D_{(1)}{}^{(type\,P2)}$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} = D_{(i-1)}{}^{(type\,k)} \quad \text{..........}Eq.18$$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P2)} = 1 \quad \text{..........}(119)$$

The second pair (u=2) is the pair of column P2 and segment. From Eq.2, the slope of column P2 and slope of the segment are calculated as the following.

$D_{(2)}{}^{(type\,P2)} = 2, R_{(2)}{}^{(type\,P2)} = 2, D_{(3)}{}^{(type\,P2)} = 1, and\ R_{(3)}{}^{(type\,P2)} = 1$

$$m_{(2)}{}^{(type\,P2)} = \frac{2}{2}\ and\ m_{(3)}{}^{(type\,P2)} = \frac{1}{1} \quad \text{..........}(120)$$

From (95), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\,k)} = D_{(2)}{}^{(type\,P2)}$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} = D_{(i-1)}{}^{(type\,k)} \quad \text{..........}Eq.18$$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,2)(type\,P2)} = 2 \quad \text{..........}(121)$$

From (119) and (121), the project duration by type P2 is determined by Eq.21 an Eq.22.

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} \times Q_A{}^{(type\,k)}\right) \quad \text{..........}Eq.\ 21$$

$$Q_A{}^{(type\,P2)} = 6 + 1 + 5 \quad \text{..........}(115)$$

$$P^{(type\,P2)} = 1 + 2 + (1 \times 12) \quad \text{..........}(121)$$

$$P^{(type\,P2)} = 15 \quad \text{..........}(122)$$

For the second trial solution of type P2, the solution provides 15 days of project duration which is lower than 18 days of the desired project duration. Therefore, the solution is a possible solution for type P2. Furthermore, the solution provides that project duration that reaches the minimum project duration of type P2 by all pairs of diverging lines. So, it is unnecessary to search for more solutions because none of them can provide the cost and time lower than the second trial solution. Thus, the second trial solution is the optimal solution type P2.

Finally, the next type is the calculation of the optimal solution for type P3 (k=3). The first trial solution is s = 1 where all $R_{(i)}{}^{(type\ P3)}$ is equal to 1. The number of total pairs (U = 2) is two and there is one set (V=1), so the first pair (u=1) is the pair of the footing P3 and column P3. The variable $N_{(set\ 1)}{}^{(type\ P3)}$ and the variable $N_{(set\ 1\to 2)}{}^{(type\ P3)}$ are the following.

$$N_{(set\ 1)}{}^{(type\ P3)} = 2, N_{(set\ 1\to 2)}{}^{(type\ P3)} = 1$$

From Eq.2, the slope of the footing P3 and slope of the column P3 are calculated as the following.

$$D_{(1)}{}^{(type\ P3)} = 3, R_{(1)}{}^{(type\ P3)} = 1, D_{(2)}{}^{(type\ P3)} = 2,\ R_{(2)}{}^{(type\ P3)} = 1$$

$$m_{(1)}{}^{(type\ P2)} = \frac{1}{3}\ and\ m_{(2)}{}^{(type\ P2)} = \frac{1}{3} \qquad (123)$$

From (123), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\ k)} = D_{(1)}{}^{(type\ P3)}$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = D_{(i-1)}{}^{(type\ k)} \qquad Eq.18$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 1)(type\ P3)} = 3 \qquad (124)$$

The second pair (u=2) is the pair of column P3 and segment. From Eq.2, the slope of column P3 and slope of the segment are calculated as the following.

$$D_{(2)}{}^{(type\ P3)} = 3, R_{(2)}{}^{(type\ P3)} = 1, D_{(3)}{}^{(type\ P3)} = 1,\ R_{(3)}{}^{(type\ P3)} = 1$$

$$m_{(1)}{}^{(type\ P3)} = \frac{1}{3}\ and\ m_{(2)}{}^{(type\ P3)} = \frac{1}{1} \qquad (125)$$

From (125), the slopes of lines are the converging case, u = U = 2, and V = 1, so $DSS_{(1)}$ is retrieved from Eq.14. There is only one set, so $DFS_{(J)}$ of set 1 is 0. $N_{(set\ 1)}{}^{(type\ P3)} = 2, m_{(3)}{}^{(type\ P3)} = \frac{1}{3}, m_{(2)}{}^{(type\ P3)} = 1, D_{(2)}{}^{(type\ P3)} = 3, R_{(2)}{}^{(type\ P3)} = 1,\ DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ 2)(type\ P3)} = 0$

$$DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ u)(type\ k)} = DSS_{(1)}{}_{(set\ 1)}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)} - 1\right) - \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)}\right) - \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \quad \ldots\ldots Eq.14$$

$$0 = DSS_{(1)}{}_{(set\ 1)}{}^{(Pair\ 2)(type\ P3)} + \left(\frac{1}{1}\right) \times (2-1) - \left(\frac{3}{1}\right) \times (2) - \left(\frac{1-1}{1}\right) \times 3 \quad \ldots(126)$$

$$DSS_{(1)}{}_{(set\ 1)}{}^{(Par\ 2)(type\ P3)} = 5 \quad \ldots\ldots(127)$$

From (124) and (127), the project duration by type P3 is determined by Eq.21 and Eq.22.

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)}{}_{(set\ 1)}{}^{(Pair\ u)(type\ k)} + \left(D_{(I)}{}^{(type\ k)} \times Q_A{}^{(type\ k)}\right) \quad \ldots\ldots Eq.\ 21$$

$$Q_A{}^{(type\ k)} = \sum_{v=1}^{v=V} \left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ v\rightarrow(v+1))}{}^{(type\ k)}\right) \quad \ldots\ldots Eq.22$$

$$Q_A{}^{(type\ P3)} = 2 + 1 \quad \ldots\ldots(128)$$

$$P^{(type\ P3)} = 3 + 5 + (1 \times 3) \quad \ldots\ldots(129)$$

$$P^{(type\ P3)} = 11 \quad \ldots\ldots(130)$$

For the first trial solution of type P3, the solution provides 11 days of project duration which achieves 18 days of the desired project duration. Moreover, the solution is the set of all decision variables is equal to 1 which has the minimum total cost of specific resources. Thus, the first trial solution is the optimal solution type P3.

From the solutions of type P1, type P2, and type P3, the project duration (P) is retrieved from Eq23. The total cost of specific resources for each type is calculated by Eq.24. The total cost of specific resources for the project is obtained from Eq.25.

$$P = Max\left(P^{(type\,1)}, P^{(type\,2)}, P^{(type\,3)}, \ldots, P^{(type\,K)}\right) \quad \text{......} Eq.\,23$$

$$P = Max\,(18, 15, 11) = 18 \quad \text{......} (131)$$

$$C_{TR}{}^{(type\,k)} = \left(R_{(1)}{}^{(type\,k)}\right)\left(C_{(1)}{}^{(type\,k)}\right) + \left(R_{(2)}{}^{(type\,k)}\right)\left(C_{(2)}{}^{(type\,k)}\right)$$

$$+\left(R_{(3)}{}^{(type\,k)}\right)\left(C_{(3)}{}^{(type\,k)}\right) + \ldots + \left(R_{(I-1)}{}^{(type\,k)}\right)\left(C_{(I-1)}{}^{(type\,k)}\right) \quad \text{......} Eq.\,24$$

$$C_{TP} = \sum_{k=1}^{k=K} \left(C_{TR}{}^{(tpe\,k)}\right) \quad \text{......} Eq.\,25$$

Table 4.7 shows the optimal solution of the second example by manual trial-and-error. The project duration is controlled by type P1 with 18 days. The total cost of specific resources for P1, P2, and P3 is 3 million baht, 3 million baht, and 2 million baht, respectively. The total cost of the specific resources of the project is 8 million baht. The LOB diagram by the general approach for the second example with the optimal solution is illustrated in Figure 4.40 below.

Table 4.7 Optimal solution of the second example by using trial-and-error

Type /Activtiy	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resource (m baht)
Type P1 (k = 1)				
Column	Formwork for Column P1	1	1	1
Footing	Formwork for Footing P1	1	2	2
Type P1	Project duration by P1	18 days	Total cost of resources	3
Type P2 (k = 2)				
Column	Formwork for Column P2	1	2	2
Footing	Formwork for Footing P2	1	1	1
Type P2	Project duration by P2	15 days	Total cost of resources	3
Type P3 (k = 3)				
Column	Formwork for Column P3	1	1	1
Footing	Formwork for Footing P3	1	1	1
Type P3	Project duration by P3	11 days	Total cost of resources	2
Project duration	18 days	Total cost of specific resource of the project		8

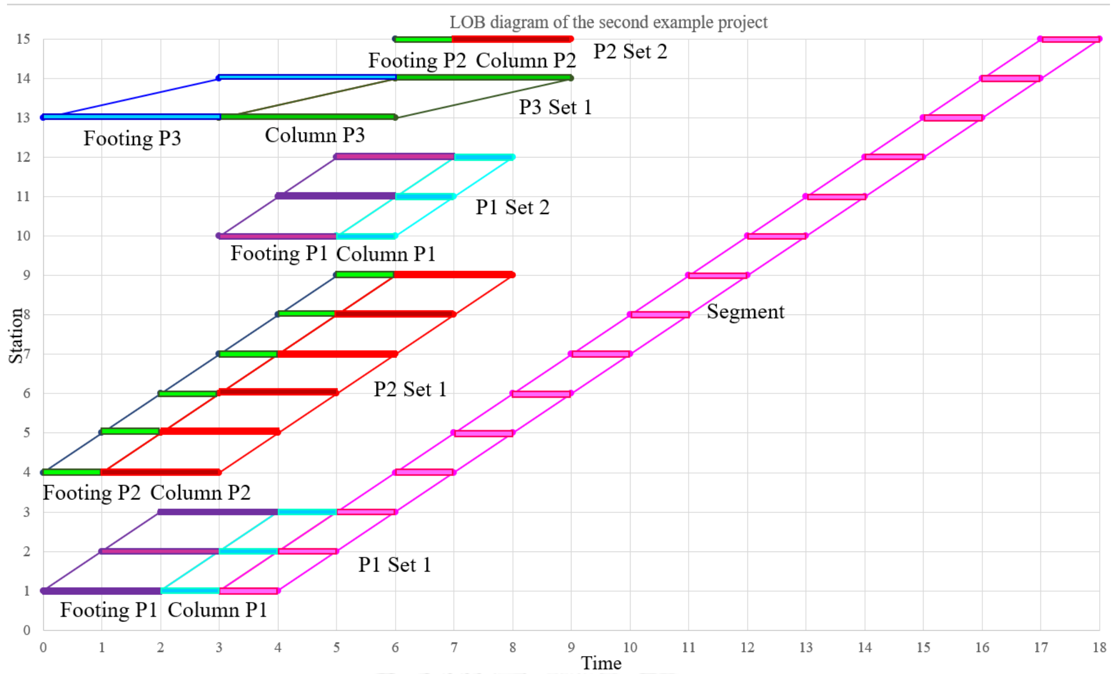


Figure 4.40 LOB diagram of the second example with the optimal solution

For the optimization model, the information of the second example from Table 4.6 is assigned to the optimization model. After the computation, the searching path for the second example is shown in Figure 4.41, 4.42, and 4.43. The result from the model shows that the optimal solution is exactly as trial-and-error shown in Table 4.8.

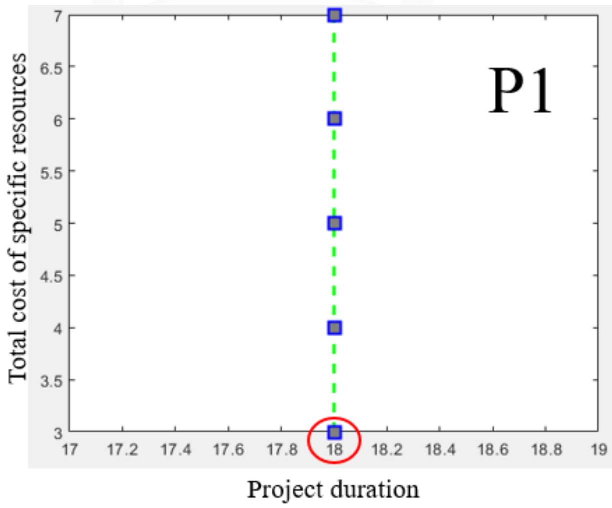


Figure 4.41 Searching path of P1 for the second example

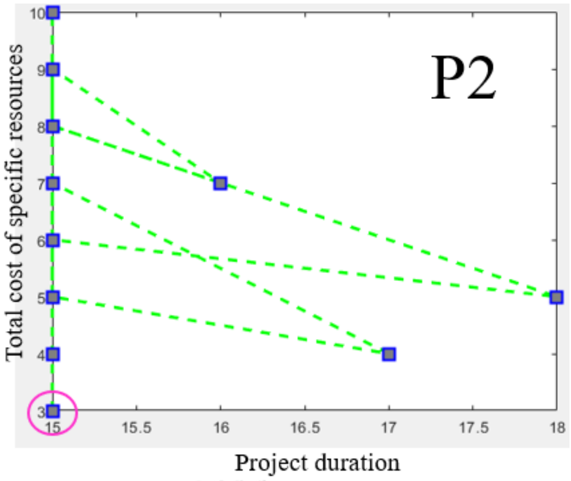


Figure 4.42 Searching path of P2 for the second example

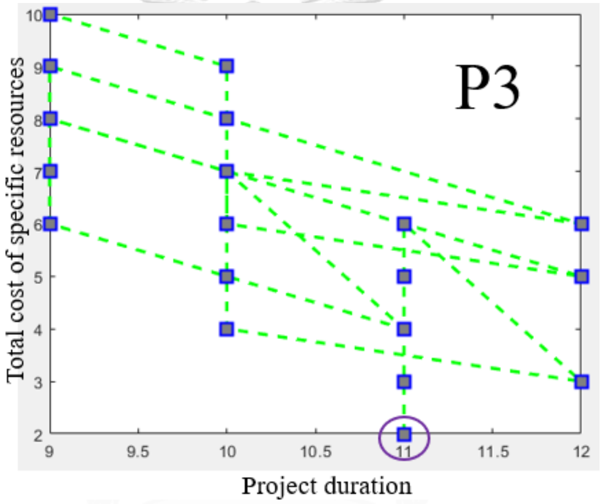


Figure 4.43 Searching path of P3 for the second example

Table 4.8 Optimal solution of the second example by using the optimization model

Type /Activtiy	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resource (m baht)
Type P1 (k = 1)				
Column	Formwork for Column P1	1	1	1
Footing	Formwork for Footing P1	1	2	2
Type P1	Project duration by P1	18 days	Total cost of resources	3
Type P2 (k = 2)				
Column	Formwork for Column P2	1	2	2
Footing	Formwork for Footing P2	1	1	1
Type P2	Project duration by P2	15 days	Total cost of resources	3
Type P3 (k = 3)				
Column	Formwork for Column P3	1	1	1
Footing	Formwork for Footing P3	1	1	1
Type P3	Project duration by P3	11 days	Total cost of resources	2
Project duration	18 days	Total cost of specific resource of the project		8

The trial-and-error process can solve the optimal solution for P1 with 18 days and 3 million baht of the total cost, for P2 with 15 days and 3 million baht of the total cost, for P3 with 11 days and 2 million baht of the total cost. With 5 maximum available resources, the optimization model searched for 25 sets of decision variables of each type. The model computes the optimal solution for P1 with 18 days and 3 million of the total cost, for P2 with 15 days and 3 million of the total cost, for P3 with 11 days and 2 million of the total cost as the trial-and-error. The verification with the second example has expressed that the optimization can solve the exact solution compared with the solution from trial-and-error. The solution shows that P1 is the control type due to the longest project duration. This result can ensure that the optimization model is capable of dealing with the project that multi-identical types of units with short activity duration.

In addition, the desired project duration in the second example is provided intently to be the minimum project duration with 18 days. This intention aims to illustrate that the optimization model can compute the minimum total cost of the project when the project is compelled to complete the minimum project duration. Thus, when the requirement of the project duration and minimum total cost occurs, the minimum project duration from the case and all pairs having diverging lines as showing in two examples can be assigned to the model to determine its minimum total cost of specific resources of the project.

4.4.3 Third example

The third example is a project with multi-identical types of units. 12 units are divided into two types which are type P1 containing four repetitive activities and type P2 consisting of five repetitive activities. The desirable project duration is 70 days. Verification with the third example aims to demonstrate the project duration can be controlled by any type depending on the number of activities, sequence, duration, specific resources, cost per unit, and pier station. The information about the third example is in Table 4.9 and Figure 4.44.

Table 4.9 Information of the third example project

Type /Activtiy	Sequence	Duration (days)	Sepcific resource	Cost per unit (m baht/unit)
Type P1 (k = 1)				
Segment	i = 4	4	-	-
Column	i = 3	6	Formwork for Column P1	2
Footing	i = 2	4	Formwork for Footing P1	1.5
Pile	i = 1	5	Casing Ø 1.5 m	1
Type P2 (k = 2)				
Segment	i = 5	4	-	-
Crossbeam	i = 4	9	ormwork for Crossbeam P	3
Column	i = 3	7	Formwork for Column P2	2
Footing	i = 2	5	Formwork for Footing P2	1
Pile	i = 1	6	Casing Ø 1.8 m	1.5

Station	Type of pier
12	TYPE (P1)
11	TYPE (P1)
10	TYPE (P2)
9	TYPE (P2)
8	TYPE (P2)
7	TYPE (P1)
6	TYPE (P1)
5	TYPE (P2)
4	TYPE (P2)
3	TYPE (P1)
2	TYPE (P1)
1	TYPE (P1)

Figure 4.44 Direction of launching gantry, station, and type of pier for the third example

From Figure 4.44, the variable $N_{(set\ v)}{}^{(type\ k)}$ and the variable $N_{(set\ v\to(v+1)}{}^{(type\ k)}$ are the following.

$$N_{(set\ 1)}{}^{(type\ P1)} = 3, N_{(set\ 2)}{}^{(type\ P1)} = 2, N_{(set\ 3)}{}^{(type\ P1)} = 2$$

$$N_{(set\ 1\to2)}{}^{(type\ P1)} = 2, N_{(set\ 2\to3)}{}^{(type\ P1)} = 3$$

$$N_{(set\ 1)}{}^{(type\ P2)} = 2, N_{(set\ 2)}{}^{(type\ P2)} = 3$$

$$N_{(set\ 1\to2)}{}^{(type\ P2)} = 2, N_{(set\ 2\to3)}{}^{(type\ P2)} = 2$$

$$Q_A{}^{(type\,k)} = \sum_{v=1}^{v=V} (N_{(set\,v)}{}^{(type\,k)} + N_{(set\,\text{v}\rightarrow(\text{v}+1))}{}^{(type\,k)}) \ldots\ldots\ldots Eq.22$$

The minimum project duration for type P1 and type P2 is calculated below.

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)}{}_{(set\,1)}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} \times Q_A{}^{(type\,k)}\right) \ldots\ldots\ldots Eq.\,21$$

Minimum $P^{(type\,P1)} = 5 + 4 + 6 + (4 \times 12) = 63$ days

Minimum $P^{(type\,P2)} = 6 + 5 + 7 + 9 + (4 \times 9) = 63$ days

For the third example, the minimum project duration is controlled by both type P1 and type P2. So, the trial-and-error will be ceased when the project duration reached 63 days or the desired project duration (70 days) and the increasing number of resources does not reduce the total cost.

The first type to find the optimal solution is type P1 (k=1). To find the optimal solution with the trial-and-error, the first trial solution is s = 1 where all $R_{(i)}{}^{(type\,P1)}$ is equal to 1. The number of total pairs (U = 3) is three pairs and there are three sets (V=3) of type P1, so the first pair (u=1) is the pair of the pile P1 and footing P1. The variable $N_{(set\,\text{v})}{}^{(type\,P1)}$ and the variable $N_{(set\,v\rightarrow(v+1))}{}^{(type\,P1)}$ are the following.

$$N_{(set\,1)}{}^{(type\,P1)} = 3, N_{(set\,2)}{}^{(type\,\text{P1})} = 2, N_{(set\,3)}{}^{(type\,\text{P1})} = 2$$

$$N_{(set\,1\rightarrow 2)}{}^{(type\,P1)} = 2, N_{(set\,2\rightarrow 3)}{}^{(type\,P1)} = 3$$

From Eq.2, the slope of the pile P1 and slope of the footing P1 are calculated as the following.

$$D_{(1)}{}^{(type\,P1)} = 5, R_{(1)}{}^{(type\,P1)} = 1, D_{(2)}{}^{(type\,P1)} = 4, R_{(2)}{}^{(type\,P1)} = 1$$

$$m_{(1)}{}^{(type\,P1)} = \frac{1}{5} \; and \; m_{(2)}{}^{(type\,P1)} = \frac{1}{4} \ldots\ldots\ldots (132)$$

From (132), the slopes of lines are the converging case and u = 1 < U, so $DSS_{(1)}$ is retrieved from Eq.17.

$$Q^{(type\,k)} = 7,\ m_{(i)}{}^{(type\,k)} = m_{(2)}{}^{(type\,P1)},\ m_{(i-1)}{}^{(type\,k)} = m_{(1)}{}^{(type\,P1)},$$

$$D_{(i-1)}{}^{(type\,k)} = D_{(1)}{}^{(type\,P1)}, R_{(i-1)}{}^{(type\,k)} = R_{(1)}{}^{(type\,P1)},$$

$$and\ DFS_{(J)_{(set\,V)}}{}^{(Pair\,u)(type\,k)} = 0$$

$$DFS_{(J)_{(set\,V)}}{}^{(Pair\,u)(type\,k)}$$

$$= DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(\frac{1}{m_{(i)}{}^{(type\,k)}}\right) \times \left(Q^{(type\,k)} - 1\right) - \left(\frac{1}{m_{(i-1)}{}^{(type\,k)}}\right) \times \left(Q^{(type\,k)}\right) - \left(\frac{R_{(i-1)}{}^{(type\,k)} - 1}{R_{(i-1)}{}^{(type\,k)}}\right) \times D_{(i-1)}{}^{(type\,k)} \quad \text{Eq.17}$$

$$0 = DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P1)} + \left(\frac{4}{1}\right) \times (7-1) - \left(\frac{5}{1}\right) \times (7) - \left(\frac{1-1}{1}\right) \times 5 \quad (133)$$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P1)} = 11 \quad (134)$$

The second pair (u=2) is the pair of the footing P1 and the column P1. From Eq.2, the slope of the footing P1 and slope of the column P1 are calculated as the following.

$$D_{(2)}{}^{(type\,P1)} = 4, R_{(2)}{}^{(type\,P1)} = 1, D_{(3)}{}^{(type\,P1)} = 6, and\ R_{(3)}{}^{(type\,P1)} = 1$$

$$m_{(2)}{}^{(type\,P1)} = \frac{1}{4}\ and\ m_{(3)}{}^{(type\,P1)} = \frac{1}{6} \quad (135)$$

From (135), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\,k)} = D_{(2)}{}^{(type\,P1)}$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} = D_{(i-1)}{}^{(type\,k)} \quad \text{Eq.18}$$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,2)(type\,P1)} = 4 \quad (136)$$

The third pair (u=3) is the pair of the column P1 and the segment. From Eq.2, the slope of the column P1 and slope of the segment are calculated as the following.

$$D_{(3)}{}^{(type\,P1)} = 6, R_{(3)}{}^{(type\,P1)} = 1, D_{(4)}{}^{(type\,P1)} = 4, and\ R_{(4)}{}^{(type\,P1)} = 1$$

$$m_{(3)}{}^{(type\,P1)} = \frac{1}{6}\ and\ m_{(4)}{}^{(type\,P1)} = \frac{1}{4} \quad (137)$$

From (137), the slopes of lines are the converging case, u = U = 3, and V = 3, so $DSS_{(1)}$ is retrieved from Eq.14 and Eq.15. First trial $DSS_{(1)}$ of set 1= $D_{(3)}^{(\text{type P1})}$= 6.

$N_{(set\ 1)}{}^{(type\ P1)} = 3, N_{(set\ 2)}{}^{(type\ \mathrm{P}1)} = 2, N_{(set\ 3)}{}^{(type\ \mathrm{P}1)} = 2, N_{(set\ 1\to 2)}{}^{(type\ P1)} = 2, N_{(set\ 2\to 3)}{}^{(type\ P1)} = 3,\ m_{(3)}{}^{(type\ P1)} = \frac{1}{6}, m_{(4)}{}^{(type\ P1)} = \frac{1}{4}, D_{(3)}{}^{(type\ P1)} = 6, R_{(3)}{}^{(type\ P1)} = 1$

$$DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ u)(type\ k)} = DSS_{(1)}{}_{(set\ 1)}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)} - 1\right) - \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)}\right) - \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \quad \text{.........} Eq.14$$

$$DFS_{(J)}{}_{(set\ v)}{}^{(Pair\ u)(type\ k)} = DFS_{(J)}{}_{(set\ v-1)}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right)\left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ (v-1)\to v)}{}^{(type\ k)}\right) - \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right)\left(N_{(set\ v)}{}^{(type\ k)}\right) \quad \text{.........} Eq.15$$

$$DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ 3)(type\ P1)} = 6 + \left(\frac{4}{1}\right) \times (3-1) - \left(\frac{6}{1}\right) \times (3) - \left(\frac{1-1}{1}\right) \times 6 \quad \text{....(138)}$$

$$DFS_{(J)}{}_{(set\ 2)}{}^{(Pair\ 3)(type\ P1)} = DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ 2)(type\ P1)} + \left(\frac{4}{1}\right)(2+2) - \left(\frac{6}{1}\right)(2) \quad \text{.........(139)}$$

$$DFS_{(J)}{}_{(set\ 3)}{}^{(Pair\ 3)(type\ P1)} = DFS_{(J)}{}_{(set\ 2)}{}^{(Pair\ 2)(type\ P1)} + \left(\frac{4}{1}\right)(2+3) - \left(\frac{6}{1}\right)(2) \quad \text{.........(140)}$$

$$DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ 3)(type\ P1)} = -4 \quad \text{.........(141)}$$

$$DFS_{(J)}{}_{(set\ 2)}{}^{(Pair\ 3)(type\ P1)} = 0 \quad \text{.........(142)}$$

$$DFS_{(J)}{}_{(set\ 3)}{}^{(Pair\ 3)(type\ P1)} = 8 \quad (143)$$

From (141), (142), and (143), $DFS_{(J)}$ of set 1 still violates the sequence logic, so trial new $DSS_{(1)}$ of set 1 with more increased value.

$$DSS_{(1)}{}_{(set\ 1)}{}^{(Pair\ 3)(type\ P1)} = 10 \quad (144)$$

$$DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ 3)(type\ P1)} = 4 \quad (145)$$

$$DFS_{(J)}{}_{(set\ 2)}{}^{(Pair\ 3)(type\ P1)} = 0 \quad (146)$$

$$DFS_{(J)}{}_{(set\ 3)}{}^{(Pair\ 3)(type\ P1)} = 12 \quad (147)$$

From (144), (145), (146), and (147), $DSS_{(1)}$ of set 1 = 10 is the minimum $DSS_{(1)}$ of set 1 which fulfills the sequence logic for set 1, 2 and 3 of type P1. To calculate the project duration by type P1, $DSS_{(1)}$ of set 1 from (134), (136) and (144) are used to determine project duration by Eq.21 and Eq.22.

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)}{}_{(set\ 1)}{}^{(Pair\ u)(type\ k)} + \left(D_{(I)}{}^{(type\ k)} \times Q_A{}^{(type\ k)}\right) \quad Eq.\ 21$$

$$Q_A{}^{(type\ k)} = \sum_{v=1}^{v=V} \left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ v\rightarrow(v+1))}{}^{(type\ k)}\right) \quad Eq.22$$

$$Q_A{}^{(type\ P1)} = 3 + 2 + 2 + 2 + 3 \quad (148)$$

$$P^{(type\ P1)} = 11 + 4 + 10 + (4 \times 12) \quad (149)$$

$$P^{(type\ P1)} = 73 \quad (150)$$

For the first trial solution of type P1, the project duration is 73 days which is longer than 70 days of the desired project duration. From (134), $DSS_{(1)}$ of pair 1 type P1 has the longest different duration with 11 days and casing 1.5m for the pile P1 has the lowest cost per unit with 1 million baht per unit. So, increasing the number of resources for the pile type P1 can reduce the project duration significantly. Thus, the second trial solution of type P1 is the set of decision variables where $R_{(1)}{}^{(type\ P1)}, R_{(2)}{}^{(type\ P1)}, R_{(3)}{}^{(type\ P1)}$ is 2, 1, 1, respectively.

From Eq.2, the slope of the pile P1 and slope of the footing P1 are calculated as the following.

$D_{(1)}{}^{(type\,P1)} = 5, R_{(1)}{}^{(type\,P1)} = 2, D_{(2)}{}^{(type\,P1)} = 4,\ R_{(2)}{}^{(type\,P1)} = 1$

$$m_{(1)}{}^{(type\,P1)} = \frac{2}{5}\ and\ m_{(2)}{}^{(type\,P1)} = \frac{1}{4} \quad\text{......}(151)$$

From (151), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\,k)} = D_{(2)}{}^{(type\,P1)}$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} = D_{(i-1)}{}^{(type\,k)} \quad\text{......}Eq.18$$

$$DSS_{(1)_{(set\,1)}}{}^{(Pair\,1)(type\,P1)} = 5 \quad\text{......}(152)$$

From (136), and (144), the number of resources in pair 2 and pair 3 stills the same. Thus, the project duration is retrieved from (136), (144) and, (156) by Eq.21 and Eq.22.

$$P^{(type\,k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} + \left(D_{(I)}{}^{(type\,k)} \times Q_A{}^{(type\,k)}\right) \quad\text{......}Eq.\,21$$

$$Q_A{}^{(type\,P1)} = 3 + 2 + 2 + 2 + 3 \quad\text{......}(148)$$

$$P^{(type\,P1)} = 5 + 4 + 10 + (4 \times 12) \quad\text{......}(153)$$

$$P^{(type\,P1)} = 67 \quad\text{......}(154)$$

For the second trial solution of type P1, the set of decision variables which $R_{(1)}{}^{(type\,P1)}, R_{(2)}{}^{(type\,P1)}, R_{(3)}{}^{(type\,P1)}$ is 2, 1, and 1 can provide 67 days of the project duration. The solution not only achieves 73 days of the desired project duration but also provides the minimum total cost of specific resources. Due to the casing 1.5m for the pile P1 has the lowest cost per unit. Only increasing the number of casing 1.5m from 1 to 2 can reduce the 6 days of the project duration from 73 days to 67 days. So, it is then concluded that the second trial solution of type P1 is the optimal solution.

Next is the calculation of the optimal solution for type P2 (k=2). The first trial solution is s = 1 where all $R_{(i)}{}^{(type\ 1)}$ is equal to 1. The number of total pairs (U = 4) is two and there are two sets (V=2), so the first pair (u=1) is the pair of the pile P2 and footing P2. The variable $N_{(set\ \mathrm{v})}{}^{(type\ P2)}$ and the variable $N_{(set\ v\rightarrow(v+1)}{}^{(type\ P2)}$ are the following.

$N_{(set\ 1)}{}^{(type\ P2)} = 2, N_{(set\ 2)}{}^{(tpe\ \mathrm{P2})} = 3$

$N_{(set\ 1\rightarrow 2)}{}^{(type\ P2)} = 2, N_{(set\ 2\rightarrow 3)}{}^{(type\ P2)} = 2$

From Eq.2, the slope of the pile P2 and slope of the footing P2 are calculated as the following.

$D_{(1)}{}^{(type\ P2)} = 6, R_{(1)}{}^{(type\ P2)} = 1, D_{(2)}{}^{(type\ P2)} = 5,\ R_{(2)}{}^{(type\ P2)} = 1$

$m_{(1)}{}^{(type\ P2)} = \frac{1}{6}\ and\ m_{(2)}{}^{(type\ P2)} = \frac{1}{5}$ ....(155)

From (155), the slopes of lines are the converging case and u = 1 < U, so $DSS_{(1)}$ is retrieved from Eq.17.

$Q^{(type\ k)} = 5,\ m_{(i)}{}^{(type\ k)} = m_{(2)}{}^{(type\ P2)},\ m_{(i-1)}{}^{(type\ k)} = m_{(1)}{}^{(type\ P2)},$

$D_{(i-1)}{}^{(type\ k)} = D_{(1)}{}^{(type\ P2)}, R_{(i-1)}{}^{(type\ k)} = R_{(1)}{}^{(type\ P2)},$

$and\ DFS_{(J)_{(set\ V)}}{}^{(Pair\ u)(type\ k)} = 0$

$$DFS_{(J)_{(set\ V)}}{}^{(Pair\ u)(type\ k)}$$

$$= DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(Q^{(type\ k)} - 1\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(Q^{(type\ k)}\right)$$

$$- \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \text{..} Eq.17$$

$0 = DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 1)(type\ P2)} + \left(\frac{5}{1}\right) \times (5-1) - \left(\frac{6}{1}\right) \times (5) - \left(\frac{1-1}{1}\right) \times 5$...(156)

$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 1)(type\ P2)} = 10$...(157)

The second pair (u=2) is the pair of the footing P2 and the column P2. From Eq.2, the slope of the footing P2 and slope of the column P2 are calculated as the following.

$$D_{(2)}{}^{(type\ P2)} = 5, R_{(2)}{}^{(type\ P2)} = 1, D_{(3)}{}^{(type\ P2)} = 7,\ R_{(3)}{}^{(tpe\ P2)} = 1$$

$$m_{(2)}{}^{(type\ P2)} = \frac{1}{5}\ and\ m_{(3)}{}^{(type\ P2)} = \frac{1}{7}\ \ldots\ldots\ldots\ldots(158)$$

From (158), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\ k)} = D_{(2)}{}^{(type\ P2)}$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = D_{(i-1)}{}^{(type\ k)}\ \ldots\ldots\ldots\ldots Eq.18$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P2)} = 5\ldots\ldots\ldots\ldots(159)$$

The third pair (u=3) is the pair of the column P2 and the crossbeam P2. From Eq.2, the slope of the column P2 and slope of the crossbeam P2 are calculated as the following.

$$D_{(3)}{}^{(type\ P2)} = 7, R_{(3)}{}^{(type\ P2)} = 1, D_{(4)}{}^{(type\ P2)} = 9,\ R_{(4)}{}^{(type\ P2)} = 1$$

$$m_{(3)}{}^{(type\ P2)} = \frac{1}{7}\ and\ m_{(4)}{}^{(type\ P2)} = \frac{1}{9}\ \ldots\ldots\ldots\ldots(160)$$

From (160), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\ k)} = D_{(3)}{}^{(type\ P2)}$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = D_{(i-1)}{}^{(type\ k)}\ \ldots\ldots\ldots\ldots Eq.18$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P2)} = 7\ldots\ldots\ldots\ldots(161)$$

The fourth pair (u=4) is the pair of the crossbeam P2 and the segment. From Eq.2, the slope of the crossbeam P2 and slope of the segment are calculated as the following.

$$D_{(4)}{}^{(type\ P2)} = 9, R_{(4)}{}^{(type\ P2)} = 1, D_{(5)}{}^{(type\ P2)} = 4,\ R_{(4)}{}^{(type\ P2)} = 1$$

$$m_{(1)}{}^{(type\ P2)} = \frac{1}{9}\ and\ m_{(5)}{}^{(type\ P2)} = \frac{1}{4}\ \ldots\ldots\ldots\ldots(162)$$

From (162), the slopes of lines are the converging case, u = U = 4, and V = 2, so $DSS_{(1)}$ is retrieved from Eq.14 and Eq.15. First trial $DSS_{(1)}$ of set 1= $D_{(4)}^{(\text{type P2})}$= 9.

$N_{(set\ 1)}{}^{(type\ P2)} = 2, N_{(set\ 2)}{}^{(type\ \mathrm{P}2)} = 3, N_{(set\ 1\rightarrow 2)}{}^{(type\ P2)} = 2,$

$N_{(set\ 2\rightarrow 3)}{}^{(type\ P2)} = 2, m_{(4)}{}^{(type\ P2)} = \frac{1}{9}\ , m_{(5)}{}^{(type\ P2)} = \frac{1}{4},$

$D_{(4)}{}^{(type\ P2)} = 9, R_{(4)}{}^{(type\ P2)} = 1$

$$DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ u)(type\ )}$$

$$= DSS_{(1)}{}_{(set\ 1)}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)} - 1\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)}\right)$$

$$- \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \ldots\ldots\ldots\ldots Eq.14$$

$$DFS_{(J)}{}_{(set\ v)}{}^{(Pair\ u)(type\ k)}$$

$$= DFS_{(J)}{}_{(set\ v-1)}{}^{(Pair\ u)(type\ k)}$$

$$+ \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ (v-1)\rightarrow v)}{}^{(type\ k)}\right)$$

$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \left(N_{(set\ v)}{}^{(type\ k)}\right) \ldots\ldots\ldots\ldots Eq.15$$

$$DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ 4)(type\ P1)} = 9 + \left(\frac{4}{1}\right) \times (2-1) - \left(\frac{9}{1}\right) \times (2) - \left(\frac{1-1}{1}\right) \times 9 \ldots (163)$$

$$DFS_{(J)}{}_{(set\ 2)}{}^{(Pair\ 4)(type\ P1)}$$

$$= DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ 2)(type\ P1)} + \left(\frac{4}{1}\right)(3+2) - \left(\frac{9}{1}\right)(3) \ldots\ldots (164)$$

$$DFS_{(J)}{}_{(set\ 1)}{}^{(Pair\ 4)(type\ P1)} = -5 \ldots\ldots\ldots\ldots (165)$$

$$DFS_{(J)}{}_{(set\ 2)}{}^{(Pair\ 4)(type\ P1)} = -12 \ldots\ldots\ldots\ldots (166)$$

From (165) and (166) $DFS_{(J)}$ still violates the sequence logic, so trial new $DSS_{(1)}$ of set 1 with more increased value.

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)} = 21 \quad (167)$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)} = 7 \quad (168)$$

$$DFS_{(J)_{(set\ 2)}}{}^{(Pair\ 4)(type\ P1)} = 0 \quad (169)$$

From (167), (168), and (169), $DSS_{(1)}$ of set 1 = 21 is the minimum $DSS_{(1)}$ of set 1 which fulfills the sequence logic for set 1 and 2 of type P2. To calculate the project duration by type P2, $DSS_{(1)}$ of set 1 from (157), (159), (161), and (167) are used to determine project duration by Eq.21 and Eq.22.

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(D_{(I)}{}^{(type\ k)} \times Q_A{}^{(type\ k)}\right) \quad Eq.\ 21$$

$$Q_A{}^{(type\ k)} = \sum_{v=1}^{v=V} (N_{(set\ v)}{}^{(type\ k)} + N_{(set\ v\rightarrow(v+1))}{}^{(type\ k)}) \quad Eq.22$$

$$Q_A{}^{(type\ P2)} = 2 + 2 + 2 + 3 \quad (170)$$

$$P^{(type\ P2)} = 10 + 5 + 7 + 21 + (4 \times 9) \quad (171)$$

$$P^{(type\ P2)} = 79 \quad (172)$$

For the first trial solution of type P2, the project duration is 79 days which is longer than 70 days of the desired project duration. To find the optimal solution, this study has tried many solutions and found that the second the set of decision variables where $R_{(1)}{}^{(type\ P2)}, R_{(2)}{}^{(type\ P2)}, R_{(3)}{}^{(type\ P2)}, R_{(4)}{}^{(type\ P2)}$ is 2, 1, 2, 2, respectively can provide the optimum solution. The calculation of the solution is as the following.

From Eq.2, the slope of the pile P2 and slope of the footing P2 are calculated as the following.

$$D_{(1)}{}^{(type\ P2)} = 6, R_{(1)}{}^{(type\ P2)} = 2, D_{(2)}{}^{(type\ P2)} = 5,\ R_{(2)}{}^{(type\ P2)} = 1$$

$$m_{(1)}{}^{(type\ P2)} = \frac{2}{6}\ and\ m_{(2)}{}^{(type\ P2)} = \frac{1}{5} \quad (173)$$

From (173), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. ${D_{(i-1)}}^{(type\ k)} = {D_{(1)}}^{(type\ P2)}$

$${DSS_{(1)}}_{(set\ 1)}^{(Pair\ u)(type\ k)} = {D_{(i-1)}}^{(type\ k)} \ldots\ldots \textit{Eq.18}$$

$${DSS_{(1)}}_{(set\ 1)}^{(Pair\ 1)(type\ P2)} = 6 \ldots\ldots (173)$$

The second pair (u=2) is the pair of the footing P2 and the column P2. From Eq.2, the slope of the footing P2 and slope of the column P2 are calculated as the following.

$${D_{(2)}}^{(type\ P2)} = 5, {R_{(2)}}^{(type\ P2)} = 1, {D_{(3)}}^{(type\ P2)} = 7,\ {R_{(3)}}^{(type\ P2)} = 2$$

$${m_{(2)}}^{(type\ P2)} = \frac{1}{5}\ \ and\ {m_{(3)}}^{(type\ P2)} = \frac{2}{7} \ldots\ldots (174)$$

From (174), the slopes of lines are the converging case and u = 1 < U, so $DSS_{(1)}$ is retrieved from Eq.17.

$$Q^{(type\ k)} = 5,\ {m_{(i)}}^{(type\ k)} = {m_{(2)}}^{(type\ P2)},\ {m_{(i-1)}}^{(type\ k)} = {m_{(1)}}^{(type\ P2)},$$

$${D_{(i-1)}}^{(type\ k)} = {D_{(1)}}^{(type\ P2)}, {R_{(i-1)}}^{(type\ k)} = {R_{(1)}}^{(type\ P2)},$$

$$and\ {DFS_{(J)}}_{(set\ V)}^{(Pair\ u)(type\ k)} = 0$$

$$\begin{aligned} {DFS_{(J)}}_{(set\ V)}^{(Pair\ u)(type\ k)} \\ &= {DSS_{(1)}}_{(set\ 1)}^{(Pair\ u)(type\ k)} + \left(\frac{1}{{m_{(i)}}^{(type\ k)}}\right) \times \left(Q^{(type\ k)} - 1\right) \\ &- \left(\frac{1}{{m_{(i-1)}}^{(type\ k)}}\right) \times \left(Q^{(type\ k)}\right) \\ &- \left(\frac{{R_{(i-1)}}^{(type\ k)} - 1}{{R_{(i-1)}}^{(type\ k)}}\right) \times {D_{(i-1)}}^{(type\ k)} \ldots\ldots \textit{Eq.17} \end{aligned}$$

$$0 = {DSS_{(1)}}_{(set\ 1)}^{(Pair\ 2)(type\ P2)} + \left(\frac{7}{2}\right) \times (5-1) - \left(\frac{5}{1}\right) \times (5) - \left(\frac{1-1}{1}\right) \times 5 \ldots (175)$$

$${DSS_{(1)}}_{(set\ 1)}^{(Pair\ 2)(type\ P2)} = 11 \ldots\ldots (176)$$

The third pair (u=3) is the pair of the column P2 and the crossbeam P2. From Eq.2, the slope of the column P2 and slope of the crossbeam P2 are calculated as the following.

$$D_{(3)}{}^{(type\ P2)} = 7, R_{(3)}{}^{(type\ P2)} = 1, D_{(4)}{}^{(type\ P2)} = 9,\ R_{(4)}{}^{(type\ P2)} = 1$$

$$m_{(3)}{}^{(type\ P2)} = \frac{2}{7}\ and\ m_{(4)}{}^{(type\ P2)} = \frac{2}{9} \ldots\ldots(177)$$

From (177), the slopes of lines are the diverging case, so $DSS_{(1)}$ is retrieved from Eq.18. $D_{(i-1)}{}^{(type\ k)} = D_{(3)}{}^{(type\ P2)}$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} = D_{(i-1)}{}^{(type\ k)} \ldots\ldots Eq.18$$

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 3)(type\ P2)} = 7 \ldots\ldots(178)$$

The fourth pair (u=4) is the pair of the crossbeam P2 and the segment. From Eq.2, the slope of the crossbeam P2 and slope of the segment are calculated as the following.

$$D_{(4)}{}^{(type\ P2)} = 9, R_{(4)}{}^{(type\ P2)} = 2, D_{(5)}{}^{(type\ P2)} = 4,\ R_{(4)}{}^{(type\ P2)} = 1$$

$$m_{(1)}{}^{(type\ P2)} = \frac{2}{9}\ and\ m_{(5)}{}^{(type\ P2)} = \frac{1}{4} \ldots\ldots(179)$$

From (162), the slopes of lines are the converging case, u = U = 4, and V = 2, so $DSS_{(1)}$ is retrieved from Eq.14 and Eq.15. First trial $DSS_{(1)}$ of set 1= $D_{(4)}{}^{(\text{type P2})}$= 9.

$$N_{(set\ 1)}{}^{(type\ P2)} = 2, N_{(set\ 2)}{}^{(type\ \mathrm{P2})} = 3, N_{(set\ 1\rightarrow 2)}{}^{(type\ P2)} = 2,$$

$$N_{(set\ 2\rightarrow 3)}{}^{(type\ P2)} = 2, m_{(4)}{}^{(type\ P2)} = \frac{2}{9}, m_{(5)}{}^{(type\ P2)} = \frac{1}{4},$$

$$D_{(4)}{}^{(type\ P2)} = 9, R_{(4)}{}^{(type\ P2)} = 2$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$$

$$= DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)} - 1\right)$$
$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right) \times \left(N_{(set\ 1)}{}^{(type\ k)}\right)$$
$$- \left(\frac{R_{(i-1)}{}^{(type\ k)} - 1}{R_{(i-1)}{}^{(type\ k)}}\right) \times D_{(i-1)}{}^{(type\ k)} \ldots\ldots Eq.14$$

$$DFS_{(J)_{(set\ v)}}{}^{(Pair\ u)(type\ k)}$$
$$= DFS_{(J)_{(set\ v-1)}}{}^{(Pair\ u)(type\ k)}$$
$$+ \left(\frac{1}{m_{(i)}{}^{(type\ k)}}\right)\left(N_{(set\ v)}{}^{(type\ k)} + N_{(set\ (v-1)\rightarrow v)}{}^{(type\ k)}\right)$$
$$- \left(\frac{1}{m_{(i-1)}{}^{(type\ k)}}\right)\left(N_{(set\ v)}{}^{(type\ k)}\right) \ldots\ldots Eq.15$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)} = 9 + \left(\frac{4}{1}\right) \times (2-1) - \left(\frac{9}{2}\right) \times (2) - \left(\frac{2-1}{2}\right) \times 9 \ldots (180)$$

$$DFS_{(J)_{(set\ 2)}}{}^{(Pair\ 4)(type\ P1)}$$
$$= DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 2)(type\ P1)} + \left(\frac{4}{1}\right)(3+2) - \left(\frac{9}{2}\right)(3) \ldots (181)$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)} = -0.5 \ldots (182)$$

$$DFS_{(J)_{(set\ 2)}}{}^{(Pair\ 4)(type\ P1)} = 6 \ldots (183)$$

From (182) and (183) $DFS_{(J)}$ still violates the sequence logic, so trial new $DSS_{(1)}$ of set 1 with more increased value.

$$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)} = 10 \ldots (184)$$

$$DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)} = 0.5 \ldots (185)$$

$$DFS_{(J)_{(set\ 2)}}{}^{(Pair\ 4)(type\ P1)} = 6.5 \ldots (186)$$

From (184), (185), and (186), $DSS_{(1)}$ of set 1 = 10 is the minimum $DSS_{(1)}$ of set 1 which fulfills the sequence logic for set 1 and 2 of type P2. To calculate the project duration by type P2, $DSS_{(1)}$ of set 1 from (173), (176), (178), and (184) are used to determine project duration by Eq.21 and Eq.22.

From (185), $DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)}$ should be zero to provide the minimum of $DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)}$. In this case, $DFS_{(J)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)}$ equal to zero, the $DSS_{(1)_{(set\ 1)}}{}^{(Pair\ 4)(type\ P1)}$ would be 9.5 days. However, from the equation of project duration Eq.21, the 9.5 days would cause the project duration to result as a decimal value. Thus, the value of $DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)}$ for any pair u type k is fixed to be integer value in this study, for example, (184), (185), and (186).

$$P^{(type\ k)} = \sum_{u=1}^{u=U} DSS_{(1)_{(set\ 1)}}{}^{(Pair\ u)(type\ k)} + \left(D_{(I)}{}^{(type\ k)} \times Q_A{}^{(type\ k)}\right) \text{.............} Eq.\ 21$$

$$Q_A{}^{(type\ k)} = \sum_{v=1}^{v=V} (N_{(set\ v)}{}^{(type\ k)} + N_{(set\ v\rightarrow(v+1))}{}^{(type\ k)}) \text{....................................} Eq.22$$

$$Q_A{}^{(type\ P2)} = 2 + 2 + 2 + 3 \text{......} (187)$$

$$P^{(type\ P2)} = 6 + 11 + 7 + 10 + (4 \times 9) \text{......} (188)$$

$$P^{(type\ P2)} = 70 \text{......} (189)$$

For the second trial solution of type P2, the solution provides 70 days of project duration which is equal to 70 days of the desired project duration. Therefore, the solution is a possible solution for type P2. The second trial solution of type P2 can be decided to be the optimal solution due to the set of decision variables. The set which $R_{(1)}{}^{(type\ P2)}, R_{(2)}{}^{(type\ P2)}, R_{(3)}{}^{(type\ P2)}, R_{(4)}{}^{(type\ P2)}$ is 2, 1, 2, and 2 can reach the desired project duration with the minimum total cost of specific resources. From the slope of lines and $DSS_{(1)}$, if the number of formwork for footing P2 ($R_{(2)}{}^{(type\ P2)}$) is equal to 2 and one of the other is one instead of 2, the project duration will be extended longer than the desired project duration. So, the second trial solution is then the optimal solution.

From the solutions of type P1 and type P2, the project duration (P) is retrieved from Eq23. The total cost of specific resources for each type is calculated by Eq.24. The total cost of specific resources for the project is obtained from Eq.25.

$$P = Max\left(P^{(type\ 1)}, P^{(type\ 2)}, P^{(type\ 3)}, \dots, P^{(type\ K)}\right) \text{......} Eq.\ 23$$

$$P = Max\ (67, 70) = 70 \text{......} (190)$$

$$C_{TR}{}^{(type\ k)} = \left(R_{(1)}{}^{(type\ k)}\right)\left(C_{(1)}{}^{(type\ k)}\right) + \left(R_{(2)}{}^{(type\ k)}\right)\left(C_{(2)}{}^{(type\ k)}\right)$$
$$+ \left(R_{(3)}{}^{(type\ k)}\right)\left(C_{(3)}{}^{(type\ k)}\right) + \dots + \left(R_{(I-1)}{}^{(type\ k)}\right)\left(C_{(I-1)}{}^{(type\ k)}\right) \text{......} Eq.24$$

$$C_{TP} = \sum_{k=1}^{k=K} (C_{TR}{}^{(type\ k)}) \text{......} Eq.\ 25$$

Table 4.10 shows the optimal solution of the second example by manual trial-and-error. The project duration is controlled by type P2 with 70 days. The total cost of specific resources for P1 and P2 is 5.5 million baht and 14 million baht, respectively. The total cost of the specific resources of the project is 19.5 million baht. The LOB diagram by the general approach for the third example with the optimal solution is illustrated in Figure 4.45.

Table 4.10 Optimal solution of the third example by using trial-and-error

Type /Activtiy	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resource (m baht)
Type P1 (k = 1)				
Column	Formwork for Column P1	2.0	1.0	2.0
Footing	Formwork for Footing P1	1.5	1.0	1.5
Pile	Casing Ø 1.5 m	1.0	2.0	2.0
Type P1	Project duration by P1	67 days	Total cost of resources	5.5
Type P2 (k = 2)				
Crossbeam	Formwork for Crossbeam P2	3.0	2.0	6.0
Column	Formwork for Column P2	2.0	2.0	4.0
Footing	Formwork for Footing P2	1.0	1.0	1.0
Pile	Casing Ø 1.8 m	1.5	2.0	3.0
Type P2	Project duration by P2	70 days	Total cost of resources	14.0
Project duration	70 days	Total cost of specific resource of the project		19.5

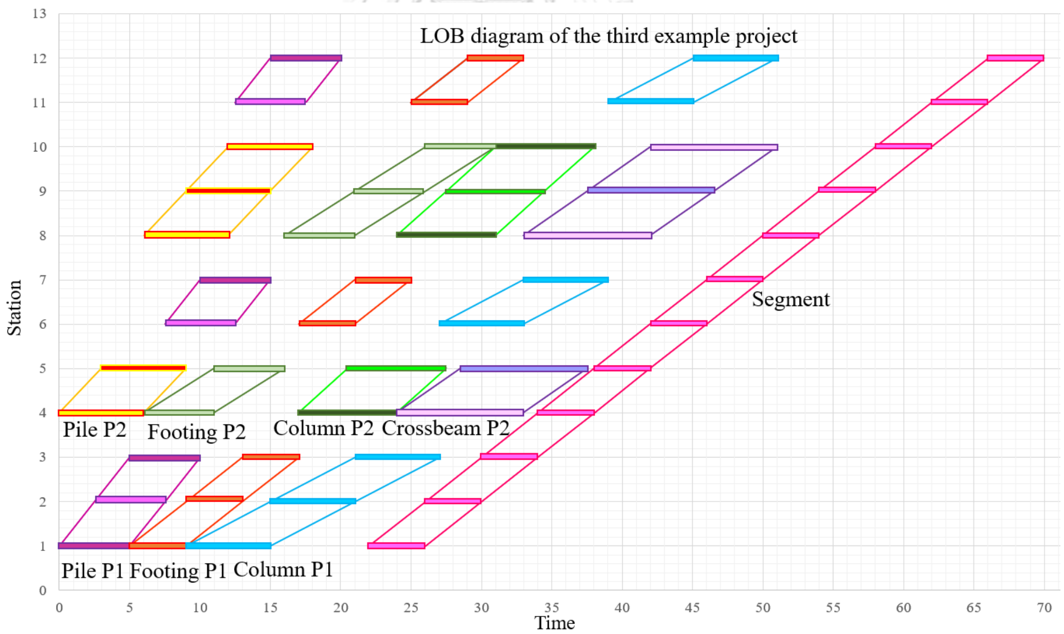
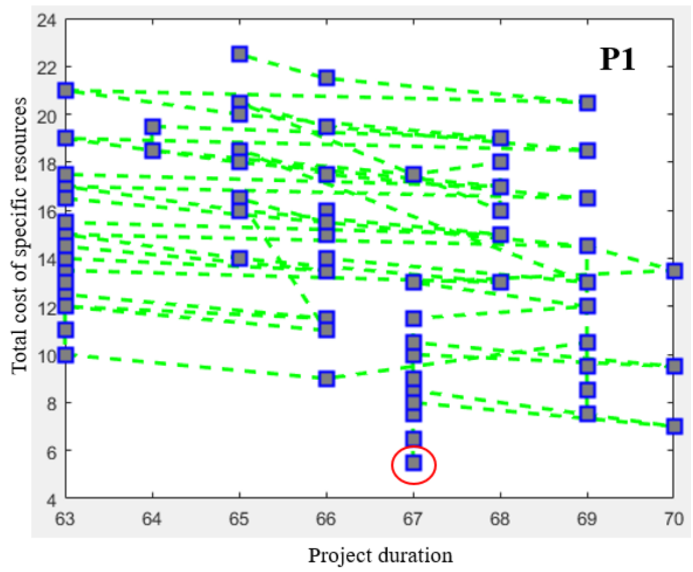


Figure 4.45 LOB diagram of the third example with the optimal solution

For the optimization model, the information of the third example from Table 4.9 is assigned to the optimization model. After the computation, the searching path for the third example is shown in Figure 4.46. The result from the model shows that the optimal solution is exactly as trial-and-error shown in Table 4.11.



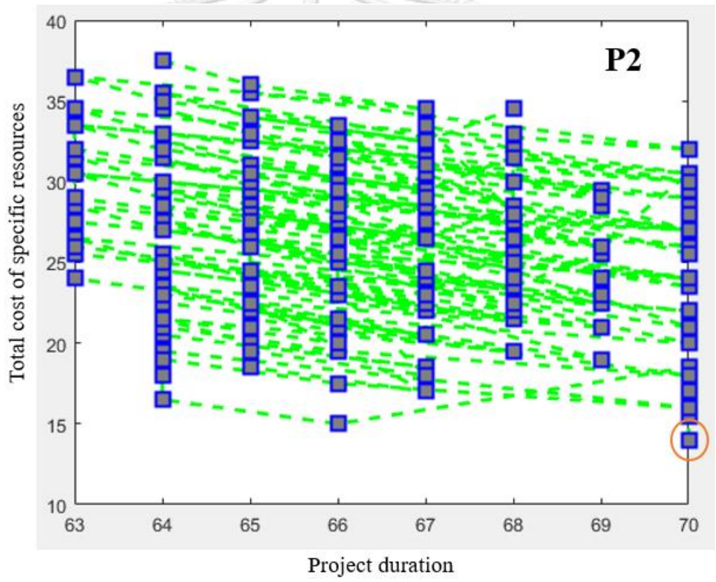


Figure 4.46 Searching path of the optimization model for the third example

Table 4.11 Optimal solution of the third example by using the optimization model

Type /Activtiy	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resource (m baht)
Type P1 (k = 1)				
Column	Formwork for Column P1	2.0	1.0	2.0
Footing	Formwork for Footing P1	1.5	1.0	1.5
Pile	Casing Ø 1.5 m	1.0	2.0	2.0
Type P1	Project duration by P1	67 days	Total cost of resources	5.5
Type P2 (k = 2)				
Crossbeam	Formwork for Crossbeam P2	3.0	2.0	6.0
Column	Formwork for Column P2	2.0	2.0	4.0
Footing	Formwork for Footing P2	1.0	1.0	1.0
Pile	Casing Ø 1.8 m	1.5	2.0	3.0
Type P2	Project duration by P2	70 days	Total cost of resources	14.0
Project duration	70 days	Total cost of specific resource of the project		19.5

The trial-and-error process can solve the optimal solution for P1 with 67 days and 5.5 million baht of the total cost, for P2 with 70 days and 14 million baht of the total cost. The model computes the optimal solution for P1 with 67 days and 5.5 million baht of the total cost, for P2 with 70 days and 14 million baht of the total cost as the trial-and-error. The verification with the third example has addressed that the optimization can solve the exact solution compared with the solution from trial-and-error. The solution shows that type P2 is the control type due to the longest project duration. This result could guarantee that the optimization model is capable of dealing with the project that multi-identical types of units. The third example has shown that the project duration can be controlled by any type depending on the location, the number of repetitive activities, number of units. Moreover, to determine the optimal solution, manual trial-and-error heavily requires human efforts and it is too complicated to guarantee the optimal solution. On the other side, When the project duration is the primary target, the proposed optimization model automatically expresses the optimal solution with the minimum total cost of specific resources.

4.4.4 Conclusion

The results of verification show that the optimization model solves the optimization problem of the three small projects correctly comparing with the trial-and-error process. This can prove the conceptual framework of the application of Line of Balance handling the scheduling problem of multi-identical types of units. With the verified conceptual framework, a large scale as the case study can be carried out. The optimal solution provided by the model includes an optimal set of specific resources, an optimal total cost of specific resources, and an optimal project duration. The solution can be used to support practitioners dealing with the resource optimization problem. However, the model cannot produce the start and finish times of all activities. Thus, the next section is the development of schedule generator which computes the start time and the finish time of every activity for the optimal solution.

4.5 Schedule generator

In the previous section, the optimization model for multi-identical types of units has been developed. The optimization model has an objective to minimize total specific resource cost under desirable duration while maintaining work continuity. After the optimal set of specific resources is acquired, the next step is to find start times and finish times of activities for the optimal set of specific resources. The process of the schedule generator runs after the optimization process has finished. Computation of start and finish times is provided to individually operates for each type. Thus, groups of start and finish times for segment erection appear depending on the number of types. This study develops two schedule generators based-on different aspects, decimal time and integer time. The generators compute start and finish time of all activities by retrieving an optimal set of specific resources from the optimization model. The generator for decimal time is designed to compute early start and early finish in decimal value as the concept of the optimization model. Thus, early start and early finish may result in decimal value such as early start day 1.33 or early finish day 5.67. This decimal result occurs when the number of resources (R) divided by activity duration (D) is an irrational divide (slope m is not an integer). Although these decimal times give accurate times in order to achieve optimum performance, the decimal times are difficult to follow in real-life construction. Therefore, this study concurrently invents the schedule generator for integer time to support practical performance. The schedules from both generators are alternative management tools. The project managers can select which one is acceptable for their projects.

4.5.1 Schedule generator for decimal time

The schedule generator for decimal time aims to provide the start and finish times as the slope line of activity. The slope used in this schedule generator is called sub-slope. The main slope is provided by the optimization model which carries out the optimal solution. For the decimal time, the main-slope and the sub-slope are the same with delay D/R in decimal value as shown in Figure 4.47.

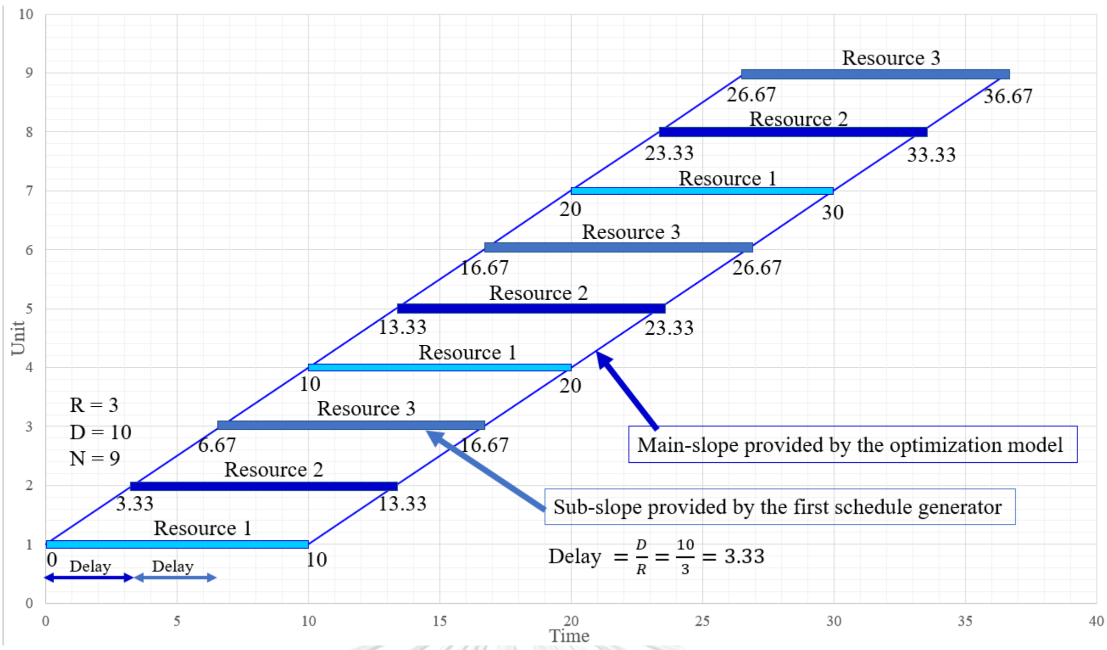


Figure 4.47 Main slope and sub-slope in the schedule generator for decimal times

To develop the decimal generator, the application of LOB is utilized including Eq.5, Eq.6, and Eq.7

$$ES_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N-1) \ldots\ldots\ldots\ldots Eq.5$$

$ES_{(j)}$ = Early Start time of activity at jth unit, m = Rate of delivery,

$ES_{(1)}$ = Early Start time of activity at 1st unit, N = Number of units j

$$EF_{(j)} = ES_{(1)} + \left(\frac{1}{m}\right) \times (N) + \left(\frac{R-1}{R}\right) \times D \ldots\ldots\ldots\ldots Eq.6$$

$EF_{(j)}$ = Early Finish time of activity at jth unit

$ES_{(j)}$ = Early Start time of activity at jth unit

$ES_{(1)}$ = Early Start time of activity at 1st unit, m = Rate of delivery

N = Number of units j, R = Number of specific resources, D = Activity duration

$$DSS_{(1)} = ES_{(i)(1)} - ES_{(i-1)(1)} \ldots\ldots\ldots\ldots Eq.7$$

$DSS_{(1)}$ = The difference time between the early start time of successor at the 1st unit and the early start time of predecessor at the 1st unit

$ES_{(i-1)(1)}$ = Early Start time of predecessor at the 1st unit

$ES_{(i)(1)}$ = Early Start time of successor at the 1st unit

In this study, the schedule generator for decimal time is designed to individually compute the start time and the finish time one type at a time. To utilize the equations for multi-identical types of units, the variables in Eq.5, Eq.6, and Eq.7 must be modified. In Eq.5 and Eq.6, the variable N is considered as the number of units only in a set of units. To cover the other sets, the variable N is changed to variable $q^{(type\ k)}$ where $q^{(type\ k)}$ is the unit in consideration of type k. The total units for type k is $Q^{(type\ k)}$ which is retrieved from Eq. 16. According to the condition of work continuity and resource synchronization, the specific resources are maintained to perform tasks without the idle time by starting from the first unit to the last unit. Therefore, the computation of start and finish time for decimal time can consider the units in sets (${N_{(set\ v)}}^{(type\ k)}$) in once with the variable $Q^{(type\ k)}$ and $q^{(type\ k)}$ as Eq.27 and Eq.28.

$$Q^{(type\ k)} = \sum_{v=1}^{v=V} {N_{(set\ v)}}^{(type\ k)} \text{......} Eq.16$$

$Q^{(type\ k)}$ = Quantity of units from the summation of all $N_{(set\ v)}$ of type k

${N_{(set\ v)}}^{(type\ k)}$= Number of units of set v of type k

Modify Eq.5 and Eq.6 in term of i, k, and q where i is activity in consideration, k is type in consideration, and q is unit in consideration.

$${ES_{(i)(q)}}^{(type\ k)} = {ES_{(i)(1)}}^{(type\ k)} + \left(\frac{1}{{m_{(i)}}^{(type\ k)}}\right) \times \left(q^{(type\ k)} - 1\right) \text{......} Eq.27$$

${ES_{(i)(q)}}^{(type\ k)}$ = Early Start time of ith activity at jth unit of type k

${ES_{(i)(1)}}^{(type\ k)}$= Early Start time of ith activity at 1st unit of type k

${m_{(i)}}^{(type\ k)}$ = Rate of delivery of ith activity of type k

$q^{(type\ k)}$ = Number of unit q at qth unit

$$EF_{(i)(q)}{}^{(type\,k)} = ES_{(i)(1)}{}^{(type\,k)} + \left(\frac{1}{m_{(i)}{}^{(type\,k)}}\right) \times \left(q^{(type\,k)}\right)$$
$$+ \left(\frac{R_{(i)}{}^{(type\,k)} - 1}{R_{(i)}{}^{(type\,k)}}\right) \times D_{(i)}{}^{(type\,k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.28$$

$EF_{(i)(q)}{}^{(type\,k)}$ = Early Finish time of i^{th} activity at j^{th} unit of type k

$ES_{(i)(1)}{}^{(type\,k)}$= Early Start time of i^{th} activity at 1^{st} unit of type k

$m_{(i)}{}^{(type\,k)}$ = Rate of delivery of i^{th} activity of type k

$q^{(type\,k)}$ = Number of units q at q^{th} unit

$R_{(i)}{}^{(type\,k)}$ = Number of specific resources

$D_{(i)}{}^{(type\,k)}$ = Duration of i^{th} activity of type k

Eq.27 and Eq.28. are used to determine the start times and finish times for repetitive activity i of type k at any unit q. In this study, the start time of the first activity at the first unit for any type k ($ES_{(1)(1)}{}^{(type\,k)}$) is provided to be zero (0). For the start time of activity any i where i >1, it can be determined by considering the value of $DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)}$ where u is the number of pairs. $DSS_{(1)}$ of all pairs type k are also the result of the optimization model after the optimizing process is completed. Thus, Eq.7 requires modification to consider in term of pair u type k as Eq. 29. So, the repetitive activity i =1 is the first consideration in the generator.

$$ES_{(i)(1)}{}^{(type\,k)} = ES_{(i-1)(1)}{}^{(type\,k)} + DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.29$$

Where u is the pair of two consecutive activities in consideration, i is the repetitive activity in consideration, and u is equal to i -1 (u = i-1)

$DSS_{(1)_{(set\,1)}}{}^{(Pair\,u)(type\,k)}$ = The difference time between the early start time of successor at the 1^{st} unit and the early start time of predecessor at the 1^{st} unit of pair u type k

$ES_{(i)(1)}{}^{(type\,k)}$ = Early Start time of successor at the 1^{st} unit of pair u, type k

$ES_{(i-1)(1)}{}^{(type\,k)}$ = Early Start time of predecessor at the 1^{st} unit of pair u, type k

From the previous section, Eq.27 and Eq.28 consider the units in any type k to compute the time. For the segment erection, Eq.27, Eq.28, and Eq.29 need more modification to cover the units of other types because the segment erection performs in every unit in the project. The Eq.27and Eq.28 for the segment erection is written by considering the q_A and Q_A. q_A is unit of the project in consideration and Q_A is the quantity of total units of the project. The quantity of total units of the project (Q_A) is retrieved from the number of total units from the first unit of the first type of project to the last unit of the project. This can be defined that Q_A is equal to the highest ${Q_A}^{(type\,k)}$ from Eq .22 among the other type k.

$${Q_A}^{(type\,k)} = \sum_{v=1}^{v=V} \left({N_{(set\,v)}}^{(type\,k)} + {N_{(set\,\mathrm{v}\rightarrow(\mathrm{v}+1))}}^{(type\,k)}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.22$$

${Q_A}^{(type\,k)}$ = the quantity of units from the 1st unit of the considering type k to the last unit of the project

${N_{(set\,v)}}^{(type\,k)}$ = Number of units of set v type k

${N_{(set\,\mathrm{v}\rightarrow(\mathrm{v}+1))}}^{(type\,k)}$= Number of units between set v and set v +1 of type k

$${ES_{(I)(q)}}^{(type\,k)} = {ES_{(I)(1)}}^{(type\,k)} + \left(\frac{1}{{m_{(I)}}^{(type\,k)}}\right) \times (q_A - 1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.30$$

${ES_{(I)(q)}}^{(type\,k)}$ = Early Start time of I^{th} activity at j^{th} unit of type k

${ES_{(I)(1)}}^{(type\,k)}$= Early Start time of I^{th} activity at 1st unit of type k

${m_{(I)}}^{(type\,k)}$ = Rate of delivery of I^{th} activity of type k

q_A = Number of units q at ${q_A}^{th}$ unit of the project

$${EF_{(I)(q)}}^{(type\,k)} = {ES_{(I)(1)}}^{(type\,k)} + \left(\frac{1}{{m_{(I)}}^{(type\,k)}}\right) \times (q_A) + \left(\frac{{R_{(I)}}^{(type\,k)} - 1}{{R_{(I)}}^{(type\,k)}}\right) \times {D_{(I)}}^{(type\,k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.31$$

${EF_{(I)(q)}}^{(type\,k)}$ = Early Finish time of I^{th} activity at j^{th} unit of type k

${ES_{(I)(1)}}^{(type\,k)}$= Early Start time of I^{th} activity at 1st unit of type k

$m_{(I)}{}^{(type\ k)}$ = Rate of delivery of I^th activity of type k

q_A = Number of units q at q^th unit of the project

$R_{(I)}{}^{(type\ k)}$ = Number of specific resources

$D_{(I)}{}^{(type\ k)}$ = Duration of I^th activity of type k

For the start time and the first unit of the segment erection, the Eq.29 can be used only to determine the start time of the first unit of a type but the segment erection starts from the first unit of the project. Moreover, any type k can be the control type of the project in which the type k also controls the performance of the segment erection. Therefore, Eq.29 is then modified for any type k controlling the performance of segment erection by considering $Q_A{}^{(type\ k)}$ and Q_A.

$$ES_{(I)(1)}{}^{(type\ k)} = DSS_{(1)_{(set\ 1)}}{}^{(Pair\ U)(type\ k)} + ES_{(I-1)(1)}{}^{(type\ k)} - (Q_A - Q_A{}^{(type\ k)}) \times D_{(I)}{}^{(type\ k)} \ldots\ldots Eq.32$$

$DSS_{(1)_{(set\ 1)}}{}^{(Pair\ U)(type\ k)}$ = The difference time between the early start time of successor at the 1^st unit and the early start time of predecessor at the 1^st unit of pair U type k

$ES_{(I)(1)}{}^{(type\ k)}$ = Early Start time of successor at the 1^st unit of pair U type k

$ES_{(I-1)(1)}{}^{(type\ k)}$ = Early Start time of predecessor at the 1^st unit of pair U type k

$Q_A{}^{(type\ k)}$ = The quantity of units from the 1^st unit of the considering type k to the last unit of the project

Q_A = The quantity of total units of the project

$D_{(I)}{}^{(type\ k)}$ = Duration of I^th activity of type k

The input of the schedule generator for decimal time are as the follows:

1) Sequences of activity of one unit for every type
2) Activity duration for every type (all $D_{(i)}{}^{(type\ k)}$)
3) Number of units in sets for every type (all $N_{(set\ v)}{}^{(type\ k)}$)

4) Number of units between sets for every type (all $N_{(set\ v \rightarrow (v+1))}^{(type\ k)}$)

5) The optimal sets of specific resources for every type (all $R_{(i)}^{(type\ k)}$)

6) DSS(1) of all pairs of optimal solution for every type (all $DSS_{(1)}^{(pair\ u)(type\ k)}$)

In Figure 4.48, the flow of the generator begins with retrieving the input from the optimization model. The total units of the project (Q_A) is computed from all $Q_A^{(type\ k)}$. Then, the flow considers the first type k where k is equal to 1. k is the type in consideration. The data of type k is taken into the loop for generating the start time and finish time. The start time of the first activity at the first unit ($ES_{(1)(1)}^{(type\ k)}$) is provided to be zero. The start time and finish time of the activity i where i is equal to 1 is determined. i is the activity in consideration. If i is equal to zero, the process gets into the loop directly to generate times from unit 1 of type k to unit $Q^{(type\ k)}$ for the first activity. If i is higher than 1 and not equal to I, the start time of activity i at the first unit is calculated before the loop of generating times, else for i equals to I, the process compute start time and finish time for activity I from unit 1 of the project to the last unit Q_A of the project. When a loop of generating times is broken the output is the start times and finish times of activity i which is then stored in the array of times of type k. Then, if i is not equal to I, the loop of i is run by increased i by 1 (i=i+1) until the times of activity I is computed. After that, the times of type k is save to the array of times of all type and if k is not equal to K, the whole loops from k equal to 1 is repeated with increased k by 1 (k = k+1). The loop of k is broken when k is equal to K and the output is the times of all types in the project.

The output of the schedule generator for decimal time:

1) Start times of all repetitive activities of all types in the project

2) Finish times of all repetitive activities of all types in the project

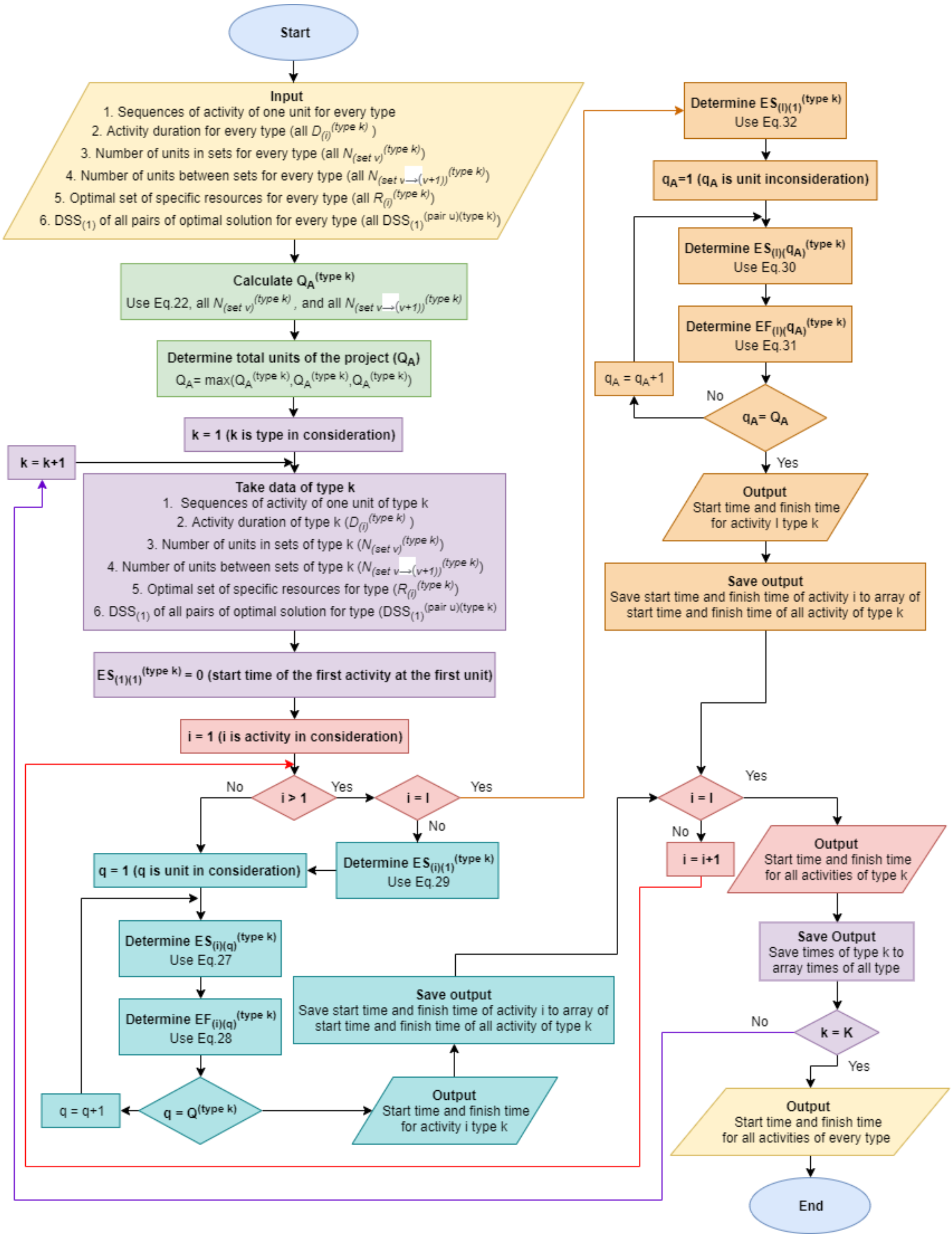
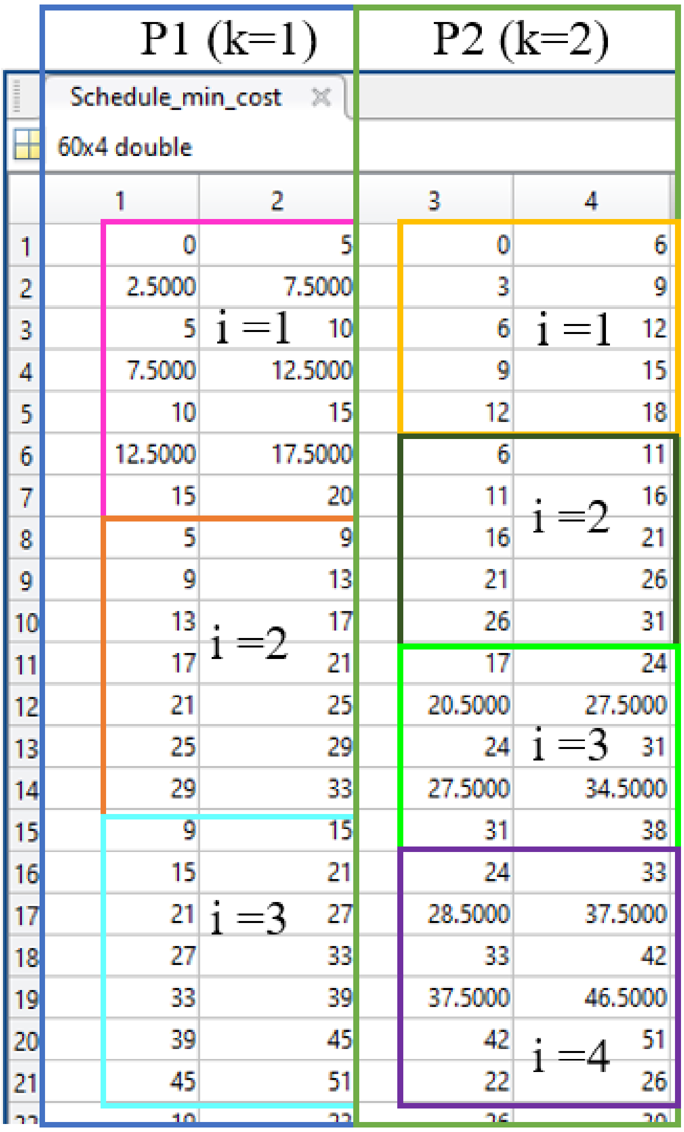


Figure 4.48 Flowchart of schedule generator for decimal time

Figure 4.49 is an example result of the decimal generator from the third example. The example project has two different types and each type has an individual number of units. The odd column is the start times and even column is finish times.



	1	2	3	4
1	0	5	0	6
2	2.5000	7.5000	3	9
3	5	10	6	12
4	7.5000	12.5000	9	15
5	10	15	12	18
6	12.5000	17.5000	6	11
7	15	20	11	16
8	5	9	16	21
9	9	13	21	26
10	13	17	26	31
11	17	21	17	24
12	21	25	20.5000	27.5000
13	25	29	24	31
14	29	33	27.5000	34.5000
15	9	15	31	38
16	15	21	24	33
17	21	27	28.5000	37.5000
18	27	33	33	42
19	33	39	37.5000	46.5000
20	39	45	42	51
21	45	51	22	26

Figure 4.49 Generated start and finish times for two types of units by the decimal generator

For example, the 3rd column is the start times of activities of type P2 from 1st unit to the last units (5th unit). The even column is the finish times of activities for a type. For example, the 4th column is the finish time of activities of type P2 from 1st unit to the last units (5th unit). The row presents the unit and repetitive activity of a type. For example, the 6th row of 3rd and 4th column provides the start time and finish time of the 2nd activity (i =2) at the 1st unit of type P2. With the schedule generator, the construction schedule of the optimal set of specific resources is automatically generated.

4.5.2 Schedule generator for integer time

This study develops an integer generator to support the practical operation by the computation start and finish dates as integer values. For any type k, the first start date of the first activity at the first unit is provided as day one of the project which is different from the decimal generator. The start and finish dates are computed by rounding-down the delay (D/R) from the slope line (m). The rounding-down the delay (D/R) aims to provide start and finish times in which a resource starts the activity as soon as possible. The rounded-down delay forces the resources starting an activity with an integer time. Unlike the crew synchronization and the sub-slope in the decimal generator, the delay with integer time maintains resource starting an activity after rounding-down the delay (D/R) relative to its preceding unit. The first resource instantly starts the activity in the next unit after finishing the current unit.

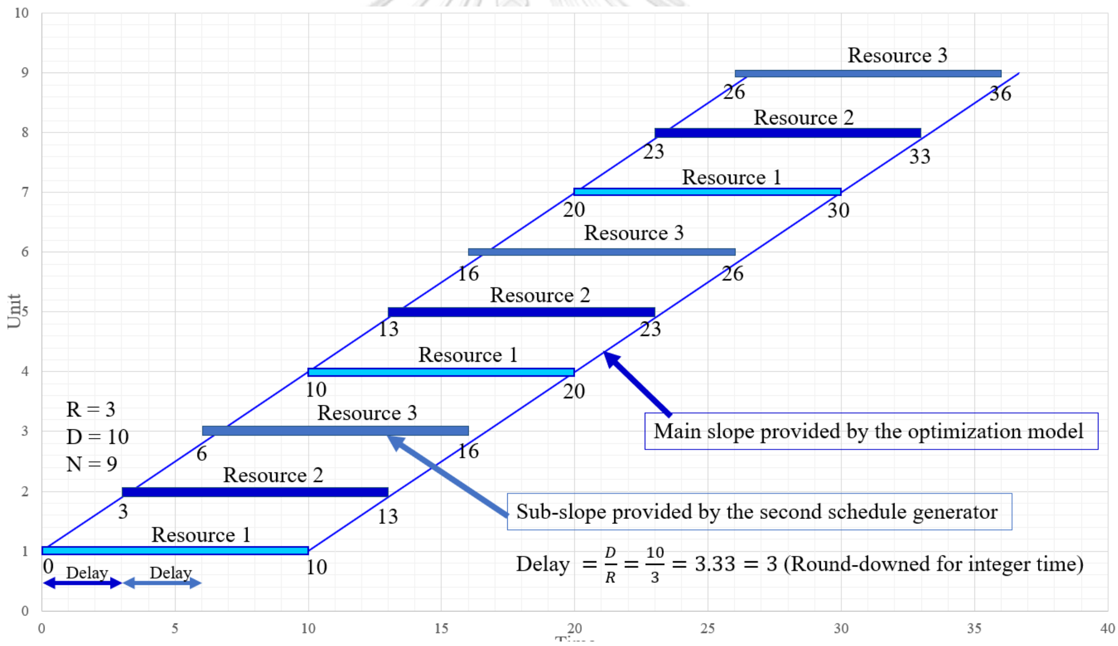


Figure 4.50 Concept of schedule generator for integer time

The rounded-down delay causes sub-slope of the integer generator that probably has a different slope compared with the main slope by the optimization model. This condition affects Eq.5 for the early start and Eq.6 for the early finish incapable of computing the start and finish due to the different slope. For the integer times, the critical point may not locate at first unit or last unit of the sets for integer start and finish dates. The verification with the representative equations as the optimization model may provide incorrect results. Thus, $DSS_{(1)}$ and $DFS_{(J)}$ from the representative equations may be insufficient. However, $DFS_{(J)}$ by considering the early start time of succeeding activity and finish time of preceding activity at any unit can verify sequence logic. The integer generator is developed by using the rounding-down delay. The flow of the integer generator is shown in Appendix B.

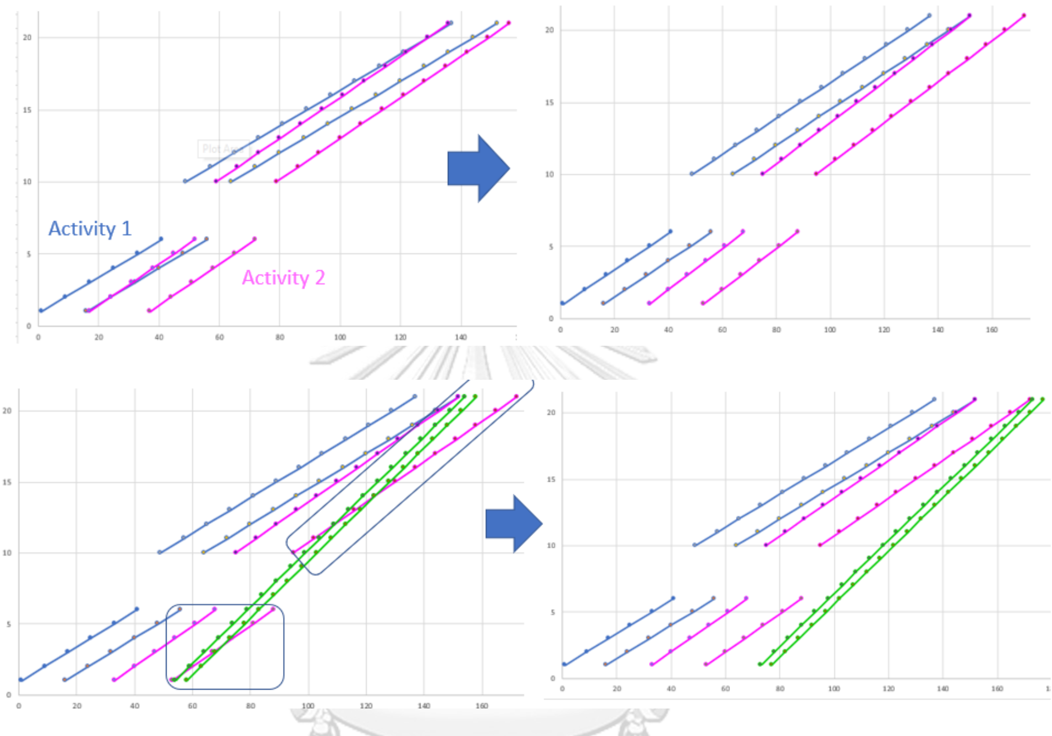
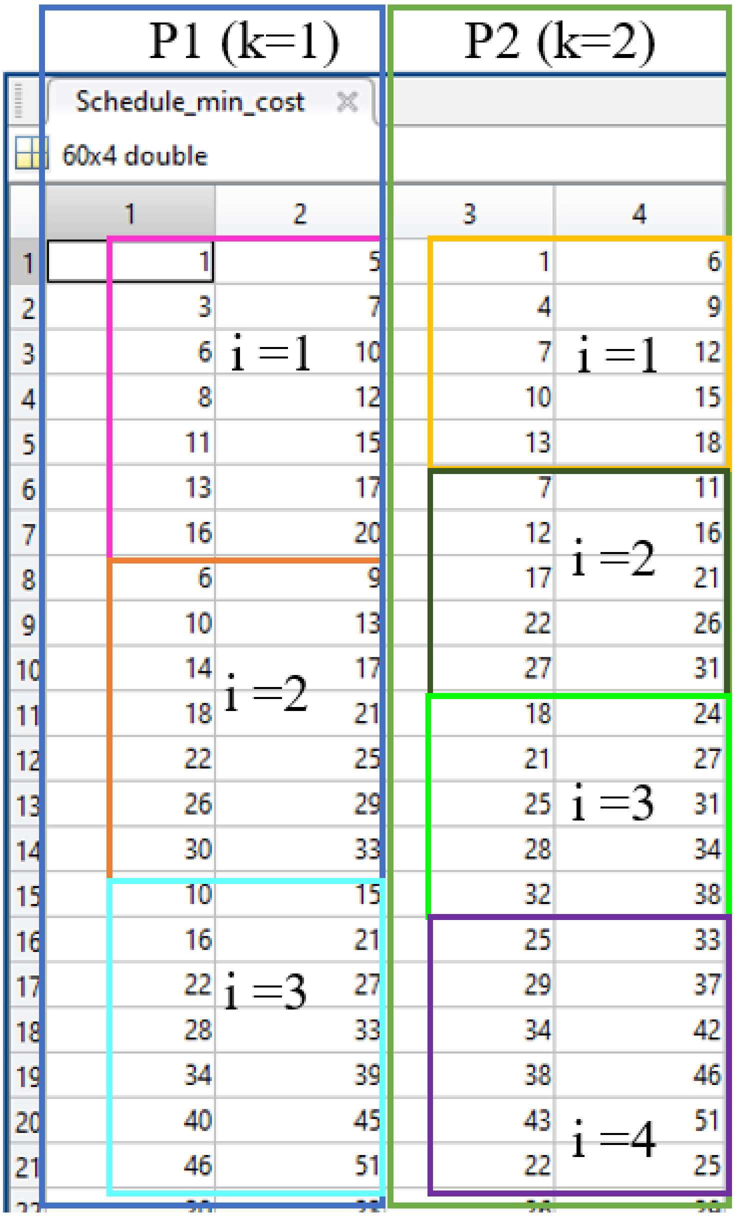


Figure 4.51 Example computation of the generator for integer time

The computation begins from the first activity. After that, it is the computation of succeeding activity. The start time at the first unit of succeeding activity is initially one day after the preceding activity at the first unit finished. The sequence logic is then verified with the value of $DFS_{(J)}$.

The integer generator checks sequence logic with the result of the difference between all start dates of successor and all finish dates of the predecessor, $DFS_{(J)}$ for station ith. If $DFS_{(J)} > 0$ in all stations the sequence logic is not violated. To find the acceptable $ES_{(1)(q)}{}^{(type\ k)}$, All $ES_{(1)(q)}{}^{(type\ k)}$ are gradually increased until the condition is achieved. The process is repeated until the last activity I. The result of the third example by the generator for integer time is illustrated in Figures 4.52.



	1	2	3	4
1	1	5	1	6
2	3	7	4	9
3	6	10	7	12
4	8	12	10	15
5	11	15	13	18
6	13	17	7	11
7	16	20	12	16
8	6	9	17	21
9	10	13	22	26
10	14	17	27	31
11	18	21	18	24
12	22	25	21	27
13	26	29	25	31
14	30	33	28	34
15	10	15	32	38
16	16	21	25	33
17	22	27	29	37
18	28	33	34	42
19	34	39	38	46
20	40	45	43	51
21	46	51	22	25

Figure 4.52 Generated start and finish times for three types by the integer generator

4.5.3 Verification of schedule generator for integer time

This section presents the verification of the integer generator. Because the integer time uses the different conditions from the optimization model, verification is required to prove the capability of the generator. This study utilizes a scheduling software called Asta Powerproject which has Line of Balance feature to verify the integer generator by comparing the result from the software and the generator. The software automatically displays the start and finish times when the workflow (number of resources) of any activity is changed. The comparison exams the start and finish dates with the same set of resources. The three example projects from section 4.4 are analyzed in order to illustrate the capability of the integer generator. The result of the verification is illustrated in the following figures.

Table 4.12 Comparison of start and finish times for the first example

Activity Name	Sequence	Station	Start date Manaul	Start date Generator	Finish date Manaul	Finish date Generator
Pile	i = 1	1	01-07-19 8:00	01.07.19	06-07-19 17:00	06.07.19
Pile	i = 1	2	04-07-19 8:00	04.07.19	09-07-19 17:00	09.07.19
Pile	i = 1	3	07-07-19 8:00	07.07.19	12-07-19 17:00	12.07.19
Pile	i = 1	4	10-07-19 8:00	10.07.19	15-07-19 17:00	15.07.19
Pile	i = 1	5	13-07-19 8:00	13.07.19	18-07-19 17:00	18.07.19
Footing	i = 2	1	07-07-19 8:00	07.07.19	10-07-19 17:00	10.07.19
Footing	i = 2	2	11-07-19 8:00	11.07.19	14-07-19 17:00	14.07.19
Footing	i = 2	3	15-07-19 8:00	15.07.19	18-07-19 17:00	18.07.19
Footing	i = 2	4	19-07-19 8:00	19.07.19	22-07-19 17:00	22.07.19
Footing	i = 2	5	23-07-19 8:00	23.07.19	26-07-19 17:00	26.07.19
Column	i = 3	1	11-07-19 8:00	11.07.19	17-07-19 17:00	17.07.19
Column	i = 3	2	18-07-19 8:00	18.07.19	24-07-19 17:00	24.07.19
Column	i = 3	3	25-07-19 8:00	25.07.19	31-07-19 17:00	31.07.19
Column	i = 3	4	01-08-19 8:00	01.08.19	07-08-19 17:00	07.08.19
Column	i = 3	5	08-08-19 8:00	08.08.19	14-08-19 17:00	14.08.19
Segment	i = 4	1	26-07-19 8:00	26.07.19	30-07-19 17:00	30.07.19
Segment	i = 4	2	31-07-19 8:00	31.07.19	04-08-19 17:00	04.08.19
Segment	i = 4	3	05-08-19 8:00	05.08.19	09-08-19 17:00	09.08.19
Segment	i = 4	4	10-08-19 8:00	10.08.19	14-08-19 17:00	14.08.19
Segment	i = 4	5	15-08-19 8:00	15.08.19	19-08-19 17:00	19.08.19

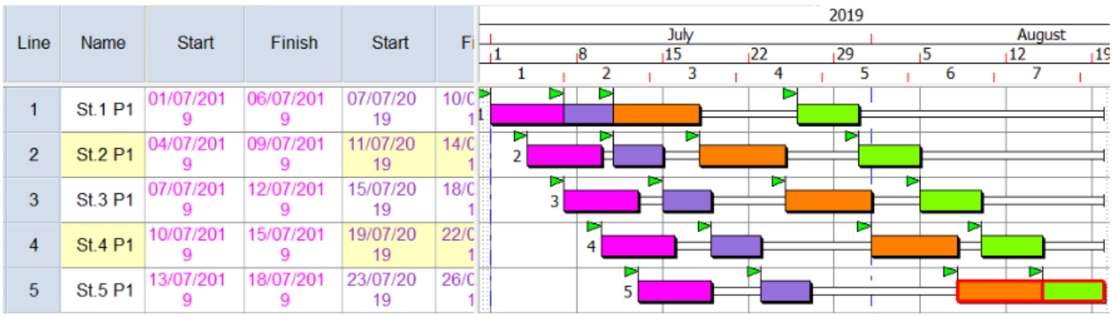


Figure 4.53 Manual creation of the first example by optimum set of resources

Table 4.13 Comparison of start and finish times for P1 in the second example

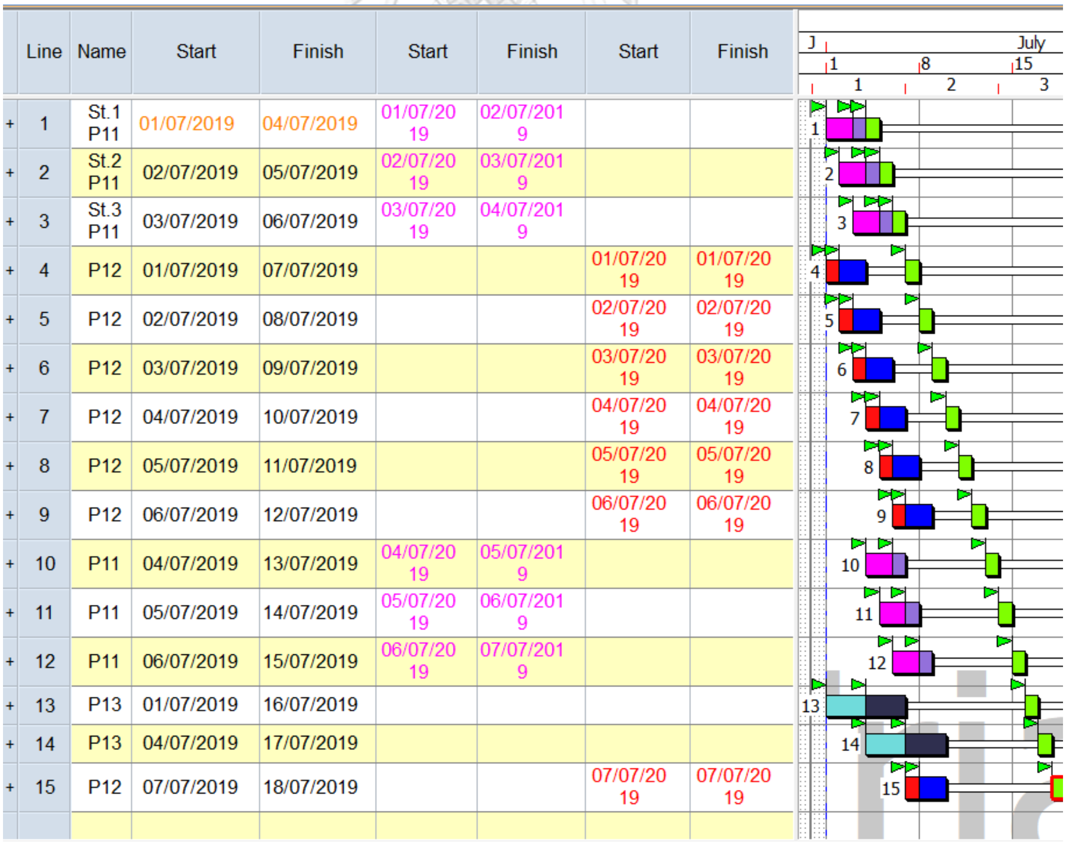
Activity Name	Sequence	Station	P1	P1	P1	P1
			Start date	Start date	Finish date	Finish date
			Manaul	Generator	Manaul	Generator
Footing	i = 1	1	01-07-19 8:00	01-07-2019	02-07-19 17:00	02-07-2019
Footing	i = 1	2	02-07-19 8:00	02-07-2019	03-07-19 17:00	03-07-2019
Footing	i = 1	3	03-07-19 8:00	03-07-2019	04-07-19 17:00	04-07-2019
Footing	i = 1	10	04-07-19 8:00	04-07-2019	05-07-19 17:00	05-07-2019
Footing	i = 1	11	05-07-19 8:00	05-07-2019	06-07-19 17:00	06-07-2019
Footing	i = 1	12	06-07-19 8:00	06-07-2019	07-07-19 17:00	07-07-2019
Column	i = 2	1	03-07-19 8:00	03-07-2019	03-07-19 17:00	03-07-2019
Column	i = 2	2	04-07-19 8:00	04-07-2019	04-07-19 17:00	04-07-2019
Column	i = 2	3	05-07-19 8:00	05-07-2019	05-07-19 17:00	05-07-2019
Column	i = 2	10	06-07-19 8:00	06-07-2019	06-07-19 17:00	06-07-2019
Column	i = 2	11	07-07-19 8:00	07-07-2019	07-07-19 17:00	07-07-2019
Column	i = 2	12	08-07-19 8:00	08-07-2019	08-07-19 17:00	08-07-2019
Segment	i = 3	1	04-07-19 8:00	04-07-2019	04-07-19 17:00	04-07-2019
Segment	i = 3	2	05-07-19 8:00	05-07-2019	05-07-19 17:00	05-07-2019
Segment	i = 3	3	06-07-19 8:00	06-07-2019	06-07-19 17:00	06-07-2019
Segment	i = 3	4	07-07-19 8:00	07-07-2019	07-07-19 17:00	07-07-2019
Segment	i = 3	5	08-07-19 8:00	08-07-2019	08-07-19 17:00	08-07-2019
Segment	i = 3	6	09-07-19 8:00	09-07-2019	09-07-19 17:00	09-07-2019
Segment	i = 3	7	10-07-19 8:00	10-07-2019	10-07-19 17:00	10-07-2019
Segment	i = 3	8	11-07-19 8:00	11-07-2019	11-07-19 17:00	11-07-2019
Segment	i = 3	9	12-07-19 8:00	12-07-2019	12-07-19 17:00	12-07-2019
Segment	i = 3	10	13-07-19 8:00	13-07-2019	13-07-19 17:00	13-07-2019
Segment	i = 3	11	14-07-19 8:00	14-07-2019	14-07-19 17:00	14-07-2019
Segment	i = 3	12	15-07-19 8:00	15-07-2019	15-07-19 17:00	15-07-2019
Segment	i = 3	13	16-07-19 8:00	16-07-2019	16-07-19 17:00	16-07-2019
Segment	i = 3	14	17-07-19 8:00	17-07-2019	17-07-19 17:00	17-07-2019
Segment	i = 3	15	18-07-19 8:00	18-07-2019	18-07-19 17:00	18-07-2019

Table 4.14 Comparison of start and finish times for P3 in the second example

Activity Name	Sequence	Station	P3	P3	P3	P3
			Start date	Start date	Finish date	Finish date
			Manaul	Generator	Manaul	Generator
Footing	i = 1	13	01-07-19 8:00	01-07-2019	03-07-19 17:00	03-07-2019
Footing	i = 1	14	04-07-19 8:00	04-07-2019	06-07-19 17:00	06-07-2019
Column	i = 2	13	04-07-19 8:00	04-07-2019	06-07-19 17:00	06-07-2019
Column	i = 2	14	07-07-19 8:00	07-07-2019	09-07-19 17:00	09-07-2019

Table 4.15 Comparison of start and finish times for P2 in the second example

Activity Name	Sequence	Station	P2	P2	P2	P2
			Start date	Start date	Finish date	Finish date
			Manaul	Generator	Manaul	Generator
Footing	i = 1	4	01-07-19 8:00	01-07-2019	01-07-19 17:00	01-07-2019
Footing	i = 1	5	02-07-19 8:00	02-07-2019	02-07-19 17:00	02-07-2019
Footing	i = 1	6	03-07-19 8:00	03-07-2019	03-07-19 17:00	03-07-2019
Footing	i = 1	7	04-07-19 8:00	04-07-2019	04-07-19 17:00	04-07-2019
Footing	i = 1	8	05-07-19 8:00	05-07-2019	05-07-19 17:00	05-07-2019
Footing	i = 1	9	06-07-19 8:00	06-07-2019	06-07-19 17:00	06-07-2019
Footing	i = 1	15	07-07-19 8:00	07-07-2019	07-07-19 17:00	07-07-2019
Column	i = 2	4	02-07-19 8:00	02-07-2019	03-07-19 17:00	03-07-2019
Column	i = 2	5	03-07-19 8:00	03-07-2019	04-07-19 17:00	04-07-2019
Column	i = 2	6	04-07-19 8:00	04-07-2019	05-07-19 17:00	05-07-2019
Column	i = 2	7	05-07-19 8:00	05-07-2019	06-07-19 17:00	06-07-2019
Column	i = 2	8	06-07-19 8:00	06-07-2019	07-07-19 17:00	07-07-2019
Column	i = 2	9	07-07-19 8:00	07-07-2019	08-07-19 17:00	08-07-2019
Column	i = 2	15	08-07-19 8:00	08-07-2019	09-07-19 17:00	09-07-2019



Line	Name	Start	Finish	Start	Finish	Start	Finish
1	St.1 P11	01/07/2019	04/07/2019	01/07/2019	02/07/2019		
2	St.2 P11	02/07/2019	05/07/2019	02/07/2019	03/07/2019		
3	St.3 P11	03/07/2019	06/07/2019	03/07/2019	04/07/2019		
4	P12	01/07/2019	07/07/2019			01/07/2019	01/07/2019
5	P12	02/07/2019	08/07/2019			02/07/2019	02/07/2019
6	P12	03/07/2019	09/07/2019			03/07/2019	03/07/2019
7	P12	04/07/2019	10/07/2019			04/07/2019	04/07/2019
8	P12	05/07/2019	11/07/2019			05/07/2019	05/07/2019
9	P12	06/07/2019	12/07/2019			06/07/2019	06/07/2019
10	P11	04/07/2019	13/07/2019	04/07/2019	05/07/2019		
11	P11	05/07/2019	14/07/2019	05/07/2019	06/07/2019		
12	P11	06/07/2019	15/07/2019	06/07/2019	07/07/2019		
13	P13	01/07/2019	16/07/2019				
14	P13	04/07/2019	17/07/2019				
15	P12	07/07/2019	18/07/2019			07/07/2019	07/07/2019

Figure 4.54 Manual creation of the second example by optimum set of resources

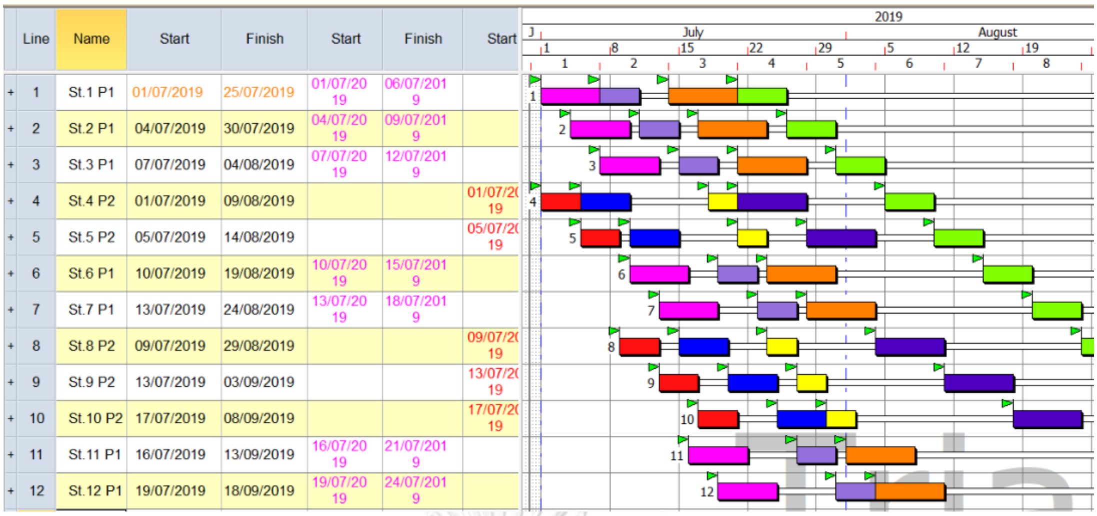


Figure 4.55 Manual creation of the third example by optimum set of resources

Table 4.16 Comparison of start and finish times for P1 in the third example

Activity Name	Sequence	Station	P1 Start date Manaul	P1 Start date Generator	P1 Finish date Manaul	P1 Finish date Generator
Pile	i = 1	1	01-07-19 8:00	01-07-2019	05-07-19 17:00	05-07-2019
Pile	i = 1	2	03-07-19 8:00	03-07-2019	07-07-19 17:00	07-07-2019
Pile	i = 1	3	06-07-19 8:00	06-07-2019	10-07-19 17:00	10-07-2019
Pile	i = 1	6	08-07-19 8:00	08-07-2019	12-07-19 17:00	12-07-2019
Pile	i = 1	7	11-07-19 8:00	11-07-2019	15-07-19 17:00	15-07-2019
Pile	i = 1	11	13-07-19 8:00	13-07-2019	17-07-19 17:00	17-07-2019
Pile	i = 1	12	16-07-19 8:00	16-07-2019	20-07-19 17:00	20-07-2019
Footing	i = 2	1	06-07-19 8:00	06-07-2019	09-07-19 17:00	09-07-2019
Footing	i = 2	2	10-07-19 8:00	10-07-2019	13-07-19 17:00	13-07-2019
Footing	i = 2	3	14-07-19 8:00	14-07-2019	17-07-19 17:00	17-07-2019
Footing	i = 2	6	18-07-19 8:00	18-07-2019	21-07-19 17:00	21-07-2019
Footing	i = 2	7	22-07-19 8:00	22-07-2019	25-07-19 17:00	25-07-2019
Footing	i = 2	11	26-07-19 8:00	26-07-2019	29-07-19 17:00	29-07-2019
Footing	i = 2	12	30-07-19 8:00	30-07-2019	02-08-19 17:00	02-08-2019
Column	i =3	1	10-07-19 8:00	10-07-2019	15-07-19 17:00	15-07-2019
Column	i =3	2	16-07-19 8:00	16-07-2019	21-07-19 17:00	21-07-2019
Column	i =3	3	22-07-19 8:00	22-07-2019	27-07-19 17:00	27-07-2019
Column	i =3	6	28-07-19 8:00	28-07-2019	02-08-19 17:00	02-08-2019
Column	i =3	7	03-08-19 8:00	03-08-2019	08-08-19 17:00	08-08-2019
Column	i =3	11	09-08-19 8:00	09-08-2019	14-08-19 17:00	14-08-2019
Column	i =3	12	15-08-19 8:00	15-08-2019	20-08-19 17:00	20-08-2019

Table 4.17 Comparison of start and finish times for P2 in the third example

Activity Name	Sequence	Station	P2 Start date Manaul	P2 Start date Generator	P2 Finish date Manaul	P2 Finish date Generator
Pile	i = 1	4	01-07-19 8:00	01-07-2019	06-07-19 17:00	06-07-2019
Pile	i = 1	5	04-07-19 8:00	04-07-2019	09-07-19 17:00	09-07-2019
Pile	i = 1	8	07-07-19 8:00	07-07-2019	12-07-19 17:00	12-07-2019
Pile	i = 1	9	10-07-19 8:00	10-07-2019	15-07-19 17:00	15-07-2019
Pile	i = 1	10	13-07-19 8:00	13-07-2019	18-07-19 17:00	18-07-2019
Footing	i = 2	4	07-07-19 8:00	07-07-2019	11-07-19 17:00	11-07-2019
Footing	i = 2	5	12-07-19 8:00	12-07-2019	16-07-19 17:00	16-07-2019
Footing	i = 2	8	17-07-19 8:00	17-07-2019	21-07-19 17:00	21-07-2019
Footing	i = 2	9	22-07-19 8:00	22-07-2019	26-07-19 17:00	26-07-2019
Footing	i = 2	10	27-07-19 8:00	27-07-2019	31-07-19 17:00	31-07-2019
Column	i = 3	4	18-07-19 8:00	18-07-2019	24-07-19 17:00	24-07-2019
Column	i = 3	5	21-07-19 8:00	21-07-2019	27-07-19 17:00	27-07-2019
Column	i = 3	8	25-07-19 8:00	25-07-2019	31-07-19 17:00	31-07-2019
Column	i = 3	9	28-07-19 8:00	28-07-2019	03-08-19 17:00	03-08-2019
Column	i = 3	10	01-08-19 8:00	01-08-2019	07-08-19 17:00	07-08-2019
Crossbeam	i = 4	4	25-07-19 8:00	25-07-2019	02-08-19 17:00	02-08-2019
Crossbeam	i = 4	5	29-07-19 8:00	29-07-2019	06-08-19 17:00	06-08-2019
Crossbeam	i = 4	8	03-08-19 8:00	03-08-2019	11-08-19 17:00	11-08-2019
Crossbeam	i = 4	9	07-08-19 8:00	07-08-2019	15-08-19 17:00	15-08-2019
Crossbeam	i = 4	10	12-08-19 8:00	12-08-2019	20-08-19 17:00	20-08-2019
Segment	i = 5	1	22-07-19 8:00	22-07-2019	25-07-19 17:00	25-07-2019
Segment	i = 5	2	26-07-19 8:00	26-07-2019	29-07-19 17:00	29-07-2019
Segment	i = 5	3	30-07-19 8:00	30-07-2019	02-08-19 17:00	02-08-2019
Segment	i = 5	4	03-08-19 8:00	03-08-2019	06-08-19 17:00	06-08-2019
Segment	i = 5	5	07-08-19 8:00	07-08-2019	10-08-19 17:00	10-08-2019
Segment	i = 5	6	11-08-19 8:00	11-08-2019	14-08-19 17:00	14-08-2019
Segment	i = 5	7	15-08-19 8:00	15-08-2019	18-08-19 17:00	18-08-2019
Segment	i = 5	8	19-08-19 8:00	19-08-2019	22-08-19 17:00	22-08-2019
Segment	i = 5	9	23-08-19 8:00	23-08-2019	26-08-19 17:00	26-08-2019

The result shows that the integer generator provides the exact dates compared to the result from Powerproject for the three example projects. This result can support the potential of the generator dealing with integer times. The start and finish dates computed by the second generator can facilitate practical performance. Although dates by the second generator are easier to follow than the decimal time, the process of the second generator may not provide project duration as the optimal solution.

4.5.4 Analysis of decimal time and integer time

From Figure 4.49, the generator for decimal time provides the start times and finish times according to the slope lines as the optimization model. On the other hand, from Figure 4.52, the decimal times are adjusted by the generator for integer times to support the operation in the project. This adjustment affects the times by shifting forward or backward depending on the rounded times and the number of resources as shown in the sub-slope in Figure 4.50. Due to the different sub-slope and the sequence verification with all $DFS_{(J)}$, the project duration provided by the generator for integer time is probably longer, shorter or equal to the project duration provided by the optimization model. This case occurs uncertainly depending on many variables such as activity duration, number of resources, number of units in sets, and sequence logic. Whenever the optimal project duration is equal to the desired project duration and the project duration provided by the integer generator exceeds the desired project duration. It implies that the project duration computed by the integer generator violates the constraint of the optimization model.

4.4.5 Alternative solution

With the issues of the integer times, decimal times, and optimal solution, this study proposes alternative solutions into three choices. The first choice is the optimal solution by the optimization model with decimal times by the decimal generator. The second choice is the optimal solution by the optimization model with integer times by the integer generator. The third choice is provided when the project duration by the integer generator can not achieve the desired project duration. The third choice is computed by repeating the whole process (the optimization process and the schedule generation) with the required project duration as shown in Figure 4.56. The required project duration is a reduced project duration from the first desired project duration. The required project duration is used temporarily to determine an acceptable project duration by the generator for integer time. For example, the third example project aims to complete within 70 days at the lowest cost. If the generator for integer time provides 71 days of project duration, so the project duration can not achieve the desired project duration. The third choice is determined by shortening the desired project duration from 70 days to 69 days and run the whole process to find a new optimal solution. In Figure 4.56, a new desired project duration is assigned, so the optimal solution is then changed from 70 days to 66 days. With this solution, the project duration by the generator for integer time provides the project duration with 67 days that achieve the first desired project duration (70 days). This process only computes the solution that can achieve the constraints by selecting the optimal solution from the shorter one. However, the total cost of specific resources is also increased by following the new optimal solution. The example of the alternative solutions is in Table 4.18.

Table 4.18 Alternative solutions of the third example project

Alternative solutions	Schedule Generator	Desired project duration	Project duration from the optimization model	Total cost (million baht)	Project duration from the schedule generator
First choice	Decimal	70	70	14.5	70
Second choice	Integer	70	70	14.5	71
Third choice	Integer	70	66	15.5	67

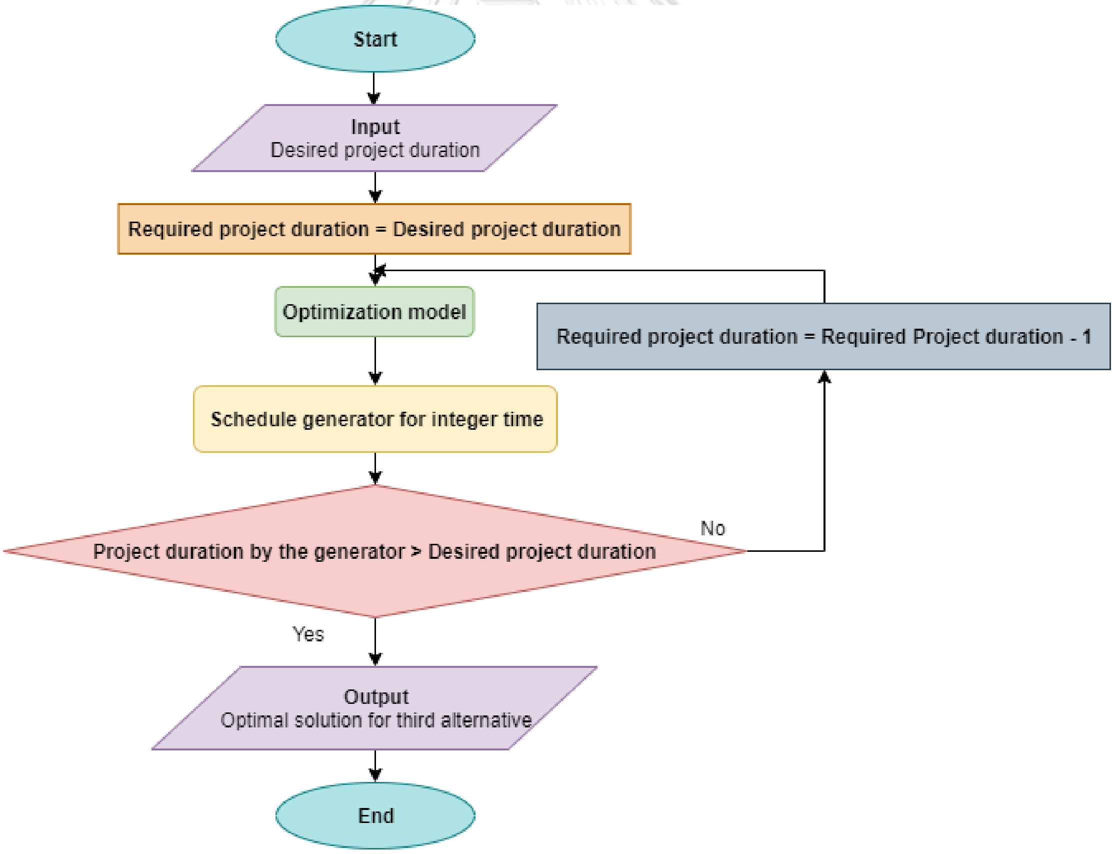


Figure 4.56 Flow of determination of the third choice for the alternative solution

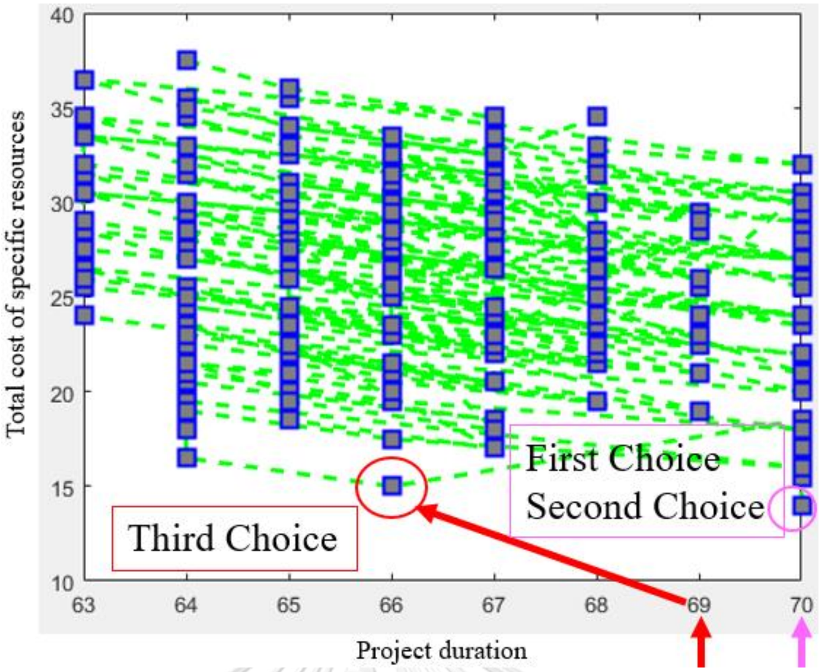


Figure 4.57 The third choice of the alternative solution

4.6 Conclusion

This chapter presents the application of LOB for the scheduling problem of the multi-identical types of units in a linear construction project. The application proposes an optimization model for multi-identical types of units with the objective of minimizing the total cost of specific resources under the desired project duration while maintaining work continuity. To develop the model, an objective function is created to determine the total cost for any set of specific resources. For the constraint function, an indirect method of project duration calculation is invented which is utilized to determine the project duration for any set of specific resources with the proposed representative equations. The method considers the slope of two consecutive activities to identify the case of lines and determine the different time between the start times at the first unit of the two consecutive activities ($DSS_{(1)}$) and the different time between the start time of succeeding activity and the finish time of preceding activity at the last unit ($DFS_{(J)}$). The value of $DSS_{(1)}$ and $DFS_{(J)}$ are used in the verification of sequence logic and also the keys to calculating the project duration. Many examples are utilized to demonstrate the procedure of the method. Finally, the method is employed to develop the proposed optimization model.

To verify the optimization model's capability, this study compares the results from the optimization model with the results from trial-and-error. Three example projects with different circumstances are used as experimental cases. According to the results of the verification, the optimization model can provide the exact optimal solutions and generates accurate schedules compared to manual scheduling.

In the last section, the schedule generators for decimal time and integer time are invented to compute start and finish times. The schedule generators for decimal time is designed to provide the times as the slope lines. The integer schedule generator is developed to produce start and finish dates as the real-world operation. Due to the different concepts, verification of the integer generator is required to examine the capability of the integer generator. A scheduling software Asta Powverproject is utilized to compare its result and the result from the integer generator. With a similar set of resources, the dates from both techniques are exactly the same. Lastly, this study proposes alternative solutions into three choices in order to cover the issue of decimal time, integer time, and the desired project duration.

Chapter 5
BIM-based Line of Balance Scheduling System

This chapter presents an application of BIM information and development of the BIM-based Line of Balance Scheduling System (BIM-LOB-SS) to solve the problem of massive input and the difficulty of visualization. This chapter includes framework of the proposed scheduling system, application of BIM information, output of management tools, and system validation.

5.1 Framework of the proposed scheduling system

The framework of the BIM-LOB-SS consists of the sources of input, BIM information transformation, optimizing and scheduling process, and output management tools as shown in Figure 5.1.

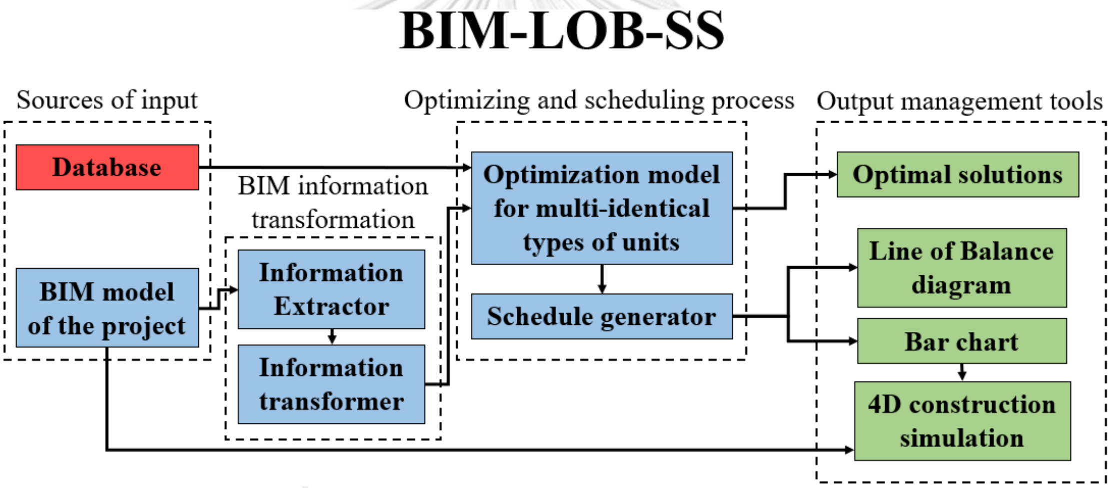


Figure 5.1 Framework of BIM-based Line of Balance Schedule System

In Chapter 4, the optimizing and scheduling process of the system is developed. The process consists of the optimization model and the schedule generator which are completely established. The input of the process consists of Sequences of activities of one unit for every type, Activity duration for every type, Number of units in sets for every type, Number of units between sets for every type, Cost per unit of each specific resource for every type, Number of maximum available resources for every type, and Desired project duration. In Chapter 4, this information has to be assigned manually into the optimization model. To reduce the input assignment, this chapter proposes the utilization of the BIM information for the optimization model.

In the proposed system, BIM-LOB-SS, the sources of input come from two sources which are a database and the BIM model of the project. The database is provided to store the input which are Activity duration for every type, Cost per unit of each specific resource for every type, and Sequence of activities of one unit for every type. For the BIM model, the information in the model is expected to provide Number of units in sets for every type and Number of units between sets for every type.

BIM information transformation consists of two components which are an information extractor for BIM information extraction, and an information transformer for parsing the information to be matched with the input of the optimizing and scheduling process.

For the desired project duration and Number of maximum available resources for every type, the user has to assign directly on Matlab in order to easily revise when he/she needs to adjust these two variables. So, all required input for the optimizing and scheduling process is successfully obtained and the process is ready to operate.

After the optimizing and scheduling process operated, the process produces the outcomes which are the optimum solution and the generated start and finish times. These outcomes are then utilized to create the output management tools for the project. Finally, the output management tools consist of an optimal set of resources, an optimal project duration, an optimal total cost of specific resources, LOB diagram, bar chart, and 4D construction simulation.

5.2 Application of Building Information Modeling

This section explains the application of BIM. The application aims to utilize the BIM model for reducing input assignments and improving visualization of the project operation. Building Information Modeling (BIM) is a process that begins with the creation of an intelligent 3D model and enables document management, coordination and simulation during the entire lifecycle of a project (plan, design, build, operation and maintenance). BIM model generally stores enriched information, for example, the volume of concrete 280 ksc strength, dimension of element, or the location of an element at coordinate x,y,z. With this information, BIM models can fulfill several requirements based on users' purposes.

5.2.1 Development of BIM model

This study develops the BIM model of the project on Autodesk Revit. The development begins with creating the BIM elements of the structures from 2D CAD and the construction drawing. Then, the BIM elements are assembled to create the BIM model according to the type of pier, the station, alignment, and elevation. Generally, gridlines and elevation are essential to present the location of the structure in a project. However, the linear infrastructure project as in the case study has a very long alignment. The alignment requires many gridlines to present the pier's location. This causes the creation of gridlines complicated and difficult to communicate. Hence, this study proposes an additional parameter called station code in order to present the location instead of gridlines. In the BIM model, the station code is attached to every BIM element. The details of the station codes will be explained in the following section.

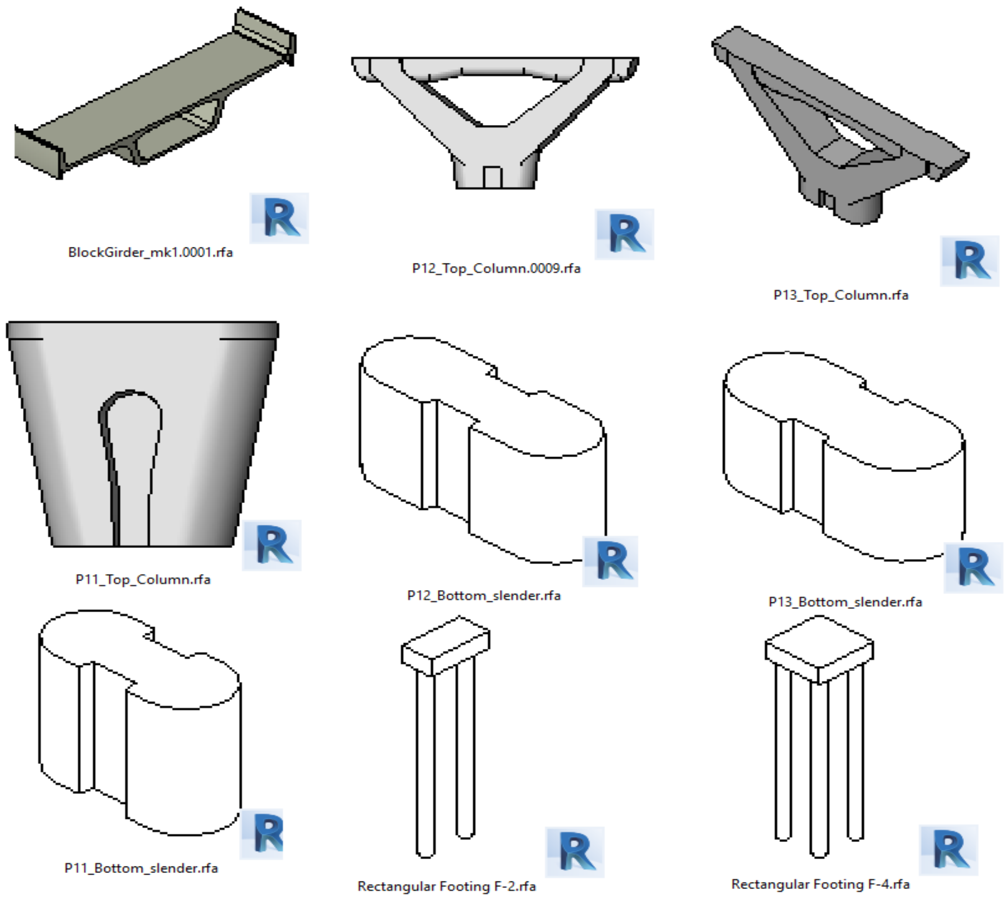


Figure 5.2 Example of BIM element

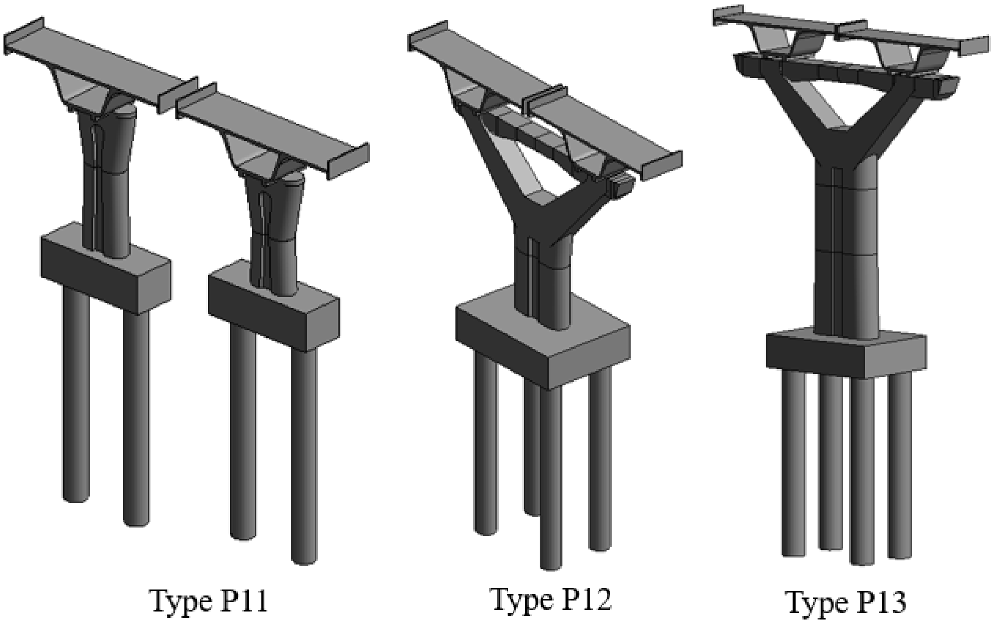


Figure 5.3 BIM model of pier type P11, P12, and P13

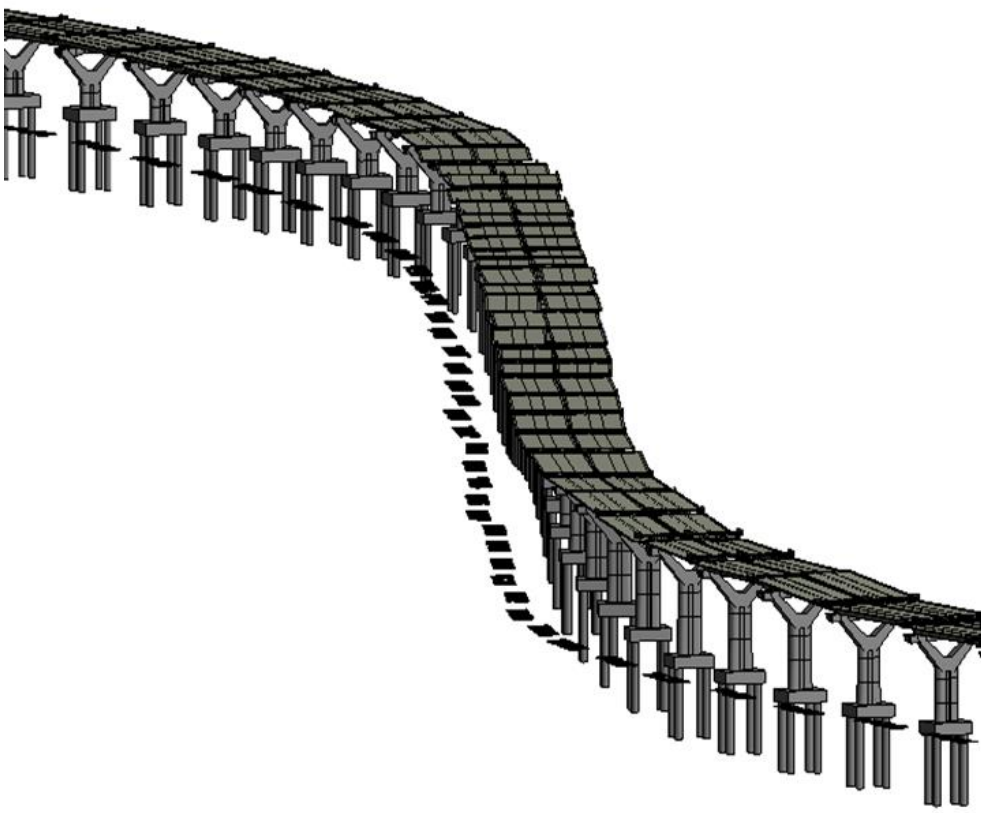


Figure 5.4 BIM model of the case study project

5.2.2 Selected BIM information

Number of units in sets for every type and Number of units between sets for every type are the input of the optimizing and scheduling process. The number of units of each set is a variable that is retrieved from the number of continuous identical units and locations of units. Thus, the information which is used to generate the input must lead to the type of pier, location, number of units, and sequence logic from unit to unit. This study selects Family & Type of elements and the station codes which store in the BIM model to generate the required input.

5.2.2.1 Family & Type of BIM element

A family is a group of elements with a common set of properties, called parameters, and a related graphical representation. All of the elements that users add to Revit projects are created with families. For example, the structural members, walls, roofs, windows, and doors that users use to assemble a BIM model, as well as the callouts, fixtures, tags, and detail components that users use to document them, are all created with families. Family & Type separate BIM elements according to its identification during model creation. This study then uses the Family in BIM to classify the types of piers in the project.

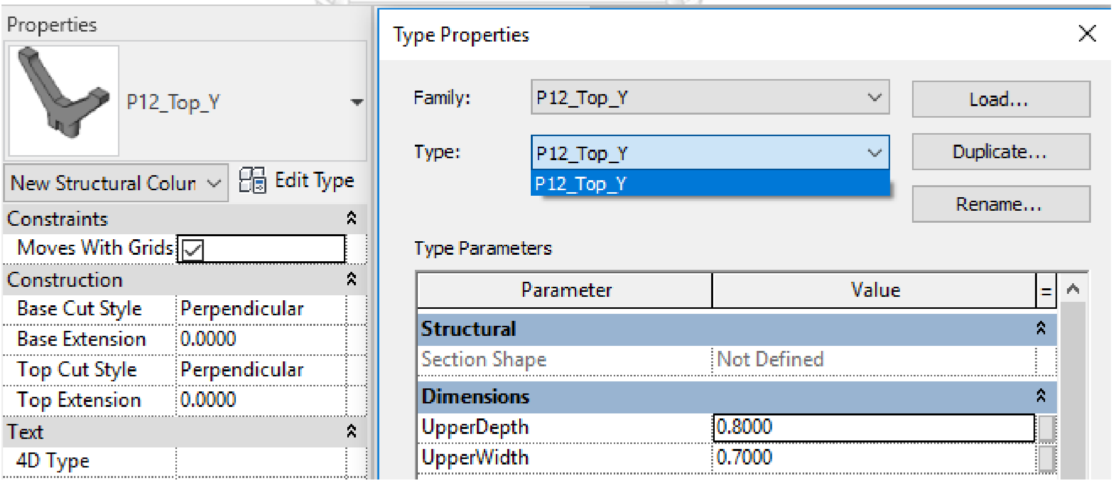


Figure 5.5 Family & Type of BIM element

5.2.1.2 Station code of BIM element

A station is a horizontal measurement along with the survey line of the project. Distances are measured and points are identified on drawing with reference to station codes. The station code is commonly provided as a combination of numbers and letters such as S1-101 or V1-002. The station code usually increases from the beginning of the project to the end of the project. This format comes from the agreement of the project participants. The project participants essentially use the code to communicate the location of the pier on the alignment instead of distance from the beginning. In the BIM model, the station code is attached to every BIM element. The figure below is an example of BIM element in which the station code V1-209 is attached in the parameter Mark.

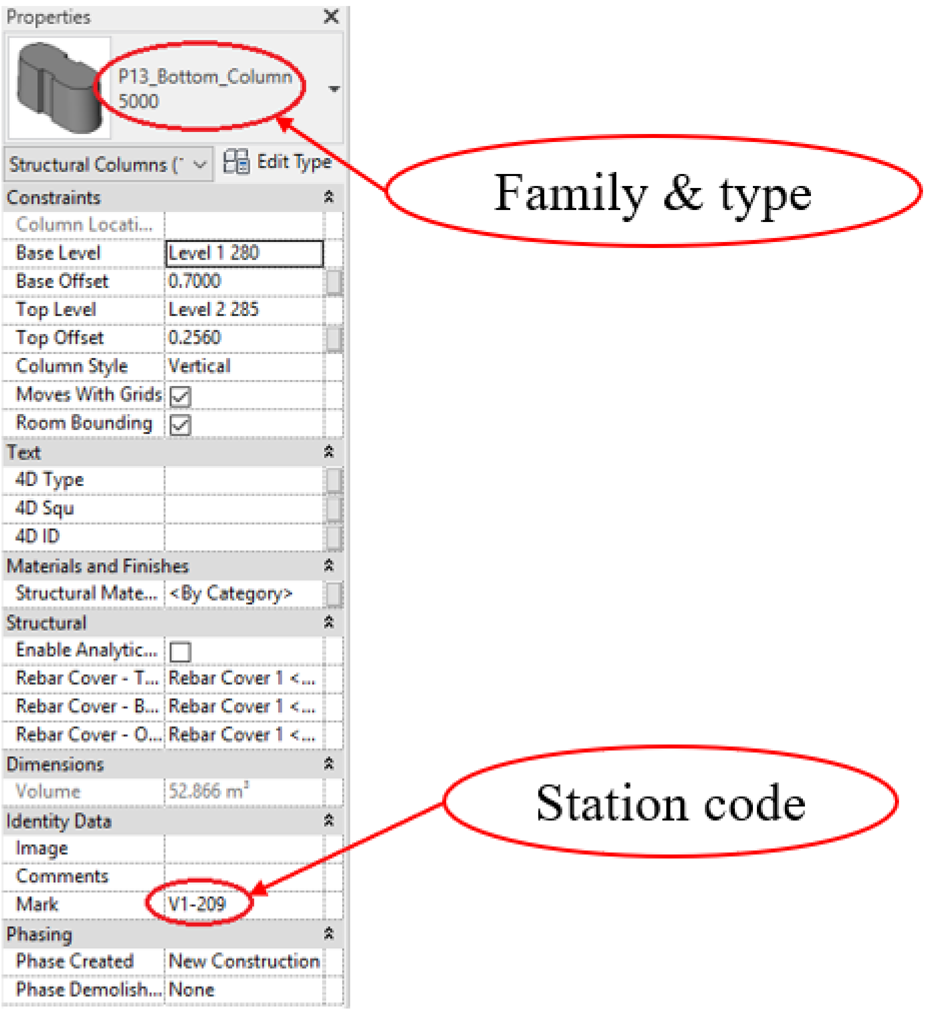


Figure 5.6 Station code in BIM element

5.2.3 BIM data transformation

The previous sections explained the definition of selected BIM information. Family & Type of BIM element can be used to classify the type of pier and the station code can present the location of pier in the project. With the selected BIM information, Number of units in sets for every type and Number of units between sets for every type can be retrieved by examining between all station codes of the project and the station codes of type k. In Figure 5.7, the first column is all station codes of the project. The second column is the type of pier. Column 3, 4, and 5 are the station codes of type P11, P12, and P13, respectively.

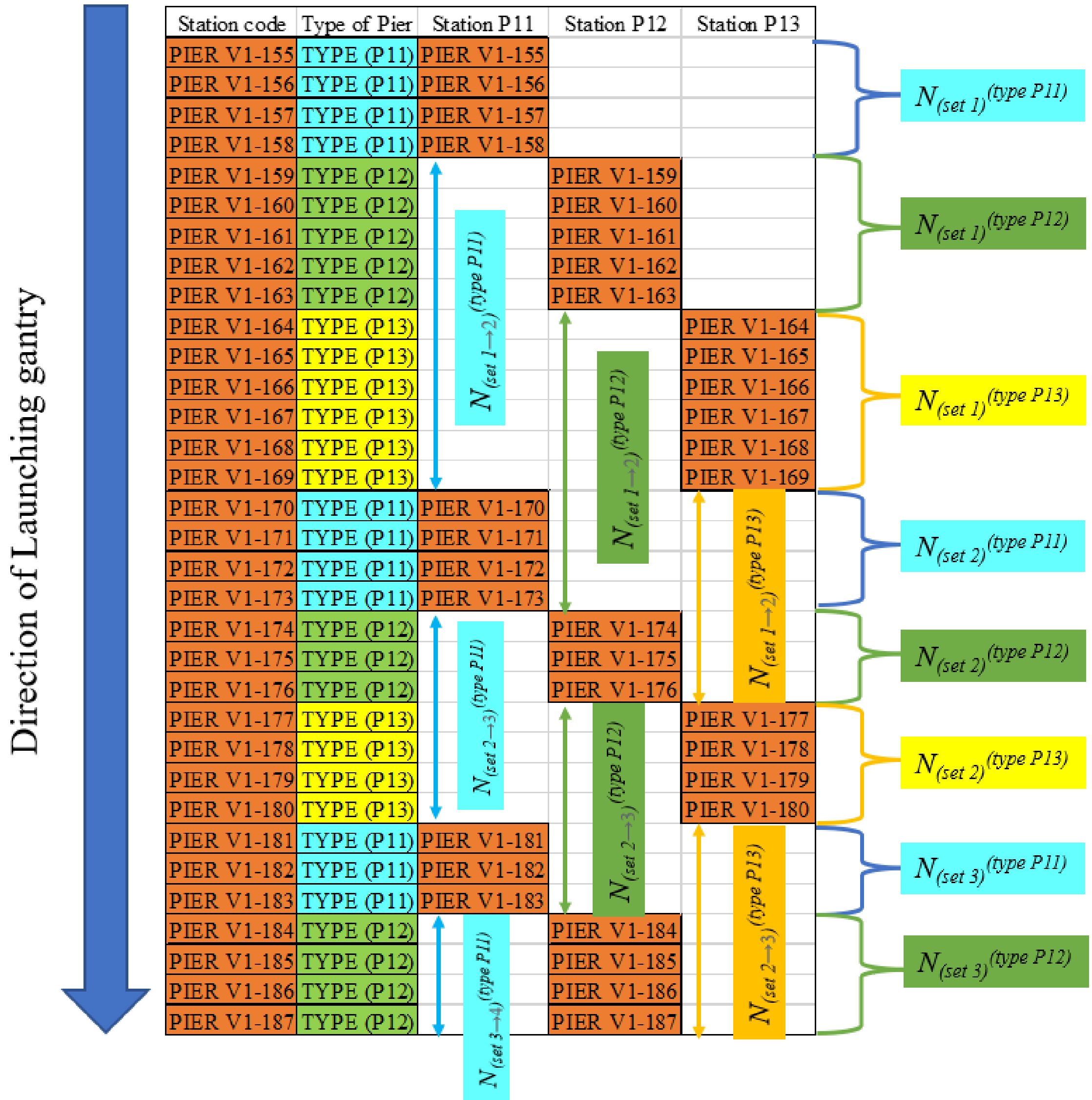


Figure 5.7 Determination of $N_{(set\ v)}^{(type\ k)}$ and $N_{(set\ v\rightarrow(v+1))}^{(type\ k)}$

$N_{(set\ v)}^{(type\ k)}$ and $N_{(set\ v \rightarrow (v+1))}^{(type\ k)}$ for type any k are determined by considering one type at a time. For example, $N_{(set\ v)}^{(type\ k)}$ of type P11 is acquired by comparing between all station codes of the project in column 1 and station codes of type P11 in column 3. A station code from column 1 is compared to station code in column 3. If the station codes are the same, the result is identified as TRUE. If they are not, the result appears as False. So, this can identify the station of type P11 that where it belongs in the project. In this analysis, TRUE has the meaning that the station codes of the project are the station codes of the type k in consideration. FALSE means the station codes of the project are not the station codes of the type in consideration as showing in Table 5.1.

Table 5.1 Concept of BIM information analyzer

Station code	Type of Pier	Station P11	Station P12	Station P13
PIER V1-155	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-156	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-157	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-158	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-159	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-160	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-161	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-162	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-163	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-164	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-165	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-166	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-167	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-168	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-169	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-170	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-171	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-172	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-173	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-174	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-175	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-176	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-177	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-178	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-179	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-180	TYPE (P13)	FALSE	FALSE	TRUE
PIER V1-181	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-182	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-183	TYPE (P11)	TRUE	FALSE	FALSE
PIER V1-184	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-185	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-186	TYPE (P12)	FALSE	TRUE	FALSE
PIER V1-187	TYPE (P12)	FALSE	TRUE	FALSE

After the comparison, counting of the continuous TRUE provides $N_{(set\ v)}^{(type\ k)}$ of type P11 from set one to any set v. On the other hand, counting of the continuous FALSE provides $N_{(set\ v\rightarrow(v+1))}^{(type\ k)}$ set one to any set v. With the same process, $N_{(set\ v)}^{(type\ k)}$ and $N_{(set\ v\rightarrow(v+1))}^{(type\ k)}$ of every type can be retrieved by considering one type at a time. The FLASE before the first TURE as row 1-4 in column 4 is not counted into $N_{(set\ v\rightarrow(v+1))}^{(type\ k)}$ because the stations are unnecessary for the optimizing and scheduling process. Thus, the FLASE is counted after the TURE for $N_{(set\ 1)}^{(type\ k)}$ determined.

This section has explained the concept of BIM information transformation to become the input of the optimizing and scheduling process. The required BIM information is the station codes of the project, and station codes of every type. So, the next section is the development of the information extractor and followed by the development of the information transformer.

5.2.3.1 Information extractor

This study develops the information extractor on Dynamo, open-source graphical programming on Revit. The extractor is developed to extract station codes from the BIM model and arrange them by Family & Type for the types of units. The required information is all station codes of the project and the station codes of every type. In the project, the viaduct segment is the element that belongs to every station. Thus, all station codes can be obtained from the BIM element of viaduct segment. The station codes of each type can be retrieved from a BIM element of a pier of each type, The output of the extractor is an Excel file containing the column of Family & Type and the row of station codes for every element in the project. To prepare the Excel file for the information transformer, the station code for each type is selected by the users from a representative BIM element of a type of unit. The element must be unique for each type in order to avoid the error from the same station codes of BIM elements of the same types.

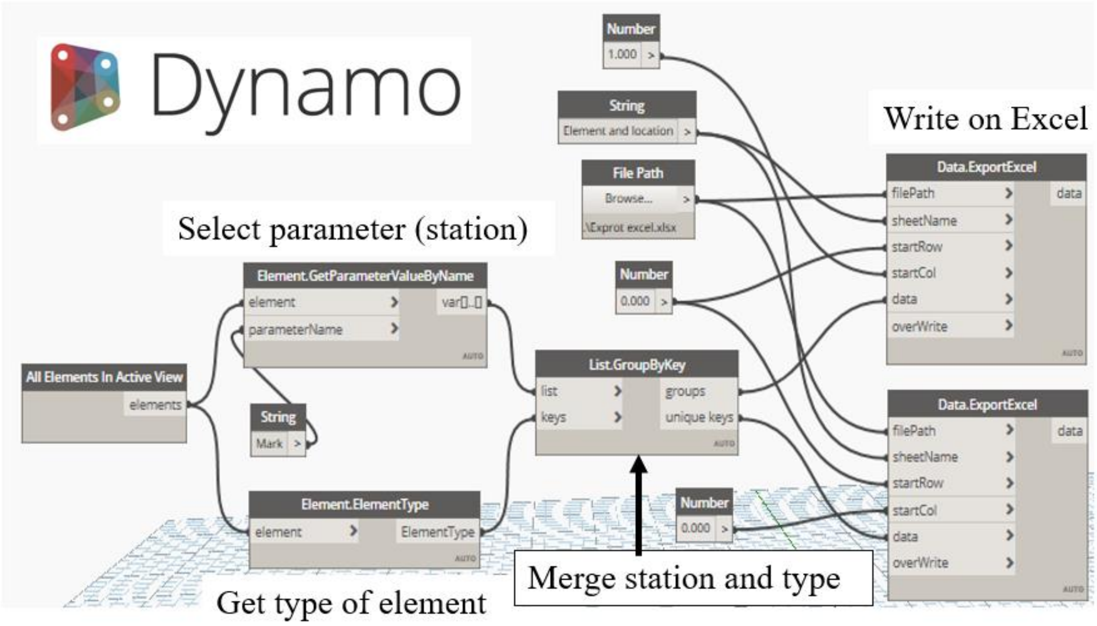
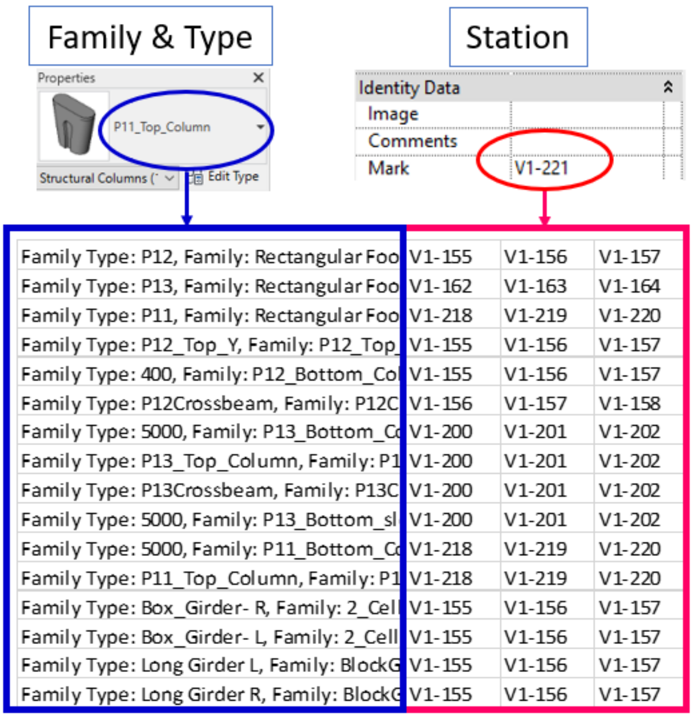


Figure 5.8 Extractor workflow on Dynamo

Figure 5.8 is the process of the extractor on Dynamo. The extractor begins with taking a BIM model as input. Family & Type of BIM element is exported to merge with the station code taken from parameter Mark. Finally, the station codes are written on an Excel file according to Family & Type as showing below.



Family Type: P12, Family: Rectangular Foo	V1-155	V1-156	V1-157
Family Type: P13, Family: Rectangular Foo	V1-162	V1-163	V1-164
Family Type: P11, Family: Rectangular Foo	V1-218	V1-219	V1-220
Family Type: P12_Top_Y, Family: P12_Top_	V1-155	V1-156	V1-157
Family Type: 400, Family: P12_Bottom_Col	V1-155	V1-156	V1-157
Family Type: P12Crossbeam, Family: P12C	V1-156	V1-157	V1-158
Family Type: 5000, Family: P13_Bottom_Co	V1-200	V1-201	V1-202
Family Type: P13_Top_Column, Family: P1	V1-200	V1-201	V1-202
Family Type: P13Crossbeam, Family: P13C	V1-200	V1-201	V1-202
Family Type: 5000, Family: P13_Bottom_sl	V1-200	V1-201	V1-202
Family Type: 5000, Family: P11_Bottom_Co	V1-218	V1-219	V1-220
Family Type: P11_Top_Column, Family: P1	V1-218	V1-219	V1-220
Family Type: Box_Girder- R, Family: 2_Cell	V1-155	V1-156	V1-157
Family Type: Box_Girder- L, Family: 2_Cell	V1-155	V1-156	V1-157
Family Type: Long Girder L, Family: BlockG	V1-155	V1-156	V1-157
Family Type: Long Girder R, Family: BlockG	V1-155	V1-156	V1-157

Figure 5.9 Output of BIM information extractor

5.2.3.2 Information transformer

After the development of the extractor, the information transformer is developed to parse the BIM information and produce the required input for the optimizing and scheduling process. This study develops the transformer from the concept of data transformation in section 5.23. For the station codes, they can be strings (letters) and ints (number). It's a disadvantage of Matlab that cannot deal with string variables. Thus, Jupyter is selected to invent the transformer. The input of the transformer is the prepared Excel file before assigning it. In Table 5.2, the first three rows are the station codes for type P11, P12, and P13, respectively. The last row is all station codes obtained from the Box_Girder (Family & Type of the segment).

Table 5.2 Example input of the information transformer

BIM Element	Station code							
Family Type: P11	V1-218	V1-219	V1-220	V1-221	V1-222	V1-223		
Family Type: P12	V1-155	V1-156	V1-157	V1-158	V1-159	V1-160	V1-161	V1-169
Family Type: P13	V1-162	V1-163	V1-164	V1-165	V1-166	V1-167	V1-168	V1-183
Family Type: Box_Girder	V1-155	V1-156	V1-157	V1-158	V1-159	V1-160	V1-161	V1-162

In figure 5.11, the flow of the information transformer starts with obtaining the input including station codes of every type, and all station codes of the project. The analyzer considers each type per loop. Hence, the number of loops is repeated according to the number of rows minus one (number of types (K)). For each type k, the comparison of the codes provides True and False in an array of type k. To find the sequence of the sets, counting of Ture and False starts from the first station of the project to the last station. Counting of Ture starts when the first Ture of the array is found. Then, the process counts the continuous True until a False is detected. The number of units in set v for type k ($N_{(set\ v)}^{(type\ k)}$) is the number of continuous True. Then, the process switches to count continuous False until a Ture is detected. The number of continuous False is the number of units between sets type k ($N_{(set\ v \rightarrow (v+1))}^{(type\ k)}$). The process of counting Ture and False will be switched whenever its opposite detected, for example, while counting the continuous Ture if the process meets False the number of continuous Ture from the start to the False is defined for the number of units of set v. Then, the process is switched to count the continuous False to determine the number of units between sets. For the variable v, it is firstly equal to 1 and is increased by one when the switching is activated. This counting is repeated to the last station counted. Then, the process changes to consider the next type k (k = k + 1). After the process has considered all types, the output $N_{(set\ v)}^{(type\ k)}$ and $N_{(set\ v\rightarrow(v+1))}^{(type\ k)}$ are generated on an Excel file that is used for the optimizing and scheduling process as shown in Figure 5.11.

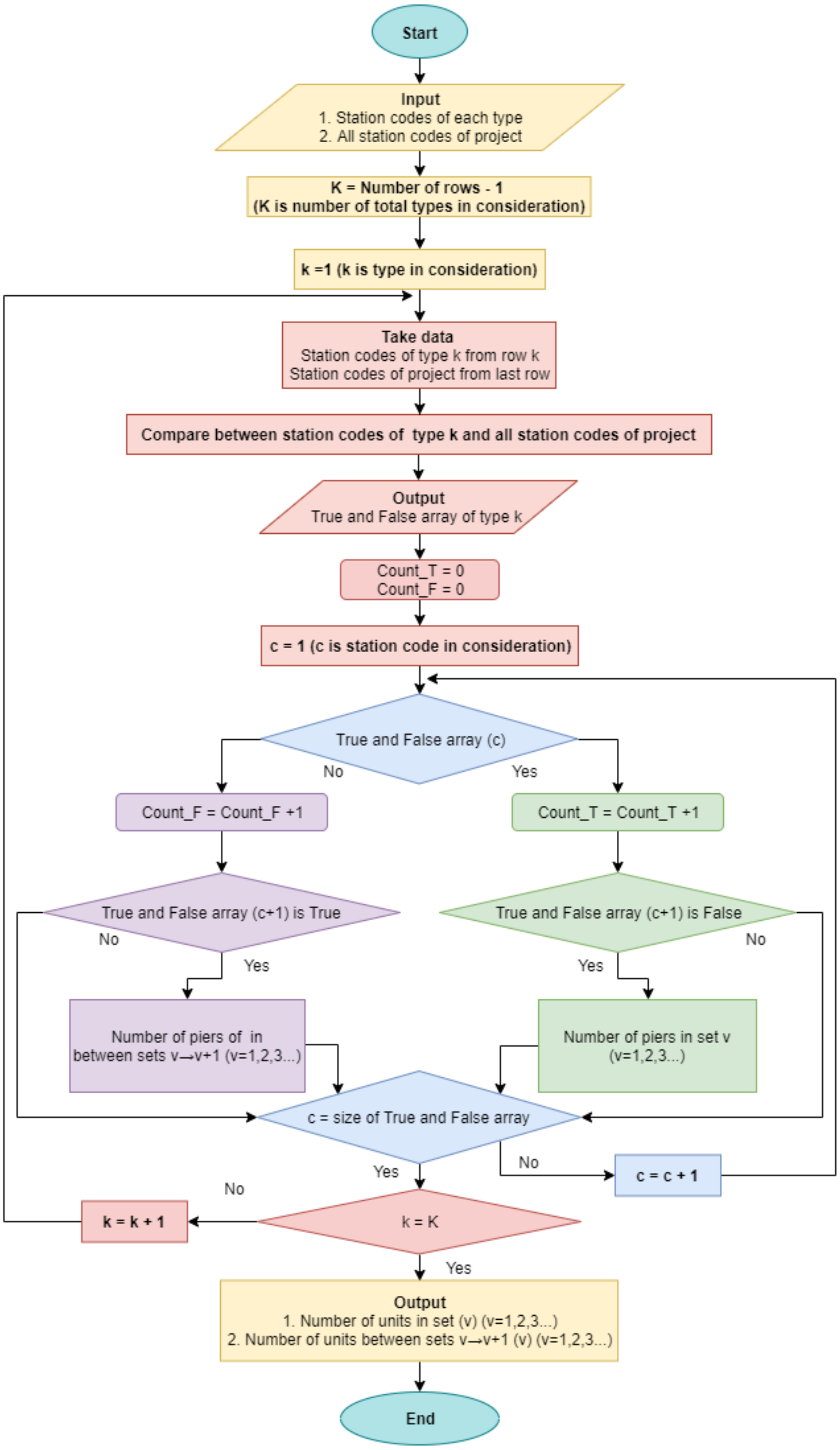


Figure 5.10 Flowchart of the information transformer

Table 5.3 The output of the information transformer

Number of units in set					Number of units between sets			
Type/set	**1**	**2**	**3**	**4**	**Type/set**	**1→2**	**2→3**	**3→4**
P11	4	4	3	0	**P11**	11	7	4
P12	5	3	4		**P12**	10	7	
P13	6	4			**P13**	7	7	

5.2.4 Summary of the source of input

This section explains the application of BIM information for the optimizing and scheduling process. While the input of the optimizing and scheduling process requires input by user assignment, the application of BIM information can reduce massive input assignments by utilizing BIM information.

Table 5.4 Sources of input for the optimizing and scheduling process

Database

Activtiy duration & Sequecnes			
Activity	Duration	Activity	Duration
Name	P1	Name	P2
Segment	1	Segment	1
Column	4	Column	6
Footing	9	Footing	9
Cost per unit of specific resource			
Activity	Cost/Unit	Activity	Cost/Unit
Name	Milion	Name	Milion
	P1		P2
Column	2.00	Column	1.00
Footing	1.00	Footing	2.00

BIM information

Number of units of in set		
Type/Set	Set 1	Set 2
P1	6	4
P2	3	0
Number of units between sets		
Type/Set	Set 1-2	Set 2-3
P1	3	0
P2	4	0

Table 5.4 shows the two sources of input stored in the Excel file. The first source comes from the database and user assignment. When the user assigns the activity duration and the specific resource cost per unit, the user must arrange them in order from the pile to the viaduct segment (bottom to top). Activity duration, specific resource cost must be ranked in order correctly based on their sequences rule. This process needs the user to recheck and assure that the input belongs in a valid position.

5.3 Output management tools

This section explains how to enhance the presentation for better communication and utilization. There are four outputs of the proposed system which are the optimal solution, LOB diagram, Bar chart on MS project, and 4D construction simulation. The first output is the optimal solution on an Excel template including optimal project duration, an optimal set of specific resources, and an optimal total cost of specific resources as shown in Table 5.5.

Table 5.5 Presentation of the optimal solution of an example project

Type /Activtiy	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resource (m baht)
Type P1 (k = 1)				
Column	Formwork for Column P1	2.0	1.0	2.0
Footing	Formwork for Footing P1	1.5	1.0	1.5
Pile	Casing Ø 1.5 m	1.0	2.0	2.0
Type P1	Project duration by P1	67 days	Total cost of resources	5.5
Type P2 (k = 2)				
Crossbeam	Formwork for Crossbeam P2	3.0	2.0	6.0
Column	Formwork for Column P2	2.0	2.0	4.0
Footing	Formwork for Footing P2	1.0	1.0	1.0
Pile	Casing Ø 1.8 m	1.5	2.0	3.0
Type P2	Project duration by P2	70 days	Total cost of resources	14.0
Project duration	70 days	Total cost of specific resource of the project		19.5

The second output is Line of Balance diagram. The generated start and finish times by the decimal generator are exported from Matlab to a prepared Excel template for LOB diagram. The creation process and example of LOB diagram is shown in Figure 5.11 and Figure 5.12.

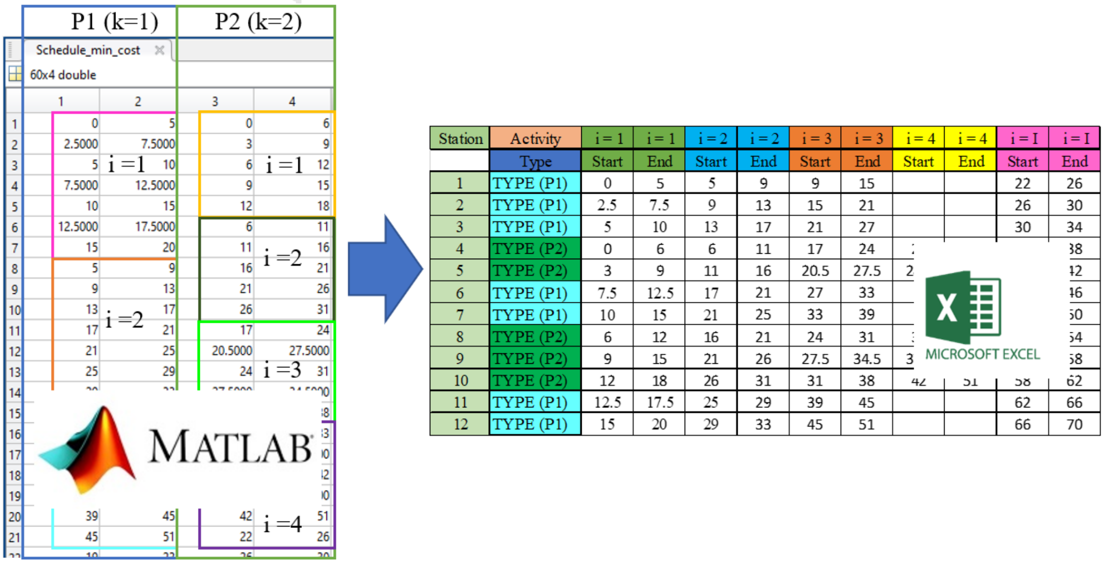


Figure 5.11 Creation of LOB diagram of the proposed system

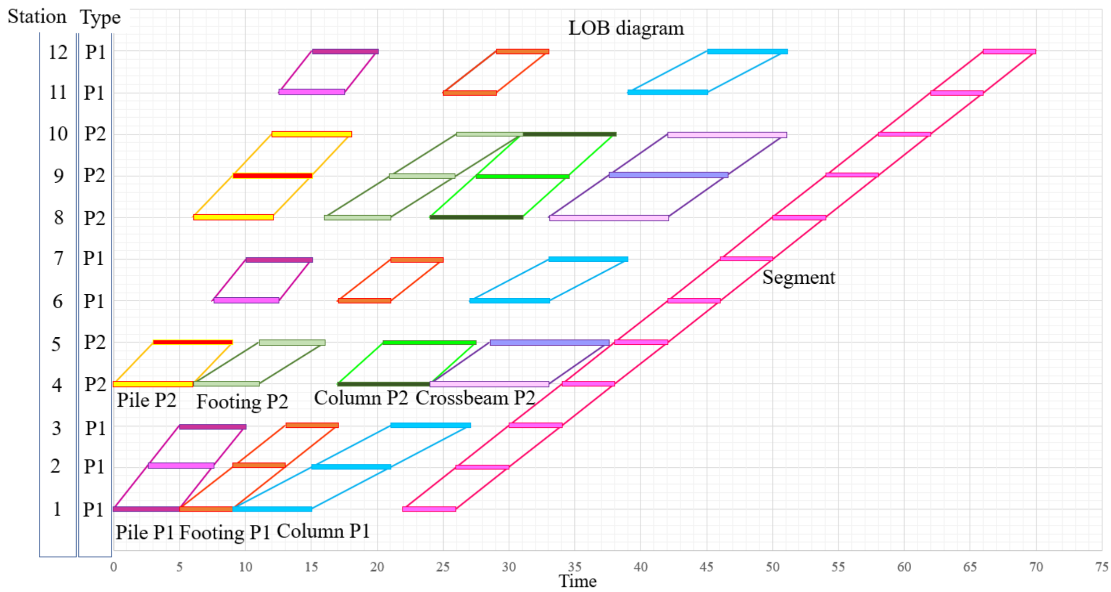


Figure 5.12 Example of LOB diagram by the proposed system

The third output is the Bar chart on MS Project. The generated start and finish dates by the integer generator can be exported to MS Project to display a bar chart by using MS Excel as a connector. In the system, the bar chart only provides the start and finish times of the activities but the sequence links are not attached. This process requires users to perform. MS Project file can enhance the use of the generated schedule. Other information that the system does not provide could be added to the schedule for the utilization in operation of the project.

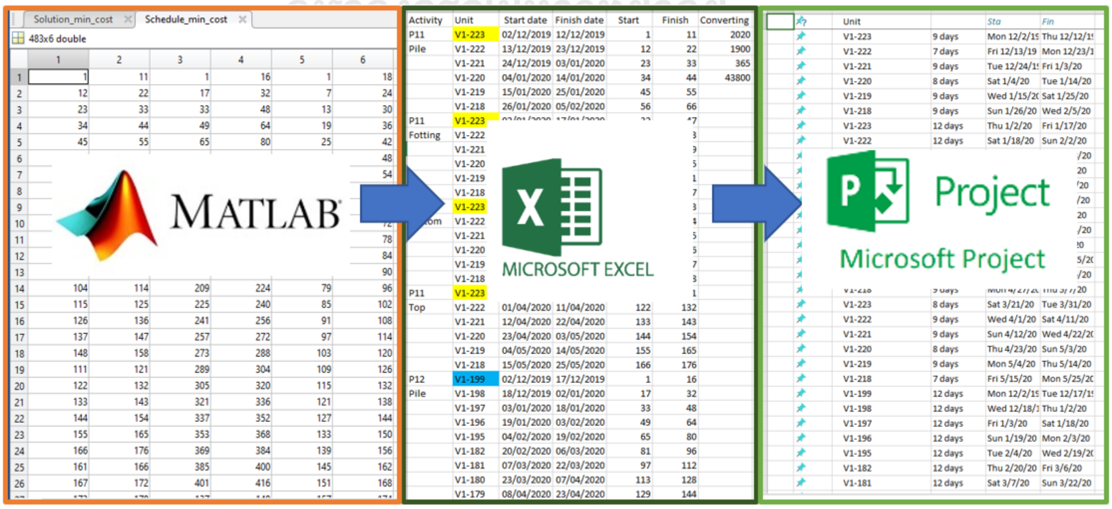


Figure 5.13 Exportation of generated start and finish dates to MS Project

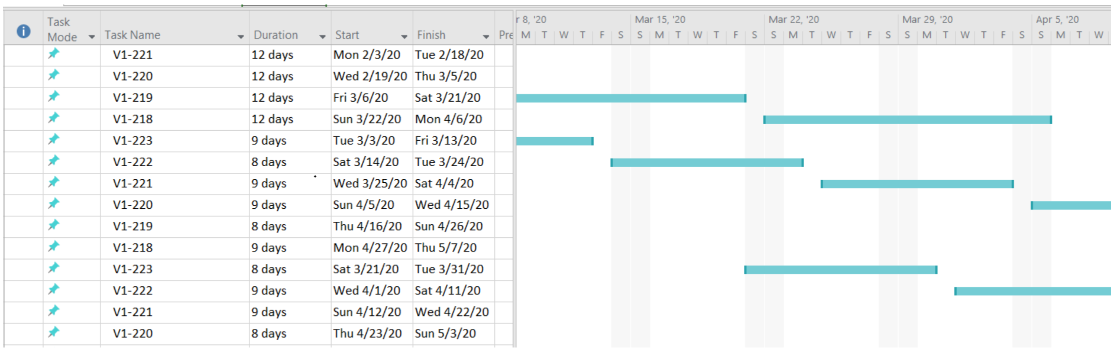


Figure 5.14 Example of Bar chart in MS Project by the proposed system

Finally, the last output is 4D construction simulation. This study uses Autodesk Navisworsk to create the 4D construction simulation of the project by merging the BIM model and the bar chart on MS project.

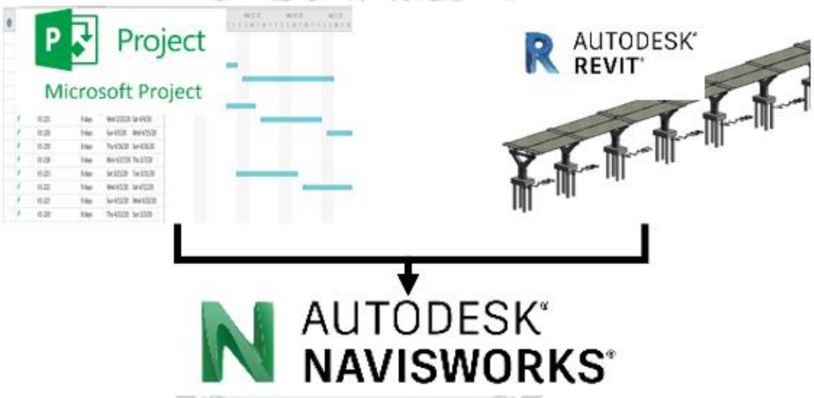


Figure 5.15 Creation of 4D construction simulation

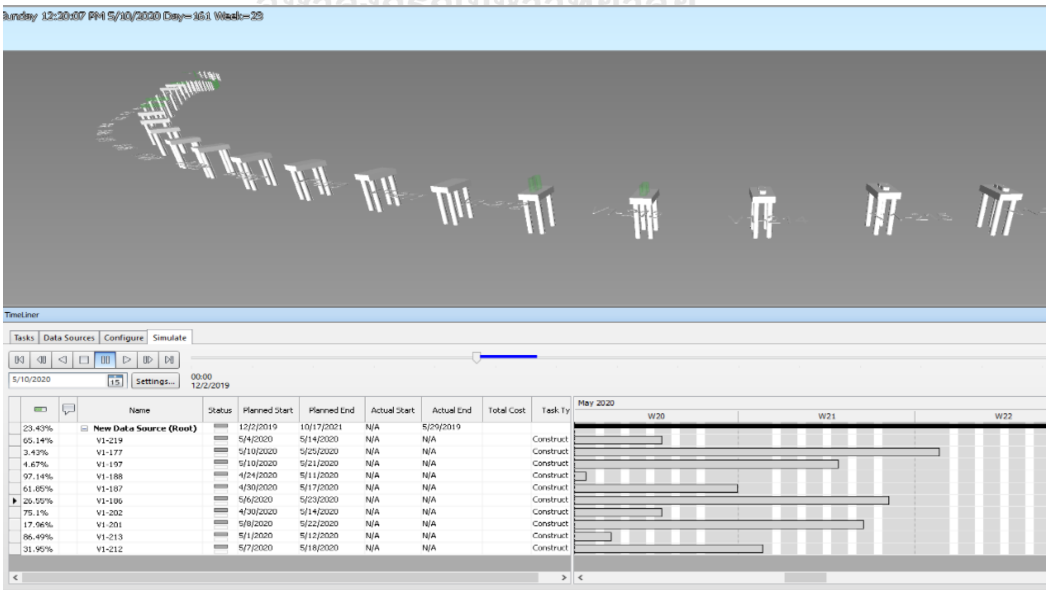


Figure 5.16 Example of 4D construction simulation by the proposed system

5.4 BIM-based Line of Balance Scheduling System (BIM-LOB-SS)

This study develops BIM-LOB-SS with a combination of several programs which are shown in Figure 5.17. The source of input consists of the database on Excel and BIM model of the project. Autodesk Revit is used to develop the BIM model. For the BIM data transformation, the information extractor is invented on Dynamo and Jupyter is employed to develop the information transformer. For the optimizing and scheduling process, this study only uses MATLAB to develop the optimization model and the schedule generator. For the management tools, the optimal solution and LOB diagram are illustrated in MS Excel. MS project is used to display the bar chart which can be imported to Autodesk Naviswork for the creation of 4D construction simulation by merging with the BIM model from Revit. These programs are connected to become BIM-LOB-SS by using MS Excel as the connectors. MS Excel is the general platform to transfer data between programs in BIM-LOB-SS. With the connection by MS Excel, the BIM-LOB-SS is illustrated in Figure 5.17. The workflow of the BIM-LOB-SS is in Figure 5.18 to illustrate the flow of the data from the beginning to the destination of the system.

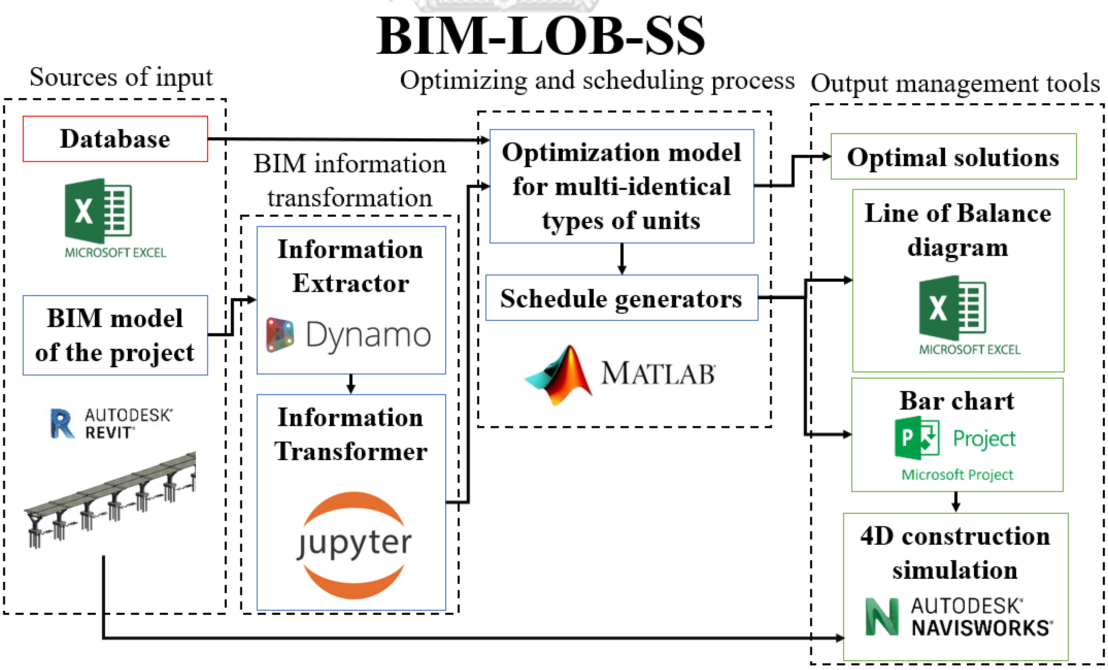


Figure 5.17 Combination of the programs in BIM-LOB-SS

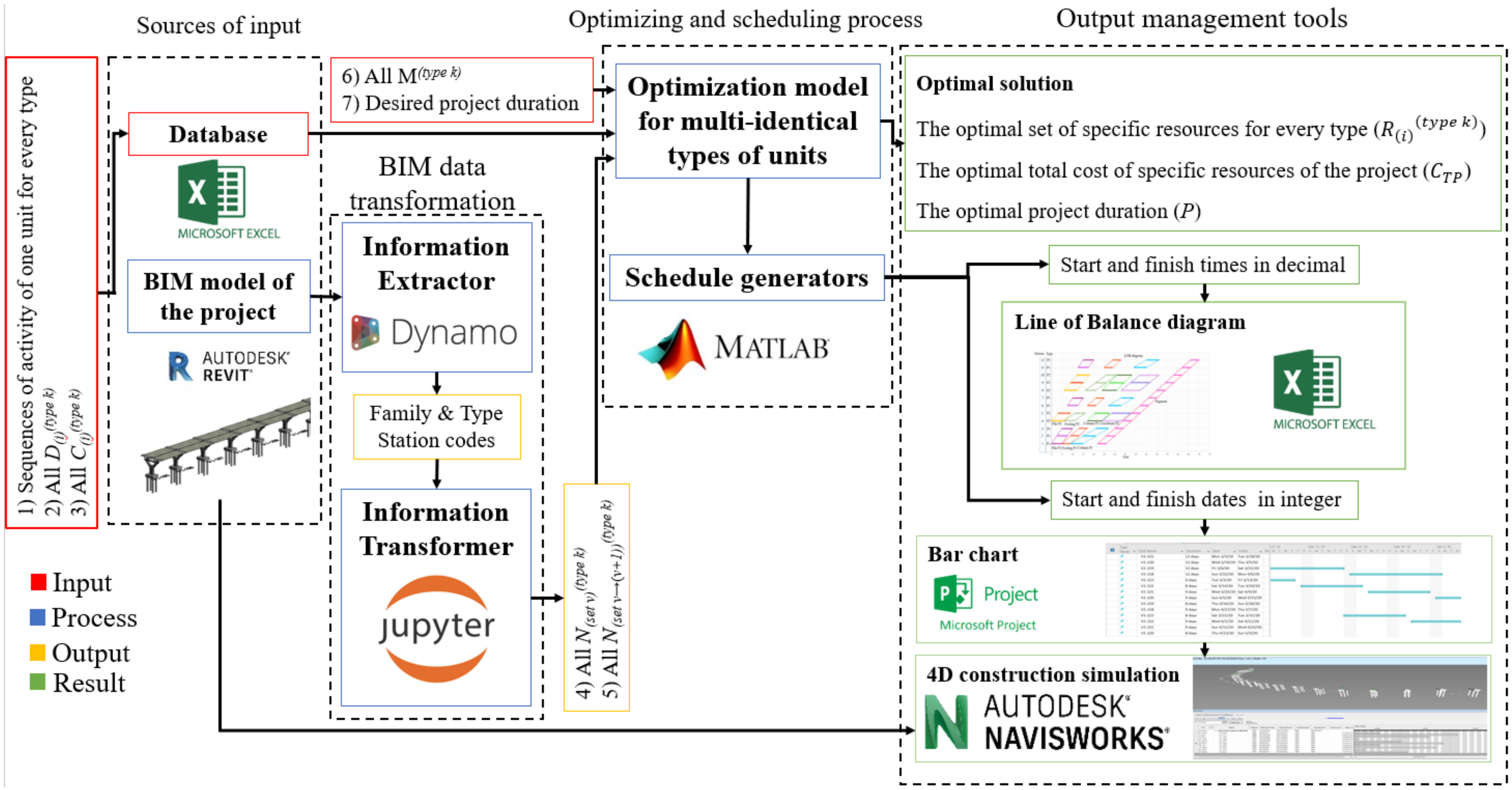


Figure 5.18 Workflow of BIM-based Line of Balance Scheduling System (BIM-LOB-SS)

5.5 System validation

After the proposed system is completely developed, this section validates BIM-LOB-SS with a part of information of the elevated highway construction project. The validation presents several parts including system demonstration, system limitation, and system discussion.

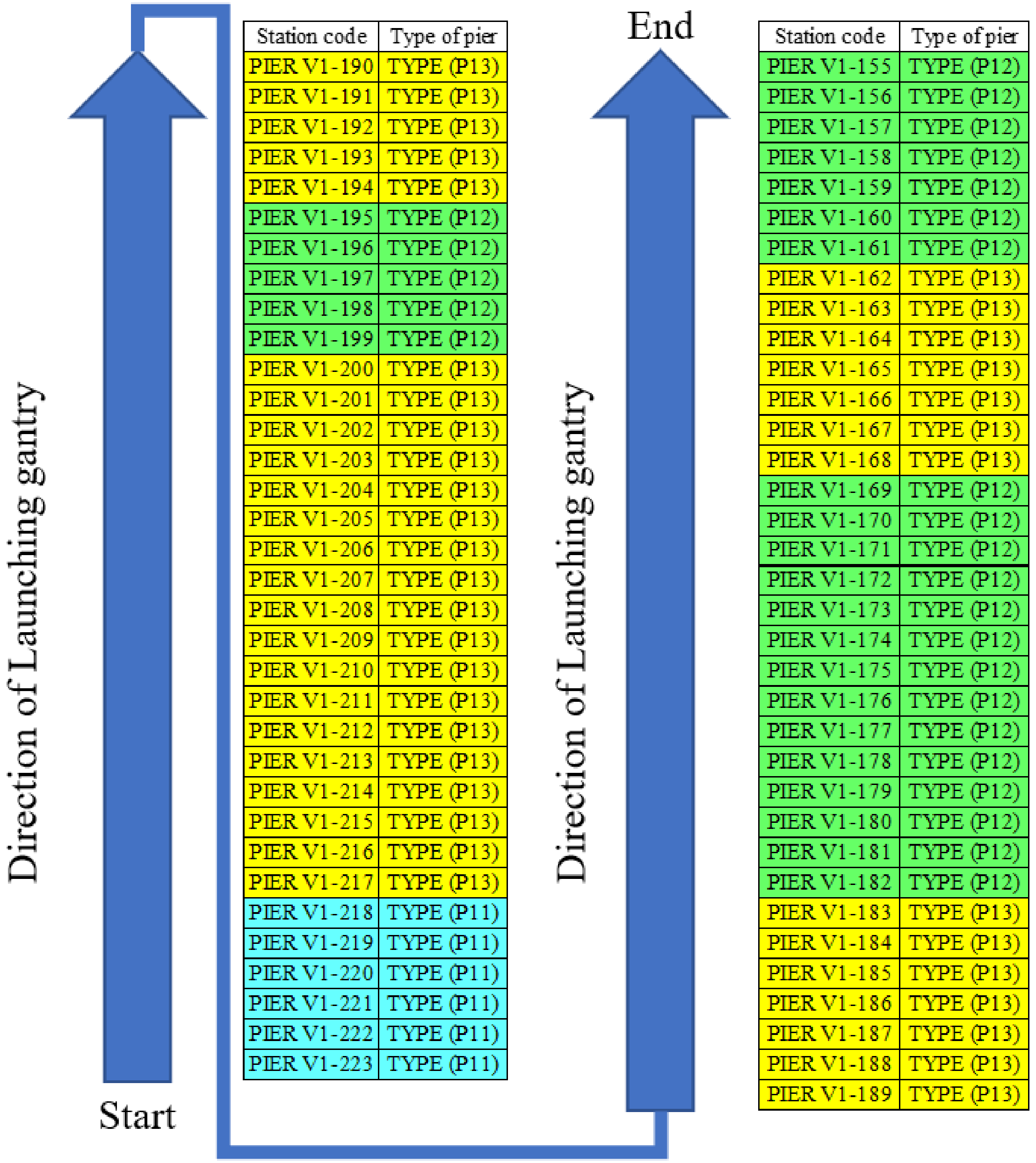
5.5.1 System demonstration

The case study project consists of three types of piers, P11, P12, and P13 representing three identical units. The first pier type P11 (k = 1) contains four structural elements including piles, footing, column, and head column. The second pier type P12 (k = 2) consists of 5 structural elements including piles, footing, column, Y-shape column, and crossbeam. The third pier type P13 (k = 3) comprises six structural elements including piles, footing, base column, column, Y-shape column, and crossbeam. The project is provided to complete within 600 days, and the maximum available resources are not exceeded than 5 teams (all $M^{(type\ k)} = 5$). The information for the database of the system is provided in the following tables.

Table 5.6 Information for the database

Type /Activtiy	Sequence	Duration (days)	Sepcific resource	Cost per unit (m baht/unit)
Type P11 (k = 1)				
Segment	i = 5	6	-	-
Head Column	i = 4	11	Formwork for Head Column P11	0.5
Column	i = 3	11	Formwork for Column P11	0.2
Footing	i = 2	16	Formwork for Footing P11	0.5
Pile	i = 1	11	Casing Ø 1.2 m	0.3
Type P12 (k = 2)				
Segment	i = 6	6	-	-
Crossbeam	i = 5	25	Formwork for Crossbeam P12	2
Y-shape column	i = 4	10	Formwork for Y-shape column P12	3
Column	i = 3	10	Formwork for Column P12	1
Footing	i = 2	12	Formwork for Footing P12	0.75
Pile	i = 1	16	Casing Ø 1.5 m	0.3
Type P13 (k = 3)				
Segment	i = 7	6	-	-
Crossbeam	i = 6	25	Formwork for Crossbeam P13	2
Y-shape column	i = 5	12	Formwork for Y-shape column P13	3
Column	i = 4	10	Formwork for Column P13	1
Base column	i = 3	12	Formwork for Base column P13	1
Footing	i = 2	15	Formwork for Footing P13	0.75
Pile	i = 1	18	Casing Ø 1.8 m	0.3

Figure 5.19 shows the station and types for the case study. The direction of launching gantry for segment erections starts at the station V1-223. It successively erects the segments from a station to the next station and ends at the station V1-155.



Station code	Type of pier
PIER V1-190	TYPE (P13)
PIER V1-191	TYPE (P13)
PIER V1-192	TYPE (P13)
PIER V1-193	TYPE (P13)
PIER V1-194	TYPE (P13)
PIER V1-195	TYPE (P12)
PIER V1-196	TYPE (P12)
PIER V1-197	TYPE (P12)
PIER V1-198	TYPE (P12)
PIER V1-199	TYPE (P12)
PIER V1-200	TYPE (P13)
PIER V1-201	TYPE (P13)
PIER V1-202	TYPE (P13)
PIER V1-203	TYPE (P13)
PIER V1-204	TYPE (P13)
PIER V1-205	TYPE (P13)
PIER V1-206	TYPE (P13)
PIER V1-207	TYPE (P13)
PIER V1-208	TYPE (P13)
PIER V1-209	TYPE (P13)
PIER V1-210	TYPE (P13)
PIER V1-211	TYPE (P13)
PIER V1-212	TYPE (P13)
PIER V1-213	TYPE (P13)
PIER V1-214	TYPE (P13)
PIER V1-215	TYPE (P13)
PIER V1-216	TYPE (P13)
PIER V1-217	TYPE (P13)
PIER V1-218	TYPE (P11)
PIER V1-219	TYPE (P11)
PIER V1-220	TYPE (P11)
PIER V1-221	TYPE (P11)
PIER V1-222	TYPE (P11)
PIER V1-223	TYPE (P11)

Station code	Type of pier
PIER V1-155	TYPE (P12)
PIER V1-156	TYPE (P12)
PIER V1-157	TYPE (P12)
PIER V1-158	TYPE (P12)
PIER V1-159	TYPE (P12)
PIER V1-160	TYPE (P12)
PIER V1-161	TYPE (P12)
PIER V1-162	TYPE (P13)
PIER V1-163	TYPE (P13)
PIER V1-164	TYPE (P13)
PIER V1-165	TYPE (P13)
PIER V1-166	TYPE (P13)
PIER V1-167	TYPE (P13)
PIER V1-168	TYPE (P13)
PIER V1-169	TYPE (P12)
PIER V1-170	TYPE (P12)
PIER V1-171	TYPE (P12)
PIER V1-172	TYPE (P12)
PIER V1-173	TYPE (P12)
PIER V1-174	TYPE (P12)
PIER V1-175	TYPE (P12)
PIER V1-176	TYPE (P12)
PIER V1-177	TYPE (P12)
PIER V1-178	TYPE (P12)
PIER V1-179	TYPE (P12)
PIER V1-180	TYPE (P12)
PIER V1-181	TYPE (P12)
PIER V1-182	TYPE (P12)
PIER V1-183	TYPE (P13)
PIER V1-184	TYPE (P13)
PIER V1-185	TYPE (P13)
PIER V1-186	TYPE (P13)
PIER V1-187	TYPE (P13)
PIER V1-188	TYPE (P13)
PIER V1-189	TYPE (P13)

Figure 5.19 Station codes, types of piers, the direction of launching gantry for the case study

From section 5.2.1, the BIM model of the case study was developed by using Autodesk Revit and construction drawing. So, the source of input is prepared, and BIM-LOB-SS is ready. After the system processed, the results of the system are shown as the following.

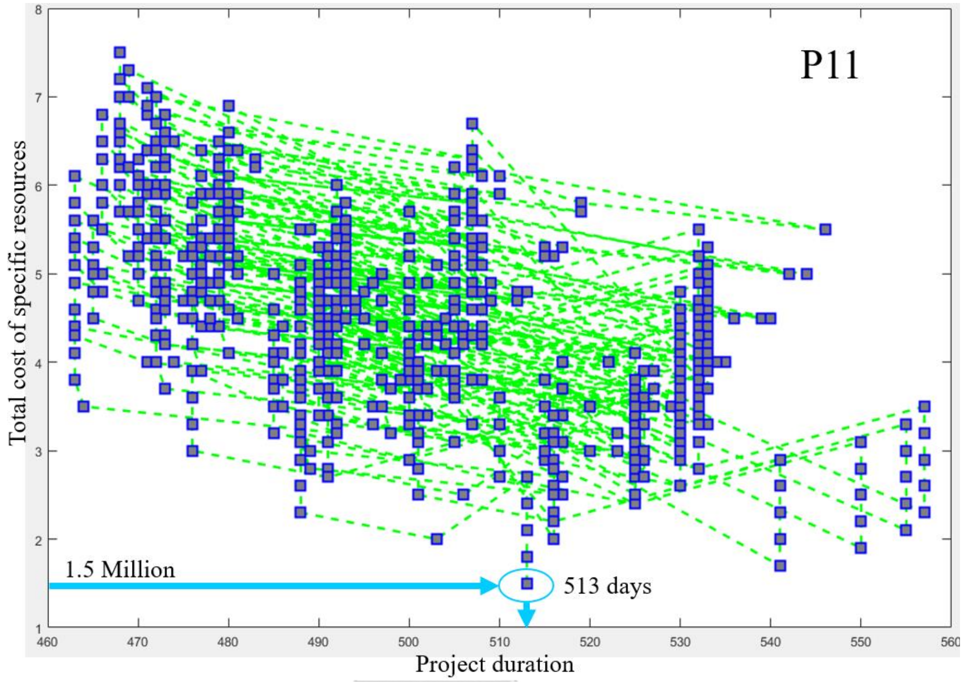


Figure 5.20 Searching path of P11 for the case study

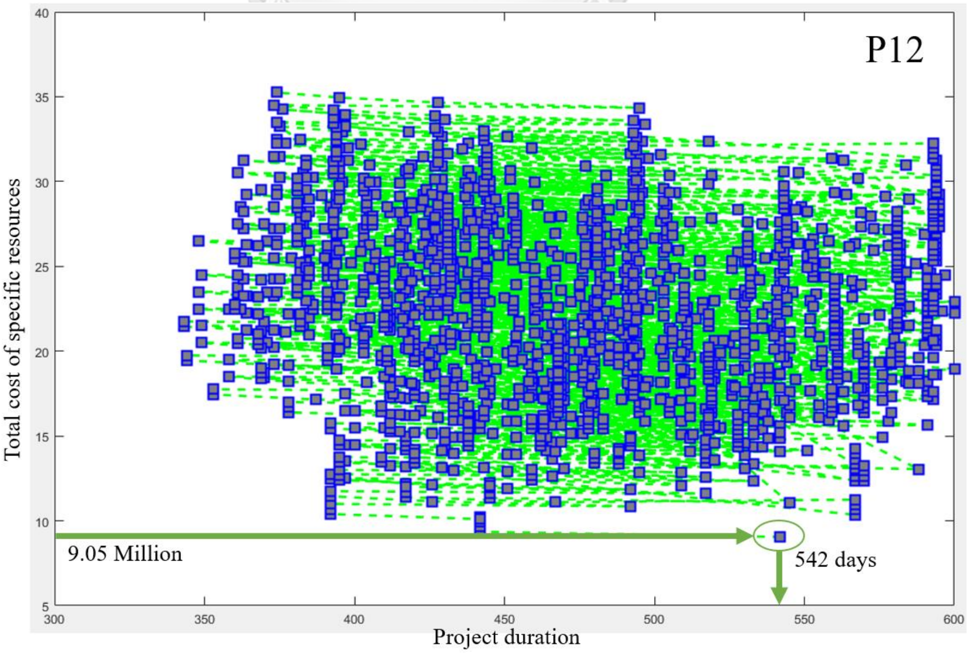


Figure 5.21 Searching path of P12 for the case study

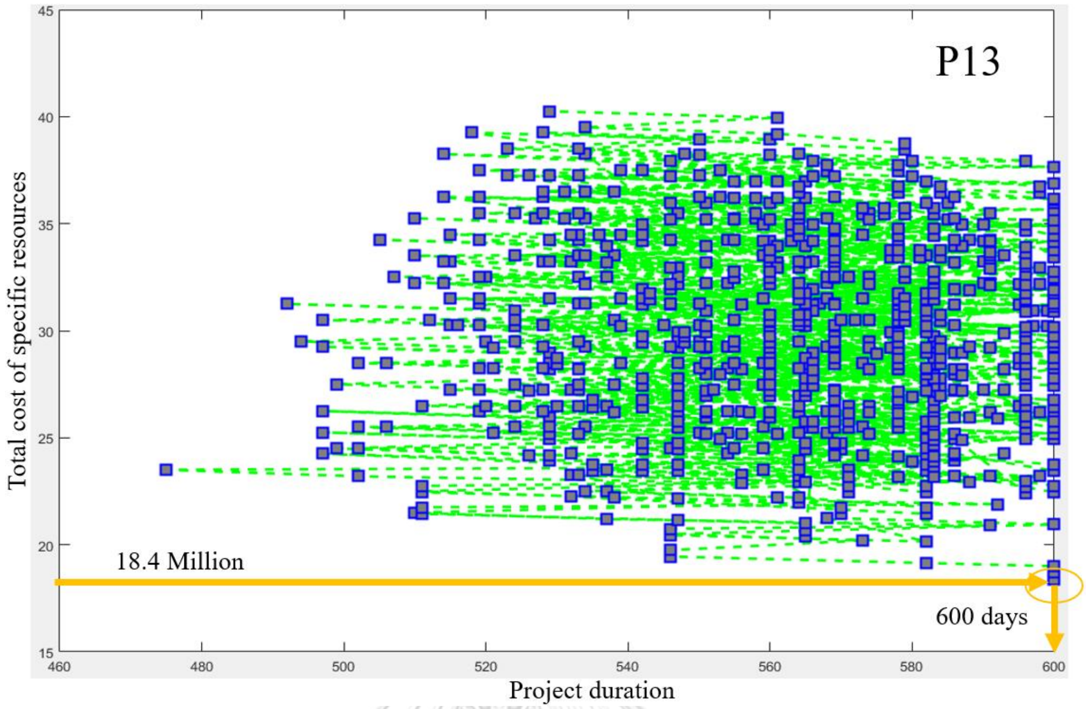


Figure 5.22 Searching path of P13 for the case study

Table 5.7 The optimal solution for the case study project

Type /Activtiy	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resources (m baht)
Type P11 (k = 1)				
Head Column	Formwork for Head Column P11	0.50	1	0.5
Column	Formwork for Column P11	0.20	1	0.2
Footing	Formwork for Footing P11	0.50	1	0.5
Pile	Casing Ø 1.2 m	0.30	1	0.3
Type P11 (k = 1)	Project duration by P11	513 days	Total cost of resources	1.5
Type P12 (k = 2)	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resources (m baht)
Crossbeam	Formwork for Crossbeam P12	2.00	2	4
Y-shape column	Formwork for Y-shape column P12	3.00	1	3
Bottom column	Formwork for Column P12	1.00	1	1
Footing	Formwork for Footing P12	0.75	1	0.75
Pile	Casing Ø 1.5 m	0.30	1	0.3
Type P12 (k = 2)	Project duration by P12	542 days	Total cost	9.05
Type P13 (k = 3)	Sepcfic Reosurce	Cost per unit (m baht/unit)	Number of resource	Cost of resources (m baht)
Crossbeam	Formwork for Crossbeam P13	2.00	3	6
Y-shape column	Formwork for Y-shape column P13	3.00	2	6
Top column	Formwork for Column P13	1.00	2	2
Bottom column	Formwork for Base column P13	1.00	2	2
Footing	Formwork for Footing P13	0.75	2	1.5
Pile	Casing Ø 1.8 m	0.30	3	0.9
Type P13 (k = 3)	Project duration by P13	600 days	Total cost	18.4
Project duration	600 days	Total cost of specific resource of the project		28.95

From Figure 5.20, 5.21, and 5.22, the searching paths show that the optimal solution is selected from the sets of specific resources that fulfill the objective of minimizing the total cost of specific resources under the desired project duration. Type P11 provides 513 days of project duration with 1.5 million baht of the total cost of specific resources. Type P12 provides 542 days of project duration with 9.05 million baht of the total cost of specific resources. Type P13 provides 600 days of project duration with 18.4 million baht of the total cost of specific resources. Thus, the control type is type P13 with 600 days due to the longest duration among three types. This causes type P11 has free-float time for 87 days and type P12 has free-float time for 58 days. The total cost of specific resources of the project is 28.95 million baht and the number of each specific resource is shown in Table 5.7.

For alternative solutions, the project duration provided by the integer generator is equal to the project duration produced by the optimization model as shown in Table 5.8. Therefore, for the 600 days of the desired project duration, the sub-slope in the integer generator does not influence the optimal solution.

Table 5.8 Alternative solutions for the case study project.

Alternative solutions	Schedule Generator	Desired project duration	Project duration from the optimization model	Total cost (million baht)	Project duration from the schedule
First choice	Decimal	600	600	28.95	600
Second choice	Integer	600	600	28.95	600
Third choice	Integer	600	600	28.95	600

The generated start and finish times by the generator for decimal time are exported to the template of LOB diagram in MS Excel to create the LOB diagram as shown in Figure 5.23. The lines of the repetitive activities of type P11, P12, and P13 are drawn by the MS Excel template from the first unit to the last unit (unit 69) depending on the sets and type of units. The last repetitive activity is the segment erection which is continuously performed by a launching gantry for every unit.

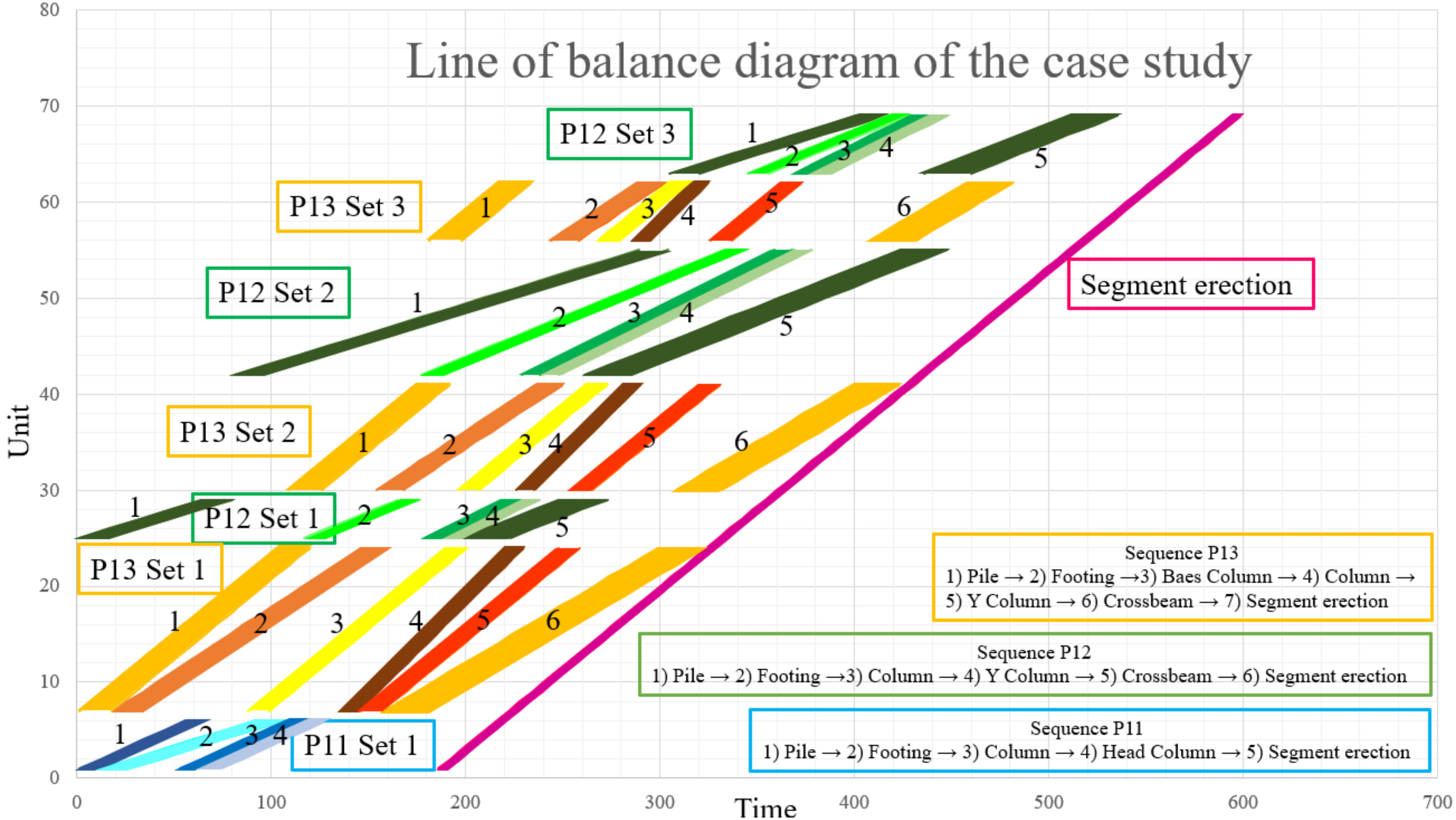
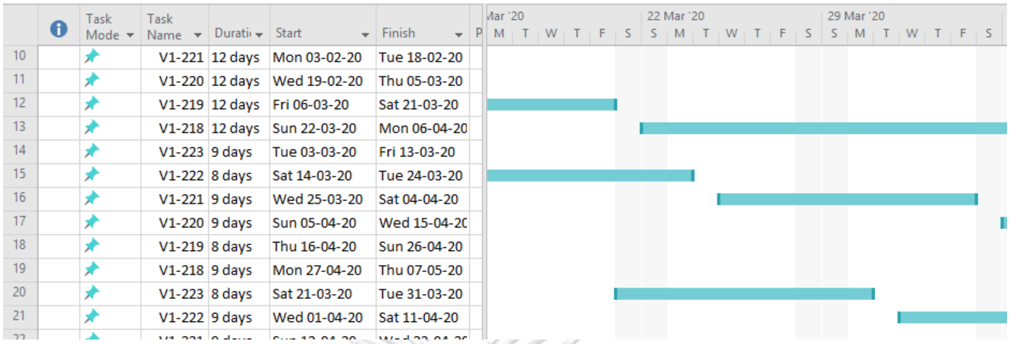


Figure 5.23 LOB diagram of the case study

The generated start and finish dates by the integer generator are used to create a bar chart in the MS project. Finally, the 4D simulation of the project is conducted.



	Task Mode	Task Name	Duration	Start	Finish
10		V1-221	12 days	Mon 03-02-20	Tue 18-02-20
11		V1-220	12 days	Wed 19-02-20	Thu 05-03-20
12		V1-219	12 days	Fri 06-03-20	Sat 21-03-20
13		V1-218	12 days	Sun 22-03-20	Mon 06-04-20
14		V1-223	9 days	Tue 03-03-20	Fri 13-03-20
15		V1-222	8 days	Sat 14-03-20	Tue 24-03-20
16		V1-221	9 days	Wed 25-03-20	Sat 04-04-20
17		V1-220	9 days	Sun 05-04-20	Wed 15-04-20
18		V1-219	8 days	Thu 16-04-20	Sun 26-04-20
19		V1-218	9 days	Mon 27-04-20	Thu 07-05-20
20		V1-223	8 days	Sat 21-03-20	Tue 31-03-20
21		V1-222	9 days	Wed 01-04-20	Sat 11-04-20

Figure 5.24 Generated schedule of the case study in MS project

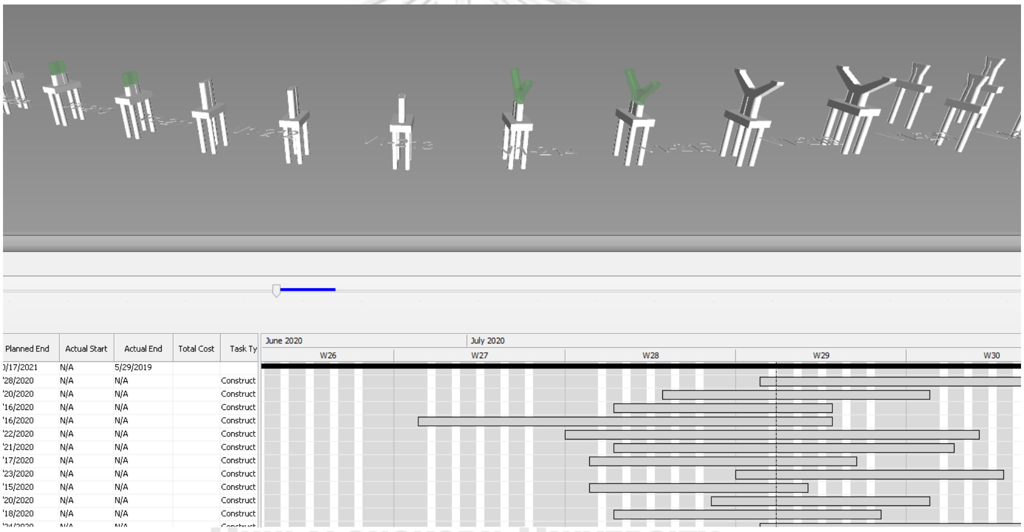


Figure 5.25 4D construction simulation of the case study

From the results, the proposed system can suggest an optimal set of specific resources that can achieve 600 days of the desired project duration while using 28.95 million baht of the total cost of specific resources. For the schedule generation, the start and finish times of 445 activities are generated in both decimal values and integer values. The LOB diagram created by the decimal times shows that there is no sequence violation occurred. The LOB diagram gives an overview of the case study with a clarified presentation. The bar chart generated by the integer dates provides the worktable schedule for the real-world operation. Finally, the 4D construction simulation with the optimal solution can be examined how the construction process will appear at different project stages.

5.5.2 System discussion

Multi-identical types of units in elevated highway construction should be examined by a more efficient methodology in that the units are separated into many types and each type requires specific resources to build the units. Conducting a proper schedule manually in optimizing the total cost of specific resources is a challenge for the existing repetitive scheduling methodologies (such as Line of Balance, MS Project, and TILOS), due to many factors related to the cost, time and quality. This research proposed a system for the project manager to create management tools more convenient with the combination of an optimization model and BIM technology. Unlike the previous optimization models, the optimization model proposed by this study is capable to deal with the scheduling problem of the multi-identical types of units when the project duration and total cost of specific resources are the primary objectives. Moreover, the schedule generators in both decimal and integer value can express the creation of the LOB diagram and bar chart which reflect the project operation with the optimal set of specific resources. The proposed system demonstrates that the BIM model is not only used for improving visualization with the 4D simulation but also illustrates the utilization of the information stored in the BIM model for the optimizing and scheduling process by getting thought the BIM data transformation. The examination of the 4D construction simulation with the optimal solution probably increases the better chances to discover the opportunities of the project. Finally, this research has established that the application of Line of Balance (LOB) and Building Information Modeling enhances the scheduling process and the use of BIM in construction more efficiency.

5.5.3 System limitation

Some limitations of the BIM-LOB-SS can be described as follows:

1) The results of the system only provide suggested solutions for decision making. The project managers essentially need to examine the solutions before using the project.

2) The BIM model is developed based on the document from the case study. For the other projects, in case the station codes undefined, the users should

consider that there may have other information which can be applied for the information transformer.

3) The application of BIM information is only designed for the variables of the proposed optimization model in this study, so the information transformer is probably incapable to cover other optimization models.

4) Some manual processes are required users to perform. The manual processes in the proposed system consist of assigning information to the database, selecting the BIM element for the information transformer, importing the optimal solution and the generated times to MS Excel, importing the generated dates to MS Project, and merging BIM model and bar chart for creating the 4D construction simulation. Therefore, the users must be careful about human-error in these processes.

Chapter 6
Conclusion

6.1 Research conclusion

Project scheduling is an essential process to carry out the project successfully with limited time and cost. A schedule is a communication tool that illustrates activities required to be done, which resource assigned to the task, and location of work. The linear infrastructure project as elevated highway construction is a large scale project and has the complexity in project management. Recently, most of the scheduling methodologies mainly rely on the project manager who may cause the human-error leading to improper schedule. Moreover, the presentations of the schedules created by the existing methodologies still insufficient to express various views of the project. Thus, an application of Line of Balance (LOB) and Building Information Modeling (BIM) is proposed.

For the scheduling problem, this study establishes an application of Line of Balance for solving the scheduling problem of the multi-identical types of units in the elevated highway construction. The application of Line of Balance aims to develop an optimization model with the objective of minimizing the total cost of specific resources under the desired project duration while maintaining work continuity. Instead of direct considering the LOB diagram, a method of project duration calculation is developed to consider the scheduling problem in terms of mathematical equations. The method examines two consecutive activities and creates the representative equations to determine two variables which are $DSS_{(1)}$ and $DFS_{(j)}$. These two variables are used to verify the sequence logic of two consecutive activities and also be the key variables of the project duration calculation. Finally, the method is used to develop a function of project duration calculation for any set of resources. After the model is completely developed, this study verifies the optimization model with three example projects by each example presenting different conditions from each other. The results show that the optimization model can provide the correct solution compared to the results from the trial-and-error.

For the manual creation of the construction schedule, the application of Line of Balance is not only used to develop the optimization model but also utilized to invent the schedule generator for computing the start and finish times of all activities in the project. There are two schedule generators are developed in this study (the schedule generator for decimal time and the schedule generator for integer time). The schedule generator for decimal time is developed to present the LOB diagram while the schedule generator for integer time is invented to create bar chart in MS project. The schedule generator for integer time is examined its capability by comparison with the result from Powerproject, a scheduling software with LOB feature. The result shows that the generator can compute the dates correctly.

For the application of Building Information Modeling, this study demonstrates the utilization of information stored in the BIM model to reduce massive input assignments. Family Type and Station codes are selected for the input of the optimization model. To make use of the BIM information, the information extractor and the information transformer are developed to extract and transform the BIM information matched with the input of the optimization model. Moreover, the BIM model is not only used for information storage but also used for the creation of the 4D construction simulation by merging the bar chart with the elements of the BIM model. The 4D construction simulation with the optimal solution provides a full overview of the project operation during the construction.

With the optimization model and BIM technology, this study developed a BIM-based Line of Balance Scheduling System (BIM-LOB-SS). The proposed system contains four main components: (1) sources of input, the BIM model and database store information of the project; (2) the BIM information transformer, which makes use of BIM information stored in BIM model for the optimizing and scheduling process; (3) the optimizing and scheduling process, which computes the optimum solution and generates the start and finish times; (4) output management tools, which are the optimum set of specific resources, the optimal project duration, and the optimal total cost of specific resources, LOB diagram, bar chart in MS Project, and 4D construction simulation.

Finally, the proposed system is validated with the case study of an elevated highway construction project. The result illustrates that the project manager can utilize the preliminary management tools to improve the project operation more efficiency. Additionally, the manager can easily adjust the results for suitable operation because the results are created with compliance of some objectives already.

6.2 Research contributions

The contributions of this research include the following aspects:

1) The method of project duration calculation for linear repetitive projects
2) The optimization model with the objective of minimizing the total cost of specific resources under desirable duration while maintaining work continuity
3) The application of BIM and the implementation of the BIM model for scheduling system
4) The prototype of BIM-based Line of Balance Scheduling System (BIM-LOB-SS) for linear infrastructure projects

6.3 Limitations and suggestions

Although the system has shown its capability dealing with the scheduling problems of the elevated highway project, the proposed system still has limitations as the following aspect:

1) The specific resource considered in this study is one time purchased. The cost per unit of resource is a fixed cost that does not vary on time. Thus, resources such as workers or rental equipment are not considered.
2) The project duration computed by the optimization process may not equal to the project duration generated by the schedule generator. The cause is the flooring-down of the delay (D/R) in the schedule generator. It moved some activities to start sooner which directly reduce the project duration. Thus, the proper solution can be selected from the alternative solutions depending on the users' proposes.

3) Some manual processes may cause human-error. So, the users must carefully recheck to ensure the correct manual processes before using the results of the system.

4) The results of the system only provide suggested solutions for decision making. The project managers need to examine the solutions before applying to the project.

6.4 The future direction of research

In order to enhance the capability of the optimization model and the scheduling system, the following directions could be explored in the future:

1) Conditions such as work interruption or multi-resource assignment could carry out the more optimal schedule. However, these conditions may lead to massive search space of decision variables where more efficient searching algorithms are required.

2) The proposed system can be improved by replacing some manual processes of the system with more automatic approaches.

REFERENCES

Adeli, H. (2001). Neural networks in civil engineering: 1989–2000. *Computer-Aided Civil and Infrastructure Engineering, 16*(2), 126-142.

Adeli, H., & Karim, A. (1997). Scheduling cost optimization and neural dynamics model for construction. *Journal of Construction Engineering and Management-Asce, 123*(4), 450-458.

Adeli, H., & Karim, A. (2001). *Construction scheduling, cost optimization and management*: CRC Press.

Arditi, D., & Albulak, Z. (1986). Line-of-balance scheduling in pavement construction. *Journal of Construction Engineering and Management-Asce, 112*(3), 411-424.

Arditi, D., Tokdemir, O. B., & Suh, K. (2002). Challenges in line-of-balance scheduling. *Journal of Construction Engineering and Management-Asce, 128*(6), 545-556.

Chen, S.-M., Griffis, F. H., Chen, P.-H., & Chang, L.-M. (2013). A framework for an automated and integrated project scheduling and management system. *Automation in construction, 35*, 89-110.

Chrzanowski, E. N., & Johnston, D. W. (1986). Application of Linear Scheduling. *Journal of Construction Engineering and Management-Asce, 112*(4), 476-491.

Damci, A., Arditi, D., & Polat, G. (2013a). Multiresource Leveling in Line-of-Balance Scheduling. *Journal of Construction Engineering and Management-Asce, 139*(9), 1108-1116.

Damci, A., Arditi, D., & Polat, G. (2013b). Resource Leveling in Line-of-Balance Scheduling. *Computer-Aided Civil and Infrastructure Engineering, 28*(9), 679-692.

Dang, D. T. P., & Tarar, M. (2012). Impact of 4D modeling on construction planning process. *Chalmers University of Technology Göteborg, Sweden*.

Daniel W., H., Gunnar, L., & Bolivar A, S. (2017). *Construction management*: John Wiley & Sons.

El-Rayes, K., & Moselhi, O. (2001). Optimizing resource utilization for repetitive construction projects. *Journal of Construction Engineering and Management-Asce, 127*(1), 18-27.

G V, B., & Shankar, S. R. (2015). Suitability of linear scheduling over CPM in scheduling highway project. *International Journal of Engineering Research & Technology 4*(3).

Georgy, M. E. (2008). Evolutionary resource scheduler for linear projects. *Automation in construction, 17*(5), 573-583.

Harris, R. B., & Ioannou, P. G. (1998). Scheduling projects with repeating activities. *Journal of Construction Engineering and Management, 124*(4), 269-278.

Hart, S. (2007). The Last Three Miles: Politics, Murder, and the Construction of America's First Superhighway. *New York: The New Press*.

Hegazy, T., & Wassef, N. (2001). Cost optimization in projects with repetitive nonserial activities. *Journal of Construction Engineering and Management, 127*(3), 183-191.

Hyari, K., & El-Rayes, K. (2006). Optimal planning and scheduling for repetitive construction projects. *Journal of Management in Engineering, 22*(1), 11-19.

Johnston, D. W. (1981). Linear scheduling method for highway construction. *Journal of the Construction Division, 107*(2), 247-261.

Kang, L. S., Park, I. C., & Lee, B. H. (2001). Optimal schedule planning for multiple repetitive construction process. *Journal of Construction Engineering and Management-Asce, 127*(5), 382-390.

Kataoka, M. (2008). Automated generation of construction plans from primitive gemometries. *Journal of Construction Engineering and Management, 134*(8), 592-600.

Kim, H., Anderson, K., Lee, S., & Hildreth, J. (2013). Generating construction schedules through automatic data extraction using open BIM (building information modeling) technology. *Automation in construction, 35*, 285-295.

Leu, S.-S., & Hwang, S.-T. (2001). Optimal repetitive scheduling model with shareable resource constraint. *Journal of Construction Engineering and Management-Asce, 127*(4), 270-280.

Liu, H., Al-Hussein, M., & Lu, M. (2015). BIM-based integrated approach for detailed construction scheduling under resource constraints. *Automation in construction, 53*, 29-43.

Liu, H., Lei, Z., Li, H. X., & Al-Hussein, M. (2014). *An Automatic Scheduling Approach: Building Information Modeling-based Onsite Scheduling for Panelized Construction*. Paper presented at the Conference: Construction Research Congress 2014, ASEC 2014.

Liu, S.-S., & Wang, C.-J. (2007). Optimization model for resource assignment problems of linear construction projects. *Automation in construction, 16*(4), 460-473.

Liu, S.-S., & Wang, C.-J. (2012). Optimizing linear project scheduling with multi-skilled crews. *Automation in construction, 24*, 16-23.

Long, L. D., & Ohsato, A. (2009). A genetic algorithm-based method for scheduling repetitive construction projects. *Automation in construction, 18*(4), 499-511.

Lumsden, P. (1968). The line-of-balance method. In *Pergamon, Tarrytown*. New york.

Lutz, J. D. H., Adib (1993). Planning repetitive construction. *Construction Management and Economics, 11*(2), 99-110.

NIBS (2015). National BIM Standard-United States ®. Retrieved from https://www.nationalbimstandard.org/

Ostenfeld, K., Hommel, D., Olsen, D., & Hauge, L. (2000). Planning of major fixed links. In: Boca Raton, FL: CRC Press.

Rosignoli, M. (2016). Bridge construction equipment. In *Innovative Bridge Design Handbook*: Elsevier.

Srisuwanrat, C., Ioannou, P. G., & Tsimhoni, O. (2008). *Simulation and optimization for construction repetitive project projects using Promodel and Simrunner.* Paper presented at the Proceedings of the 40th Conference on Winter Simulation.

Vries, B. d., & Harink, J. M. J. (2007). Generation of a construction planning from a 3D CAD model. *Automation in construction, 16*(1), 13-18.

Wu, I.-C., BorrmannBeissert, A., Beißert, U., & König, M. (2010). Bridge construction schedule generation with pattern-based construction methods and constraint-based simulation. *Advanced Engineering Informatics, 24*(4), 379-388.

Yamin, R., & Harmelink, D. J. (2001). Comparison of linear scheduling model (LSM) and critical path method (CPM). *Journal of Construction Engineering and Management-Asce, 127*(5), 374-381.

Zoli, T. P. T. (2012). A Bridge by the People, for the People. *Civil Engineering Magazine Archive, 82*(6), 49-57.

APPENDIX

Appendix A

Notation of variables

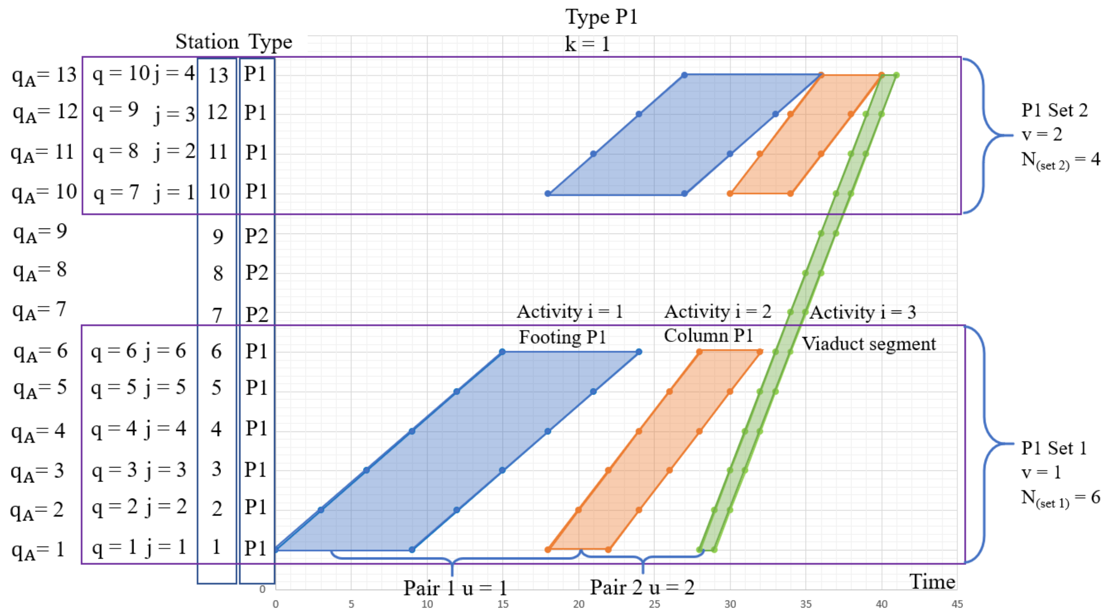


Figure A1 Example notation of variables by type P1

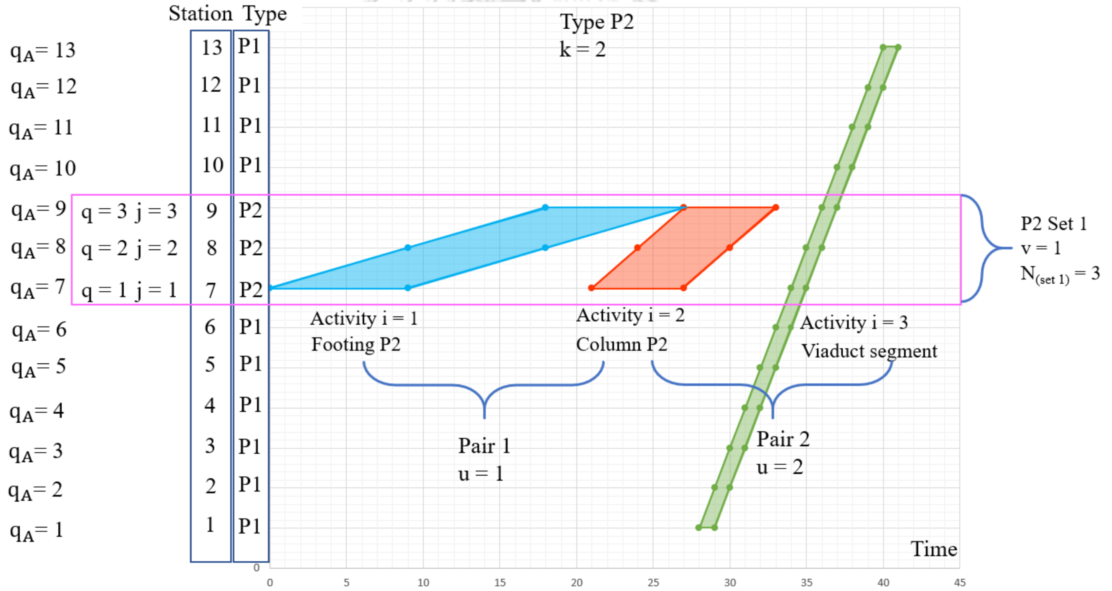


Figure A2 Example notation of variables by type P2

1) i is repetitive activity of type k, 2) j is unit in set of type k, 3) k is type of units,

4) u is pair of consecutive activities, 5) v is set of units of type k, 6) q is unit of type k,

7) q_A is unit in project, 8) $N_{(set\ v)}$ is number of units in set v type k

Appendix B

Flowchart of schedule generator for integer time

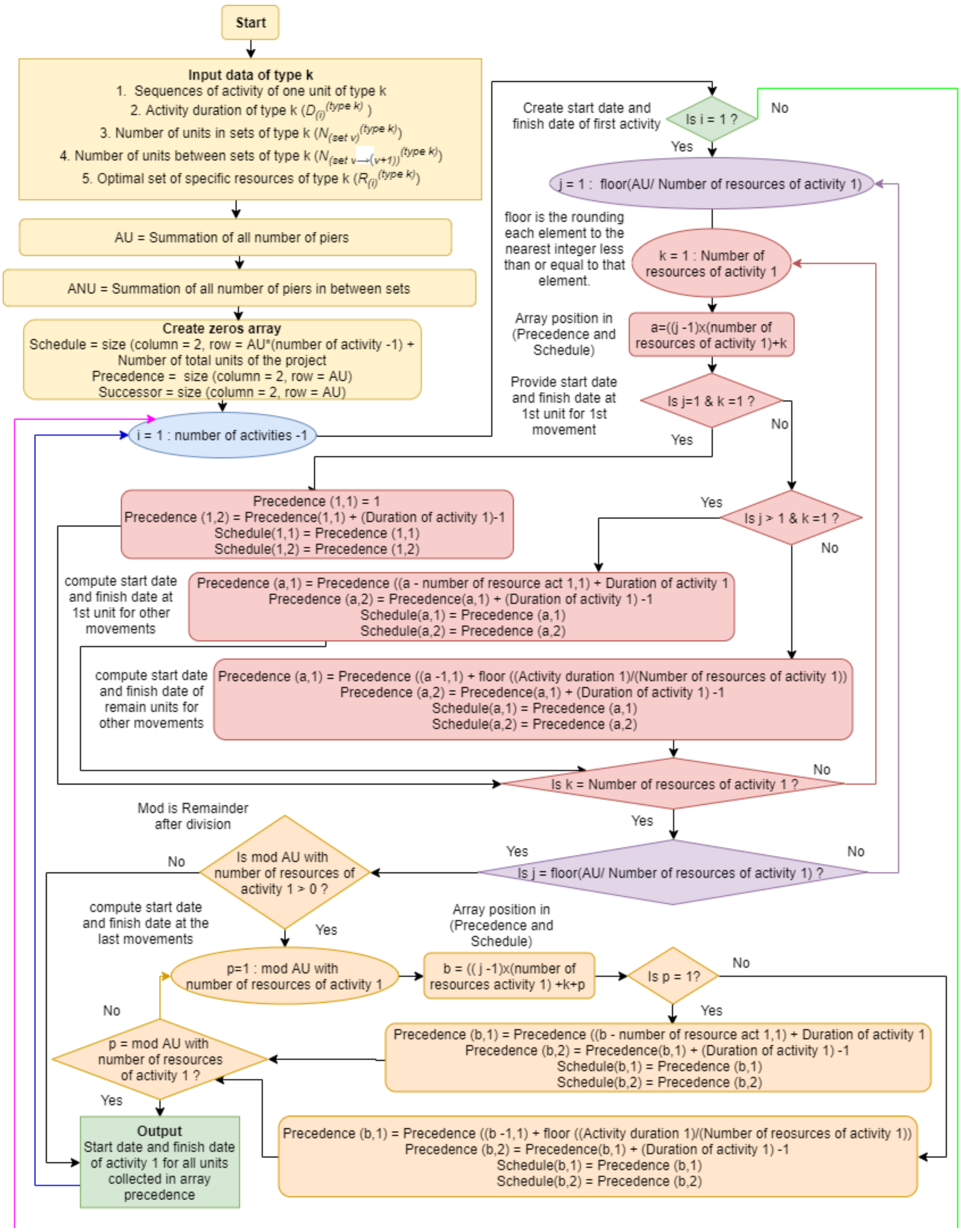


Figure B1 Flowchart of schedule generator for integer time (1/3)

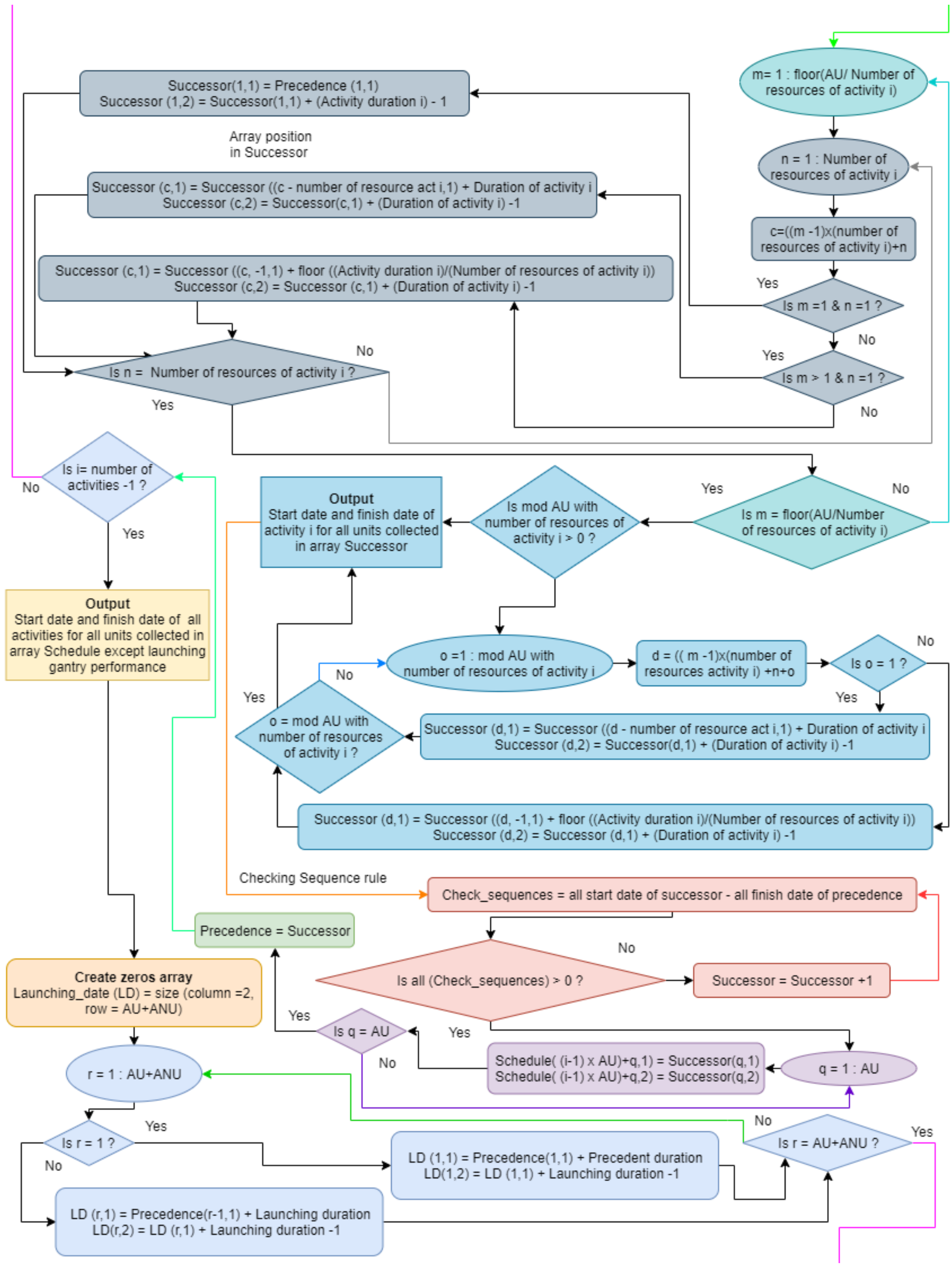


Figure B2 Flowchart of schedule generator for integer time (2/3)

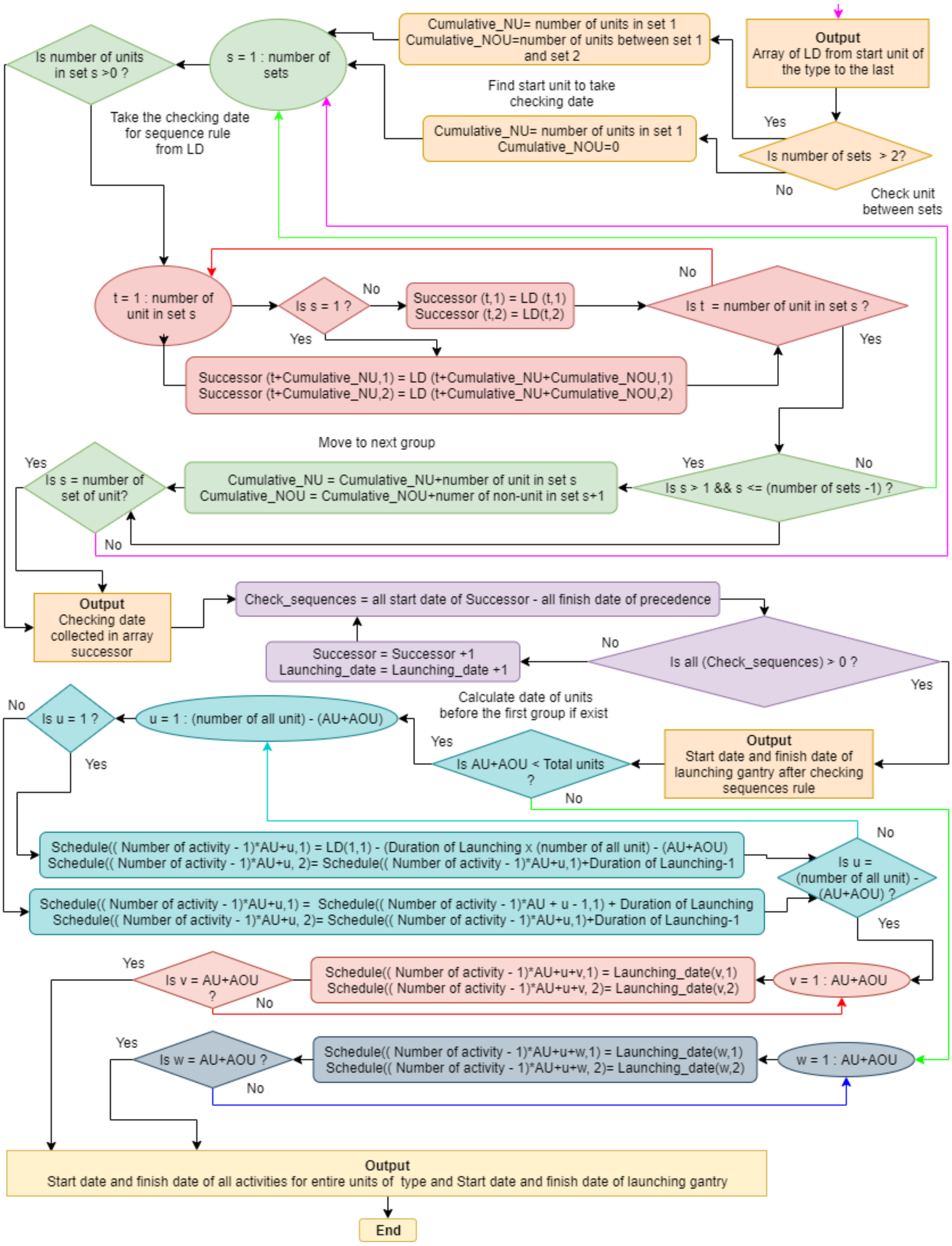


Figure B3 Flowchart of schedule generator for integer time (3/3)

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