Chapter 1

Introduction

In the classical case, the theory of uniform distribution of sequences of real numbers is well-known and has varied applications. The three main concepts in this theory are : uniform distribution modulo 1, uniform distribution modulo m, where m is any positive integer > 1 and uniform distribution in $\mathbb{Z}$. Roughly speaking, a sequence $(x_n)_{n=1}^{\infty}$ with values in $[0,1)$ is said to be uniformly distributed modulo 1 if *each subinterval of* $[0,1)$ *asymptotically receives its fair proportion of values of* x_n.

The formal definition of uniform distribution modulo 1 was first given by Hermann Weyl. The notions of uniform distribution modulo m and uniform distribution in $\mathbb{Z}$ were first introduced by Niven [17]. An excellent account of the theory uniform distribution can be found in the book of Kuipers and Niederreiter [8], which also gives a wide list of references covering the field. For a complete survey of uniform distribution modulo m and uniform distribution in $\mathbb{Z}$, we refer to Narkiewicz [15].

In the next chapter, we introduce the concept of this theory and some of its applications, including the well-known Weyl's uniform distribution criterion, Van der Corput's difference theorem, some applications in theory of power series and uniform distribution modulo 1 in the multidimensional case. Our principal reference for this chapter is [8]. Our contribution here is in Theorem 2.2.1 and 2.2.3 which are slight extensions of Theorem 1 and 2 in Newman [16].

There are many investigations about uniform distribution of sequences of real numbers; at the same time the idea of uniform distribution was generalized into several directions. For example, we can develop the theory of uniform distribution in more abstract setting of an arbitrary compact Hausdorff space with a countable base. Furthermore, adding the algebraic structure as the real numbers have, we can also develop the theory of uniform distribution in compact topological groups. A survey of uniform distribution in compact Hausdorff spaces with a countable base and in compact topological groups can be found in Kuipers and Niederreiter [8]. For a reference of uniform distribution of g-adic numbers, we refer to Meijer [11],[13] and the concept of uniform distribution of sequences of algebraic integers can be found in Lo and Niederreiter [9]. For recent developments in the uniform distribution of sequences, we refer to G. Myerson [14].

In this thesis we introduce the theory of uniform distribution of sequences in $\mathbb{F}_q[x]$, the ring of polynomials over an arbitrary finite field $\mathbb{F}_q$, and in $\mathbb{F}_q((x^{-1}))$, the field of formal Laurent series over any finite field $\mathbb{F}_q$. The concept of uniform distribution modulo 1 in $\mathbb{F}_q((x^{-1}))$ was first defined by Carlitz (1952) [3] and studied further by Dijksma (1969) [4],[5], Hodges (1970) [7], Meijer and Dijksma (1970) [12] and Webb (1973) [19]. Related results on P.V. numbers can be found in Bateman and Duquette [1]. For the case of $\mathbb{F}_q[x]$, the concept of uniform distribution of sequences was introduced by Hodges(1966) [6]. Further references are Hodges (1970) [7], Meijer and Dijksma(1970) [12], and Dijksma (1969) [4],[5]. We present ,in the last chapter, a relatively complete investigation of the basic part of the theory, following the direction set out in Chapter 2. Our contributions include self-contained proofs of

1) Theorem 3.2.1 different from that in Carlitz [2],
2) Theorem 3.3.1 extending the one in Hodges [6],
3) new Theorem 3.3.2,
4) Theorem 3.3.3 simplifying the one in Hodges [7],
5) Theorem 3.4.5 extending the one in Dijksma [5],
6) new Theorem 3.4.6 and 3.4.9.