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Chapter 1
Bose-Einstein Condensation: An Overview

In 1924 the young physicist named Satyendra Nath Bose from Decca University in India sent Albert Einstein a paper, in which Planck distribution law for photons was derived by entirely statistical arguments [1] without resorting to results from classical electrodynamics. Einstein apart from realized its significance to the description of light, he also extended Bose's idea applying to the matter as well. He came up with the papers in 1924 and 1925 [2, 3, 4] which fully described the quantum theory for bosonic particles.

In the second paper, Einstein predicted that at sufficient low temperature the particles would be stayed together in the lowest single-particle state. This phenomenon was called Bose-Einstein condensation (BEC) after Bose and Einstein in honour of their contribution. BEC is a phase transition that differ from any phase transitions because BEC is possible even in a perfectly noninteracting gas, while other phase transitions occur because of interations between the constituent particles.

The dynamical behavior of a gas at room temperature is not affected by the fact that one atom cannot be distinguished from another. But at sufficiently low temperature, the atomic de Broglie waves of neighboring atoms in an assembly start overlapping, giving rise to the quantum statistical effects that discriminate between fermions and bosons. Pauli exclusion principle prohibits any two fermions from occupying exactly the same quantum state. On the other hand, bosons are not limited in this way, and an arbitrary number of bosons can occupy the same quantum state.

The process of condensation is driven by the quantum statistics as follow (see e.g. [5])-: each collision between atoms results in scattering into any pair of states allowed by energy and momentum conservation. When, however, one takes quantum statistics into account, one finds that scattering is enhanced into those states that already have some atoms in them. And since at low temperature the total energy to be distribute between the atoms is small the accessible states is then reduced. The preferential energy distribution is that most of atoms occupy the ground state and the remaining atoms have more energy at their disposal.

Einstein's original of BEC was in an ideal gas but the first experiments in BEC were in superfluid helium, a liquid. The beautiful and startling experiment on viscosity, vortices, and heat-flow in liquid helium, and the ground-breaking theory those experiments inspired, more or less defined the field of Bose-Einstein condensation for four decades and more. However, liquid interact quite strongly. It masks the clear understanding of quantum statistical nature of BEC phase transition. So the realization of BEC in system of weakly interacting gas had long been one of the major goals in atomic physics. Because of weakly interacting feature, they can be immediately applied the perturbation theory to the system of ideal gas.

Figure 1.1 is a qualitative phase-diagram which shows the general features common to any atomic system. The solid line is a phase boundary. At low densities and high temperatures, everything is vapor. At high densities and lower temperatures, everything is condensed, into solid or liquid form. Under the solid line is a thermodynamical forbidden region. And BEC is always deep down in the forbidden region. So liquid helium is the only substance that can be Bose condensed under normal thermal equilibrium conditions.

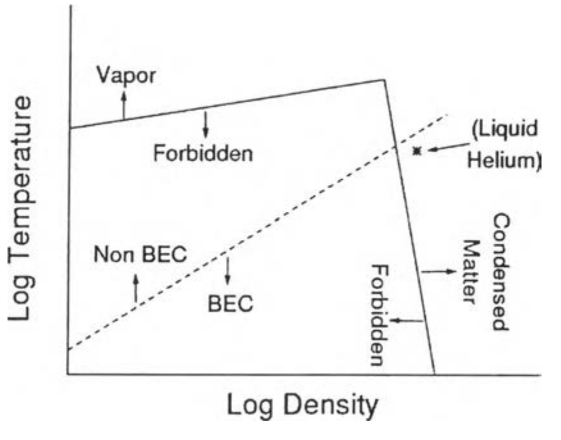

Figure 1.1: Generic phase-diagram common to all atoms. The dotted line shows the boundary between non-BEC and BEC. The solid line shows the boundary between allowed and forbidden regions of the temperature-density space. Note that at low and intermediate densities, BEC exists only in the thermodynamically forbidden regime.

To enter in the forbidden region, we have to avoid the atoms to start forming the molecule, which led the system to a condensed phase, in the absence of nucleation site, the system will reach a metastable phase allowing to realize BEC. The need to maintain metastability require very low densities, hence low temperature. This stringent conditions make BEC in pure form so elusive and require 70 years since it first propose by Einstein.

Efforts to make a dilute BEC in an atomic gas were spurred by provocative papers by Hecht [6] and Stwalley and Nosanow [7]. They argues on the basis of the quantum theory of corresponding states that spin-polarized hydrogen would remain a gas down to zero temperature, and thus would be a great candidate for making an weakly interacting BEC, In 1980, spin-polarized hydrogen was first stabilized by Silvera and Walraven [8], and by mid-80s spin-polarized hydrogen had been brought within a factor of 50 of condensing [9]. These experiments were performed in a dilution refrigerator, in a cell whose walls were coated with superfluid liquid helium. Hydrogen is first cooled there, then trapped by a magnetic field and further cooled by evaporation, This approach has come very close to observing Bose-Einstein condensation, but is limited by the recombination of individual atoms to form molecules and by detection efficiency.

At ultra cold temperature, atoms stick to all surfaces so for cooling to work efficiently they must be thermally isolated from their surroundings. This is done by trapping atoms with magnetic fields or with laser light inside ultrahigh vacuum chambers. Precooling is a requisite for trapping. Atom traps can confine particles only with a maximum energy of one Kelvin at best, and in many cases the trap depth is just a few millikelvin. The precooling is done by laser cooling. This technique was first suggested by Wineland and Dehmelt [10], Hänsch and Schawlow [11], Letokhov [12]. A full account of this development is given in [13, 14, 15] and in the Nobel lecture of Chu, Cohen-Tannoudji and Phillips [16, 17, 18]. In Ketterle group with sodium the sample was reduced the temperature from 600 kelvin (correspond to 800 m/s) to 1 kelvin (correspond to 30 m/s). This is sufficiently cold to capture atoms in a magneto-optical trap (MOT) [19], which consist of six laser beams intersecting at the center of the trap (see Figure

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1.2). The laser beams provide restoring forces and compress the atoms to a small cloud of about two millimeter in size. At the same time, they cool the atoms about 1 millikelvin.

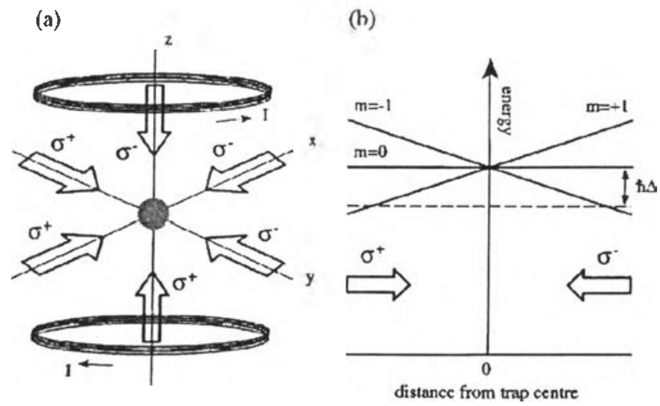

Figure 1.2: (a) A typical magneto-optical trap configuration. Three pairs of counter-propagating laser beam with opposite circular polarizations (σ^+ and σ^-) and a frequency tuned slightly below the atomic resonance (by the amount Δ) are superimposed on a magnetic quadrupole field produced by a pair of anti-Helmholtz coils. As shown in (b) (in one dimension), the Zeeman sublevels of an atom are shifted by the local magnetic field in such a way that (due to selection rules) the atom tunes into resonance with the laser field propagating in the opposite direction to the atom's displacement from the origin;hence, the net force on the atom is always towards the origin. In practice, additional repumping laser beams are required to maintain the atoms in the correct hyperfine levels and these beams are spatially distributed so as to create a "dark spot" in the center of the cloud where atoms are "hidden" from the trapping beams in an uncoupled hyperfine level;this reduces trap loss and heating due to light scattering allowing higher densities to be attained before transfer to a purely magnetic trap.

By 1990 it was clear that there were fairly strict limits to both the temperature and density obtainable with laser cooling. Theory caught up with experiment and showed that the sub-Doppler temperatures were due to a combination of light-shifts and optical pumping that became known as Sysiphus cooling. Random momentum fluctuations from rescattered photons limit the ultimate temperature to about a factor of ten above the recoil limit [20]. The rescattered photons are also responsible for a density limit - the outward flux of scattered photons which results in a strong repulsive light pressure force (weakening the trapping potential) [21]. Momentum fluctuations and trap loss arising from light-assisted collisions limited temperature and density as well [22]. The product of the coldness limit and the density limit works out to a phase-space density of about 10^{-5}, which is five orders of magnitude too low for BEC.

To overcome this, an additional cooling step is applied. All the laser beams are then switched off - the missing phase-space density are gained in using evaporative cooling in a magnetic trap (see Fig. 1.3 and Fig. 1.4).

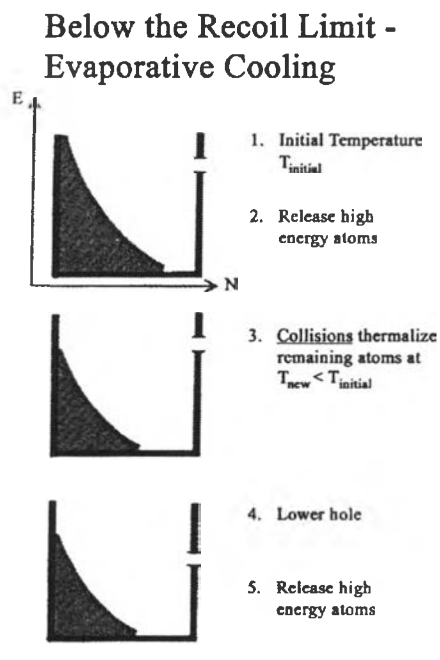

Figure 1.3: Schematic of evaporative cooling.

Evaporative cooling is done by continuously removing the high-energy tail of the thermal distribution from the trap. The evaporated atoms carry away more than the average energy, which means that the temperature of the remaining

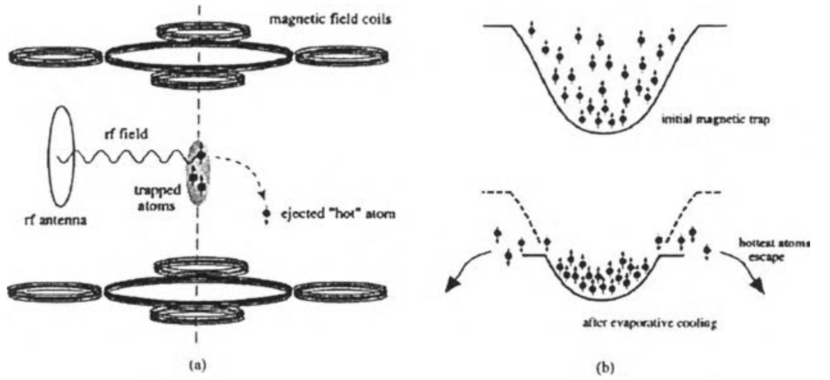

Figure 1.4: Schematics of the evaporative cooling process in a magnetic trap configuration. (a) Cloverleaf configuration of trapping coils used by Mewes et. al. [23]. The central (outer) coils provide axial (radial) confinement. The rf field induces spinflips of hot atoms. By adjusting the frequency of the rf field, the effective depth of the trap is altered, facilitating evaporative cooling a depicted in (b).

atoms decreases. The high energy tail is constantly repopulated by collisions, thus maintaining the cooling process. In an inhomogeneous trap, decreasing temperature in turn means decreasing occupied volume. One can actually increases the density of the remaining atoms even though the total number of confined atoms decreases. Evaporative cooling is a common phenomenon in daily life - it's how hot water cools down in a bathtub or in a cup of coffee. The technique of evaporative cooling was developed at MIT by Greytak, Kleppner and collaborators as a method for cooling atomic hydrogen which had precooled by cryogenic method [24]. The essential condition for this technique is a lifetime of the atomic sample longer than the collisional thermalization time. This requires a trap with tight confinement, since this allows high densities and hence fast rethermalization times.

In these experiments, the evaporation of atoms was controlled by rf radiation (see Figure 1.4). This technique was proposed by Pritchard [25] and Walraven

[26] and first realized by Ketterle's group [27]. The radio frequency radiation flips the spin of the atom. As a result, the attractive trapping force turns into a repulsive force and expels the atoms from the trap. This scheme is energy-selective because the resonance frequency is proportional to the magnetic field and therefore to the potential energy.

Evaporative cooling in a purely optical trap was realized in Stanford in the summer of 1995 [28]. The combination of laser cooling and evaporative cooling was the most important single step towards Bose-Einstein condensation in dilute atomic gases because it closed the gap between laser cooling which only work at low density and evaporative cooling which requires high density.

The successful in combining laser cooling and evaporative cooling led to a realization of BEC in ^{87}Rb by C. E. Wieman and E. A. Cornell's group at JILA in Boulder in June 1995 [29]. A few months later Ketterle's group also obtained BEC in ^{23}Na [30]. Prior to the realization, much discussion was devoted to the question of an unambiguous signature for the presence of BEC. When the breakthrough came, no doubt remained: as can be seen in Figures 1.5 and 1.6 , the appearance of a central condensed atomic cloud is manifest.

In condensed matter theory, ground state is one of the main interest. Thus we propose to use Feynman path integration, as a tool, to derive the ground state properties of BEC in the weakly dilute gas trapped in an inhomogeneous magnetic field. Also we consider only to the repulsive interaction between the particles. This system, at ultracold temperature, is dominated by the two-body collisions (good collisions) which prevail the formation of molecule nucleation caused by three-body collisions (bad collisions). Hence the system remain in a metastable

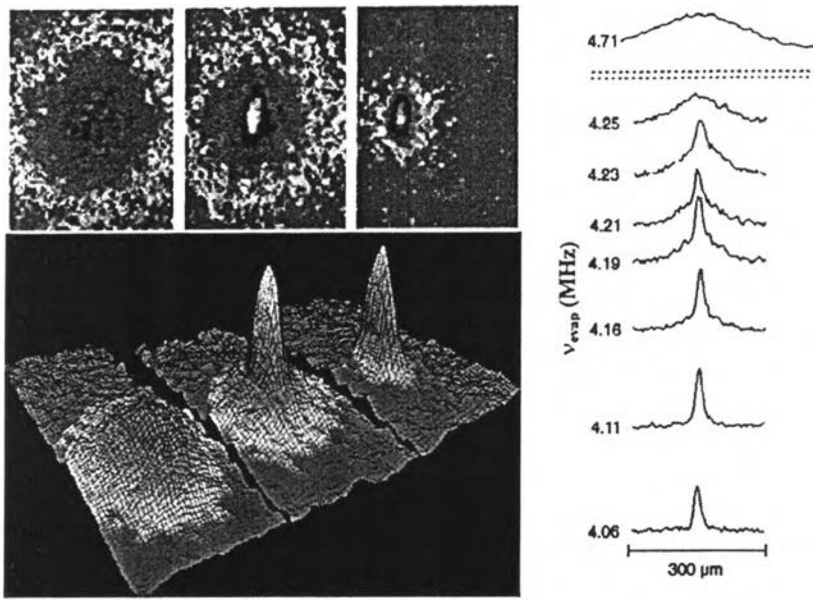

Figure 1.5: Observation of BEC in rubidium by JILA group. The upper left sequence of pictures shows the shadow created by absorption in the expanding atomic cloud released from the trap. Below, the same data are shown in another representation, where the distribution of the atoms in the cloud is depicted. In the first frame to the left, we see the situation just before the condensation sets in, in the middle a condensate peak with a thermal background is observed, whereas the third figure shows the situation where almost all atoms participate in the condensate. The thermal cloud is seen as a spherically symmetric broad background, whereas the sharp peak describing the condensate displays the squeezed shape expected in an asymmetric trap. The diagram to the right cuts through the atomic cloud when it is cooled by more and more atoms being evaporated. The figures are from publication [29].

state for a long time (compare to the rethermalized time). The condition for dilution allows us to approximate two-body collision as a hard sphere interaction. The model of hard sphere allows us to highlight that interaction between atoms made a significant role in the phenomenon. And prove to give a much more accurate results than the noninteracting particles proposed by Einstein. Consequently, this give a confidence to the theorist that the approximation and the theory are acceptable and predictable.

This thesis is organized as follows: Chapter 2 contains the explanation of the Feynman path integration. Also it was demonstrated that this formalism is

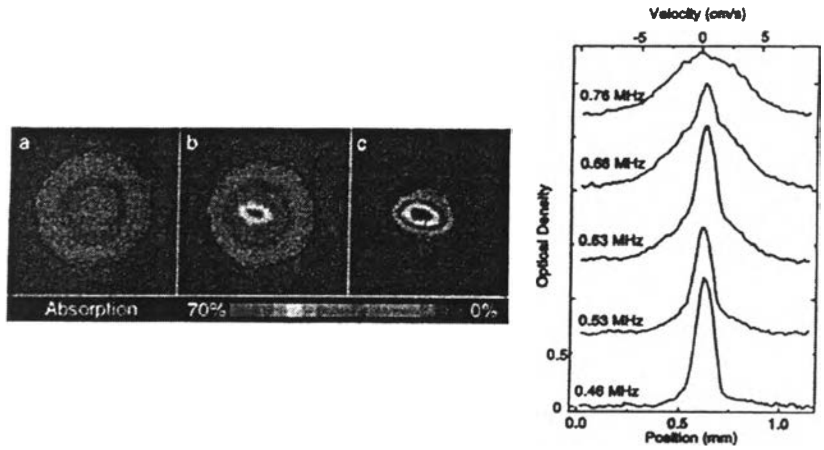

Figure 1.6: Observations of BEC in sodium atoms achieved in the MIT group. These pictures are obtained as those presented in Fig. 1.5. The left-hand side shows shadow images as in Figure 1.5, where the density of the condensate is seen to grow with decreasing temperature from left to right. The right-hand diagrams show cuts through the density as the condensate develops. The figure is from publication [30].

equivalent to the more conventional formalism. The examples of the free particle and harmonic oscillator are shown in the Section 2.2. Section 2.3 the relation between path integration and statistical mechanics is explained. The variational method in path integral approach is presented in Section 2.4.

Chapter 3 contains the theoretical account for BEC in a dilute weakly interacting bose gas. This chapter is based on the mean field approach which shown to work quite well. In Section 3.1 the ideal case is shown. In Section 3.2 the interaction case is shown to be more accurate than the ideal case. In Chapter 4 the detail of calculation is presented. In the final chapter conclusions and discussions are given.