Chapter 1

Spherical Shock Acceleration

Overview

1.1 Introduction

Cosmic rays are high energetic particles or radiation from outside the Earth. [1] Cosmic rays were discovered several decades ago in 1912 by Victor Hess. This was a starting point to stimulate human interest to explore deeper and deeper space around the planet called "the Earth." At present, the most exiting and fundamental ideas in particle theory involve energy scales well beyond the reach of conventional terrestrial accelerators where the highest energies achieved in accelerators laboratories are only about 1000 GeV (10^{12} eV), while the measured energy scale for cosmic rays is in excess of 10^{20} eV (Hayashida *et al.*, 1994; Bier-

[1] More details of cosmic ray history can be found in Longair (1997).

mann *et al.*, 1996; Yoshida *et al.*, 1997). At the time cosmic rays were discovered, the questions about the cosmic ray sources and the appropriate physical processes that energize them to such a high energy level consequentially arose. This is why some of the important fundamental ideas on the origin of cosmic rays were proposed. The theories therein to explain the possible cosmic ray sources and acceleration mechanisms have been investigated and developed deeper and deeper in detail.

Recently, the topic of particle acceleration is one of the most important and interesting in plasma physics as well as in cosmic ray astrophysics. The most important and well-established, known particle acceleration process is "shock acceleration." A shock is defined as a discontinuity in the physical properties (pressure, temperature, density, fluid speed, magnetic field, *etc.*) (adapted from the Dictionary of Astronomy edited by Illingworth 1994; an interested reader is suggested to read Landau and Lifshitz, 1963; Parks, 1991; Longair, 1997).

The concept of diffusive acceleration of particles in space plasmas has been around for nearly five decades. Fermi (1949) first postulated that cosmic ray acceleration could result via diffusion due to collisions with interstellar clouds (more details will be found in Appendix A). If such clouds have random directions of motion, the different rates of head-on and following collisions between the cosmic rays and the cloud would lead to a net acceleration. The model is so successful that it has become known as "Fermi acceleration." The mechanism of diffusive Fermi acceleration at collisionless plasma shock waves is widely invoked in astrophysics to explain the appearance of non-thermal particle populations in a variety of environments, including sites of cosmic ray production. Recently, the ideas of Fermi are widely exploited to describe the acceleration mechanism of charged particles in the vicinity of a shock, and the theory of cosmic ray acceleration at planar shock waves is well established (Krymsky 1977; Axford, Leer and Skadron, 1977).

For a more realistic study in this work we will take the effects of shock curvature into account. By this point of view, we are considering a "non-planar" shock wave front. The most straightforward simplification assumption for a non-planar geometry is a spherically symmetric wave front.

In the following section, we will review the current status of investigations into various aspects of the ideas presented of mechanisms of spherical shock acceleration of cosmic rays.

1.2 Spherically Symmetric Shocks

Most attempts to deal with the acceleration of energetic particle by non-planar shocks have been restricted to cases of spherical symmetry. The essential difficulty is that the flow on either side of the shock is divergent ($\vec{\nabla} \cdot \vec{V} \neq 0$), in contrast to the one-dimensional case, so that energy change occurs due to adiabatic expansion and compression in addition to the acceleration associated with the shock. Furthermore, in a non-planar geometry, energetic particles may diffuse towards the shock from downstream and also escape upstream relatively easily, whereas in plane shocks convection usually dominates the behavior at large distance, both upstream and downstream. It is not surprising then, that the energy/momentum spectra produced by non-planar shocks are not in general power laws and that the acceleration produced by the shock should in many cases be partly undone by adiabatic expansion in the downstream region (Axford, 1981).

Following Axford (1981), in his invited paper presented for the 17^{th} International Cosmic Ray Conference (ICRC), we may classify *stationary*, spherical shocks into two categories:

Stellar wind shocks where a stationary inward-facing terminal shock is surrounded by an extended region of the subsonic flow extending to infinity. The natural events with this situation are the stellar wind termination shocks, solar wind termination shock, galactic wind termination shocks, *etc.* (Jokipii, 1968; Krymsky and Petukhov, 1980; Axford, 1981; Forman, 1981; Völk, 1981; Petukhov *et al.*, 1985; Webb *et al.*, 1985; Jokipii, 1986; Jokipii and Morfill, 1987; Lee and Axford, 1988; Potgieter and Moraal, 1988; Markiewicz *et al.*, 1990; Ziemkiewicz, 1994; Achterberg and Ball, 1994; Vandas, 1994, 1995a, b; Klepach, Ptuskin and Zirakashvili, 2000).

Stellar accretion flow shocks with a stationary outward-facing shock which is the reverse of the first case. This case relies on the accretion flow around black holes or neutron stars (*i.e.*, compact objects), which may be found as isolated objects within a galaxy or in an active galactic nucleus (AGN) (Axford, 1981; Cowsik and Lee, 1981; Webb and Bogdan, 1987; Kazanas, 1988; Spruit, 1988; Schneider and Bogdan, 1989; Becker, 1992; Siemieniec-Oziębło and Ostrowski, 2000).

The configurations are shown schematically in Figure (1.1). In the notation of Axford (1981), K is the diffusion coefficient and the arrows indicate the magnitude and the direction of the fluid flow.

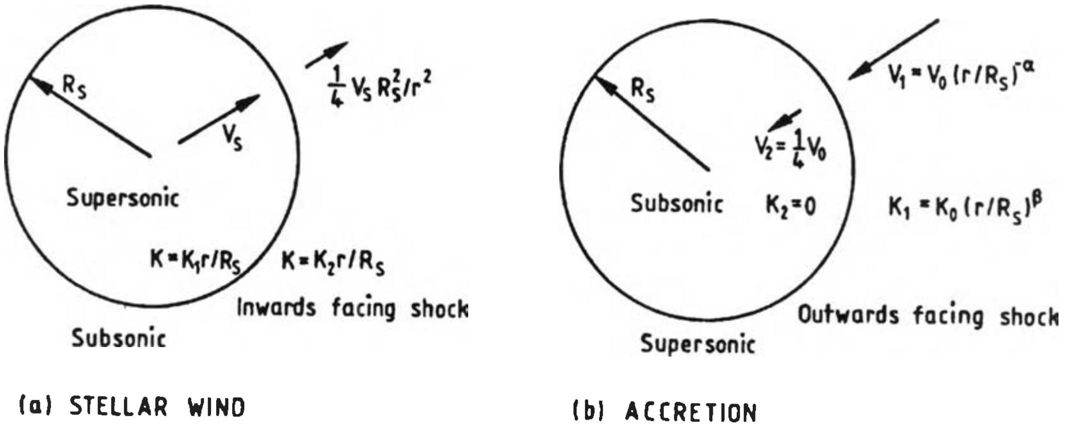

Figure 1.1: Schematic diagrams depicting two cases of shock flow, (a) a stellar wind termination shock, and (b) an accretion shock, where V_0 is the wind speed at $r \to \infty$, α and β are arbitrary parameters, R_s denotes the shock radius and the circles indicate the shocks boundary (from Axford, 1981).

In both cases the energy/momentum spectrum has a given form and the radial current vanishes at $r = \infty$, while particles may also be injected at the shock ($r = R_s$).

It is well known that low energy cosmic rays (up to a few hundred MeV) known as anomalous cosmic rays originate from the solar wind boundary (Fisk, Kozlovsky and Ramaty, 1974). More energetic cosmic rays may possibly be generated from a termination shock of a stellar wind where the mechanism is very similar to the low energy case but possibly on a larger scale (Axford, 1994; Rosner and Bodo, 1996; Stanev, 1997). That is the size of the acceleration site itself will limit the energy range. Ultra-high energy (hereafter UHE) cosmic rays, for which the possible energy range may exceed 10^{20} eV, should be energized by an enormous acceleration structure (Becker, 1992).

Indeed, in nature spherical shocks are not only be found in stationary as described above but also in a moving (time-dependent) manner. Hence, the next type of spherical shock is:

Non-stationary shocks which comprise all of instantaneous moving shock front whether inward or outward facing. The examples of this case are interplanetary shocks, supernova shocks, *etc.* (e.g., Prischep and Ptuskin, 1981; Klepach *et al.* 2000).

The main acceleration mechanism (nonthermal process) concerned is Fermi acceleration as mentioned in the above section. To avoid confusion about the discussion of the two cases of stationary, spherical shocks, we will describe them separately in detail.

1.2.1 Accretion Flow Shocks

The theory of particle transport in spherically symmetric media is a primary component in studies of the energization of particles in astrophysical winds and accretion flows. Most such studies are based on the fundamental Fokker-Planck equation describing the effects of first-order Fermi acceleration, spatial diffusion and bulk advection, derived by Parker (1965), Gleeson and Axford (1967), and Skilling (1975). For a review of the subject, see Gleeson and Webb (1980).

The accretion problem has been addressed by Axford (1981), in the steady state for a wind speed $V_1 = V_0(r/R_s)^{-\alpha}$, a diffusion coefficient chosen to be independent of energy/momentum, $\kappa_1 = \kappa_0(r/R_s)^{\beta}$, and assuming injection at the shock such that $Q \propto \delta(r - R_s)\delta(p - p_1)$ [the parameters were described in Figure (1.1 b)]. However, the solutions have been obtained only for the case $\alpha + \beta = 1$ with an asymptotic form at high energies. Although in general, the spectrum is essentially a power law, the spectral index is affected by the diffusion coefficient through η, where $\eta = V_0 R_s/\kappa_0$ is a modulation parameter. For $\eta \to \infty$, the spectral index, η, approaches that of the planar case as expected, but for finite η the spectrum is harder as a result of the additional acceleration associated with the convergence of the flow ahead of the shock. (In this case, the particle pressure becomes infinite at the shock, requiring a non-linear treatment.) For small η the shock produces less efficient acceleration and the spectrum is very soft, such that $n \to 3(1 + \beta)/4\eta$.

Cowsik and Lee (1982) obtained an exact solution to the Fokker-Planck equation describing the transport of monoenergetic particles injected at a stationary stand-off shock in a spherical accretion flow with a momentum-independent diffusion coefficient. Webb and Bogdan (1987) extended the results of Cowsik and Lee by adding a collisional loss term proportional to both the particle momentum and the divergence of the bulk flow velocity. The diffusion coefficient employed by Cowsik and Lee (1982) and Webb and Bogdan (1987) vanished downstream from the shock, and equals the product of the bulk flow velocity and the radius in the upstream region. The corresponding diffusion velocity equals either zero ($r < r_{shock}$) or the bulk flow velocity ($r > r_{shock}$). Unfortunately, this prescription for the diffusion coefficient precluded the interesting physical effects associated with finite but unequal diffusion and bulk flow velocities.

Schneider and Bogdan (1989) remedied this shortcoming by allowing the diffusion coefficient to vary as a power law in radius and particle momentum. The momentum loss term appearing in the transport equation studied by Webb and Bogdan (1987) and Schneider and Bogdan (1989) is directly proportional to the particle momentum, and therefore provides an appropriate description of the losses suffered by cosmic-ray protons due to nuclear collisions with a freely falling background plasma. However, relativistic electrons lose energy primarily via synchrotron and inverse Compton emission, with a rate proportional to the *square* of the particle momentum. Becker (1992) examined the consequences of replacing the linear loss-term studied previously with a quadratic dependence, relevant for the transport of relativistic electrons. He obtained the exact solution for the Green's function describing the transport of relativistic particles in the steady state, spherically symmetric background flow, including the effects of first-order Fermi acceleration, spatial diffusion, bulk advection and losses proportional to the square of the particle momentum. The flow velocity of the background (scattering) plasma and the spatial diffusion coefficient are assumed to vary as $v(r) \propto r^{-\alpha}$ and $\kappa(p, r) \propto r^{\beta} K(p)$, respectively, where r is the radius and $K(p)$ is an arbitrary function of the particle momentum p. The analysis can accommodate any separable diffusion coefficient with a power-law radial dependence and essentially represents the quadratic analog to the linear-loss theory developed by Schnieder and Bogdan (1989).

More recently, Siemienniec-Oziębło and Ostrowski (2000) demonstrated that the diffusive acceleration in the accretion flow onto a galaxy supercluster can provide an extremely hard spectrum of accelerated UHE protons, by using a 1D steady-state symmetric model. They assumed that the seed particles were provided for the cosmic ray acceleration mechanism by the galaxies concentrated near the central plane of a flattened supergalactic structure. On both sides the structure is accompanied by *planar* waves. They claimed that the model can be applied to the case of spherical accretion shock flow. Symmetry will cause the acceleration process to be more efficient than in the planar case and the maximum energy can reach a value of 10^{20} eV.

1.2.2 Stellar-Wind Flow Shocks

Acceleration of cosmic rays at the solar wind terminal shock wave was first proposed by Jokipii and Parker (1967). The first consideration of the spherical geometry of the shock was by Jokipii (1968), in order understand the effects of a curved geometry or a diverging wind on the shock acceleration of energetic particles. (Infact, the case considered in that work was the further acceleration of energetic galactic cosmic rays, which is now believed to be negligible.) Following the paper the acceleration is significant if the particle mean free path, λ, is around 1 AU (Astronomical Unit, or the mean distance from the Sun to the Earth) beyond the solar wind termination boundary, particularly for low-energy particles with energies $\lesssim 1$ GeV. The possible acceleration of solar cosmic rays is indicated. Analytic solutions of the steady state corresponding to either a monoenergetic galactic spectrum or a constant monoenergetic source of solar particles were proposed. The particle densities are formed in a power-law spectrum with exponent $a = -(1 + 3\kappa/2V_0R)$, where V_0 is the solar-wind velocity, κ is the diffusion coefficient and R is the radial distance to the wind boundary. If κ were much larger than V_0R, the accelerated particle spectrum would be very soft; physically this is because the particles would diffuse away before gaining appreciable energy.

However, during the 1970's the anomalous enhancements in the low-energy spectra of cosmic ray helium, nitrogen and oxygen were discovered (Hovestadt *et al.*, 1973; Garcia-Munoz *et al.*, 1973; McDonald *et al.*, 1974). Following this discovery, Fisk, Kozlovsky and Ramaty (1974) proposed a theory for the origin of the so-called "anomalous component" (AC) of cosmic rays – what are more recently called "anomalous cosmic rays (ACR)" as mentioned earlier. In this model the AC comprise interstellar neutral atoms that have been swept into the heliosphere, ionized by the solar wind or solar UV, and then accelerated to energies of $\sim$ 10 MeV/nuc and greater, probably at the solar wind termination shock (Pesses, Jokipii and Eichler 1981).

Cassé and Paul (1980), using the previous assumption of Jokipii (1968) of the possible acceleration of cosmic rays at the termination shock of the solar wind, suggested that young stars in OB associations could produce cosmic rays.

At about the same time, Krymsky and Petukov (1980) solved the time-dependent transport equation for spherically symmetric stellar wind shock flow. The analytical solutions for the particle density upstream and downstream were expressed in the forms of power laws, where the energy spectral index explicitly depended on the shock geometery and compression ratio – the ratio of the fluid speed between upstream and downstream.

Following the summary paper of Axford (1981), the solution of the terminal shock problem for which $F_0(r,p) \rightarrow \delta(p - p_1)$ (monoenergetic source) as $r \rightarrow \infty$ and $r^2 S = 0$ at $r = 0$ (S is the particle flux) can be written in the form of a two-term function of $V_1 R_s/\kappa_1$, $V_2 R_s/\kappa_2$ and V_2/V_1. One term is associated with shock acceleration, being a power law as in the planar case, but with a spectral index no longer dependent only on V_2/V_1 (the compression ratio of the fluid from upstream and downstream). The other one represents the effects of deceleration with the stellar wind region (*i.e.*, modulation) which causes some particles to lose energy but in general also lead to a net acceleration.

Petukhov, Turpanov and Nikolaev (1985) again examined the idea of a cosmic ray source from stellar and solar wind termination shocks. The study found that the shock acceleration is efficient enough to transfer a significant fraction of the inflow plasma kinetic energy to the accelerated particles. For a solar wind energy density E_k near the shock, the numerical calculation indicates that the accelerated particles ($T \geq 10$ keV) contain 10% of E_k.

Another model was proposed by Forman, Webb and Axford (1981) and Webb, Forman and Axford (1985) for a spherically symmetric shock wave. The steady state was studied analytically, with particular emphasis on monoenergetic source solutions.

For the case of cosmic ray acceleration at a stellar wind termination shock, where the shock is located at radius $r = R$ from the star, the cosmic rays undergo convection, diffusion, and adiabatic deceleration inside the shock ($0 < r < R$), where the flow is supersonic. This is the classical modulation process in the solar wind. In addition, the model includes the effect of shock acceleration at the termination shock. First-order Fermi acceleration also occurs at the shock because the cosmic rays scatter in the downstream region, outside the shock ($r > R$). Steady state, spherically symmetric, analytic solutions of the cosmic ray transport equation are applicable to this situation, with particular emphasis on monoenergetic source solutions (see Figure (1.2)).

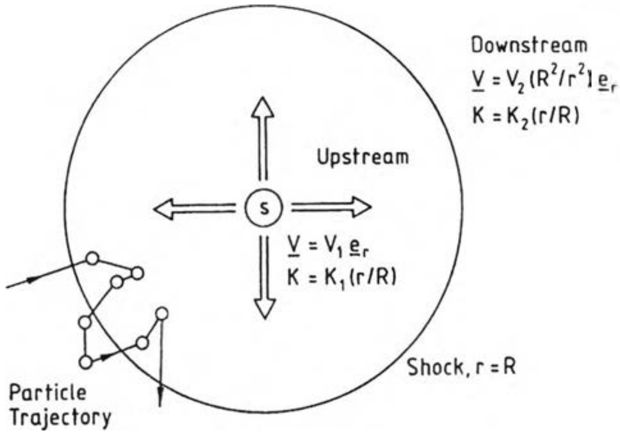

Figure 1.2: Schematic diagram of a spherical shock from the stellar wind, with a particle trajectory encountering the termination shock, scattering back and forth and gaining kinetic energy. (Webb *et al.*, 1985)

The solutions show a characteristic power-law momentum spectrum for accelerated particles and the more complex spectrum of particles that are decelerated in the stellar wind. The power-law spectral index depends on the compression ratio of the shock and on the modulation parameters characterizing propagation conditions in the upstream and downstream regions of the shock. Solutions of the transport equations for the total density N (integrated over all energies), pressure P_c, and energy flux F_c of Galactic cosmic rays interacting with the stellar wind and shock are also studied. The density $N(r)$ increases with radius r, and for strong shocks with large enough modulation parameters there may be a significant enhancement of the pressure of weakly relativistic particles near the shock compared to the cosmic-ray background pressure P_∞. The emergent energy flux at infinity is of the order of $4\pi R^2 V_1 P_\infty$ (V_1 is the wind velocity upstream of the shock, and R is the shock radius). This is also of the order of the mechanical power $\frac{1}{2}\dot{M}V^2$ ($\dot{M}$ is mass loss rate) in the wind. On this basis, early-type stars could supply a significant fraction of the 3×10^{40} ergs s^{-1} required to account for Galactic cosmic rays.

Note that the solution basically depends on three dimensionless parameters:

$$\eta_1 = \frac{V_1 R}{\kappa_1}, \qquad \eta_2 = \frac{V_2 R}{\kappa_2}, \quad \text{and} \quad r_c = \frac{V_1}{V_2}. \tag{1.1}$$

The modulation parameters η_1 and η_2 determine how strongly the cosmic rays are affected by the wind in the upstream and downstream regions, and r_c is the compression ratio of the shock. All particles undergo both acceleration and deceleration in the stellar wind. The solution consists of a spectrum of particles which have a net energy loss and a power-law spectrum of particles with spectral index μ_c which have a net energy gain. The characteristic power law index μ_c depends on the modulation parameters (Equation (1.1)), in addition to the shock compression ratio r_c. As $\eta_1 \to \infty$ and $\eta_2 \to \infty$ ($R \to \infty$, the plane shock limit), $\mu_c \to 3r_c/(r_c - 1)$, which is the classical spectral index obtained by Krymsky (1977); Axford, Leer, and Skadron (1977); Bell (1978a, b); and Blandford and Ostriker (1978) for the case of plane shocks. As $\eta_1 \to 0$ and $\eta_2 \to 0$, $\mu_c \to \infty$, so that there are few accelerated particles for a highly curved shock or one in which the coupling between the fluid and particles is weak.

Jokipii (1986) numerically studied the main features of a heliospheric termination shock on the modulation of cosmic rays. He solved the time dependent, two-dimensional (in radial distance and polar angle) cosmic-ray transport equation, advancing the solution in time until a time-asymptotic steady state was reached. The results of this model seem consistent with some of the observed features of the anomalous cosmic ray component. For example, the drift effects in the model shift the maxima of the calculated spectra toward higher energies after the solar polarity reversal of 1980, as observed by Cummings, Stone and Webber (1986).

The numerical solution of a combined shock acceleration/modulation model is a difficult problem, because particles migrate both up and down in energy. This is why Jokipii (1986) used a time-dependent model and then searched for a time-asymptotic steady state. Potgieter and Moraal (1988) demonstrated that it is in fact possible to handle this problem as a purely steady-state one. The technique was first introduced by Fisk and Lee (1980), who applied it to acceleration in corotating interaction regions. The accelerated spectra are the expected power laws, provided that the diffusion length scale is much less than the shock radius.

Vandas (1994, 1995a, b) presented analytical models of acceleration by cylindrical and spherical shocks with zero thickness and non-zero thickness. He has calculated the energy gain of electrons at these shocks. Electrons were divided, according to their interactions, into three groups: reflected, transmitted, and reflected through the tangent point. The energy gain of reflected electrons is strongly affected by the curvature and is generally lower than that at a corresponding planar shock. The energy gain of transmitted electrons is approximately the same as that at a corresponding planar shock. The energy gain of the third group is closer to that of the transmitted electrons. The initial and final magnetic moments of electrons approximately coincide. These studies followed Terasawa (1979) and Hudson (1965) by directly calculating the particle orbits.