CHAPTER V

QUANTUM TRANSPORT

AT LONG CORRELATION LENGTH

Introduction

In this chapter we would like to add some imperfections into our system resulting from previous chapter we known that the longitudinal conductivity is absent and there is only the transverse one which contradicts to the real experimental results[9] which still have the longitudinal component at the change of plateaus regions. This is discussed by Halperin[23] and is generally accepted. Schematic diagrams of the quantum Hall conductivities are shown in fig.(1.3).

As we have discussed in Chapter II that the imperfections, we have added into our system creates the potential $V(r)$ for an electron. Then the Hamiltonian of our system will become

$$H = \frac{1}{2m}\left(p+\frac{e}{c}A\right)^2 + V(r) - f\cdot r \tag{141}$$

The imperfections are modeled to be fixed impurities located randomly in the system. The interactions of an electron with these impurities are supposed to be two-body interactions and are modeled to be a gaussian function. The interaction appears in the Hamiltonian of eq.(141) the becomes

$$V(r) = \sum_{i=1}^{N} v(r-R_i) \tag{142}$$

where $v(r - R_i)$ is the two-body interaction between an electron and impurities at the site R_i. In Bezak's model of random gaussian model[18], the interaction has properties

$$
\begin{aligned}
< V(r_t) > &= 0 \\
< V(r_t)V(r'_t{}') > &= W(r_t - r_{t'}), \qquad (143)
\end{aligned}
$$

where $W(r_t - r_{t'})$ is called the *potential correlation function* and modeled to be a gaussian function

$$
W(r_t - r_{t'}) = \xi_L^2 e^{-(r_t - r_{t'})^2/L^2} \qquad (144)
$$

where ξ_L and L are called the *fluctuation parameter* and correlation length , respectively. The averaging symbol appeared in eq.(143) is the impurity sites averaging and was discussed in Chapter II.

In this disorder model, the impurities act as fixed scatterers and have no dynamical part in the expression of the Hamiltonian in eq.(141), then in practical process we will take only their effects on the dynamical motion of an electron. The process of averaging over disorder discussed in Chapter II result in the averaged propagator and density matrix, eq.(72) and eq.(77), respectively.

Density of States

From the eq.(72) of Chapter II, the averaged propagator was taken into the form

$$\begin{aligned} K^{av}(r_T, r_0; t) &= \int \mathcal{D}[r_t] \exp\{\frac{i}{\hbar}\int_0^T dt \mathcal{L}(r, \dot{r}; f) \\ &\quad -\frac{1}{2\hbar^2}\int_0^T dt \int_0^T dt' W(r_t - r_{t'})\} \end{aligned} \tag{145}$$

where the $\mathcal{L}(r, \dot{r}; f)$ is the Lagrangian corresponding to the Hamiltonian in eq.(141). To treat this problem in some limit of the disorder, since it is impossible to get an exact results from the disordered systems, we would like to make more approximation of the potential correlation function, $W(r_t - r_{t'})$, that at *long correlation length* , $L \to \infty$, then it can be expanded into a series as

$$W(r_t - r_{t'}) = \xi_L^2 \left(1 - \frac{(r_t - r_{t'})^2}{L^2} + \ldots\right) \tag{146}$$

We would like to keep only the first two terms, then our averaged propagator in eq.(145) becomes

$$\begin{aligned} K^{av}(r_T, r_0; T) &= e^{-\frac{1}{2}\xi_L^2 T^2/\hbar^2 L^2} \int \mathcal{D}[r_t] \exp\{\frac{i}{\hbar}\int_0^T dt \mathcal{L}(r, \dot{r}; f) \\ &\quad +\frac{\xi_L^2}{2\hbar^2 L^2}\int_0^T dt \int_0^T dt' (r_t - r_{t'})^2\}. \end{aligned} \tag{147}$$

Let us consider the last term of the exponent on the right hand side of eq.(147) in details, we find that it becomes an oscillating part and square of the integration part. If we define the complex time dependent frequency

$$\omega^2 = \frac{2i}{m\hbar}\left(\frac{\xi_L^2}{L^2}\right) T \tag{148}$$

then our propagator in eq.(147) becomes

$$K^{av}(r_T, r_0; T) = e^{-\frac{1}{2}\xi_L^2 T^2/\hbar^2 L^2} \int \mathcal{D}[r_t] \exp\{\frac{i}{\hbar}\int_0^T dt \mathcal{L}(r, \dot{r}; f) - \frac{i}{\hbar}\frac{1}{2}m\omega^2 \int_0^T dt r_t^2 - \frac{\xi_L^2}{\hbar^2 L^2}\int_0^T dt \int_0^T dt' r_t r_{t'}\} \quad (149)$$

Applying Hubbard-Stratonovich transformation to the last exponential of eq.(149) results in the transformed propagator $\tilde{K}(r_T, r_0; T)$

$$K^{av}(r_T, r_0; T) = C^{-1}\int_{-\infty}^{\infty} d\alpha e^{-(L^2/4\xi_L^2)\alpha\alpha} \tilde{K}(r_T, r_0; T) \quad (150)$$

where C is the constant of gaussian integration which equals to $(4\pi\xi_L^2/L^2)$ and $\tilde{K}(r_T, r_0; T)$ takes to be

$$\tilde{K}(r_T, r_0; T) = e^{-\frac{1}{2}\xi_L^2 T^2/\hbar^2 L^2} \int \mathcal{D}[r_t] \exp\left\{\frac{i}{\hbar}\int_0^T dt \tilde{\mathcal{L}}(r, \dot{r}; f + \alpha)\right\} \quad (151)$$

The Lagrangian $\tilde{\mathcal{L}}(r, \dot{r}; f + \alpha)$ in eq.(151) has the form

$$\tilde{\mathcal{L}}(r, \dot{r}; f + \alpha) = \frac{1}{2}\dot{r}m\dot{r} + \frac{eB}{2c}\dot{r}\epsilon r - \frac{1}{2}m\omega^2 r^2 + r(f + \alpha) \quad (152)$$

which is quadratic in form. Then the propagator of eq.(151) can be taken into the exact form

$$\tilde{K}(r_T, r_0; T) = \tilde{F}(T) \exp\left\{\frac{i}{\hbar}\tilde{S}_{cl}(r; f + \alpha)\right\} \quad (153)$$

where the corresponding classical action $\tilde{S}_{cl}(r, f + \alpha)$ is

$$\begin{aligned} \tilde{S}_{cl}(r; f + \alpha) = & \frac{m\mu}{2sin(\mu T)}\left((r_T^2 + r_0^2)cos(\mu T) - 2r_T e^{\Omega_c T/2} r_0\right) \\ & + \frac{r_T}{sin(\mu T)}\int_0^T e^{\Omega_c(T-t)/2} sin(\mu t)(f + \alpha)dt \\ & + \frac{r_0}{sin(\mu T)}\int_0^T e^{-\Omega_c t/2} sin(\mu(T - t))(f + \alpha)dt \\ & + \frac{1}{m}\int_0^T dt \int_0^T dt'(f + \alpha)_t G(t, t')(f + \alpha)_{t'} \end{aligned} \quad (154)$$

where we have used the notation for frequency

$$\mu^2 = \frac{\Omega^2}{4} + \omega^2 \tag{155}$$

and $G(t,t')$ in the last term of eq.(154) is the Green's function, which has the form

$$\begin{aligned} G(t,t') &= -\frac{1}{\mu sin(\mu T)} e^{\Omega_c(t-t')/2} \{ sin(\mu(T-t)) sin(\mu t') H(t-t') \\ &\quad + sin(\mu t) sin(\mu(T-t')) H(t'-t) \} \end{aligned} \tag{156}$$

With the classical action in eq.(154), resulting in the prefactor function

$$\tilde{F}(T) = \frac{m\mu}{2\pi i\hbar sin(\mu T)} \tag{157}$$

Since we are only interested in the propagator at the origin, then we have

$$\begin{aligned} \tilde{K}(0,0;T) &= \left(\frac{m\mu}{2\pi i\hbar sin(\mu T)} \right) e^{-\frac{1}{2}\xi_L^2 T^2/\hbar^2 L^2} \\ &\cdot \exp\{ \frac{i}{\hbar m} \int_0^T dt \int_0^T dt' f_t G(t,t') f_{t'} \\ &+ \frac{2i}{\hbar m} \int_0^T dt f_t G(t,T) \alpha + \frac{i}{\hbar m} \alpha G(T) \alpha \} \end{aligned} \tag{158}$$

where we have used the notations $G(t,T) = \int_0^T dt' G(t,t')$ and $G(T) = \int_0^T dt \int_0^T dt' G(t,t')$. Putting this expression back into eq.(151) and making the α-integration, resulting to the averaged propagator $K^{av}(0,0;T)$ as

$$\begin{aligned} K^{av}(0,0;T) &= \left(\frac{m\mu}{2\pi i\hbar sin(\mu T)} \right) \frac{1}{(1-(4i/\hbar)(\xi_L^2/L^2)G(T))} \\ &\cdot \exp\left\{ -\frac{1}{2}\frac{\xi_L^2 T^2}{\hbar^2 L^2} + \frac{i}{\hbar m} \int_0^T dt \int_0^T dt' f_t G(t,t') f_{t'} \right\} \\ &\cdot \exp\left\{ -\frac{4\xi_L^2}{\hbar^2 m^2 L^2} \frac{(f,G)^2}{(1-(4i/\hbar)(\xi_L^2/L^2)G(T))} \right\} \end{aligned} \tag{159}$$

where we have used the notation $(f, G) = \int_0^T dt f_t G(t, T)$. We also find that

$$1 - \frac{4i}{\hbar}\frac{\xi_L^2}{L^2}G(T) = \frac{2\mu}{\omega^2}\frac{(cos(\Omega T/2) - cos(\mu T))}{sin(\mu T)} \tag{160}$$

Then $K^{av}(0,0;T)$ in eq.(159) becomes

$$\begin{aligned} K^{av}(0,0;T) &= \frac{m\omega^2 T}{4\pi i\hbar}\frac{1}{(cos(\Omega T/2) - cos(\mu T))} \\ &\cdot \exp\{-\frac{1}{2}\frac{\xi_L^2 T^2}{\hbar^2 L^2} - \frac{2\xi_L^2\omega^2 T}{\hbar^2 m^2 L^2}\frac{sin(\mu T)}{(cos(\Omega T/2) - cos(\mu T))}(f, G)^2 \\ &+ \frac{i}{\hbar m}\int_0^T dt \int_0^T dt' f_t G(t, t') f_{t'}\} \end{aligned} \tag{161}$$

where we have used for the applied force $f = -e\mathcal{E}$, which is constant in time. Then eq.(161) becomes

$$\begin{aligned} K^{av}(0,0;T) &= \frac{m\omega^2 T}{4\pi i\hbar}\frac{1}{(cos(\Omega T/2) - cos(\mu T))} \\ &\cdot \exp\{-\frac{1}{2}\frac{\xi_L^2 T^2}{\hbar^2 L^2} - \frac{2\xi_L^2\omega^2 T}{\hbar^2 m^2 L^2}\frac{sin(\mu T)}{(cos(\Omega T/2) - cos(\mu T))}(-e\mathcal{E})^2 G(T)^2 \\ &+ \frac{i}{\hbar m}(-e\mathcal{E})^2 G(T)^2\} \end{aligned} \tag{162}$$

This equation shows the full expression of the resulting propagator at the origin in the long correlation length limit. We see that we do not have any physics of this system in this regime. In real practical situation of the quantum Hall effect, the electric field is very weak compare to the magnetic field, then we can neglect the last two terms on the exponent of the right hand side of eq.(162). and this results to

$$K^{av}(0,0;T) = \frac{m\omega^2 T}{4\pi i\hbar}\frac{1}{(cos(\Omega T/2) - cos(\mu T))}e^{-\frac{1}{2}\xi_L^2 T^2/\hbar^2 L^2} \tag{163}$$

The corresponding density of states of this problem then becomes

$$n(E) = \left(\frac{A}{\pi l_B^2}\right)\int_{-\infty}^{\infty}\frac{m\omega^2 T}{4\pi i\hbar}\frac{e^{-\frac{1}{2}\xi_L^2 T^2/\hbar^2 L^2}}{(cos(\Omega T/2) - cos(\mu T))} \tag{164}$$

Using the identity $cos(\Omega T/2) - cos(\mu T)) = -2sin(T(\Omega/2 - \mu)/2)sin(T(\Omega/2 + \mu)/2))$ and the fact that $\omega^2 = -(\Omega/2 + \mu)(\Omega/2 - \mu)$, let us define the function $W_\pm(T)$ as

$$\begin{aligned}\frac{1}{2}(\mu \pm \frac{\Omega}{2}) &= \frac{1}{2}\left(\sqrt{\frac{\Omega^2}{4} + \omega^2} \pm 1\right) \\ &= \frac{\Omega}{2}\frac{1}{2}\left(\sqrt{1 + \frac{8i\xi_L^2 T}{m\hbar\Omega^2 L^2}} \pm 1\right) \\ &= \frac{\Omega}{2}W_\pm(T) \end{aligned} \tag{165}$$

From the dimensionless parameters defined in Chapter IV, we get

$$W_\pm(\tau) = \sqrt{1 + 8i\delta\tau} \pm 1 \tag{166}$$

where $\delta = (\beta/\alpha)^2$ and the density of states in eq.(164) become

$$n(E') = \frac{1}{2\pi}\left(\frac{A}{\pi l_B^2}\right)\int_{-\infty}^{\infty} d\tau \frac{\frac{\tau}{2}W_+(\tau)\frac{\tau}{2}W_-(\tau)e^{-\frac{1}{2}\beta^2\tau^2 + iE'\tau}}{\tau sin(\frac{\tau}{2}W_+(\tau))sin(\frac{\tau}{2}W_-(\tau))}. \tag{167}$$

The integration in the expression of $n(E')$ in this equation can be done by Chauchy's integrals, it was studied in details bye Spies[25]. The resulting $n(E')$ are plotted in Fig.(5.1).

In the real qunatum Hall situation, the magnetid field is very large (of the order of 10 Tesla). We can consider this situation for eq.(167). In the large magnetic field limit, $\Omega \to \infty$, it approaches the form

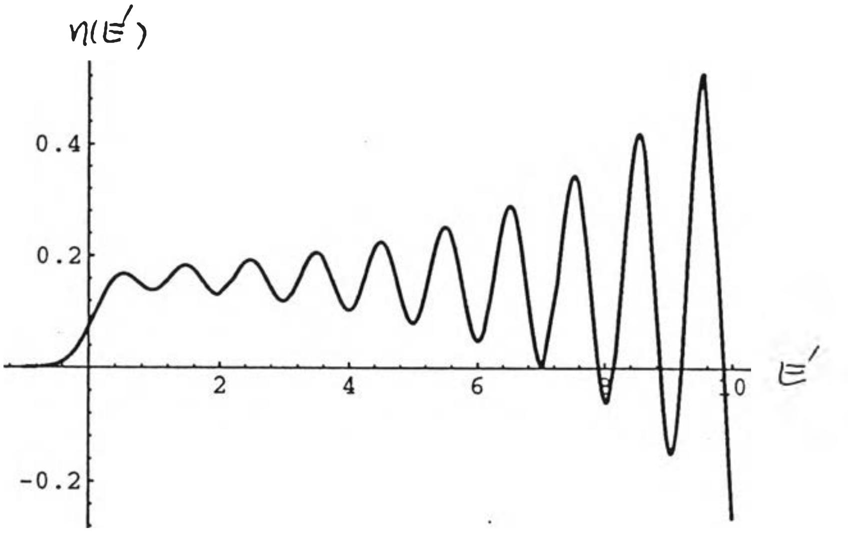
$$\begin{aligned} n(E') &= \frac{1}{4\pi^2}\left(\frac{A}{\pi l_B^2}\right)\sum_{n=0}^{\infty}\int_{-\infty}^{\infty} d\tau e^{-\frac{1}{2}\beta^2\tau^2 + i(E'-(n+1/2))\tau} \\ &= \frac{1}{2\pi}\left(\frac{A}{\pi l_B^2}\right)\sum_{n=0}^{\infty}\frac{e^{-(E'-(n+1/2))^2/2\beta^2}}{\beta^2} \end{aligned} \tag{168}$$

which results in the gaussian function and they show that in the limit of small fluctuation parameter, $\beta \longrightarrow 0$, the density of states becomes a delta-function ,

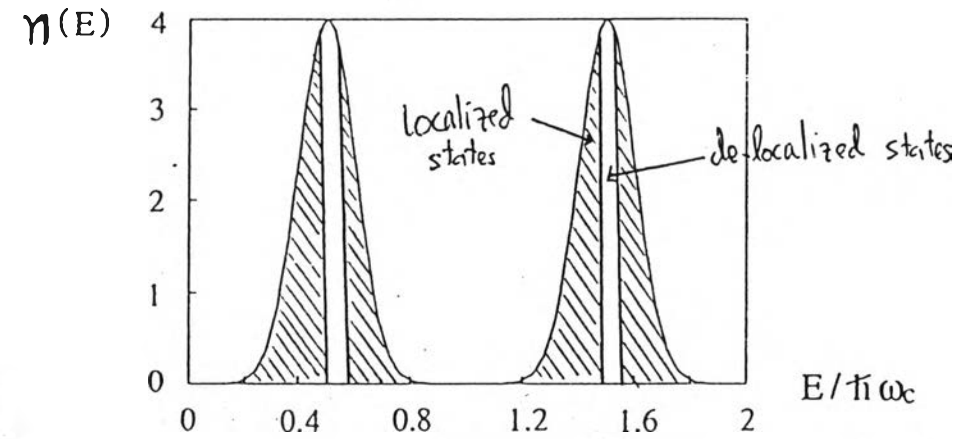
$$n(E') \quad \longrightarrow \quad \frac{1}{2\pi}\left(\frac{A}{\pi l_B^2}\right)\sum_{n=0}^{\infty}\delta(E'-(n+1/2)) \tag{169}$$

as in the case of the free electron under the applied magnetic field in a two-dimensional system without imperfections.

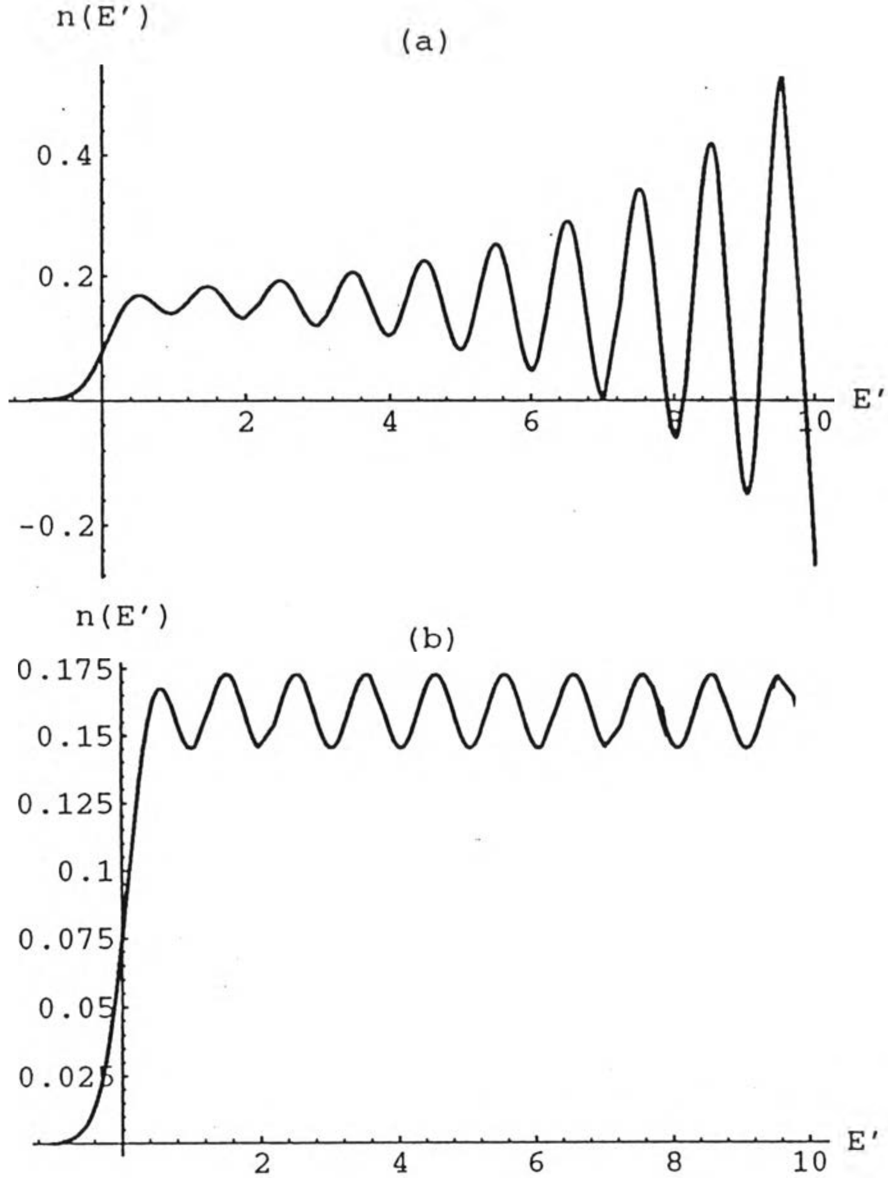
The obtained density of states, fig.(5.1), shows the broadening of Landau levels caused by the imperfections presented in the system. This broadening is different from the one caused by the applied electric field which is discussed in chapter IV, since all states in the broaden levels are delocalized states but most states in the case of the presence of disorderes are localized, as discussed by Halperin[23]. The schematic diagram of the region of localized-delocalized states is shown in fig.(5.2). We see from the figure that nearly all states in the energy spectrum of this system are localized. The conduction of an electron in the localized states is diffusive type[21], and at very low temperatures is dependent on the distribution of electrons in the system, i.e. on the Fermi function. Plots of the degradation of $n(E)$ at diferent correlation strengths are shown in figure(5.3).



Figure(5.1). Density of states ,$n(E')$, at long correlation length is plotted versus its energy E'. Using $\alpha = 6$ and $\beta = 0.4$.



Figure(5.2). Schematic diagram of localized-delocalized states in case of the presence of impurities.



Figure(5.3). Density of states, $n(E')$, is plotted at any strength of correlation length, (a) $\alpha = 6$, and (b) $\alpha = 24$.

Transport

To determine the transport properties of this system, we start by considering the averaged density matrix, eq.(77),

$$\begin{aligned}\rho^{av}(r_T, r'_T) &= \int dr_0 \int dr'_0 \rho(r_0, r'_0) \\ &\cdot \int \mathcal{D}[r_t] \int \mathcal{D}[r'_t] \exp\left\{\frac{i}{\hbar}\int_0^T dt\, (\mathcal{L}(r, \dot{r}; f) - \mathcal{L}(r'; \dot{r}'; f))\right\} \\ &\cdot \exp\{-\frac{1}{2\hbar^2}\int_0^T dt \int_0^T dt'(W(r_t - r_{t'}) \\ &-2W(r_t - r'_{t'}) + W(r'_t - r'_{t'}))\} \end{aligned} \tag{170}$$

The density matrix is averaged over all impurity configurations resulting to the impurity influence functional phase $\phi(r, r')$,

$$\begin{aligned}\frac{i}{\hbar}\phi(r, r') &= -\frac{1}{2\hbar^2}\int_0^T dt \int_0^T dt'(W(r_t - r_{t'}) \\ &-2W(r_t - r'_{t'}) + W(r'_t - r'_{t'}))\end{aligned} \tag{171}$$

In general, the action function in eq.(170) can be written in the form $S[r; f] - S[r'; f] + \phi(r, r')$ then its corresponding equation of motion of the path r_t at time $t = \tau$ is

$$m\langle \ddot{r}\rangle_\tau + \frac{eB}{c}\epsilon\langle \dot{r}\rangle_\tau - f = \langle \nabla_r \phi(r, r')\rangle_\tau \tag{172}$$

The averaging $< .. >$ is made with respect to the density matrix eq.(170). At the steady state, $< \ddot{r} >_\tau = 0$ then eq.(172) becomes

$$\frac{eB}{c}\epsilon\langle \dot{r}\rangle_\tau + f = \langle \nabla_r \phi(r, r')\rangle_\tau \tag{173}$$

This equation displays the balance between the momentum gained from the applied fields, the left hand side, and the momentum lose into the system, the right hand side, by scatterings with impurities. Let us consider in more details of the right hand side of eq.(173), from eq.(171),

$$\begin{aligned}\langle\nabla_r\phi(r,r')\rangle_\tau &= \frac{i}{2\hbar}\int_0^T dt\int_0^T dt' < \nabla_r(W(r_t-r_{t'}) \\ &\quad -2W(r_t-r'_{t'})+W(r'_t-r'_{t'}))>_\tau \end{aligned} \tag{174}$$

Let us write the potential correlation function in its Fourier transformed form,

$$W(r_t-r_{t'}) = \frac{1}{A}\sum_k W(k)e^{ik(r_t-r_{t'})}, \tag{175}$$

then eq.(174) becomes

$$\langle\nabla_r\phi(r,r')\rangle_\tau = -\frac{1}{\hbar}\frac{1}{A}\sum_k W(k)k\int_0^\tau d\sigma\left\langle\left(e^{ik(r_\tau-r'_\sigma)}-e^{ik(r_\tau-r'_\sigma)}\right)\right\rangle \tag{176}$$

Using the symmetric property of the Fourier component of the potential correlation function,

$$W(k) = W(-k), \tag{177}$$

which arises from the model function of $W(r_t-r_{t'})$ we chose, and the fact that in the averaging procedure we have the condition $\delta(r_\tau-r'_\tau)$,

$$\langle O(r)\rangle = \int dr_\tau\int dr'_\tau\delta(r_\tau-r'_\tau)O(r)\rho(r_\tau,r'_\tau). \tag{178}$$

Then we can write eq.(176) in the form

$$\langle\nabla_r\phi(r,r')\rangle_\tau = -\frac{2}{\hbar}\mathrm{Re}\frac{1}{A}\sum_k W(k)k\int_0^\tau d\sigma\left\langle e^{ik(r_\tau-r_\sigma)}\right\rangle \tag{179}$$

With this expression, eq.(173) will take into the form

$$\frac{eB}{c}\epsilon < \dot{r} >_\tau + f \quad = \quad -\frac{2}{\hbar}\mathrm{Re}\frac{1}{A}\sum_k W(k)k \int_0^\tau d\sigma \left\langle e^{ik(r_\tau - r_\sigma)}\right\rangle \tag{180}$$

To determine its transport coefficient, we would like to move onto a moving frame, R, that moves with the same velocity as the drift velocity of an electron at the steady state, v_D. That is we choose $\dot{R} = v_D$. This transformation can be done by writing $r_t = R_t + u_t$, where u_t is the fluctuation of the electronic path by the moving frame resulted from interactions with impurities. Suppose that no consideration is paid on the dynamical properties of these fluctuations, u and u', this will make eq.(180) become the form

$$\frac{eB}{c}\epsilon v_D + f \quad = \quad -\frac{2}{\hbar}\mathrm{Re}\frac{1}{A}\sum_k W(k)k \int_0^\tau d\sigma e^{ikv_D(\tau-\sigma)} \left\langle e^{ik(u_\tau - u_\sigma)}\right\rangle \tag{181}$$

Now the averaging process in this equation is made with respect to the density matrix of an electron on the moving frame. From eq.(151), the density matrix will take the form

$$\begin{aligned}\rho(u_\tau, u'_\tau) \quad = \quad & \int du_0 \int du'_0 \rho(u_0, u'_0) \\ & \cdot \exp\left\{\frac{i}{\hbar}(u_t - u'_t)(mv_D + \frac{eB}{2c}\epsilon R_t)|_0^\tau\right\} \\ & \cdot \int \mathcal{D}[u_t] \int \mathcal{D}[u'_t] e^{\frac{i}{\hbar}(S[u;f_L] - S[u';f_L] + \phi(u,u'))}\end{aligned} \tag{182}$$

where f_L is the Lorentz force;

$$f_L \quad = \quad f + \frac{eB}{c}\epsilon v_D = -e\mathcal{E} + \frac{eB}{c}\epsilon v_D. \tag{183}$$

In the long correlation length limit, the impurity influence functional becomes

$$\frac{i}{\hbar}\phi(u,u') \quad = \quad -\frac{1}{\hbar^2}\frac{\xi_L^2}{\mathrm{L}^2}\int_0^T dt \int_0^T dt' (u_t - u'_t)(u_{t'} - u'_{t'}) \tag{184}$$

Now, let us consider the average $\langle e^{ik(u_\tau - u_\sigma)}\rangle$ by first writing

$$
\begin{aligned}
ik(u_\tau - u_\sigma) &= \frac{i}{\hbar}\int_0^T dt\hbar k u_t(\delta(t-\tau) - \delta(t-\sigma)) \\
&= \frac{i}{\hbar}\int_0^T dt f_{\delta t} u_t
\end{aligned}
\tag{185}
$$

then the average is being a kind of generating functional of the delta function force $f_{\delta t}$, $g(f_\delta)$. From the density matrix of eq.(182), then we get

$$
\begin{aligned}
g(f_{\delta_s}) &= \int du_2 du_2' \delta(u_2 - u_2') \int du_1 \int du_1' \rho(u_1, u_1') \\
&\cdot \exp\left\{\frac{i}{\hbar}(u_t - u_t')(mv_D + \frac{eB}{c}\epsilon R_t)|_0^T\right\} \\
&\cdot \int \mathcal{D}[u_t] \int \mathcal{D}[u_t'] \exp\left\{\frac{i}{\hbar}\int_0^T dt\left(\mathcal{L}(u,\dot{u};F_L + f_\delta) - \mathcal{L}(u',\dot{u}';f_L)\right)\right\} \\
&\cdot \exp\left\{-\frac{1}{\hbar^2}\frac{\xi_L^2}{L^2}\int_0^T dt\int_0^T dt'(u_t - u_t')(u_{t'} - u_{t'}')\right\}
\end{aligned}
\tag{186}
$$

Applying Hubbard-Stratonovich transformation to the last term on the exponent of the right hand side of eq.(186) , resulting in

$$
g(f_\delta) = C^{-1}\int d\alpha e^{-\frac{L^2}{4\xi_L^2}\alpha^2}\tilde{\rho}(f_\delta, \alpha) \tag{187}
$$

where the transformed generating functional is

$$
\begin{aligned}
\tilde{g}(f_\delta;\alpha) &= \int du_2 \int du_2' \delta(u_2 - u_2') \\
&\cdot \exp\left\{\frac{i}{\hbar}\int_0^T dt(u_t u_t')(mv_D + \frac{eB}{c}\epsilon R_t)|_0^T\right\} \int \mathcal{D}[u_t] \int \mathcal{D}[u_t'] \\
&\cdot \exp\left\{\frac{i}{\hbar}\int_0^t dt \int_0^T dt'\left(\mathcal{L}(u,\dot{u};F) - \mathcal{L}(u',\dot{u}';F')\right)\right\}
\end{aligned}
\tag{188}
$$

where we have defined the forces F and F' in the form

$$
F = f_L + f_\delta + \alpha \quad \text{and} \quad F' = f_L + \alpha \tag{189}
$$

The detailed calculation of eq.(188) is done in the appendix , with the initial condition of the ground state of the oscillator, results in the form

$$
\begin{aligned}
\tilde{g}(f_\delta;\alpha) &= \exp\{-\frac{2iv_D}{\Omega\hbar}\int_0^t dte^{\Omega\epsilon t/2}sin(\Omega t/2)(F_t - F'_t) \\
&\quad -\frac{1}{m\Omega\hbar}\int_0^T dt\int_0^T dt'(F_t - F'_t)e^{\Omega\epsilon(t-t')/2} \\
&\quad \cdot\left(F_{t'}e^{-i\Omega(t-t')/2} - F'_{t'}e^{i\Omega(t-t')/2}\right)\}. \qquad (190)
\end{aligned}
$$

The expressions of the forces F and F', we see that they are all constant in time. Let us also define some functions for convenience;

$$
\begin{aligned}
L_1(\tau,\sigma) &= \frac{2}{\Omega}\int_0^T dte^{-\frac{\Omega}{2}\epsilon t}sin(\Omega t/2)(\delta(t-\tau) - \delta(t-\sigma)) \\
B_0(\tau,\sigma) &= \frac{1}{m\Omega}\int_0^T dt\int_0^T dt'(\delta(t-\tau) - \delta(t-\sigma))e^{\Omega\epsilon(t-t')/2} \\
&\quad \cdot\left(e^{-i\Omega(t-t')'2} - e^{i\Omega(t-t')/2}\right)(\delta(t'-\tau) - \delta(t'-\sigma)) \qquad (191) \\
B_1(\tau,T) &= \frac{2}{m\Omega}\int_0^T dt\int_0^T dt'e^{\Omega\epsilon(t-t')/2}sin(\Omega(t-t')/2) \\
&\quad \cdot(\delta(t'-\tau) - \delta(t'-\sigma)).
\end{aligned}
$$

Eq.(190) becomes

$$
\begin{aligned}
\tilde{g}(f_\delta;\alpha) &= \exp\{-iv_D L_1(\tau,\sigma) - i(f_\delta+\alpha)(B_1(\tau,T) - B_1(\sigma,T))k \\
&\quad -i\hbar k^2 B_0(\tau,\sigma)\}. \qquad (192)
\end{aligned}
$$

Inserting this equation back into eq.(187), and making the α -integration, results in

$$
\begin{aligned}
g(f_\delta) &= \exp\{-iv_D L_1(\tau,\sigma)k - if_L(B_1(\tau,T) - B_1(\sigma,T))k \\
&\quad -i\hbar k^2 B_0(\tau,\sigma) - \frac{\xi_L^2}{L^2}(B_1(\tau,T) - B_1(\sigma,T))^2\} \qquad (193)
\end{aligned}
$$

With this resulting generating functional, the equation of motion, eq.(181) becomes

$$\frac{eB}{c}\epsilon v_D + f = -\frac{2}{\hbar}\mathrm{Re}\frac{1}{A}\sum_k W(k)k\int_0^\tau d\sigma e^{ikv_D(\tau-\sigma)}$$
$$\cot\exp\{-iv_DL_1(\tau,\sigma)k - if_L(B_1(\tau,\sigma)-B_1(\sigma,T))k$$
$$-i\hbar k^2B_0(\tau,\sigma) - \frac{\xi_L^2}{L^2}(B_1(\tau,T)-B_1(\sigma,T))^2\} \quad (194)$$

This equation is very complicated, we would like to evaluate it in our condition for this chapter, they are long correlation length and large magnetic field, then eq.(193) reduce to

$$\frac{eB}{c}\epsilon v_D + f = -\frac{2}{\hbar}\mathrm{Re}\frac{1}{A}\sum_k W(k)k\int_0^\tau d\sigma e^{ikv_D(\tau-\sigma)-i\hbar k^2B_0(\tau,\sigma)} \quad (195)$$

Now let us use the fact that $\frac{1}{A}\sum_k \to \frac{1}{(2\pi)^2}\int dk^{(2)}$, and the Fourier component of the potential correlation function is

$$W(k) = W_Le^{-\frac{k^2L^2}{4}}$$

where $W_L = \xi_L^2\pi L^2$. Putting this expression into the right hand side of eq.(195), we get

$$\frac{eB}{c}\epsilon v_D + f = -\frac{2}{\hbar}\mathrm{Re}\frac{1}{(2\pi)^2}\int dk^{(2)}ke^{-k^2(\frac{L^2}{4}+i\hbar B_0(\tau,\sigma))+ikv_D(\tau-\sigma)} \quad (196)$$

The $dk^{(2)}$-integration is a gaussian integral, we can evaluate it and result in

$$\frac{eB}{c}\epsilon v_D + f = \frac{1}{8\pi}\mathrm{Re}\frac{\xi_L^2L^2}{\hbar}v_Di\int_0^\tau d\sigma(\tau-\sigma)\frac{e^{-\frac{v_D^2(\tau-\sigma)^2}{4(\frac{L^2}{4}+i\hbar B_0(\tau,\sigma))}}}{(\frac{L^2}{4}+i\hbar B_0(\tau,\sigma))^2} \quad (197)$$

From this expression, we would like to define the relaxation time τ_L from the equation

$$\frac{eB}{c}\epsilon v_D + f = mv_D\frac{1}{\tau_L} \quad (198)$$

then, from eq.(197), we get

$$\frac{1}{\tau_L} = \frac{1}{8\pi}\mathrm{Re}\frac{\xi_L^2 L^2}{m\hbar}i\int_0^{\tau} d\sigma(\tau-\sigma)\frac{e^{-\frac{v_D^2(\tau-\sigma)^2}{4(\frac{L^2}{4}+i\hbar B_0(\tau,\sigma))}}}{(\frac{L^2}{4}+i\hbar B_0(\tau,\sigma))^2} \quad (199)$$

From eq.(198), we wolud like to write it in the matrix form as

$$(-e)\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} = m\begin{pmatrix} \frac{1}{\tau_L} & -\Omega \\ \Omega & \frac{1}{\tau_L} \end{pmatrix}\begin{pmatrix} v_{Dx} \\ v_{Dy} \end{pmatrix} \quad (200)$$

which can define the mobility μ to be

$$\mu_L = \frac{(-e)}{m}\begin{pmatrix} \frac{1}{\tau_L} & -\Omega \\ \Omega & \frac{1}{\tau_L} \end{pmatrix}^{-1} \quad (201)$$

or express in its components as

$$\mu_{xx} = \frac{(-\epsilon)}{m}\frac{\tau_L}{[1+(\Omega\tau_L)^2]} \quad (202)$$

$$\mu_{yx} = \frac{(-e)}{m}\frac{\Omega\tau_L^2}{[1+(\Omega\tau_L)^2]} \quad (203)$$

The conductivity σ_L is related to the mobility from the expression

$$\sigma_L = n(-e)\mu_L \quad (204)$$

where n is the density of electrons at energy E. At this point, we put *ad hoc* the many-electrons effect in terms of Fermi function

$$f(E) = \frac{1}{(1+e^{(E-E_F)/kT})} \quad (205)$$

The electrons at finite temperature have different contributions to the longitudinal and transverse components of conductivity. For the longitudinal component, it takes the form

$$\sigma_{xx} = (-e)\int dE\left(-\frac{\partial F(E)}{\partial E}\right)n(E)\mu_{xx}(E), \quad (206)$$

and the transverse component will take the form

$$\sigma_{yx} = (-e) \int dE \frac{f(E)}{kT} n(E) \mu_{yx}(E). \tag{207}$$

From the resulting mobility of eqs.(202) and (203), we can have the expressions of the two components of conductivity. Numerical results of them are shown in next section.

Numerical Results

To determine our results of eqs.(200), (203), (204) and (207), we would like to write them in terms of dimensionless parameters defined in chapter IV. From the definition of $B_0(\tau,\sigma)$ in eq.(192), we can show that

$$B_0(\tau,\sigma) = \frac{2}{m\Omega} sin^2(\Omega(\tau-\sigma)/2). \tag{208}$$

Then the relaxation time in eq.(200) becomes

$$\frac{1}{\tau_L} = \frac{1}{8}\frac{\xi_L^2 L^2}{m\hbar} \mathrm{Re} i \int_0^\tau d\sigma(\tau-\sigma) \frac{e^{-\frac{v_D^2(\tau-\sigma)^2}{4(\frac{L^2}{4}+\frac{2i\hbar}{m\Omega}sin^2(\Omega(\tau-\sigma)/2))}}}{\left(\frac{L^2}{4}+\frac{2i\hbar}{m\Omega}sin^2(\Omega(\tau-\sigma)/2)\right)^2}. \tag{209}$$

Let us define $x = (\tau - \sigma)$ and then $x' = x\Omega$, then we have an expression of dimensionless relaxation time $\tau_L' = \Omega\tau_L$ as

$$\frac{1}{\tau_L'} = 2\beta^2\alpha^2 \mathrm{Re} i \int_0^{X'} dx'x' \frac{e^{-\frac{2E_v'\alpha^2}{(1+8i\alpha^2 sin(x'/2))}}}{(1+8i\alpha^2 sin^2(x'/2))^2} \tag{210}$$

where E_v' is defined to be equal to $(1/2)mv_D^2/E_\Omega$. From eqs.(203) and (204) we have

$$\mu_{xx} = \frac{(-e)}{m\Omega}\frac{\tau_L'}{(1+\tau_L'^2)} \tag{211}$$

and

$$\mu_{yx} = \frac{(-e)}{m\Omega} \frac{{\tau'_L}^2}{(1+{\tau'_L}^2)}. \tag{212}$$

From eq.(207), the longitudinal component of the conductivity will take the form

$$\sigma_{xx} = (-e) \int_0^\infty dE' \left(-\frac{\partial f(E')}{\partial E'}\right) n(E')\mu_{xx}(E') \tag{213}$$

and from the expression of $n(E')$ and $\mu_{xx}(E')$ in eqs.(168) and (211), respectively, we can write σ_{xx} in the form

$$\frac{\sigma_{xx}}{A} = \frac{1}{E'_{kT}} \frac{1}{\pi} \frac{1}{\beta^2} \sum_{n=0}^{\infty} \left(\frac{\tau'_L}{1+{\tau'_L}^2}\right) \int_0^\infty dE' \frac{e^{(E'-E'_F)/E_{kT}} e^{-(E'-(n+1/2))^2/2\beta^2}}{\left(1+e^{(E'-E'_F)/E'_{kT}}\right)^2} \tag{214}$$

From eq.(208), the transverse component of the conductivity will take the form

$$\sigma_{yx} = \frac{(-e)}{kT} \int_0^\infty dE' f(E') n(E') \mu_{yx}(E') \tag{215}$$

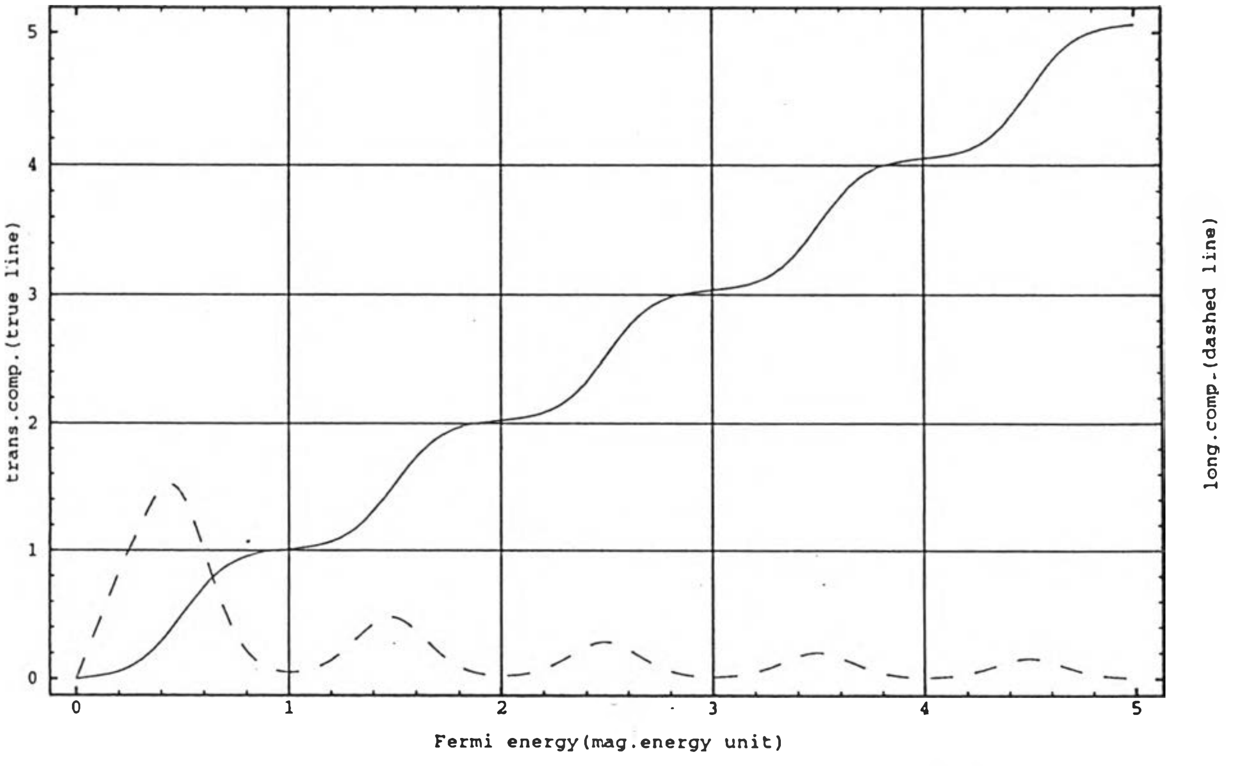
and from the expression of $n(E')$ and μ_{yx} in eqs.(168) and (212), respectively, we can write σ_{yx} in the form

$$\frac{\sigma_{yx}}{A} = \frac{1}{E'_{kT}} \frac{1}{\pi} \frac{1}{\beta^2} \sum_{n=0}^{\infty} \left(\frac{\tau'_L}{1+{\tau'_L}^2}\right) \cdot \int_0^\infty dE' \frac{e^{-(E'-(n+1/2))^2/2\beta^2}}{(1+e^{(E'-E'_F)/E'_{kT}})} \tag{216}$$

Plots of σ_{xx} and σ_{yx} are shown in fig.(5.4).

Discussions

The resulting density of states, $n(E)$, see fig.(5.1), show that the Landau levels are broaden by the effects of disorder. But these states in the broadened level are not all delocalized, see fig.(5.2), this causes the appearance of the longitudinal component of conductivity, see fig.(5.4) as a peak-like function at the region of plateau-plateau transitions. This shows that in the region that an electron unlocalized by the disordering can drift in the longitudinal direction caused by scatterings with impurities and give the contribution to the longitudinal component of conductivity.



Figure(5.4). Conductivities are plotted versus the Fermi energy, E'_F; σ_{xx} (dashed line), and σ_{yx} (true line). The calculation is made at the values of $\alpha = 6$, $\beta = 0.4$, and $E'_{kT} = 0.1$.