CHAPTER III

The Propagator

In this chapter we will evaluate the propagator for the problem of our interest which contain an electron moving in two dimensions under the influence of a constant magnetic field presented in perpendicular direction to the plane of electronic motion and with the presence of random potential of that plane and also involving the time varying external force field. In this evaluation we use the configuration space representation rather than phase space representation. The random potential we use here is the soluble model introduced by Sa-yakanit [10] in his model of disordered system. It contains the oscillator with the memory term known as a nonlocal harmonic oscillator.

III.1 Feynman's propagator.

Let us denote the plane of an electronic motion by (x,y), the lagrangian of the electron subjects to the potentials mentioned above with the symmetric gauge of the magnetic field, the vector potential $\vec{A} = (-yB, xB, 0)$, is given by

$$\mathcal{L}\left\{\dot{x},x;\dot{y},y;\tau\right\}$$

$$= \frac{m}{2}\left[\left(\dot{x}^2+\dot{y}^2\right)+\Omega\left(x\dot{y}-y\dot{x}\right)\right]$$

$$-\frac{\nu^2}{4t}\int_0^t\left[\left[x(\tau)-x(\sigma)\right]^2+\left[y(\tau)-y(\sigma)\right]^2\right]d\sigma$$

$$+f_x(\tau)\,x(\tau)+f_y(\tau)\,y(\tau) \qquad (\text{III}.1)$$

where $\Omega = \frac{eB}{mc}$ is the cyclotron frequency, ν denotes the nonlocal oscillator frequency, $f_x(\tau)$ and $f_y(\tau)$ are presented generally for the time varying external force field in x- and y- direction respectively. The required propagator can be written down in the Feynman path integral form as

$$K(a,b) = \int \mathcal{D}[x(\tau)] \int \mathcal{D}[y(\tau)] \exp\left\{ \frac{i}{\hbar} S[x,y] \right\} , \qquad (\text{III}.2)$$

where

$$S[x,y] = \int_0^t \mathcal{L}\{\dot{x}, x; \dot{y}, y; \tau\} \, d\tau \qquad (\text{III}.3)$$

is the action function, $\mathcal{D}[x(\tau)]$ and $\mathcal{D}[y(\tau)]$ denote the path integrationto be carried out with the boundary conditions $x(0) = x_a$, $x(t) = x_b$, $y(0) = y_a$ and $y(t) = y_b$. The propagator in Eq. (III.2) can be written as

$$\begin{aligned} K(a,b) = & \int_a^b \mathcal{D}[x(\tau)] \int_a^b \mathcal{D}[y(\tau)] \\ & \times \exp\left\{ \frac{i}{\hbar} \left[\frac{m}{2} \int_0^t \left(\dot{x}^2 + \dot{y}^2 + \Omega[x\dot{y} - y\dot{x}] \right) d\tau \right.\right. \\ & \qquad - \frac{m\nu^2}{4t} \int_0^t \int_0^t \left[(x(\tau) - x(\sigma))^2 + (y(\tau) - y(\sigma))^2 \right] d\tau \, d\sigma \\ & \qquad \left.\left. + \int_0^t \left[f_x(\tau) x(\tau) + f_y(\tau) y(\tau) \right] d\tau \right] \right\} \\ = & \int_a^b \mathcal{D}[x(\tau)] \int_a^b \mathcal{D}[y(\tau)] \\ & \times \exp\left\{ \frac{i}{\hbar} \left[\frac{m}{2} \int_0^t \left([\dot{x}^2 + \dot{y}^2] + \Omega[x\dot{y} - y\dot{x}] - \nu^2 [x^2 + y^2] \right) d\tau \right.\right. \end{aligned}$$

$$+ \int_0^t \left[f_x(\tau) x(\tau) + f_y(\tau) y(\tau) \right] d\tau \Big] \Big\}$$

$$\times \exp\left\{ \frac{i m \nu^2}{2\hbar t} \left(\left[\int_0^t x(\tau) d\tau \right]^2 + \left[\int_0^t y(\tau) d\tau \right]^2 \right) \right\} .$$

(Ⅲ.3)

The awkward parts of the path integrals of Eq.(Ⅲ.3) appear in the last exponential functional which involve the square of the integrations $\int_0^t x(\tau) d\tau$ and $\int_0^t y(\tau) d\tau$ respectively. These lead the action function of such a system to be rather complicated and the path integration cannot be carried out directly. However, the difficulty can be overcome as follows.

We generate these functions through averaging the linear exponential functionals involving the auxiliary force F_x and F_y in two dimensions which independent of the time τ. We follow Stratonovich's work by using the identity

$$\exp\left\{ \frac{i m \nu^2}{2\hbar t} \left(\left[\int_0^t x(\tau) d\tau \right]^2 + \left[\int_0^t y(\tau) d\tau \right]^2 \right) \right\}$$

$$= \left\langle \exp\left\{ \frac{i}{\hbar} \left[F_x \int_0^t x(\tau) d\tau + F_y \int_0^t y(\tau) d\tau \right] \right\} \right\rangle_{F_x, F_y} . \qquad (Ⅲ.4)$$

where $\langle \ldots \rangle_{F_x, F_y}$ denotes the gaussian average defined by

$$\langle O \rangle_{F_x, F_y} = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dF_x dF_y \, O \exp\left\{ \frac{-it}{2m\hbar\nu^2} (F_x^2 + F_y^2) \right\}}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dF_x dF_y \exp\left\{ \frac{-it}{2m\hbar\nu^2} (F_x^2 + F_y^2) \right\}} . \qquad (Ⅲ.5)$$

Applying Eq.(Ⅲ.4) into Eq.(Ⅲ.3) , we have

$$K(a,b) = \left\langle K_{eff}(a,b) \right\rangle_{F_x,F_y} \qquad (\text{III}\cdot 6)$$

where $K_{eff}(a,b)$ is the effective propagator corresponds to the system of the harmonically bounded charge under the magnetic field and external time varying force field. It is given by

$$K_{eff}(a,b) = \int_a^b \mathcal{D}[x(\tau)] \int_a^b \mathcal{D}[y(\tau)] \exp\left\{ \frac{i}{\hbar} \left[\frac{m}{2} \int_0^t \left(\dot{x}^2 + \dot{y}^2 + \Omega(x\dot{y} - y\dot{x}) - \nu^2(x^2+y^2) \right) d\tau + \int_0^t (F_x + f_x(\tau))\, x\, d\tau + \int_0^t (F_y + f(\tau))\, y\, d\tau \right] \right\}. \qquad (\text{III}.7)$$

This transformed propagator contains only the local action function. Thus, following from Feynman's formalism of the path integral theory the effective propagator can be expressed in term of its action function $S_{cl}(a,b)$ of an electronic motion bounded by the harmonic oscillator potential under the magnetic field and the external time varying force field. From Eq. $(\text{II}\cdot 38)$, we get

$$K_{eff}(a,b) = F_{eff}(0,t) \exp\left\{ \frac{i}{\hbar} S_{cl}(a,b) \right\} \qquad (\text{III}.8)$$

where $F_{eff}(0,t)$ is called the pre-exponential factor for the effective system which is the function of the time interval t only.

III.2 Effective Propagator.

We shall now concerned with an electron moving in a harmonic oscillator bowel

$$\frac{m}{2}\nu^2\left(x^2+y^2\right) \qquad (\mathrm{III}.9)$$

under the constant magnetic field $\vec{B}$, the external time varying force fields $f_x(\tau)$ and $f_y(\tau)$ and in the random forces F_x and F_y . From Eq. $(\mathrm{III}.7)$, the effective lagrangian is

$$\mathcal{L}_{eff}(x,y) = \frac{m}{2}\left\{ \dot{x}^2 + \dot{y}^2 + \Omega\left(x\dot{y} - y\dot{x}\right) - \nu^2\left(x^2+y^2\right)\right\} + \left(F_x + f_x(\tau)\right)x + \left(F_y + f_y(\tau)\right)y \,. \qquad (\mathrm{III}.10)$$

Note that this lagrangian appears in quadratic form, so that Van Vleck-Pauli's result can be used. We will get the pre-exponential factor in Eq. $(\mathrm{II}.8)$, $F_{eff}(t,0)$, associated with the effective action function

$$S_{el}^{eff}(a,b) = \int_0^t \mathcal{L}_{eff}(x,y)\,d\tau \qquad (\mathrm{III}.11)$$

and which is evaluated in Eq. $(\mathrm{II}.40)$ by the formula

$$F_{eff}(t,0) = \left\{ \det\left[\frac{i}{2\pi\hbar} \frac{\partial^2 S_{el}^{eff}(a,b)}{\partial(x_a,y_a)\,\partial(x_b,y_b)} \right]\right\}^{1/2} \qquad (\mathrm{III}.12)$$

where we use the notation $\frac{\partial}{\partial(x_a,y_a)}$ by means of the 2 x 1 matrix elements,

$$\frac{\partial\, O(x_a,y_a)}{\partial(x_a,y_a)} = \begin{vmatrix} \dfrac{\partial\, O(x_a,y_a)}{\partial x_a} \\[2ex] \dfrac{\partial\, O(x_a,y_a)}{\partial y_a} \end{vmatrix} \,. \qquad (\mathrm{III}.13)$$

We now wish to evaluate the effective classical action function, $S_{cl}^{eff}(a,b)$, corresponding to an effective Lagrangian, $\mathcal{L}_{eff}(x,y)$. To simplify this problem, we introduce the 2 x 2 matrix, from Papadopoulos's work, such as

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} , \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (\mathrm{III}.14a)$$

which obey the relation

$$J^2 = -I . \qquad (\mathrm{III}.14b)$$

That is the matrix J plays the role of 2 x 2 imaginary matrix. Let us also denote the 2 x 1 matrices for the axis components and for the force fields, those are

$$\gamma(\tau) = \begin{bmatrix} x(\tau) \\ y(\tau) \end{bmatrix} \quad \text{and} \quad g(\tau) = f(\tau) + F = \begin{bmatrix} f_x(\tau) + F_x \\ f_y(\tau) + F_y \end{bmatrix} . \qquad (\mathrm{III}.15)$$

From these notations, the effective Lagrangian becomes

$$\mathcal{L}_{eff}(\gamma, \dot{\gamma}) = \frac{m}{2}\left\{ \dot{\gamma}^T\dot{\gamma} + \Omega\dot{\gamma}^T J\gamma - \nu^2\gamma^T\gamma \right\} + \gamma^T g . \qquad (\mathrm{III}.16)$$

where the notation $(-)^T$ means the matrix transposition. Using Hamilton's theory for this Lagrangian,

$$\frac{d}{d\tau}\left[\frac{\partial \mathcal{L}_{eff}(\gamma,\dot{\gamma})}{\partial \dot{\gamma}^T}\right] = \left[\frac{\partial}{\partial \gamma^T}\mathcal{L}_{eff}(\gamma,\dot{\gamma})\right] \qquad (\mathrm{III}.17a)$$

it leads to the equation of motion,

$$\ddot{r} + \Omega J \dot{r} + \nu^2 r = \frac{g(\tau)}{m} \qquad (\text{III}.17b)$$

Subject to the boundary conditions,

$$r(0) = r_a \quad \text{and} \quad r(t) = r_b \, . \qquad (\text{III}.17c)$$

The algebra for the solution of the equation of motion in Eq. (III.17b) can be significantly evaluated by the method of Green's function and details of the evaluation are contained in appendix D. We confine ourselves to write down the results of the classical solution

$$r_{cl} = r_{cl}^{c} + r_{cl}^{p} \qquad (\text{III}.18a)$$

where

$$r_{cl}^{c} = \frac{1}{\sin(\omega t)} e^{-J\frac{\Omega}{2}\tau} \left\{ e^{J\frac{\Omega}{2}t} \sin(\omega\tau) r_b + \sin(\omega[t-\tau]) r_a \right\} \qquad (\text{III}.18b)$$

and

$$r_{cl}^{p} = \frac{1}{m} \int_0^t G(\tau,\tau') g(\tau') d\tau' \qquad (\text{III}.18c).$$

when $\omega^2 = \frac{\Omega^2}{4} + \nu^2$ and $G(\tau,\tau')$ is the Green's function. It can be evaluated exactly to be

$$G(\tau,\tau') = -\frac{1}{\omega \sin(\omega t)} e^{-J\frac{\Omega}{2}(\tau-\tau')}$$

$$\times \Big[\sin(\omega[t-\tau]) \sin(\omega\tau') H(\tau-\tau')$$

$$+ \sin(\omega[t-\tau']) \sin(\omega\tau) H(\tau'-\tau) \Big] \qquad (\text{III}.19a)$$

when $H(\cdots)$ being the Heaviside step function obeying the relation

$$H(\tau-\tau') = \begin{cases} 1 , & \tau > \tau' \\ 0 , & \tau' > \tau \end{cases} \qquad (\text{III}.19b)$$

The details of evaluation of the Green's function are contained in appendix E.

We now focus our attention on the effective classical action function

$$S_{el}^{eff}(a,b) = \int_0^t \mathcal{L}_{eff}(\gamma,\dot{\gamma})\,d\tau$$

$$= \int_0^t \frac{m}{2}\left\{ \dot{\gamma}^T\dot{\gamma} + \Omega\dot{\gamma}^T J\gamma - \nu^2\gamma^T\gamma \right\} d\tau + \int_0^t \gamma^T g(\tau)\,d\tau \, . \qquad (\text{III}.20)$$

Integrating by part on the first integration;

$$S_{el}^{eff}(a,b) = \frac{m}{2}\left[\dot{\gamma}_{cl}^T(\tau)\,\gamma_{cl}(\tau) \right]_{\tau=0}^{t}$$

$$- \frac{m}{2}\int_0^t \left\{ \ddot{\gamma}_{cl}^T - \Omega\dot{\gamma}_{cl}^T J + \nu^2\gamma_{cl}^T \right\} \gamma_{cl}\,d\tau + \int_0^t \gamma_{cl}^T g(\tau)\,d\tau$$

$$(\text{III}.21a)$$

and using the equation of motion from Eq.(III.17b), we obtain

$$S_{el}^{eff}(a,b) = \frac{m}{2}\left[\dot{\gamma}_{cl}^T(\tau)\,\gamma_{el}(\tau) \right]_{\tau=0}^{t} + \frac{1}{2}\int_0^t \gamma_{o}^T(\tau)\,g(\tau)\,d\tau$$

$$= \frac{m}{2}\left[\dot{\gamma}_{el}^T(t)\,\gamma_t - \dot{\gamma}_{el}^T(0)\,\gamma_a \right] + \int_0^t \gamma_{el}^{cT}(\tau)\,g(\tau)\,d\tau + \frac{1}{2m}\int_0^t\!\!\int_0^t g^T(\tau)\,G(\tau,\tau')$$

$$(\text{III}.21b) \qquad \times g(\tau')\,d\tau\,d\tau' .$$

The complete solution for an effective classical action function is obtained to be

$$S_{cl}^{eff}(a,b) = \frac{m\omega}{2\sin(\omega t)}\left\{\cos(\omega t)\left[r_b^2 + r_a^2\right] - 2r_a^T e^{J\frac{\Omega}{2}t} r_b\right\}$$
$$+\int_0^t r_{cl}^{cT}(\tau) g(\tau) d\tau + \frac{1}{2m}\int_0^t\int_0^t g^T(\tau) G'(\tau,\tau') g(\tau') d\tau d\tau'. \qquad (\text{III}.22a)$$

For convenient, we denote for the classical action $S_{cl}^{0*}(a,b)$ such as

$$S_{cl}^{0*}(a,b) = \frac{m\omega}{2\sin(\omega t)}\left\{\cos(\omega t)\left[r_b^2 + r_a^2\right] - 2r_a^T e^{J\frac{\Omega}{2}t} r_b\right\}. \qquad (\text{III}.22b)$$

The pre- exponential factor associated with the effective propagator can be evaluated exactly by using Eqs. (III.12) and (III.13). It is founded that

$$F_{eff}(t,0) = \left\{\det\left[\frac{i}{2\pi\hbar}\frac{\partial^2 S_{cl}^{eff}(a,b)}{\partial r_a^T \partial r_b}\right]\right\}$$
$$= \left\{\det\left[\frac{i}{2\pi\hbar}\frac{\partial^2 S_{cl}^{0}(a,b)}{\partial r_a^T \partial r_b}\right]\right\}$$
$$= \frac{m\omega}{2\pi i\hbar\sin(\omega t)}. \qquad (\text{III}.23)$$

The details of the evaluation are put in <u>appendix F</u>.

From Eqs. (III.8) , (III.22) and (III.23) ,we obtain the effective propagator such as

$$K_{eff}(a,b) = \left[\frac{m\omega}{2\pi i\hbar\sin(\omega t)}\right]\exp\left\{\frac{i}{\hbar}\left[S_{cl}^{0*}(a,b) + \int_0^t r_{cl}^{cT}(\tau) g(\tau) d\tau + \frac{1}{2m}\int_0^t\int_0^t g^T(\tau) G(\tau,\tau') g(\tau') d\tau d\tau'\right]\right\}$$

or

$$K_{eff}(a,b) = \left[\frac{m\omega}{2\pi i\hbar \sin(\omega t)}\right] \exp\left\{\frac{i}{\hbar}\left[\frac{m\omega}{2\sin(\omega t)}\right.\right.$$

$$\times \left[\cos(\omega t)\left(\gamma_b^2 + \gamma_a^2\right) - 2\gamma_a^T e^{J\frac{\Omega}{2}t}\gamma_b\right]$$

$$+ \frac{1}{\sin(\omega t)}\int_0^t \left[\gamma_b^T \sin(\omega\tau) e^{-J\frac{\Omega}{2}t} + \gamma_a^T \sin(\omega[t-\tau])\right] e^{J\frac{\Omega}{2}\tau} g(\tau)d\tau$$

$$- \frac{1}{2m\omega\sin(\omega t)}\int_0^t\int_0^t g^T(\tau)\exp\left(-J\frac{\Omega}{2}[\tau-\tau']\right)$$

$$\times \left[\sin(\omega\tau)\sin(\omega[t-\tau'])H(\tau-\tau')\right.$$

$$\left.\left.\left. + \sin(\omega\tau')\sin(\omega[t-\tau])H(\tau'-\tau)\right] g(\tau')d\tau d\tau'\right]\right\}.$$

(III.24)

III.3 Exact Propagator

Let us now go back to the original propagator in Eq. (III.6), we have

$$K(a,b) = \left\langle K_{eff}(a,b)\right\rangle_{F_x, F_y} \qquad (\text{III}.6)$$

By splitting $g(\tau)$ into the two parts

$$g(\tau) = f(\tau) + F$$

$$= \begin{bmatrix} f_x(\tau) \\ f_y(\tau) \end{bmatrix} + \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$

then the propagator in Eq. (III.24) becomes

$$K_{eff}(a,b) = \left[\frac{m\omega}{2\pi i\hbar\sin(\omega t)}\right] \exp\left\{\frac{i}{\hbar}\left[S_{el}^{0*}(a,b)\right.\right.$$

$$\left.\left. + \int_0^t \gamma_{el}^{cT}(\tau) f(\tau) d\tau + \frac{1}{2m}\int_0^t\int_0^t f^T(\tau) G'(\tau,\tau') f(\tau') d\tau d\tau'\right]\right\}$$

$$\times \exp\left\{\frac{i}{\hbar}\left[\int_0^t \gamma_{el}^{cT}(\tau) d\tau + \frac{1}{m}\int_0^t\int_0^t f^T(\tau) G'(\tau,\tau') d\tau d\tau'\right] F\right.$$

$$\left. + \frac{iF^T}{2\hbar m}\int_0^t\int_0^t G'(\tau,\tau') d\tau d\tau' F\right\} . \quad (\text{III}.25)$$

Then we perform the gaussian average of Eq. (III.6) , we will obtain the desired exact propagator, in the short form,

$$K(a,b) = F(t,0) \exp\left\{\frac{i}{\hbar} S_{el}(a,b)\right\}$$

$$= F(t,0) \exp\left\{\frac{i}{\hbar}\left[S_{el}^{0}(a,b) + \int_0^t R_{el}^{cT}(\tau) f(\tau) d\tau\right.\right.$$

$$\left.\left. + \frac{1}{2m}\int_0^t\int_0^t f^T(\tau) G(\tau,\tau') f(\tau') d\tau d\tau'\right]\right\} \quad (\text{III}.26)$$

where

$$S_{el}^{0}(a,b) = S_{el}^{0*}(a,b) + \frac{m\nu^4 \sin(\omega t)}{4\omega\left[\cos\left(\frac{\Omega}{2}t\right) - \cos(\omega t)\right]}\int_0^t\int_0^t \gamma_{el}^{cT}(\tau)\gamma_{el}^{d}(\tau') d\tau d\tau'$$

$$= \frac{m\omega}{2\sin(\omega t)}\left[\cos(\omega t)\left[\gamma_b^2 + \gamma_a^2\right] - 2\gamma_a^T \exp\left(J\frac{\Omega}{2}t\right)\gamma_b\right]$$

$$+ \frac{m\omega}{4\sin(\omega t)}\left\{\left[\cos\left(\frac{\Omega}{2}t\right) - \cos(\omega t)\right](\gamma_b + \gamma_a)^2\right.$$

$$- \frac{4}{\omega}\left[\omega\sin\left(\frac{\Omega}{2}t\right) - \frac{\Omega}{2}\sin(\omega t)\right]\gamma_b^T J \gamma_a$$

$$\left. + \frac{1}{\omega^2}\frac{\left[\omega\sin\left(\frac{\Omega}{2}t\right) - \frac{\Omega}{2}\sin(\omega t)\right]^2}{\left[\cos\left(\frac{\Omega}{2}t\right) - \cos(\omega t)\right]}(\gamma_b - \gamma_a)^2\right\} ,$$

$$(\text{III}.27)$$

$$R_d^{cT}(\tau) = r_d^{cT}(\tau) + \frac{\nu^4 \sin(\omega t)}{2\omega\left[\cos(\frac{\Omega}{2}t) - \cos(\omega t)\right]}$$

$$\times \int_0^t\int_0^t r_d^{cT}(\tau)\, G(\tau,\tau')\, d\tau\, d\tau'$$

$$= \frac{1}{\sin(\omega t)}\left\{ \gamma_a^T \sin(\omega[t-\tau]) + \gamma_b^T \sin(\omega\tau)\, e^{-J\frac{\Omega}{2}t} \right\} e^{J\frac{\Omega}{2}\tau}$$

$$- \frac{1}{2\sin(\omega t)}\left\{ (\gamma_b^T + \gamma_a^T) - \frac{1}{\omega}\, \frac{\left[\omega \sin(\frac{\Omega}{2}t) - \frac{\Omega}{2}\sin(\omega t)\right]}{\left[\cos(\frac{\Omega}{2}t) - \cos(\omega t)\right]}\right.$$

$$\left.\times\left[\gamma_b^T - \gamma_a^T\right]\right\}\left\{\left[\sin(\omega[t-\tau]) + \sin(\omega\tau)\exp\left(-J\frac{\Omega}{2}t\right)\right]\right.$$

$$\left.\times \exp\left(J\frac{\Omega}{2}\tau\right) - \sin(\omega t)\right\}, \qquad (\text{III}.28)$$

$$\mathcal{G}(\tau,\tau') = G(\tau,\tau') + \frac{\nu^4 \sin(\omega t)}{2\omega\left[\cos(\frac{\Omega}{2}t) - \cos(\omega t)\right]}$$

$$\times \int_0^t\int_0^t G(\tau,\tau'')\, G(\tau'',\tau')\, d\tau'' d\tau''$$

$$= -\frac{1}{\omega\sin(\omega t)}\left\{ \sin(\omega[t-\tau])\sin(\omega\tau')\, H(\tau-\tau')\right.$$

$$\left. + \sin(\omega\tau)\sin(\omega[t-\tau'])\, H(\tau'-\tau)\right\} e^{-J\frac{\Omega}{2}(\tau-\tau')}$$

$$+ \frac{1}{2\omega\left[\cos(\frac{\Omega}{2}t) - \cos(\omega t)\right]}\left\{ e^{-J\frac{\Omega}{2}\tau}\left[\sin(\omega[t-\tau])\right.\right.$$

$$\left.\left. + \sin(\omega\tau)\, e^{J\frac{\Omega}{2}t}\right] - \sin(\omega t)\right\}\left\{ e^{J\frac{\Omega}{2}\tau'}\left[\sin(\omega[t-\tau'])\right.\right.$$

$$\left.\left. + \sin(\omega\tau')\, e^{-J\frac{\Omega}{2}t}\right] - \sin(\omega t)\right\}. \qquad (\text{III}.29)$$

and

$$F(t,0) = \frac{m t \nu^2}{4\pi i \hbar \left[\cos\left(\frac{\Omega}{2} t\right) - \cos(\omega t) \right]}$$

$$= \frac{m t \nu^2}{8\pi i \hbar \sin\left(\left[\omega + \frac{\Omega}{2}\right] t/2\right) \sin\left(\left[\omega - \frac{\Omega}{2}\right] t/2\right)} \qquad (\text{III}.30)$$

The details of the evaluation of the results appear in Eqs. (III.26), (III.27), (III.28), (III.29) and (III.30) are contained in appendix G.