CHAPTER V

Discussion and Conclusion.

In this thesis we have calculated exactly the propagator for the two dimensional electronic system under the influence of the constant magnetic field, the external time varying force field and the nonlocal harmonic oscillator potential as shown in chapter III. The method of calculation we use here follows Stratonovich by transforming the nonlocal problem into the local one and we have also used the 2x2 matrix introduced by Papadopoulos for handing the magnetization of the harmonically bound charge. The main results are given in Eqs. (III.26), (III.27) (III.28) , (III.29) and (III.30).

Up to this stage we then discuss our results by the way to apply it into the system of an electronic motion in two dimensions under the constant magnetic field which presented in perpendicular direction to the plane of motion and in the presence of the random potential and generally in the external force field. As mentioned above, we use the nonlocal harmonic oscillator for calculating the propagator which, following Sa-yakanit's work [10], is corresponding to the zeroth-order of approximation which is done by the method of cumulant expansion. [21] To carry out the path integral for the propagator

for the higher order of approximation, we follow Feynman's method by using Edward's model [22] of random potential which its action function is defined as

$$S'[\mathbf{r}(\tau)] = \int_0^t \left\{ \frac{m}{2}\left[\dot{\mathbf{r}}^T\dot{\mathbf{r}} + \Omega\dot{\mathbf{r}}^T J\mathbf{r}\right] + \frac{i}{2\hbar}\rho\eta^2\int_0^t d\tau' W[\mathbf{r}(\tau)-\mathbf{r}(\tau')]\right\} d\tau \qquad \text{(V.1)}$$

where the parameter η denoting the weakness of the scattering potential which is explicitly written here to indicate the dimensions involved and ρ denoting the density of the scattering centers. $W[\gamma(\tau)-\gamma(\tau')]$ denotes the correlation function, defined as

$$W[\gamma(\tau)-\gamma(\tau')] = \int dz\, V[\gamma(\tau)-z]\, V[\gamma(\tau')-z] \qquad (V.2)$$

where $V[\cdots]$ correspond to the scattering potential which may be the screened Coulomb potential,

$$V_{sc}[\gamma-z] = \frac{\exp\{-\lambda|\gamma-z|\}}{|\gamma-z|}, \qquad (V.3)$$

or the gaussian potential,

$$V_g[\gamma-z] = (\pi\ell^2)^{-1} \exp\left\{-\frac{|\gamma-z|^2}{\ell^2}\right\}, \qquad (V.4)$$

for the two dimensional system. The infinite orders of approximation for the propagator becomes

$$\mathbb{K}_{AV.}(a,b) = \overset{o}{K}(a,b) \left\langle \exp\left\{-\frac{\rho\eta^2}{2\hbar^2}\int_0^t\int_0^t W(\gamma(\tau)-\gamma(\tau'))\, d\tau d\tau'\right\}\right\rangle_{S_0} \qquad (V.5).$$

where $\overset{o}{K}(a,b) = K_0(a,b)$ and $S_0 = \overset{o}{S}_{cl}$,

$$K_0(a,b) = \int_a^b \mathcal{D}[\gamma] \exp\left\{\frac{i}{\hbar} S_0[\gamma(\tau)]\right\}. \qquad (\text{III}.26)$$

when $S_0[\gamma(\tau)] = \int_0^t \mathcal{L}[f=0]d\tau$ which is found in Eq. (III.3) and the average over S_0, $\langle \cdots \rangle_{S_0}$, is defined as

$$\left\langle O \right\rangle_{S_0} = \frac{\int_a^b \mathcal{D}[\gamma] \exp\left\{\frac{i}{\hbar} S_0[\gamma]\right\} O}{\int_a^b \mathcal{D}[\gamma] \exp\left\{\frac{i}{\hbar} S_0[\gamma]\right\}} \qquad (V.6)$$

Approximating Eq. (V.5) by the first cumulant, we get

$$K^1_{AV.}(a,b) = K^0(a,b) \exp\left\{-\frac{\rho \hbar^2}{2\hbar^2} \int_0^t\int_0^t \left\langle W(\gamma(\tau)-\gamma(\tau')) \right\rangle_{S_0} d\tau\, d\tau' \right\}. \qquad (V.7)$$

To obtained $K^1_{AV.}(a,b)$ we have to find $K^0(a,b)$ and the average $\left\langle W(\gamma(\tau)-\gamma(\tau')) \right\rangle_{S_0}$. Sa-yakanit have shown that the average $\left\langle W(\gamma(\tau)-\gamma(\tau')) \right\rangle_{S_0}$ can be expressed solely in terms of the following averages $\left\langle \gamma_{cl} \right\rangle_{S_0}$ and $\left\langle \gamma^T_{cl}(\tau)\, \gamma_{cl}(\tau') \right\rangle_{S_0}$. Such averages can be obtained from the characteristic functional of $\left\langle \exp\left\{\frac{i}{\hbar}\int_0^t d\tau\, h^T(\tau)\gamma(\tau)\right\} \right\rangle_{S_0}$. From Feynman and Hibbs the characteristic functional can be expressed as

$$\left\langle \exp\left\{\frac{i}{\hbar}\int_0^t d\tau\, h^T(\tau)\gamma(\tau)\right\} \right\rangle_{S_0} = \exp\left\{\frac{i}{\hbar}\left\langle S_{cl} - S^0_{cl} \right\rangle_{S_0}\right\} \qquad (V.8)$$

where S_{cl} and S^0_{cl} : found in Eq. (III.26) and (III.27) respectively.. We can differentiate the expression in Eq. (V.8) with respect to $h(\tau)$ to obtain

$$\left\langle \gamma(\tau) \right\rangle_{S_0} = \frac{\delta S_{cl}(a,b)}{\delta h}\Bigg|_{h(\tau)=0} \qquad (V.9)$$

where the symbol $\Big|_{h=0}$ implies that after the differentiating, we must set $h(\tau) = 0$. Continuing the differentiation, we get

$$\langle \gamma^T(\tau)\gamma(\tau')\rangle_{S_0} = \frac{\hbar}{i}\left\{ \frac{\delta^2 S_{cl}(a,b)}{\delta h^T(\tau)\,\delta h(\tau')} + \frac{\delta S_{cl}(a,b)}{\delta h^T(\tau)}\,\frac{\delta S_{cl}(a,b)}{\delta h(\tau')} \right\}_{h=0} . \qquad (V.10)$$

Following this scheme and using the Fourier transform of $W[\gamma(\tau)-\gamma(\tau')]$ [19], we will get $K^1_{av.}(a,b)$.

Starting from this expression, Eq. (V.7) , a number of physical quantities can be studied . For instance , the density of states can be obtained by taking the trace of $K^1_{av}(a,b)$ and then performing the Fourier transform according to the standard formula [10]

$$n(E) = \frac{1}{2\pi\hbar}\int dt\ \mathrm{Tr}\, K^1_{av.}(a,b)\exp\left\{\frac{i}{\hbar}Et\right\} \qquad (V.11)$$

where Tr denotes the trace

Finally, it is interesting to note that the problem presented here is quite similar to the recent problem treated by Klinert [23] on the path integral for the second derivative lagrangian which can be compared to the nonlocal harmonic oscillator presented in this work when both terms are translationally invariant.