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Electrical Analogy Method for Torsion of Prismatic Shaft

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หัวข้อวิทยานิพนธ์ การหาแรงบิดในเพลลาปริม โดยวิธีการเปรียบเทียบทางไฟฟ้า

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บทคัดย่อ

วิทยานิพนธ์นี้ศึกษาปัญหาเกี่ยวกับทอร์ชัน (Torsion) ของเพลลาโดยอาศัยการกระจายของศักดาไฟฟ้าบนแผ่นตัวนำ เนื่องจากปรากฏการณ์ทั้งสองเป็นอนาล็อกกัน (Analogous) ในวิทยานิพนธ์นี้ใช้แผ่นกระดาษนำไฟฟ้า (Conducting paper) เป็นตัวกลาง (medium) นำแผ่นกระดาษนำไฟฟ้าที่ตัดตามรูปร่างหน้าตัดของเพลลา มาทำให้เกิดศักดาไฟฟ้าบริเวณขอบนอก (Boundary potential) ในลักษณะเดียวกับสภาวะขอบนอก (Boundary condition) ของคอนจูเกตฟังก์ชัน (Conjugate function) ในปัญหาการบิดแปลงกาของศักดาไฟฟ้าที่กระจายบนแผ่นกระดาษตัวนำเป็นคอนจูเกตฟังก์ชัน แล้วคำนวณหาค่าอื่น ๆ เช่น เชียร์สเตรส (Shear Stress) จากสมการซึ่งแสดงความสัมพันธ์ของคอนจูเกตฟังก์ชัน กับปริมาณอื่น ๆ การทดลองอาจแยกได้เป็น ๒ ส่วน ส่วนแรกทดลองกับสี่เหลี่ยมจัตุรัสหรือสี่เหลี่ยมผืนผ้า เพื่อหาวิธีการนี้ได้ผลถูกต้องเพียงไร ในกรณีทั่วไป ส่วนที่สองทดลองกับรูปตัวไอ (I) เพื่อหาความถูกต้องของวิธีการนี้ เมื่อใช้หาค่าเชียร์สเตรสตรงมุมในของส่วนที่เว้าเข้าของรูป (Re-entrant corner) ซึ่งเกิดบริเวณที่สเตรสสูงขึ้นอย่างฉับพลัน (Stress concentration) ผลลัพธ์ที่ได้จากการหาค่าด้วยวิธีนี้ได้มาไปเทียบกับค่าที่ได้จากการวิเคราะห์ (Analytical value) และค่าโดยประมาณ (Approximate value)

ผลการทดลอง ปรากฏว่าค่าที่ได้จากการทดลองในกรณีทั่วไปซึ่งรวมทั้งรูปตัวไอด้วย มีค่าใกล้เคียงกับค่าที่ได้จากการวิเคราะห์หรือการประมาณ แต่บริเวณที่เกิดสเตรสสูงขึ้นโดยฉับพลัน ความผิดพลาดค่อนข้างสูง

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#### ABSTRACT

This thesis is about the torsion problem of shaft solved by means of the distribution of electrical potential on a conducting medium sheet, this can be done because both systems are analogous.

In this thesis, the conducting paper is used as the medium. The conducting paper is cut similar to the cross-section of the shaft and then it is supplied on its boundary a form of boundary potential similar to the boundary condition of the conjugate function in the torsion problem. The potential distributed on the conducting paper is transformed to conjugate function. Then other values such as shear stress, are calculated from the equation which relate the conjugate function to that value.

The experiment may be divided into two parts, the first is concerned with the square and the rectangular cross-section to verify the reliability of the method in normal case. The second is concerned with the I cross-section shaft to investigate the shear stress at the reentrance corner where stress concentration occurs. The values obtained by this method are compared with the analytical value or the approximate values.

Experimental results show that the values obtained in normal case including the I cross-section are closed to the analytical values or the approximate values, but in the region where stress concentration occurs, the error is rather high.

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## NOMENCLATURE



$x, y, z$	Cartesian co - ordinate axes
$V$	Electrical potential
$i_x, i_y$	Components of current density in x and y direction
$\rho$	Resistivity
$\theta$	Twisting angle
$\alpha$	Twist per unit length
$u, v, w$	Components of displacement in x, y, z direction
$\varphi$	Torsion function or warping function
$\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}$	Strain components
$\delta_{xy}, \delta_{xz}, \delta_{yz}$	Shear strain components
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	Stress components
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	Shear stress components
$E$	Modulus of elasticity
$\eta$	Poisson's ratio
$\mu$	Shear modulus
$X, Y, Z$	Components of body force per unit volume of element
$\bar{X}, \bar{Y}, \bar{Z}$	Components of surfaces force per unit area
$n, s$	Normal and tangential axes
$\cos(n, x), \cos(n, y)$	Direction cosines of the normal axes
$\cos(n, z)$	
$R$	Region of cross section of shaft
$R'$	Region of conducting sheet
$C$	Boundary of cross section of shaft

$C'$	Boundary of conducting sheet
$M$	Twisting moment or torsional stiffness
$\psi$	Conjugate function
$\phi$	Shearing stress function
$\tau_{zz}, \tau_{zs}$	Shear stress components in normal and tangential axes
$\tau$	Resultant shear stress
$S_1$	Coefficient to express potential in Volt
$c_1$	Constant to express potential in Volt
$S_2$	Coefficient to relate conjugate function to $\psi$
$c_2$	Constant to relate conjugate function to $V$
$a$	The width of the rectangular shaft in analytical equation
$b$	The length of the rectangular shaft in analytical equation
$k_n$	A constant in the analytical equation of rectangular shaft equal to $(2n + 1) \pi/a$
$t$	Thickness
$t_1$	Thickness of web of I - cross section
$t_2$	Thickness of flange of I - cross section
$b_1$	Width of flange of I - cross section
$b_2$	Width of web of I - cross section
$r$	Radius of fillet
$K$	Dimensionless factor of maximum shear stress

$K_1$	Dimensionless factor of torsional stiffness
$u, v$	Cartesian co - ordinate axes in $w$ - region (use in chapter 6 conclusion only)
$m$	Constant of the transformation from $z$ - region to $w$ - region
$k_i$	Constant of boundary conjugate function of holes
$C_i$	Boundary of holes in the shaft
$\delta$	Angle between tangent of the line of current flow at the point on curve and the tangent of closed curve
$\gamma$	Angle between tangent of curve and the $x$ - axis
$\beta$	Angle between the tangent of the line of current flow at the point on curve and the $x$ - axis
$\Delta S_i$	Portion of closed curve
$I$	Total current flow across boundary