CHAPTER III

PRINCIPLES FOR ANALYZING GENERAL FLUID NETWORKS

3.1 Introduction

A general fluid network is formulated from a number of n nodes, each of which is identified by an index such as i or j.

The type of node i is specified as T_i, which assumes one of the following integral values:

0 - Pressure unspecified at node i.

1 - Pressure specified at node i.

2 - Injection or withdrawal rate specified at node i.

3 - Terminal node i with a specified injection or withdrawal rate.

The nodal connection are connected directly to others nodes by a pipeline or equipment. A connection matrix is established with possible values for a representative element C_{ij}, as follows:

1 - Node i and node j are joined by a pipeline.

2 - Centrifugal pump that pumps from node i to node j.

3 - Centrifugal compressor that compresses from node i to node j.

For nodal connection $C_{ij} = 1$, the pipeline diameter D_{ij}, length L_{ij}, and roughness ε_{ij}, are symmetrical joining node i and node j.

Nodes at which the specified injection rate is represented positive value or withdrawal rate as negative value.

Note:

For nodal connection across an equipment such as pump or compressor, if $C_{ij} = 2$ or $C_{ij} = 3$ then always $C_{ji} = 0$ because it can not operate in reverse flow.

The program always considers node i as the receiving node. Therefore, the flow rate within pipeline connection given as "Q_{ji}" is positive value for flow from node j to node i and negative for the reverse direction.

3.2 Flow in Pipelines

3.2.1 For Liquid

The flow rate from node j to node i is given by:

$$Q_{ji} = \sqrt{y/(\alpha_{ji} f_{F_{ji}})} \qquad \text{for} \qquad y > 0 \tag{3.1}$$

The flow rate from node i to node j is given by:

$$Q_{ij} = -\sqrt{-y/(\alpha_{ji} f_{F_{ji}})} \qquad \text{for} \qquad y < 0 \tag{3.2}$$

Here:

$$\alpha_{ji} = \frac{32\rho L_{ji}}{\pi^2 D_{ji}^5}, \qquad \beta = \rho g \tag{3.3}$$

$$y = p_j - p_i + \beta(z_j - z_i) \tag{3.4}$$

3.2.2 For Gas

Inclined Flow$\left(z_i \neq z_j\right)$:

The flow rate from node j to node i is given by:

$$Q_{sc-ji} = \frac{\lambda_{ji}}{\psi_{sc}} \sqrt{\frac{w}{\delta_{ji}\left(\phi_{ji} - 1\right)}} \qquad \text{for} \quad w > 0 \tag{3.5}$$

The flow rate from node i to node j is given by:

$$Q_{sc-ij} = -\frac{\lambda_{ji}}{\psi_{sc}} \sqrt{\frac{-w}{\delta_{ji}\left(\phi_{ji} - 1\right)}} \qquad \text{for} \quad w < 0 \tag{3.6}$$

Here:

$$w = p_j^2 - \phi_{ji} p_i^2 \tag{3.7}$$

$$\lambda_{ji} = \left(\frac{M}{Z_{avg} RT}\right)\left(\frac{\pi D_{ji}^2}{4}\right) \tag{3.8}$$

$$\delta_{ji} = \left(\frac{2 f_{F_{ji}} L_{ji}}{g D_{ji}\left(z_i - z_j\right)}\right) \tag{3.9}$$

$$\phi_{ji} = \exp\left(\frac{2Mg\left(z_i - z_j\right)}{Z_{avg} RT}\right) \tag{3.10}$$

$$\psi_{sc} = \frac{p_{sc} M}{Z_{sc} RT_{sc}} \tag{3.11}$$

Horizontal Flow$\left(z_i = z_j\right)$:

The flow rate from node j to node i is given by:

$$Q_{sc-ji} = \frac{A_{ji}}{\psi_{sc}} \sqrt{\xi_{ji}\left(p_j^2 - p_i^2\right)} \qquad \text{for} \quad p_j > p_i \tag{3.12}$$

The flow rate from node i to node j is given by:

$$Q_{sc-ij} = -\frac{A_{ji}}{\psi_{sc}}\sqrt{-\xi_{ji}\left(p_j^2 - p_i^2\right)} \qquad \text{for} \qquad p_j < p_i \tag{3.13}$$

Here:

$$A_{ji} = \frac{\pi D_{ji}^2}{4} \tag{3.14}$$

$$\xi_{ji} = \left(\frac{M}{Z_{avg}RT}\right)\left(\frac{D_{ji}}{4f_{F_{ji}}L_{ji}}\right) \tag{3.15}$$

$$\psi_{sc} = \frac{p_{sc}M}{Z_{sc}RT_{sc}} \tag{3.16}$$

3.3 Flow in Equipment

3.3.1 For Liquid

There are two separate cases to be considered for each of three possibilities as follows:

1. $Q_{ji} > 0$ for flow across the pump from node j to node i:

$$Q_{ji} = 0 \qquad \text{for } p_i + \beta z_i > p_j + \beta z_j + a_{ji} \tag{3.17}$$

$$Q_{ji} = \sqrt{a_{ji}/b_{ji}} \qquad \text{for } p_j + \beta z_j > p_i + \beta z_i \tag{3.18}$$

$$Q_{ji} = \sqrt{\left(p_j - p_i + a_{ji} + \beta\left(z_j - z_i\right)\right)/b_{ji}} \qquad \text{otherwise} \tag{3.19}$$

2. $Q_{ij} < 0$ for flow across the pump from node i to node j:

$$Q_{ij} = 0 \qquad \text{for } p_j + \beta z_j > p_i + \beta z_i + a_{ij} \qquad (3.20)$$

$$Q_{ij} = -\sqrt{a_{ij}/b_{ij}} \qquad \text{for } p_i + \beta z_i > p_j + \beta z_j \qquad (3.21)$$

$$Q_{ij} = -\sqrt{\left(p_i - p_j + a_{ij} + \beta\left(z_i - z_j\right)\right)/b_{ij}} \qquad \text{otherwise} \qquad (3.22)$$

Here:

$$\beta = \rho g \qquad (3.23)$$

3.3.2 For Gas

The flow rate across the compressor from node j to node i is:

$$Q_{sc-ji} = \left(\frac{1}{\psi_{sc}}\right) \frac{W_{c-ji}}{\left\{\frac{k}{k-1}\zeta T_j\left[\left(\frac{p_i}{p_j}\right)^{(k-1)/k} - 1\right] + \omega_{ji}\right\}} \qquad (3.24)$$

The flow rate across the compressor from node i to node j is:

$$Q_{sc-ij} = \left(\frac{1}{\psi_{sc}}\right) \frac{-W_{c-ij}}{\left\{\frac{k}{k-1}\zeta T_i\left[\left(\frac{p_j}{p_i}\right)^{(k-1)/k} - 1\right] + \omega_{ij}\right\}} \qquad (3.25)$$

Here:

$$\psi_{sc} = \frac{p_{sc} M}{Z_{sc} R T_{sc}} \qquad (3.26)$$

$$\zeta = \frac{Z_{avg} R}{M} \qquad (3.27)$$

$$\omega_{ji} = g\left(z_i - z_j\right) \qquad (3.28)$$

$$\omega_{ij} = g\left(z_j - z_i\right) \qquad (3.29)$$

$$k = \frac{C_P}{C_V} \tag{3.30}$$

3.4 Pipeline Flow with Partial Derivatives

In the followings relatively small variations of the Fanning friction factor are ignored.

The partial derivatives for the Newton-Raphson method with respect to p_j and p_i are given as follows:

3.4.1 For Liquid

Table 3.1 Liquid flow rate with partial derivatives

$y = p_j - p_i + \beta(z_j - z_i)$	$Q_{ji} = \sqrt{y/(\alpha_{ji} f_{F_{ji}})}$ $y > 0$	$Q_{ij} = -\sqrt{-y/(\alpha_{ji} f_{F_{ji}})}$ $y < 0$
$\frac{\partial Q}{\partial p_j}$	$\frac{\partial Q_{ji}}{\partial p_j} = 0.5\sqrt{1/(\alpha_{ji} f_{F_{ji}} y)}$	$\frac{\partial Q_{ij}}{\partial p_j} = 0.5\sqrt{-1/(\alpha_{ji} f_{F_{ji}} y)}$
$\frac{\partial Q}{\partial p_i}$	$\frac{\partial Q_{ji}}{\partial p_i} = -0.5\sqrt{1/(\alpha_{ji} f_{F_{ji}} y)}$	$\frac{\partial Q_{ij}}{\partial p_i} = -0.5\sqrt{-1/(\alpha_{ji} f_{F_{ji}} y)}$

3.4.2 For Gas

Inclined Flow $\left(z_i \neq z_j\right)$:

Table 3.2 Inclined flow rate with partial derivatives

$Q_{sc-ji} = \frac{\lambda_{ji}}{\psi_{sc}} \sqrt{\frac{w}{\delta_{ji}\left(\phi_{ji}-1\right)}}$ In which: $w = p_j^2 - \phi_{ji} p_i^2$, and $w > 0$	$Q_{sc-ij} = -\frac{\lambda_{ji}}{\psi_{sc}} \sqrt{\frac{-w}{\delta_{ji}\left(\phi_{ji}-1\right)}}$ In which: $w = p_j^2 - \phi_{ji} p_i^2$, and $w < 0$
$\frac{\partial Q_{sc-ji}}{\partial p_j} = \frac{\lambda_{ji}}{\psi_{sc}\delta_{ji}\left(\phi_{ji}-1\right)} \frac{p_j}{\sqrt{\frac{w}{\delta_{ji}\left(\phi_{ji}-1\right)}}}$	$\frac{\partial Q_{sc-ij}}{\partial p_j} = \frac{\lambda_{ji}}{\psi_{sc}\delta_{ji}\left(\phi_{ji}-1\right)} \frac{p_j}{\sqrt{\frac{-w}{\delta_{ji}\left(\phi_{ji}-1\right)}}}$
$\frac{\partial Q_{sc-ji}}{\partial p_i} = \frac{\lambda_{ji}}{\psi_{sc}\delta_{ji}\left(\phi_{ji}-1\right)} \frac{-\phi_{ji} p_i}{\sqrt{\frac{w}{\delta_{ji}\left(\phi_{ji}-1\right)}}}$	$\frac{\partial Q_{sc-ij}}{\partial p_i} = \frac{\lambda_{ji}}{\psi_{sc}\delta_{ji}\left(\phi_{ji}-1\right)} \frac{-\phi_{ji} p_i}{\sqrt{\frac{-w}{\delta_{ji}\left(\phi_{ji}-1\right)}}}$

Horizontal Flow$\left(z_i = z_j\right)$:

Table 3.3 Horizontal flow rate with partial derivatives

$Q_{sc-ji} = \frac{A_{ji}}{\psi_{sc}}\sqrt{\xi_{ji}\left(p_j^2 - p_i^2\right)}$ In Case: $p_j > p_i$	$Q_{sc-ij} = -\frac{A_{ji}}{\psi_{sc}}\sqrt{-\xi_{ji}\left(p_j^2 - p_i^2\right)}$ In Case: $p_j < p_i$
$\frac{\partial Q_{sc-ji}}{\partial p_j} = \left(\frac{A_{ji}}{\psi_{sc}}\right)\frac{\xi_{ji} p_j}{\sqrt{\xi_{ji}\left(p_j^2 - p_i^2\right)}}$	$\frac{\partial Q_{sc-ij}}{\partial p_j} = \left(\frac{A_{ji}}{\psi_{sc}}\right)\frac{\xi_{ji} p_j}{\sqrt{-\xi_{ji}\left(p_j^2 - p_i^2\right)}}$
$\frac{\partial Q_{sc-ji}}{\partial p_j} = \left(\frac{A_{ji}}{\psi_{sc}}\right)\frac{-\xi_{ji} p_i}{\sqrt{\xi_{ji}\left(p_j^2 - p_i^2\right)}}$	$\frac{\partial Q_{sc-ij}}{\partial p_i} = \left(\frac{A_{ji}}{\psi_{sc}}\right)\frac{-\xi_{ji} p_i}{\sqrt{-\xi_{ji}\left(p_j^2 - p_i^2\right)}}$

3.5 Equipment Flow with Partial Derivatives

The partial derivatives of non-zero Q_{ji} and Q_{ij} for the Newton-Raphson method with respect to p_j and p_i respectively are given as follows:

3.5.1 For Liquid

Table 3.4 Non-zero liquid flow rate across a pump with partial derivatives

$Q_{ji} = \sqrt{w_{ji}/b_{ji}}$ $w_{ji} = p_j - p_i + a_{ji} + \beta(z_j - z_i)$ In which: $C_{ji} = 2$	$Q_{ij} = -\sqrt{w_{ij}/b_{ij}}$ $w_{ij} = p_i - p_j + a_{ij} + \beta(z_i - z_j)$ In which: $C_{ij} = 2$
$\frac{\partial Q_{ji}}{\partial p_j} = \frac{1}{2}\sqrt{1/(b_{ji} w_{ji})}$	$\frac{\partial Q_{ij}}{\partial p_j} = \frac{1}{2}\sqrt{1/(b_{ij} w_{ij})}$
$\frac{\partial Q_{ji}}{\partial p_i} = -\frac{1}{2}\sqrt{1/(b_{ji} w_{ji})}$	$\frac{\partial Q_{ij}}{\partial p_i} = -\frac{1}{2}\sqrt{1/(b_{ij} w_{ij})}$

3.5.2 For Gas

Table 3.5 Gas flow rate across a compressor with partial derivatives

$Q_{sc-ji} = \left(\frac{1}{\psi_{sc}}\right)\frac{W_{c-ji}}{CR_{ji}}$ $CR_{ji} = \frac{k}{k-1}\zeta T_j\left[\left(\frac{p_i}{p_j}\right)^{(k-1)/k} - 1\right] + \omega_{ji}$ In which: $C_{ji} = 3$	$Q_{sc-ij} = \left(\frac{1}{\psi_{sc}}\right)\frac{-W_{c-ij}}{CR_{ij}}$ $CR_{ji} = \frac{k}{k-1}\zeta T_i\left[\left(\frac{p_j}{p_i}\right)^{(k-1)/k} - 1\right] + \omega_{ij}$ In which: $C_{ij} = 3$
$\frac{\partial Q_{sc-ji}}{\partial p_j} = \frac{T_j\zeta}{p_j\psi_{sc}}\frac{W_{c-ji}}{CR_{ji}^2}\left(\frac{p_i}{p_j}\right)^{(k-1)/k}$	$\frac{\partial Q_{sc-ij}}{\partial p_j} = \frac{T_i\zeta}{p_j\psi_{sc}}\frac{W_{c-ij}}{CR_{ij}^2}\left(\frac{p_j}{p_i}\right)^{(k-1)/k}$
$\frac{\partial Q_{sc-ji}}{\partial p_i} = \frac{T_j\zeta}{p_i\psi_{sc}}\frac{-W_{c-ji}}{CR_{ji}^2}\left(\frac{p_i}{p_j}\right)^{(k-1)/k}$	$\frac{\partial Q_{sc-ij}}{\partial p_i} = \frac{T_i\zeta}{p_i\psi_{sc}}\frac{-W_{c-ij}}{CR_{ij}^2}\left(\frac{p_j}{p_i}\right)^{(k-1)/k}$

3.6 Conversion Units

Table 3.6 British and SI units

Quantity	British units	SI units
p, p_{sc}	psig	bar
Q	gpm	m^3/hr
Q_{sc}	MMscfd	MMscmd
ρ	lb_m/ft^3	kg/m^3
μ	centipoise	mPa – s
L	ft	m
D	inch	mm
z	ft	m
ε	ft	mm
g	ft/sec^2	m/sec^2
T, T_{sc}	^0F	^0C
M	lb_m	kg
R	ft lb_f/lb mole ^0R	J/kmole ^0K
Z_{avg}, Z_{sc}	none	none
f_F	none	none
Re	none	none

3.6.1 For Liquid

Table 3.7 Conversion units for α_{ji}, β and Re_{ji}

British units	SI units
$\alpha_{ji} = \frac{32*(12)^5*\rho L_{ji}}{\pi^2*144*32.2*(7.48*60)^2*D_{ji}^5}$	$\alpha_{ji} = \frac{32*10^{15}*\rho L_{ji}}{\pi^2*(3600)^2*1.01325*10^5*D_{ji}^5}$
$\beta = \frac{\rho}{144}$	$\beta = \frac{9.81*\rho}{1.01325*10^5}$
$Re_{ji} = \frac{4*12*10^5*\rho Q_{ji}}{7.48*60*32.2*2.089*\pi\mu D_{ji}}$	$Re_{ji} = \frac{4*10^6*\rho Q_{ji}}{3600*\pi\mu D_{ji}}$

3.6.2 For Gas

Inclined Flow $(z_i \neq z_j)$

Table 3.8 Conversion units for λ_{ji}, δ_{ji} and ϕ_{ji}

British units	SI units
$\lambda_{ji} = \left(\frac{24*60*60}{(10)^6*(12)^2*Z_{avg}T}\right)\left(\frac{\pi D_{ji}^2}{4}\right)$	$\lambda_{ji} = \left(\frac{24*3600}{(10)^6*(10)^6*Z_{avg}T}\right)\left(\frac{\pi D_{ji}^2}{4}\right)$
$\delta_{ji} = \left(\frac{2*f_{F_{ji}}L_{ji}}{(32.2*12)^3*D_{ji}(z_i - z_j)}\right)$	$\delta_{ji} = \left(\frac{2*10^3*f_{F_{ji}}L_{ji}}{9.81*(1.01325*10^5)^2*D_{ji}(z_i - z_j)}\right)$
$\phi_{ji} = \exp\left(\frac{2*M(z_i - z_j)}{1545.3*Z_{avg}T}\right)$	$\phi_{ji} = \exp\left(\frac{2*9.81*M(z_i - z_j)}{8314.3*Z_{avg}T}\right)$
$\psi_{sc} = \frac{32.2*(12)^2 p_{sc}}{Z_{sc}T_{sc}}$	$\psi_{sc} = \frac{1.01325*10^5*p_{sc}}{Z_{sc}T_{sc}}$

Horizontal Flow $(z_i = z_j)$

Table 3.9 Conversion units for A_{ji}, ξ_{ji} and ψ_{sc}

British units	SI units
$A_{ji} = \frac{24*60*60*\pi D_{ji}^2}{(10)^6*4*144}$	$A_{ji} = \frac{24*3600*\pi D_{ji}^2}{(10)^6*4*10^6}$
$\xi_{ji} = \left(\frac{32.2*(12)^3*MD_{ji}}{4*1545.3*Z_{avg}Tf_{F_{ji}}L_{ji}}\right)$	$\xi_{ji} = \left(\frac{(1.01325*10^5)^2 MD_{ji}}{4*8314.3*10^3*Z_{avg}Tf_{F_{ji}}L_{ji}}\right)$
$\psi_{sc} = \frac{(12)^2*p_{sc}M}{1545.3*Z_{sc}T_{sc}}$	$\psi_{sc} = \frac{1.01325*10^5*p_{sc}M}{8314.3*Z_{sc}T_{sc}}$

Note:

$^\circ R = {}^\circ F + 459.67$, $\quad ^\circ K = {}^\circ C + 273.15$

$p_{absolute} = p_{gauge} + 14.73$, $\quad p_{avg-ji} = \frac{2}{3}\left(\frac{p_j^3 - p_i^3}{p_j^2 - p_i^2}\right)$

British unit:

$$Re_{ji} = \left(\frac{4*12*10^5*(12)^2*(10)^6*Q_{sc-ji}}{32.2*2.089*1545.3*24*3600*\pi\mu D_{ji}}\right)\left(\frac{p_{avg-ji}M}{Z_{avg}T}\right)$$

SI unit:

$$Re_{ji} = \left(\frac{4*10^6*1.01325*(10)^5*(10)^6*Q_{sc-ji}}{8314.3*24*3600*\pi\mu D_{ji}}\right)\left(\frac{p_{avg-ji}M}{Z_{avg}T}\right)$$

3.7 Nodal Material Balance Equations

The nodal material balance equations for all nodes i at which the pressure p_i, is not specified (for $T_i \neq 1$) can be described as follows:

For steady-state, the sum of the flows into any node i must be zero. That is:

$$F_i(\mathbf{P}) = 0, \tag{3.31}$$

Here:

$F_i(\mathbf{P})$ is the net flow into any node i.

$\mathbf{P} = [p_1, p_2, \ldots, p_n]^t$

3.7.1 For Liquid

$F_i(\mathbf{P}) =$ injection rate (or withdrawal rate) (3.32)

+ net flow in from neighboring nodes to i by pipeline

+ net flow in from neighboring nodes to i from pumps

- net flow out to neighboring nodes from i through pumps

$= 0$

The equation for $F_i(\mathbf{P})$ becomes:

$$\begin{aligned} F_i(\mathbf{P}) = {} & V_i(\text{injection (positive) or withdrawal (negative) rate}) \\ & + \sum_{j,C_{ji}=1} Q_{ji}(\text{two pipeline cases}) \\ & + \sum_{j,C_{ji}=2} Q_{ji}(\text{three cases for pumping in}) \\ & + \sum_{j,C_{ij}=2} Q_{ji}(\text{three cases for pumping out}) \end{aligned} \tag{3.33}$$

3.7.2 For Gas

$$F_i(\mathbf{P}) = \text{injection rate (or withdrawal rate)} \tag{3.34}$$

+ net flow in from neighboring nodes to i by pipeline

+ net flow in from neighboring nodes to i from compressors

- net flow out to neighboring nodes from i through compressors

$= 0$

The equation for $F_i(\mathbf{P})$ becomes:

$$\begin{aligned} F_i(\mathbf{P}) = {} & V_i(\text{injection(positive) or withdrawal (negative) rate}) \\ & + \sum_{j, C_{ji}=1} Q_{ji}(\text{two pipeline cases for inclined flow}) \\ & + \sum_{j, C_{ji}=1} Q_{ji}(\text{two pipeline cases for horizontal flow}) \\ & + \sum_{j, C_{ji}=3} Q_{ji}(\text{the case for compressor coming in}) \\ & + \sum_{j, C_{ij}=3} Q_{ji}(\text{the case for compressor going out}) \end{aligned} \tag{3.35}$$

3.8 Newton-Raphson Method

The simultaneous nonlinear equations in the unknown pressures that are obtained from nodal material balances at all nodes i are solved by the iterative Newton-Raphson method as follows:

1. Suppose we have an initial estimate of $f_{F_{ji}}$ for all connections between node j and node i such that $C_{ji} = 1$.

2. Suppose we also know the approximate and specified pressure p_i, at all nodes i.

3. The next step is to find the appropriate partial derivatives of the functions $F_i(\mathbf{P})$, $(i = 1, 2, \ldots, n)$ with respect to p_j, $(j = 1, 2, \ldots, n)$ which are then stored as the elements of the left hand side coefficient matrix, $\boldsymbol{\Phi}$ of the simultaneous linear equations:

$$\boldsymbol{\Phi}(\mathbf{P})\delta\mathbf{P} = -\mathbf{F}(\mathbf{P}) \qquad (3.36)$$

In Eqn. (3.36), the right hand side vector is defined as:

$$\mathbf{F}(\mathbf{P}) = \left[F_1(\mathbf{P}),\ F_2(\mathbf{P}),\ F_3(\mathbf{P}),\ \ldots,\ F_n(\mathbf{P})\right]^t \qquad (3.37)$$

where the correction vector $\delta\mathbf{P}$ is the solution of the simultaneous linear equations and a representative element of the coefficient matrix is:

$$\boldsymbol{\Phi}(\mathbf{P}) = F_{ij}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_j}, \qquad 1 \le i, j \le n \qquad (3.38)$$

3.8.1 <u>For Liquid</u>

The partial derivative of $F_i(\mathbf{P})$ with respect to p_j is given by one of following forms:

$$F_{ij}(\mathbf{P}) = \frac{\partial Q_{ji}}{\partial p_j} = \left\{ \begin{array}{ll} 0.5\sqrt{1/\left(\alpha_{ji} f_{F_{ji}} y\right)} & \text{for } y>0 \\ 0.5\sqrt{-1/\left(\alpha_{ji} f_{F_{ji}} y\right)} & \text{for } y<0 \end{array} \right\} \text{ if } C_{ji} = 1$$

$$F_{ij}(\mathbf{P}) = \frac{\partial Q_{ji}}{\partial p_j} = \frac{1}{2}\sqrt{1/\left(b_{ji} w_{ji}\right)} \qquad \text{if } C_{ji} = 2$$

$$F_{ij}(\mathbf{P}) = \frac{\partial Q_{ji}}{\partial p_j} = \frac{1}{2}\sqrt{1/\left(b_{ij} w_{ij}\right)} \qquad \text{if } C_{ij} = 2$$

Here $(i \neq j)$:

$$F_{ij}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_j}, \qquad 1 \le i, j \le n \qquad (3.39)$$

The partial derivatives of $F_i(\mathbf{P})$ with respect to p_i are given by summation as follows:

$$\begin{aligned} F_{ii}(\mathbf{P}) = & \sum_{j,C_{ji}=1} \frac{\partial Q_{ji}}{\partial p_i}(\text{two pipeline cases}) \\ & + \sum_{j,C_{ji}=2} \frac{\partial Q_{ji}}{\partial p_i}(\text{three cases for pumping in}) \\ & + \sum_{j,C_{ij}=2} \frac{\partial Q_{ji}}{\partial p_i}(\text{three cases for pumping out}) \end{aligned} \qquad (3.40)$$

$$\begin{aligned} F_{ii}(\mathbf{P}) = & \begin{cases} \sum_{j,C_{ji}=1} -0.5\sqrt{1/\left(\alpha_{ji} f_{F_{ji}} y\right)} & \text{for} \quad y > 0 \\ \sum_{j,C_{ji}=1} -0.5\sqrt{-1/\left(\alpha_{ji} f_{F_{ji}} y\right)} & \text{for} \quad y < 0 \end{cases} \\ & + \sum_{j,C_{ji}=2} -\frac{1}{2}\sqrt{1/\left(b_{ji} w_{ji}\right)} \\ & + \sum_{j,C_{ij}=2} -\frac{1}{2}\sqrt{1/\left(b_{ij} w_{ij}\right)} \end{aligned} \qquad (3.41)$$

Here:

$$F_{ii}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_i}, \qquad 1 \le i \le n \qquad (3.42)$$

3.8.2 For Gas

The partial derivative of $F_i(\mathbf{P})$ with respect to p_j is given by one of following forms:

$$F_{ij}(\mathbf{P})=\frac{\partial Q_{sc-ji}}{\partial p_j}=\begin{cases} z_i\neq z_j\begin{cases}\dfrac{\lambda_{ji}}{\psi_{sc}\delta_{ji}(\phi_{ji}-1)}\dfrac{p_j}{\sqrt{\dfrac{w}{\delta_{ji}(\phi_{ji}-1)}}} & \text{for } w>0\\ \dfrac{\lambda_{ji}}{\psi_{sc}\delta_{ji}(\phi_{ji}-1)}\dfrac{p_j}{\sqrt{\dfrac{-w}{\delta_{ji}(\phi_{ji}-1)}}} & \text{for } w<0\end{cases}\\ z_i=z_j\begin{cases}\left(\dfrac{A_{ji}}{\psi_{sc}}\right)\dfrac{\xi_{ji}p_j}{\sqrt{\xi_{ji}(p_j^2-p_i^2)}} & \text{for } p_j>p_i\\ \left(\dfrac{A_{ji}}{\psi_{sc}}\right)\dfrac{\xi_{ji}p_j}{\sqrt{-\xi_{ji}(p_j^2-p_i^2)}} & \text{for } p_j<p_i\end{cases}\end{cases} \quad \text{if } C_{ji}=1$$

$$F_{ij}(\mathbf{P})=\frac{\partial Q_{sc-ji}}{\partial p_j}=\frac{T_j\zeta}{p_j\psi_{sc}}\frac{W_{c-ji}}{\left\{\frac{k}{k-1}\zeta T_j\left[\left(\frac{p_i}{p_j}\right)^{(k-1)/k}-1\right]+\omega_{ji}\right\}^2}\left(\frac{p_i}{p_j}\right)^{(k-1)/k} \quad \text{if } C_{ji}=3$$

$$F_{ij}(\mathbf{P})=\frac{\partial Q_{sc-ji}}{\partial p_j}=\frac{T_i\zeta}{p_j\psi_{sc}}\frac{W_{c-ij}}{\left\{\frac{k}{k-1}\zeta T_i\left[\left(\frac{p_j}{p_i}\right)^{(k-1)/k}-1\right]+\omega_{ij}\right\}^2}\left(\frac{p_j}{p_i}\right)^{(k-1)/k} \quad \text{if } C_{ij}=3$$

The partial derivatives of $F_i(\mathbf{P})$ with respect to p_i are given by summation as follows:

$$F_{ii}(\mathbf{P})=\sum_{j,C_{ji}=1}\frac{\partial Q_{ji}}{\partial p_i}(\text{two pipeline cases, for both inclined and horizontal flow})$$

$$+\sum_{j,C_{ji}=3}\frac{\partial Q_{ji}}{\partial p_i}(\text{the case for compressor coming in})$$

$$+\sum_{j,C_{ij}=3}\frac{\partial Q_{ji}}{\partial p_i}(\text{the case for compressor going out})$$

$$
F_{ii}(\mathbf{P}) = \sum_{j,C_{ji}=1}
\begin{cases}
z_i \neq z_j \begin{cases}
\dfrac{\lambda_{ji}}{\psi_{sc}\delta_{ji}\left(\phi_{ji}-1\right)} \dfrac{-\phi_{ji} p_i}{\sqrt{\dfrac{w}{\delta_{ji}\left(\phi_{ji}-1\right)}}} & \text{for } w>0 \\[2ex]
\dfrac{\lambda_{ji}}{\psi_{sc}\delta_{ji}\left(\phi_{ji}-1\right)} \dfrac{-\phi_{ji} p_i}{\sqrt{\dfrac{-w}{\delta_{ji}\left(\phi_{ji}-1\right)}}} & \text{for } w<0
\end{cases} \\[4ex]
z_i = z_j \begin{cases}
\dfrac{A_{ji}}{\psi_{sc}} \dfrac{-\xi_{ji} p_i}{\sqrt{\xi_{ji}\left(p_j^2 - p_i^2\right)}} & \text{for } p_j > p_i \\[2ex]
\dfrac{A_{ji}}{\psi_{sc}} \dfrac{-\xi_{ji} p_i}{\sqrt{-\xi_{ji}\left(p_j^2 - p_i^2\right)}} & \text{for } p_j < p_i
\end{cases}
\end{cases}
$$

$$
+ \sum_{j,C_{ji}=3} \frac{T_j}{p_i} \frac{\zeta}{\psi_{sc}} \frac{-W_{c-ji}}{\left\{\frac{k}{k-1}\zeta T_j\left[\left(\frac{p_i}{p_j}\right)^{(k-1)/k} - 1\right] + \omega_{ji}\right\}^2} \left(\frac{p_i}{p_j}\right)^{(k-1)/k}
$$

$$
+ \sum_{j,C_{ij}=3} \frac{T_i}{p_i} \frac{\zeta}{\psi_{sc}} \frac{-W_{c-ij}}{\left\{\frac{k}{k-1}\zeta T_i\left[\left(\frac{p_j}{p_i}\right)^{(k-1)/k} - 1\right] + \omega_{ij}\right\}^2} \left(\frac{p_j}{p_i}\right)^{(k-1)/k}
$$

4. Use LU decomposition of the Gaussian elimination method with column pivoting only to solve the simultaneous linear equation with $\Phi(\mathbf{P})$ as the left hand side coefficient matrix.

5. Back substitution to find out the correction vector $\delta\mathbf{P}$. Also using a mathematical technique to improve the stability of the method at all nodes i by factor σ_i, in the correction as follows:

$$\delta p_i = \delta p_i^{*} \sigma_i \tag{3.43}$$

Where:

δp_i is the value of the correction actually applied.

δp_i^* is the value of the correction computed from the Newton-Raphson method.

It is recommended that $\sigma_i = 0.5$ is the best value to use in order to ensure convergence for the first iteration. In subsequent iterations, the value of "σ_i" is determined as below:

For	$A_i \le -1$	$\sigma_i = 0.5\lvert A_i \rvert$
For	$-1 < A_i < 0$	$\sigma_i = 0.4 - 0.15\lvert A_i \rvert$
For	$0 < A_i < 1$	$\sigma_i = 0.4 + 0.15\lvert A_i \rvert$
For	$A_i \ge 1$	$\sigma_i = 0.5$

Here A_i is computed by using the δp_i for the current and previous iterations as follows:

$$A_i = \frac{\delta p_i^{k+1}}{\delta p_i^k} \qquad i = 1,\ 2,\ \ldots,\ n \qquad (3.44)$$

In which:

δp_i^{k+1} is the correction to p_i for the current iteration.

δp_i^k is the correction of p_i for the previous iteration.

Note:

The user has to do some experimentation to obtain the coefficients of c_1, c_2, c_3, c_4, c_5 and c_6 for the factor σ_i, in accordance with his or her own system. (generally, $0.0 \le c_1,\ c_2,\ c_3,\ c_4,\ c_5,\ c_6 \le 1.0$)

6. Check for convergence after the corrections δp_i have been made at all nodes i (improved by the factor to avoid instability in item 5. if necessary) according to some criterion such as:

$$\lvert \delta p_i \rvert < \lambda \qquad i = 1,\ 2,\ \ldots,\ n \qquad (3.45)$$

7. If the corrections $\delta\mathbf{P}$ do not satisfy the convergence condition, the current vector of pressures is modified according to:

$$\mathbf{P}_{k+1} = \mathbf{P}_k + \delta\mathbf{P}_k \tag{3.46}$$

Here:

$\mathbf{P}_k$ is the current vector (or set) of pressures.

$\delta\mathbf{P}_{k+1}$ is the updated set of pressures for use the next iteration.

$\delta\mathbf{P}_k$ is the set of pressure corrections just computed.

8. With these new pressures $\mathbf{P}_{k+1}$, from equation (3.46), the updated flow rates Q_{ji}, can be calculated with the old Fanning friction factor $f^k_{F_{ji}}$, for all pipeline segments as follows:

For Liquid:

$$Q_{ji} = \sqrt{y / \left(\alpha_{ji} f_{F_{ji}}\right)} \quad \text{for} \quad y > 0 \tag{3.47}$$

$$Q_{ij} = -\sqrt{-y / \left(\alpha_{ji} f_{F_{ji}}\right)} \quad \text{for} \quad y < 0 \tag{3.48}$$

For Gas:

Inclined Flow$\left(z_i \neq z_j\right)$:

$$Q_{sc-ji} = \frac{\lambda_{ji}}{\psi_{sc}} \sqrt{\frac{w}{\delta_{ji}\left(\phi_{ji} - 1\right)}} \quad \text{for} \quad w > 0 \tag{3.49}$$

$$Q_{sc-ij} = -\frac{\lambda_{ji}}{\psi_{sc}} \sqrt{\frac{-w}{\delta_{ji}\left(\phi_{ji} - 1\right)}} \quad \text{for} \quad w < 0 \tag{3.50}$$

Horizontal Flow$\left(z_i = z_j\right)$:

$$Q_{sc-ji} = \frac{A_{ji}}{\psi_{sc}} \sqrt{\xi_{ji}\left(p_j^2 - p_i^2\right)} \quad \text{for} \quad p_j > p_i \tag{3.51}$$

$$Q_{sc-ij} = -\frac{A_{ji}}{\psi_{sc}} \sqrt{-\xi_{ji}\left(p_j^2 - p_i^2\right)} \quad \text{for} \quad p_j < p_i \tag{3.52}$$

9. The Reynolds numbers Re_{ji}, are computed for all pipeline segments as follows:

For Liquid:

$$Re_{ji} = \frac{4 * 12 * 10^5 * \rho Q_{ji}}{7.48 * 60 * 32.2 * 2.089 * \pi \mu D_{ji}} \quad \text{for British units}$$

$$Re_{ji} = \frac{4 \times 10^6 \rho Q_{ji}}{3600 \pi \mu D_{ji}} \quad \text{for SI units}$$

For Gas:

British units:

$$Re_{ji} = \frac{4 * 12 * 10^5 * (12)^2 * Q_{ji}}{7.48 * 60 * 32.2 * 2.089 * 1545.3 * \pi \mu D_{ji}} \frac{p_{avg-ji} M}{Z_{avg} T}$$

SI units:

$$Re_{ji} = \frac{4 * 10^6 * 1.01325 * 10^5 * Q_{ji}}{3600 * 8314.3 * \pi \mu D_{ji}} \frac{p_{avg-ji} M}{Z_{avg} RT}$$

10. The program updates the Fanning friction factors $f^k_{F_{ji}}$, as functions of the Reynolds number and roughness ratio in all pipeline segments as follows:

For turbulent flow $\left(Re_{ji} > 4000\right)$:

$$f_{F_{ji}} = \left\{ -1.737 \ln\left[0.269 \frac{\varepsilon_{ji}}{D_{ji}} - \frac{2.185}{Re_{ji}} \ln\left(0.269 \frac{\varepsilon_{ji}}{D_{ji}} + \frac{14.5}{Re_{ji}} \right) \right] \right\}^{-2} \quad (3.53)$$

For laminar flow $\left(Re_{ji} \le 2000\right)$:

$$f_{F_{ji}} = \frac{16}{Re_{ji}} \quad (3.54)$$

11. The sequence of calculations given above is repeated for successive iterations k = 1, 2, 3 .... until convergence occurs according to some predetermined criterion such as:

$$\left|(\mathbf{P}_{k+1})_i - (\mathbf{P}_k)_i\right| < \lambda \qquad \text{for all } i = 1, 2, \ldots n \qquad (3.55)$$

or until a specified maximum number of iterations k_{max}, has been exceed.

12. If the ith node type is $T_i = 1$ (pressure specified), it can be included in the Newton-Raphson method by using it as an unknown in the simultaneous linear equations and setting its correction $\delta \mathbf{P}$, to zero. This is achieved by setting:

$$\left.\begin{aligned} F_i(\mathbf{P}) &= 0 \\ F_{ii}(\mathbf{P}) &= 1 \\ F_{ij}(\mathbf{P}) &= 0 \end{aligned}\right\} \text{ For } T_i = 1 \qquad (3.56)$$

3.9 Terminal Node with Specified Injection Rate

Consider the special case of a terminal node i with a specified injection rate V_i, as follows:

For Liquid:

The net flow into terminal node i must equal zero, so that:

$$Q_{ji} = -V_i \qquad (3.57)$$

In the case of pipeline connection, the flow rates from Eqns. (3.1) and (3.2) can be represented by one equation instead of two as follows:

$$y = \pm\alpha_{ji} f_{F_{ji}} Q_{ji}^2 \quad (3.58)$$

$$y = -\alpha_{ji} f_{F_{ji}} V_i |V_i| \quad (3.59)$$

Define:

$$F_i(\mathbf{P}) = -\alpha_{ji} f_{F_{ji}} V_i |V_i| - y = 0 \quad (3.60)$$

Therefore:

$$-F_i(\mathbf{P}) = \alpha_{ji} f_{F_{ji}} V_i |V_i| + y \quad (3.61)$$

The partial derivatives of the function $F_i(\mathbf{P})$ with respect to p_j and p_i are given by:

$$F_{ij}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_j} = -1 \quad (3.62)$$

$$F'_{ii}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_i} = 1 \quad (3.63)$$

For Gas:

The net flow into terminal node i at standard conditions must equal zero, thus:

$$Q_{sc-ji} = -V_i \quad (3.64)$$

Inclined Flow $(z_i \neq z_j)$:

For pipeline connection, the flow rates from Eqns. (3.5) and (3.6) can be placed by one equation as follows:

$$Q_{sc-ji}^2 = \pm\frac{\lambda_{ji}^2}{\psi_{sc}^2} \frac{w}{\delta_{ji}(\phi_{ji} - 1)} \quad (3.65)$$

$$-V_i|V_i| = \left(\frac{\lambda_{ji}}{\psi_{sc}}\right)^2 \frac{w}{\delta_{ji}(\phi_{ji} - 1)} \quad (3.66)$$

Rearrangement gives:

$$-\psi_{sc}^2 V_i|V_i| = \frac{\lambda_{ji}^2 w}{\delta_{ji}(\phi_{ji}-1)} \tag{3.67}$$

Define:

$$F_i(\mathbf{P}) = -\psi_{sc}^2 V_i|V_i| - \frac{\lambda_{ji}^2 w}{\delta_{ji}(\phi_{ji}-1)} = 0 \tag{3.68}$$

Therefore:

$$-F_i(\mathbf{P}) = \psi_{sc}^2 V_i|V_i| + \frac{\lambda_{ji}^2 w}{\delta_{ji}(\phi_{ji}-1)} \tag{3.69}$$

The partial derivatives of the function $F_i(\mathbf{P})$ with respect to p_j and p_i are given by:

$$F_{ij}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_j} = \frac{-2\lambda_{ji}^2 p_j}{\delta_{ji}(\phi_{ji}-1)} \tag{3.70}$$

$$F_{ii}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_i} = \frac{2\lambda_{ji}^2 \phi_{ji} p_i}{\delta_{ji}(\phi_{ji}-1)} \tag{3.71}$$

Horizontal Flow$(z_i = z_j)$:

In the same manner of inclined flow, the flow rates from Eqns. (3.12) and (3.13) can be reduced to one equation as follows:

$$Q_{sc-ji} = \pm \frac{A_{ji}^2}{\psi_{sc}^2} \xi_{ji}(p_j^2 - p_i^2) \tag{3.72}$$

$$-V_i|V_i| = \left(\frac{A_{ji}}{\psi_{sc}}\right)^2 \xi_{ji}(p_j^2 - p_i^2) \tag{3.73}$$

Rearrangement gives:

$$-\psi_{sc}^2 V_i|V_i| = A_{ji}^2 \xi_{ji}(p_j^2 - p_i^2) \tag{3.74}$$

Define:

$$F_i(\mathbf{P}) = -\psi_{sc}^2 V_i|V_i| - A_{ji}^2\xi_{ji}\left(p_j^2 - p_i^2\right) = 0 \tag{3.75}$$

Thus:

$$-F_i(\mathbf{P}) = \psi_{sc}^2 V_i|V_i| + A_{ji}^2\xi_{ji}\left(p_j^2 - p_i^2\right) \tag{3.76}$$

The partial derivatives of the function $F_i(\mathbf{P})$ with respect to p_j and p_i are given by:

$$F_{ij}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_j} = -2p_j\xi_{ji}A_{ji}^2 \tag{3.77}$$

$$F_{ii}(\mathbf{P}) = \frac{\partial F_i(\mathbf{P})}{\partial p_i} = 2p_i\xi_{ji}A_{ji}^2 \tag{3.78}$$

3.10 FORTRAN Language

A FORTRAN program (Power Station Version 1.0) is written to accept the above information concerning any network of nodes i and use the Newton-Raphson iterative technique to compute the unknown nodal pressures at all nodes i of node type $T_i = 0$ or $T_i = 3$. The output displays a set of matrices containing the internodal flow rates Q_{ji}, nodal pressures and the Fanning friction factors $f_{F_{ji}}^k$, for all pipeline segments.

3.11 Program Description

A general flow diagram of the program is shown in Fig. 3.1 Subroutine SGEM is used to solve the simultaneous linear equations generated at each new iteration of Newton-Raphson method. Subroutine UP is implemented to generate the next estimates of fanning friction factor after no convergence test.

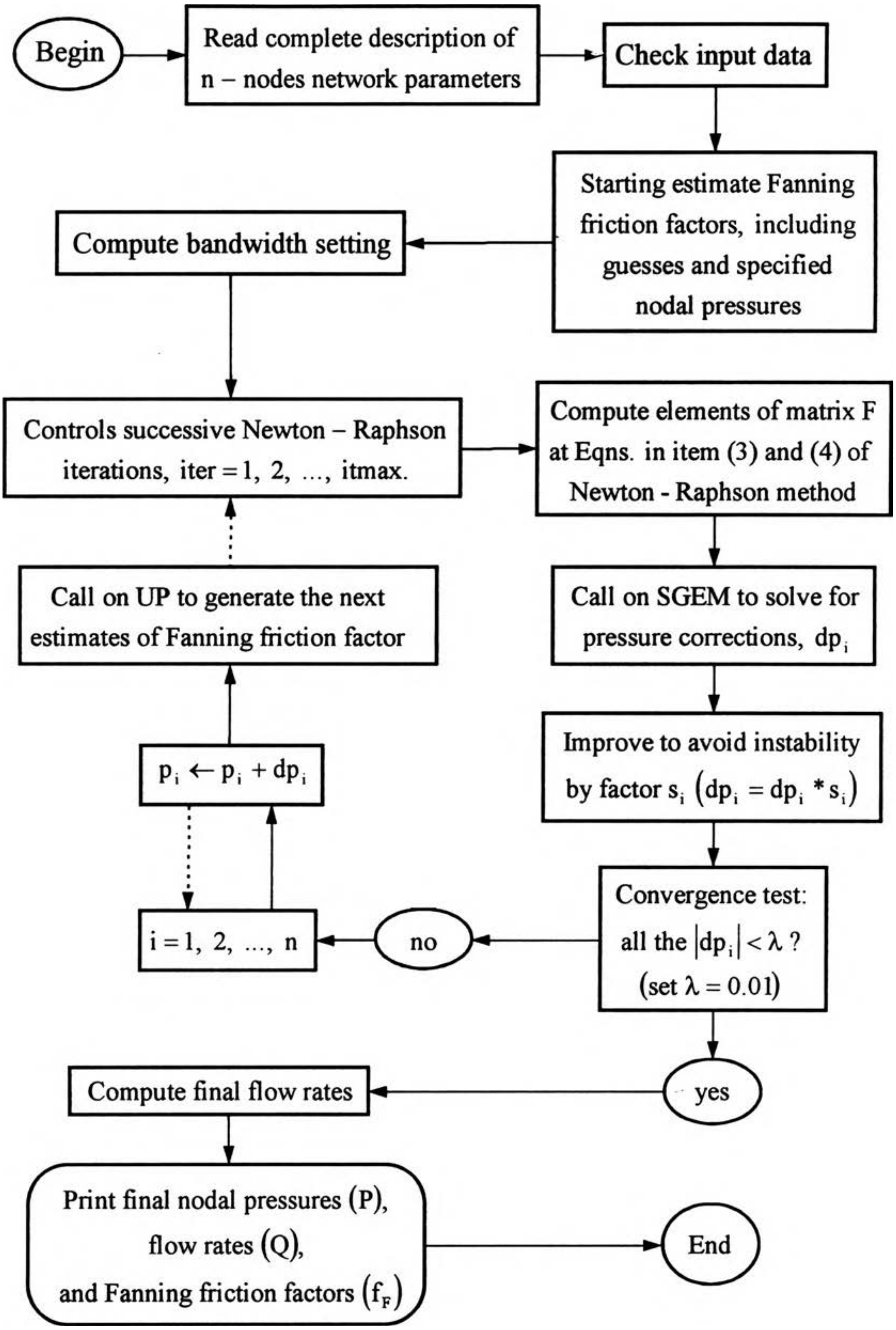

Fig. 3.1 A general flow diagram for fluid network analysis program.