CHAPTER I
INTRODUCTION

Linear partial differential equations (PDEs) of the second order are frequently classified as of elliptic, hyperbolic or parabolic type. Such a classification is possible if the equation has been transformed to the following form.

$$\sum_{i=1}^{n} A_i \frac{\partial^2 u}{\partial x_i^2} + \sum_{i=1}^{n} B_i \frac{\partial u}{\partial x_i} + Cu + D = 0 \tag{1.1}$$

In eq.(1.1) the coefficients A_i , evaluated at the point $(x_1,x_2,\ldots,x_n)$, may be 1 , -1 or zero. Here, u is the dependent variable, and the x_i are the independent variables. The following are the main possibilities of interest :

1. If all the A_i are nonzero and have the same sign, the PDE is of elliptic type.
2. If all the A_i are nonzero and have, with one exception, the same sign, the PDE is of hyperbolic type.
3. If one (e.g. A_k) A_i is zero and the remaining A_i are nonzero with the same sign, and if the coefficient B_k is nonzero, the PDE is parabolic type.

Elliptic and parabolic equations are partial differential equations which are used for modeling engineering problems such as heat transfer, boundary-layer flow, diffusion, etc.

Examples of Partial Differential Equations are

1. Unsteady Heat-Conduction Equation in a flat plate is shown below;

$$\rho c_p \frac{\partial T}{\partial t} = k[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}] \tag{1.2}$$

where T denotes temperature and k, ρ, and Cp are the thermal conductivity, density, and specific heat of the plate, material, x and y are the space variables, and t is time.

2. Vorticity Transport Equation

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = \nu \nabla^2 \zeta \tag{1.3}$$

Here, ζ is vorticity, u and v are the x and y velocity components, and ν is kinematic viscosity.

3. Laminar Flow Heat-Exchanger Equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho u C_p}{k}\frac{\partial T}{\partial z} \tag{1.4}$$

Here T is temperature, ρ. Cp. and k are the density, specific heat, and thermal conductivity of the fluid, and the axial velocity u is a known function of radius,r.

4. Steady state two-dimensional heat conduction in solids for which the PDE is known as Laplace's Equation, is shown in eq.(1.5).

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1.5}$$