CHAPTER IV
PROGRAM TESTING

4.1 Test Program I - Unsteady-state of Heat Conduction in an Infinite, Parallel-sided Slab (Parabolic Problem)

4.1.1 Problem

This problem is to solve for the time-dependent temperature of an infinite parallel-sided slab ($0 \leq x \leq L$) with the initial uniform temperature,θ_0. Both sides of the slab are subsequently maintained at a constant temperature,θ_1. The temperature, θ, inside the slab is found at different time and position.

The heat conduction equation is

$$\alpha \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial \theta}{\partial t} \tag{4.1}$$

where α is the thermal diffusivity, $k/\rho C_p$

By defining the dimensionless variables

$$T = \frac{\theta - \theta_0}{\theta_1 - \theta_0} \tag{4.2}$$

$$\tau = \frac{\alpha t}{L^2} \tag{4.3}$$

$$X = \frac{x}{L} \tag{4.4}$$

The problem can be rewritten as that of solving

$$\frac{\partial^2 T}{\partial X^2} = \frac{\partial T}{\partial \tau} \tag{4.5}$$

initial condition $\tau = 0 :\ T = 0$ for $0 \leq X \leq 1$

boundary condition $\tau > 0 :\ T = 1$ at $X = 0$ and $X = 1$

4.1.2 Method of Using Program

- Select one dimensional parabolic equation

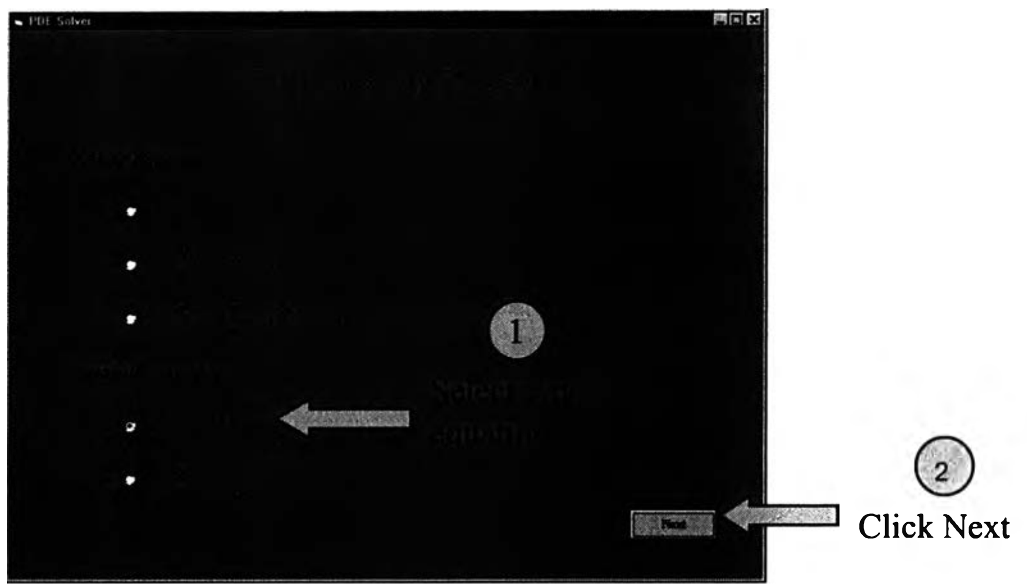


Figure 4.1 Selecting the type of equation for test program I.

- Select a method and enter all necessary values

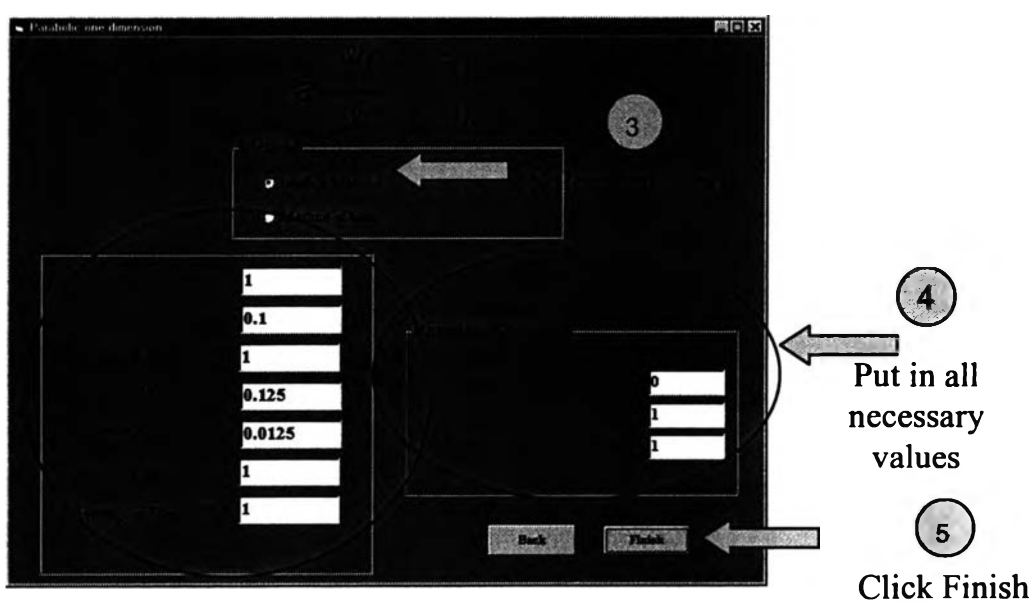
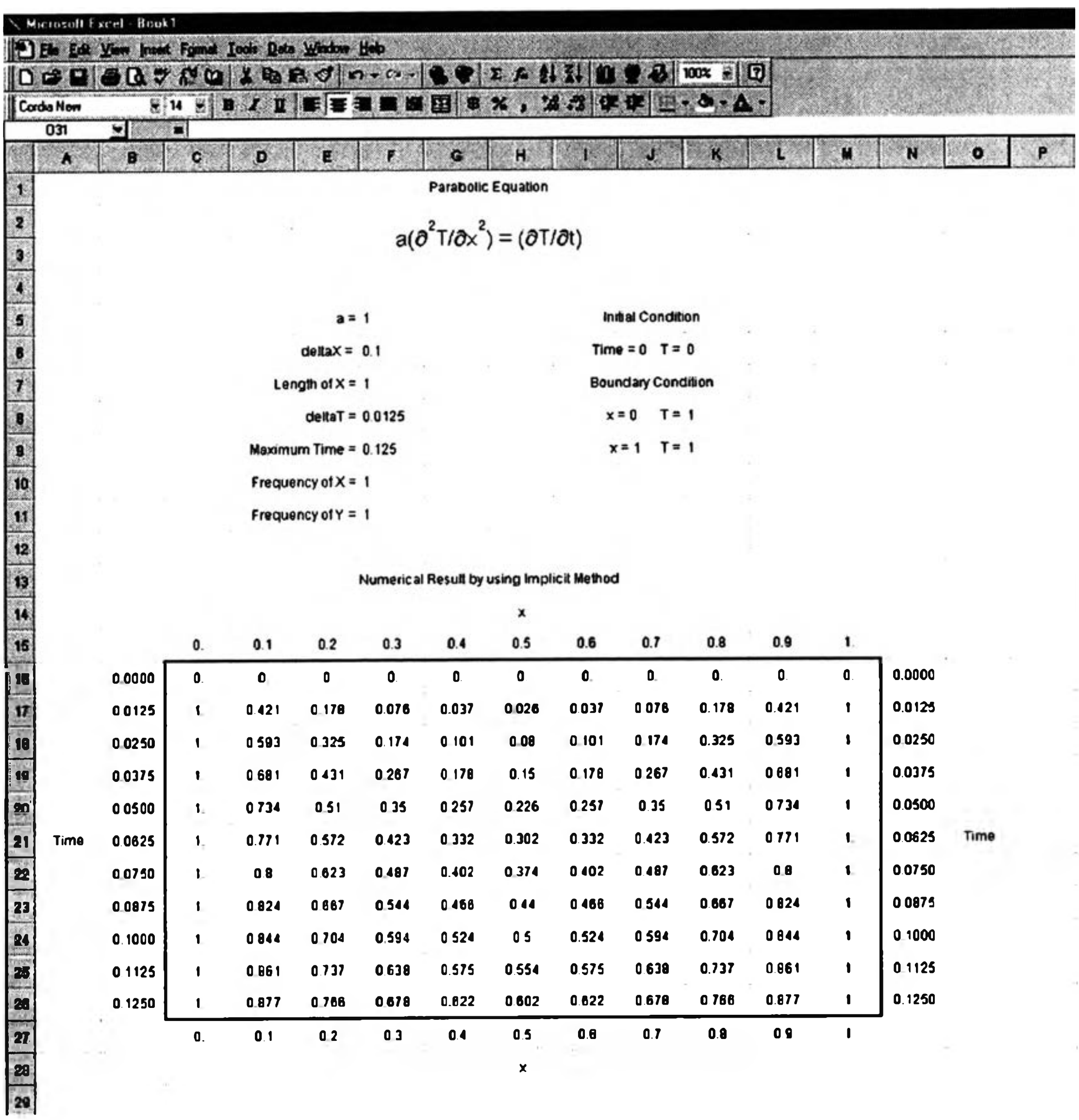


Figure 4.2 Selecting a method and entering all the necessary values for test program I.

4.1.3 <u>Results</u>

4.1.3.1 Numerical Results

The numerical results are shown below in the tabular form.


Parabolic Equation

$$a(\partial^2 T/\partial x^2) = (\partial T/\partial t)$$

a = 1
deltaX = 0.1
Length of X = 1
deltaT = 0.0125
Maximum Time = 0.125
Frequency of X = 1
Frequency of Y = 1

Initial Condition
Time = 0 T = 0
Boundary Condition
x = 0 T = 1
x = 1 T = 1

Numerical Result by using Implicit Method

Time \ x	0.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.	Time
0.0000	0.	0	0	0	0	0	0	0	0	0	0	0.0000
0.0125	1	0.421	0.178	0.076	0.037	0.026	0.037	0.076	0.178	0.421	1	0.0125
0.0250	1	0.593	0.325	0.174	0.101	0.08	0.101	0.174	0.325	0.593	1	0.0250
0.0375	1	0.681	0.431	0.267	0.178	0.15	0.178	0.267	0.431	0.681	1	0.0375
0.0500	1	0.734	0.51	0.35	0.257	0.226	0.257	0.35	0.51	0.734	1	0.0500
0.0625	1	0.771	0.572	0.423	0.332	0.302	0.332	0.423	0.572	0.771	1	0.0625
0.0750	1	0.8	0.623	0.487	0.402	0.374	0.402	0.487	0.623	0.8	1	0.0750
0.0875	1	0.824	0.667	0.544	0.466	0.44	0.466	0.544	0.667	0.824	1	0.0875
0.1000	1	0.844	0.704	0.594	0.524	0.5	0.524	0.594	0.704	0.844	1	0.1000
0.1125	1	0.861	0.737	0.638	0.575	0.554	0.575	0.638	0.737	0.861	1	0.1125
0.1250	1	0.877	0.766	0.678	0.622	0.602	0.622	0.678	0.766	0.877	1	0.1250
	0.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	

Figure 4.3 The Microsoft Excel numerical results for test program I.

4.1.3.2 Graphical Results

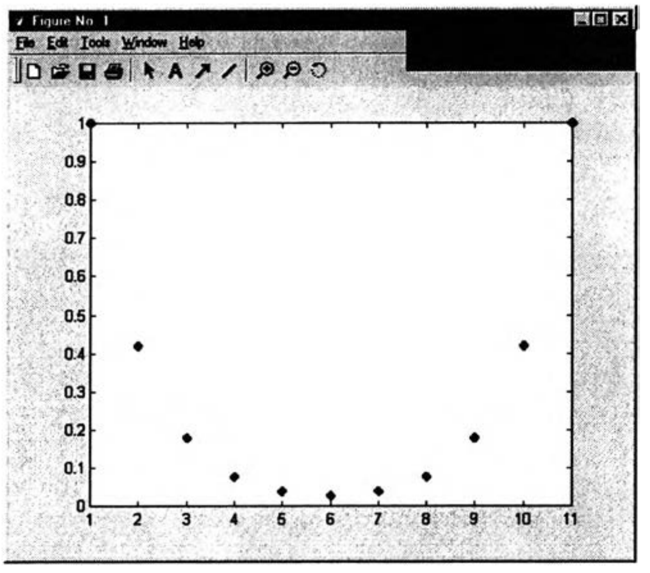


Figure 4.4 The MATLAB graphical results for test program I at time t = 0.0125 sec.

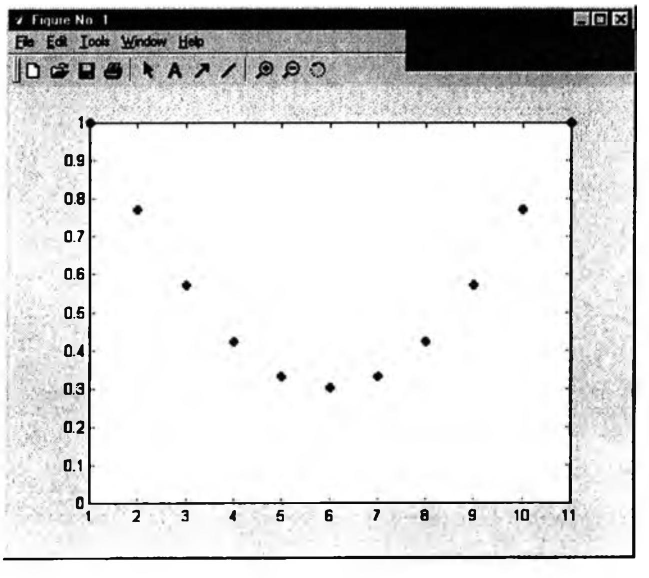


Figure 4.5 The MATLAB graphical results for test program I at time t = 0.0625 sec.

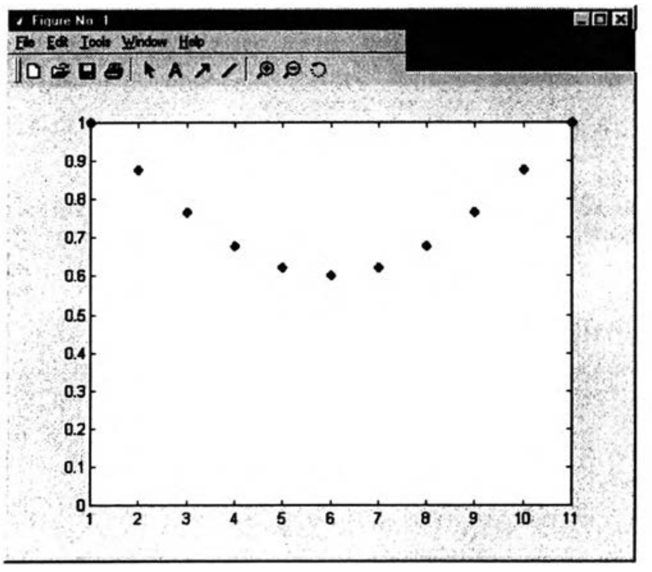


Figure 4.6 The MATLAB graphical results for test program I at time t = 0.1250.

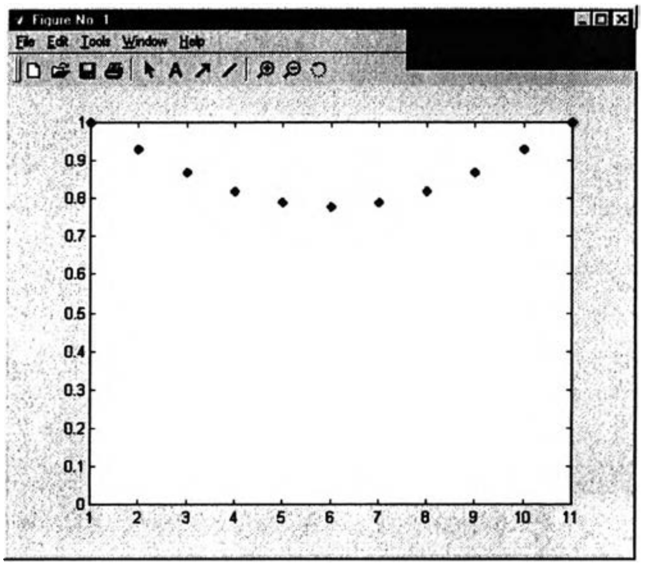


Figure 4.7 The MATLAB graphical results for test program I at time t = 0.1875 sec.

4.2 Test Program II - Unsteady-state Heat Conduction in a Long Bar of Square Cross Section (Parabolic Problem)

4.2.1 Problem

This problem is to solve for the temperature profile of an infinitely long bar having a square cross section with the side of length 2a. The initial uniform temperature is θ_0 and then suddenly the side surface is maintained at a temperature, θ_1. The subsequent temperatures, $\theta(x,y,t)$ inside the bar will be solved for. If dimensionless distances x, times τ, and temperatures T are defined by

$$T = \frac{\theta - \theta_0}{\theta_1 - \theta_0} \tag{4.6}$$

$$X = \frac{x}{a} \tag{4.7}$$

$$Y = \frac{y}{a} \tag{4.8}$$

$$\tau = \frac{\alpha t}{L^2} \tag{4.9}$$

It may be shown that the unsteady-state of conduction is governed by

$$\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} = \frac{\partial T}{\partial \tau} \tag{4.10}$$

Let the initial condition be:

$\tau = 0$: $T = 0$ throughout the region

Let boundary condition be:

$\tau > 0$: $T = 1$ along the sides $X = 1$ and $Y = 1$

$\partial T/\partial X = 0$ and $\partial T/\partial Y = 0$ along the sides $X = 0$ and $Y = 0$, respectively

4.2.2 Method of Using Program

- Select the two-dimensional parabolic equation.

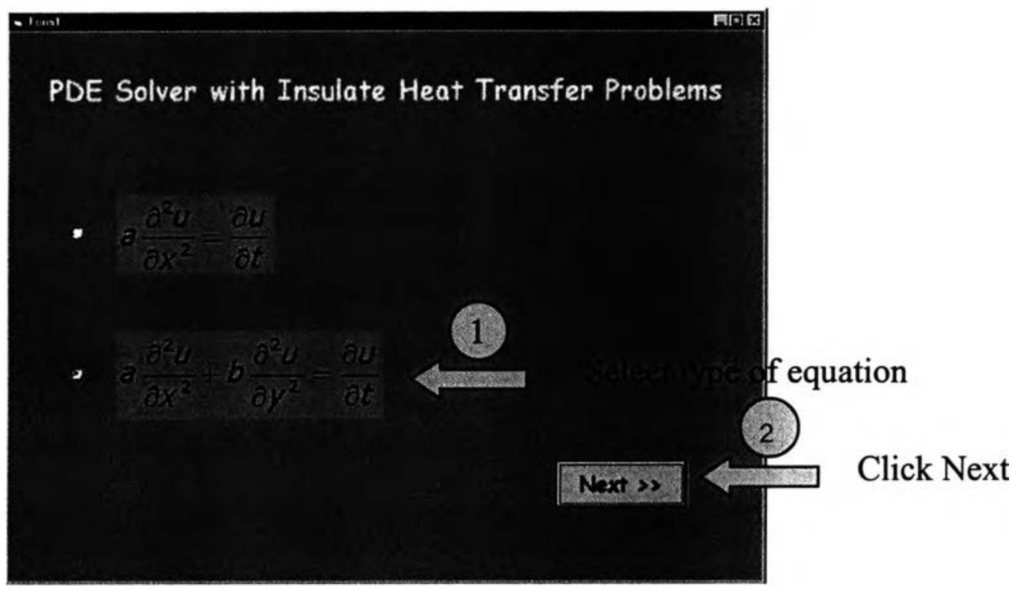


Figure 4.8 Selecting the type of equations for test program II.

- For test program II, there are two insulated sides.

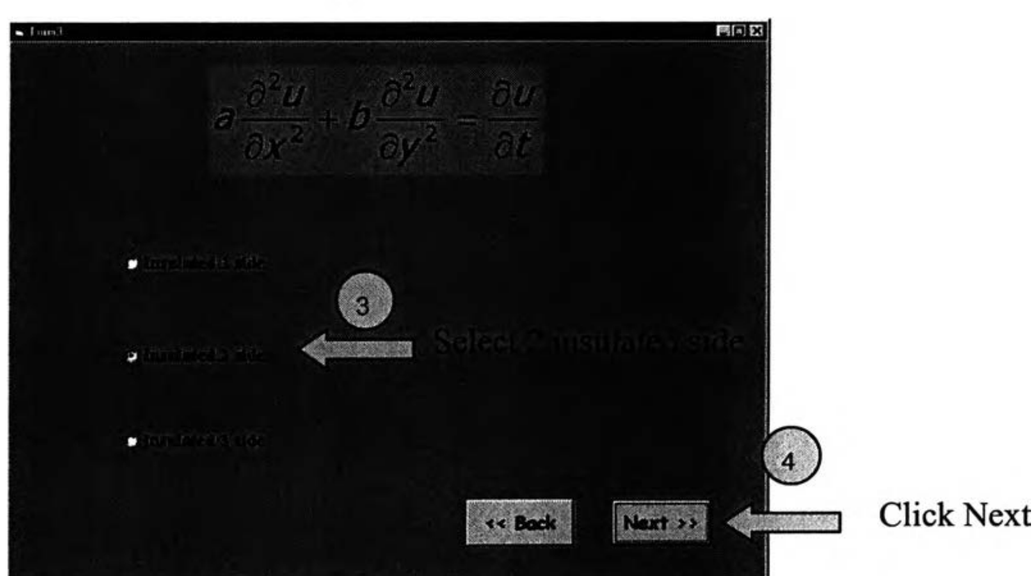


Figure 4.9 Selecting two insulated sides.

- Select the position of two insulated sides at x = 0 and y = 0

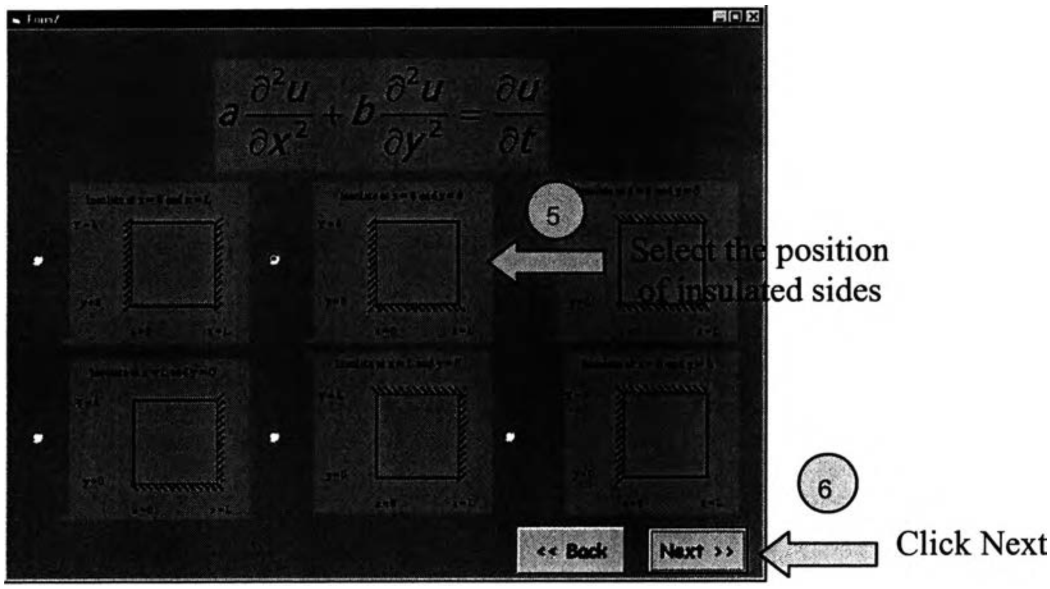


Figure 4.10 Selecting the position of two insulated sides.

- Enter all necessary values and boundary conditions

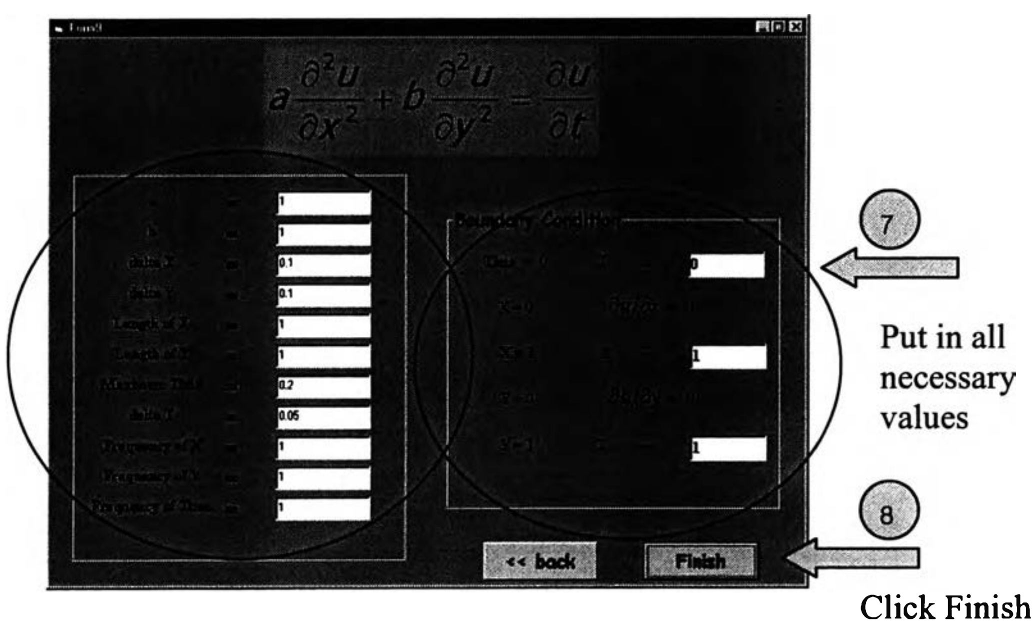


Figure 4.11 Entering all necessary values for test program II.

4.2.3 <u>Results</u>

4.2.3.1 Numerical Results

The numerical results are shown below in the tabular form.

Time = .05

y \ x	0.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.	y
0.	0.01579	0.01737	0.02271	0.03398	0.05566	0.09644	0.17262	0.31467	0.57943	1.07278	1.	0.
0.1	0.01737	0.01894	0.02428	0.03552	0.05717	0.09788	0.17394	0.31577	0.5801	1.07266	1.	0.1
0.2	0.02271	0.02428	0.02959	0.04077	0.0623	0.10279	0.17844	0.31949	0.58238	1.07227	1.	0.2
0.3	0.03398	0.03552	0.04077	0.05183	0.07311	0.11313	0.1879	0.32734	0.5872	1.07143	1.	0.3
0.4	0.05566	0.05717	0.0623	0.07311	0.09391	0.13304	0.20613	0.34243	0.59646	1.06983	1.	0.4
0.5	0.09644	0.09788	0.10279	0.11313	0.13304	0.17047	0.24041	0.37083	0.61389	1.06682	1.	0.5
0.6	0.17262	0.17394	0.17844	0.1879	0.20613	0.24041	0.30445	0.42387	0.64644	1.06118	1.	0.6
0.7	0.31467	0.31577	0.31949	0.32734	0.34243	0.37083	0.42387	0.52279	0.70714	1.05068	1	0.7
0.8	0.57943	0.5801	0.58238	0.5872	0.59646	0.61389	0.64644	0.70714	0.82028	1.0311	1.	0.8
0.9	1.07278	1.07266	1.07227	1.07143	1.06983	1.06682	1.06118	1.05068	1.0311	0.99462	1.	0.9
1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
x	0.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.	

Figure 4.12 The Microsoft Excel numerical results for test program II at Time t = 0.05 sec.

4.2.3.2 Graphical results

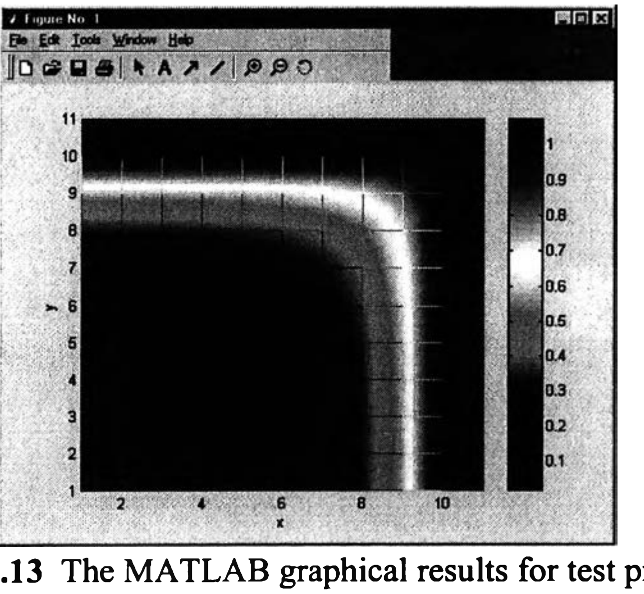


Figure 4.13 The MATLAB graphical results for test program II at Time t = 0.05 sec.

4.3 Test Program III - Steady-state Heat Conduction in a Square Plate (Elliptic Problem)

4.3.1 Problem

The problem is to solve the steady-state temperature distribution in a square plate, one side of which is maintained at 100^0C with the other three sides maintained at 0^0C as shown in Figure 4.8

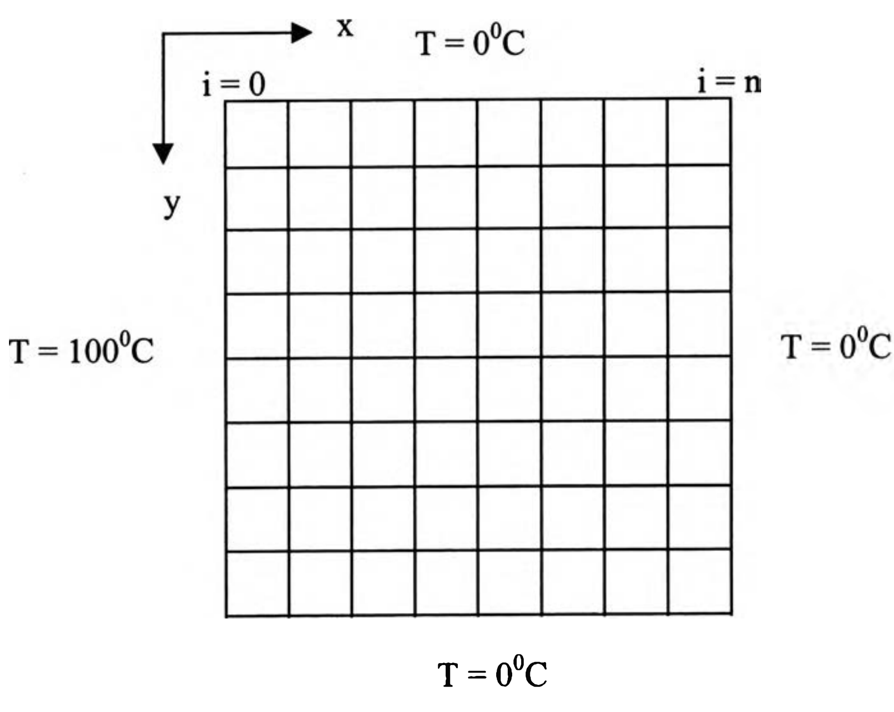


Figure 4.14 Heat conduction in a square plate.

4.3.2 Method of Using Program

- Select two dimensions of the elliptic equation.

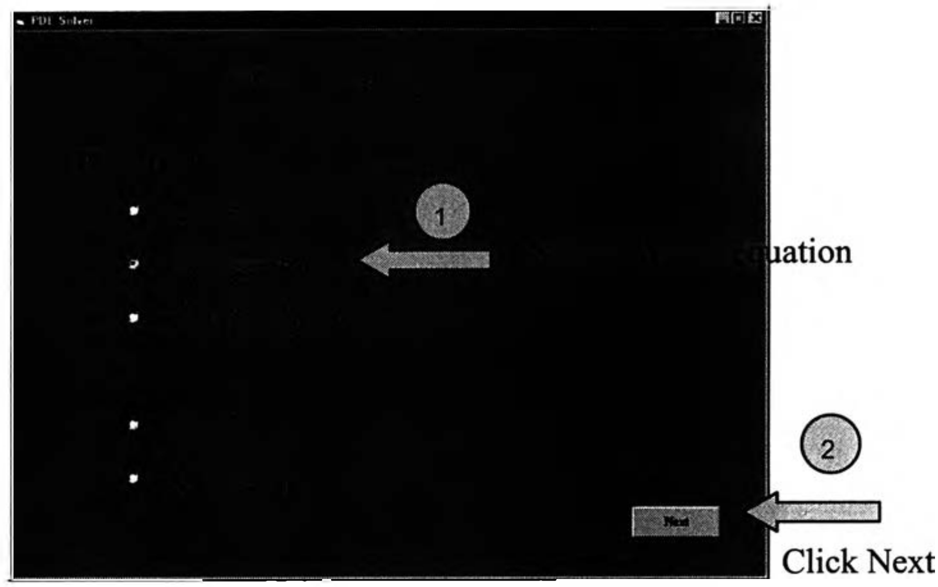


Figure 4.15 Selecting the type of equation for test program III

- Select the method and input all values.

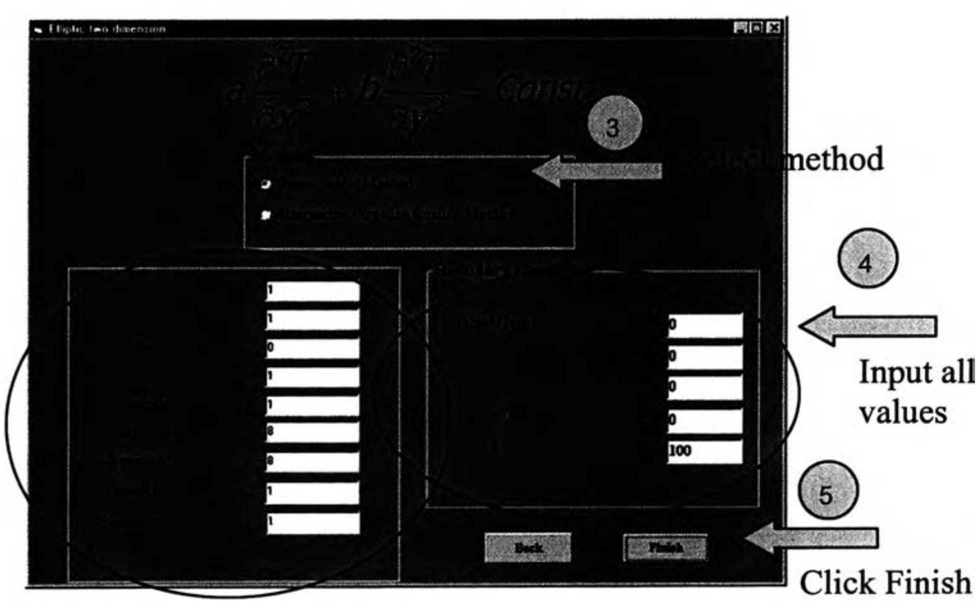
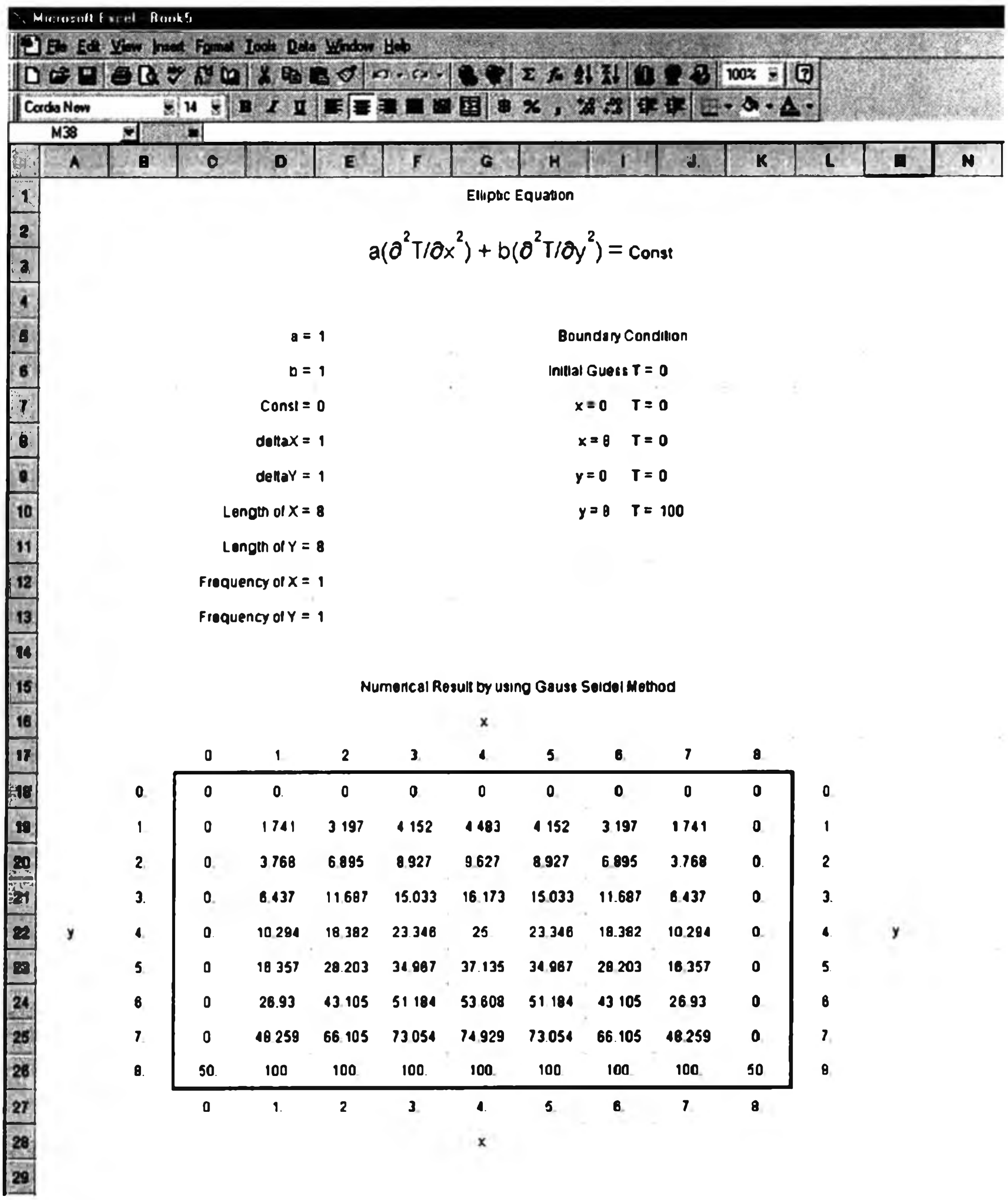


Figure 4.16 Selecting a method and entering in all the necessary values for test program III.

4.3.3 Results

4.3.3.1 Numerical Results

The numerical results are shown below in tabular form.



Elliptic Equation

$$a(\partial^2T/\partial x^2) + b(\partial^2T/\partial y^2) = \text{Const}$$

a = 1
b = 1
Const = 0
deltaX = 1
deltaY = 1
Length of X = 8
Length of Y = 8
Frequency of X = 1
Frequency of Y = 1

Boundary Condition
Initial Guess T = 0
x = 0 T = 0
x = 8 T = 0
y = 0 T = 0
y = 8 T = 100

Numerical Result by using Gauss Seidel Method

y \ x	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1.741	3.197	4.152	4.483	4.152	3.197	1.741	0
2	0	3.768	6.895	8.927	9.627	8.927	6.895	3.768	0
3	0	6.437	11.687	15.033	16.173	15.033	11.687	6.437	0
4	0	10.294	18.382	23.346	25	23.346	18.382	10.294	0
5	0	18.357	28.203	34.987	37.135	34.987	28.203	18.357	0
6	0	26.93	43.105	51.184	53.608	51.184	43.105	26.93	0
7	0	48.259	66.105	73.054	74.929	73.054	66.105	48.259	0
8	50	100	100	100	100	100	100	100	50

Figure 4.17 The Microsoft Excel numerical results for test program III

4.3.3.2 Graphical Results

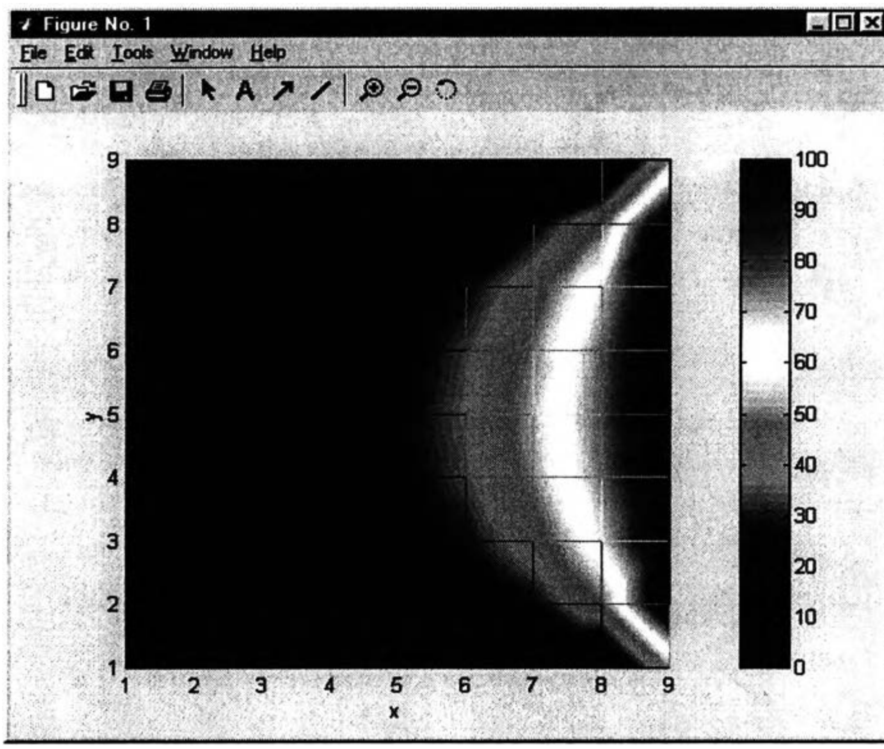


Figure 4.18 The MATLAB graphical results for test program III.