CHAPTER II

Theory of Superconductivity

In this chapter, we review the status of both experimental and theoretical works of superconductivity.

2.1 The Basic Phenomena

2.1.1 Zero Resistance and the Critical Temperature

The electromagnetic properties of the superconducting state were the first to be observed experimentally. The most remarkable of these is the sudden disappearance of the resistance of certain metals, alloys and compounds below certain critical temperatures, e.g., T_c = 4.2 K, 23 K and 125 K for Hg, Nb_3Ge and $Tl_2Ca_2Ba_2Cu_2O_{10}$ respectively (See Fig. 2.1). It is this absence of resistance, or better, the existence of persistant electrical currents (infinite conductivity) that gives the phenomenon, its name - superconducting phase transition or simply superconductivity.

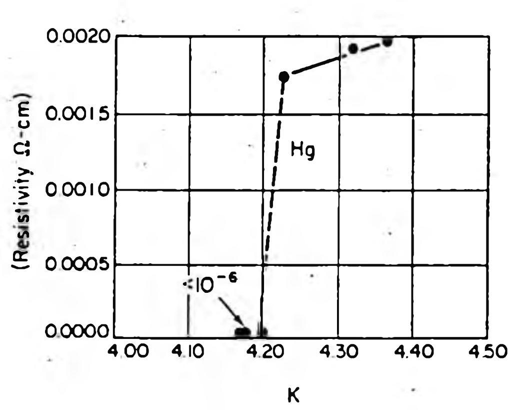


Figure 2.1 Kamerlingh Onnes's data on the superconductivity of mercury (Hg).

However , infinite conductivity imposes certain conditions on the magnetic field which were not subsequently observed. The relation between the electric current density $\vec{J}$, and the applied electric field $\vec{E}$, in a superconductor is given by Ohm's law (34) ,

$$\vec{J} = \sigma\vec{E} \qquad (2.1)$$

where σ is the conductivity. The electric field $\vec{E}$ is related to the magnetic field B by Maxwell's equation

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad (2.2)$$

If we substitute Eq. (2.1) into Eq (2.2) , we see that for infinite conductivity and $\partial\vec{B}/\partial t = 0$ thus implies that the magnetic field $\vec{B}$ remains constant for any medium with infinite conductivity because $\vec{E}$ vanishes inside the material. In particular , consider a superconductor that is cooled below T_c in zero magnetic field. The above result shows that $\vec{B}$ remains zero even if a field is subsequently applied (See Fig 2.2).

2.1.2 Perfect Diamagnetism , or the Meissner Effect.

The magnetic properties exhibited by superconductors are as dramatic as their electrical properties. The magnetic properties cannot be accounted for by the assumption that the superconducting state is characterized properly by zero electrical resistance. It is an experimental fact that a bulk superconductor in a weak magnetic field will act as a perfect diamagnet, with zero magnetic induction in the interior. When a specimen is cooled and becomes superconductiving, experiments first performed by Meissner and Ochsenfeld demonstrate that all magnetic flux is expelled from the interior (see Fig. 2.2). This is called the Meissner effect.

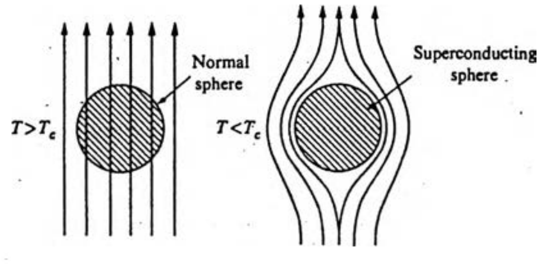


Figure 2.2 Meissner effect in superconductors : The magnetic flux is expelled from a superconductor that is , for $T < T_c$.

The consequence of the Meissner effect can be descibed using magnetostatic theory (34). The magnetic field $\vec{H}$, the magnetic induction $\vec{B}$, and the magnetization $\vec{M}$ are related by

$$\vec{B} = \vec{H} + 4\pi \vec{M} \tag{2.3}$$

At temperatures above T_c , most superconducting materials are only weakly magnetic so that $\vec{B} = \vec{H}$ in the material. At temperatures below T_c , $\vec{B} = 0$ in the material (i. e., a magnetization $\vec{M} = -\vec{H} / 4\pi$ opposite to the applied field is induced). Therefore, the magnetic moment $\vec{m}$ of a superconductor is given by

$$\vec{m} = -\left(\frac{\vec{H}}{4\pi}\right) V \qquad (2.4)$$

where V is the volume of the superconducting material.

The relationship between $\vec{H}$ and $\vec{M}$ in Eq. (2.3) is a unique property of a given material. This relationship is characterized by the magnetic susceptibility X , defined as

$$\vec{M} = X \cdot \vec{H} \qquad (2.5)$$

Since $\vec{B} = 0$ in the superconducting state , it follows that

$$\vec{M} = -\left(\frac{1}{4\pi}\right) \vec{H} \qquad (2.6)$$

from Eq. (2.5) and Eq. (2.6) then implies that $X = -1/4\pi < 0$, Such a condition in which the magnetization cancels external field exactly , is referred to as perfect diamagnetism.

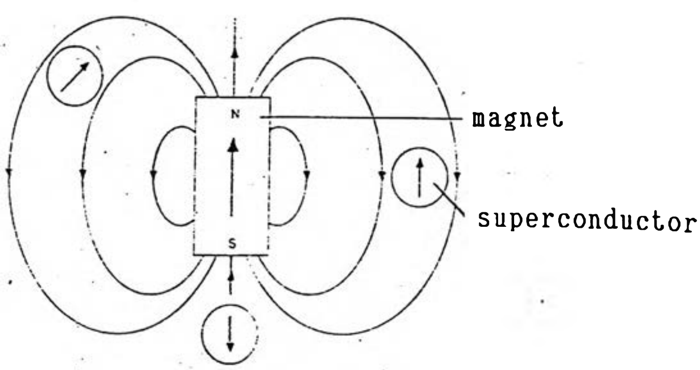


Figure 2.3 Three possible positions of a superconductor in the neighborhood of a permanent magnet. In each case, the induced moment of the superconductor , indicated by arrows is in a direction opposite to the field.

To picturize what happens when a superconductor is brought near a permanent magnet (a material with a permanent magnetic moment), with a north and south pole,we refer to Fig. 2.3 , three possible positions of a superconductor in relation to a permanent magnet, along with the magnetic field lines of the magnet and the induced magnetic moments of the superconductor are shown.

Notice that the magnetic moment of the superconductor are in the direction opposite to the magnetic field at the position of the superconductor. Therefore, the like poles of the permanent magnet and the superconductor are closer than the opposite poles , so there is always a repulsive force between then.

2.1.3 The Critical Magnetic Field

Shortly after Onnes first observed superconductivity , it was found that superconductivity can be destroyed by the application of amagnetic field. If a strong enough magnetic field $H > H_c(T)$, called the critical field , is applied to a superconducting specimen , it becomes normal and reconers its normal resistance even at below critical teemperature , We consider a long cylinder of pure superconductor in a parallel applied field H , where there are no demagnetizing effects. If the sample is superconducting at temperature T in zero field , there is a uniqe critical field $H_c(T)$ above which the sample becomes normal. This transition is reversible , for superconductivity reappears as soon as H is reduced below H_c (T). Experiments on pure superconductors shows that the curve H_c (T) is roughly parabolic (See Fig. 2.4) . This critical field $H_c(T)$ is a function of the temperature is given approximately by Tuyn's law (35) as

$$H_c(T) = H_c(0)[1 - (T/T_c)^2] \quad (2.7)$$

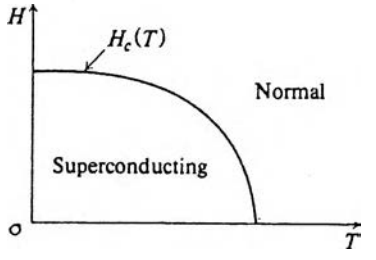


Figure 2.4 Phase diagram in H - T plane , showing superconducting and normal regions , and the critical curve $H_c(T)$ or $T_c(T)$ them.

2.2 London - Pippard Phenomenological Theory

The London equations (6) provided the first theory description of the Meissner effect. Although these equations are a pair of phenomenological constitutive relations describing the response of the supercurrent J to applied electric and magnetic fields , they may also be derived from the following simple model.

2.2.1 Derivation of London Equations

If the superelectrons are consider as an incompressible nonviscous charged fluid with velocity field $\vec{v}(x,t)$ then the supercurrent is given by

$$\vec{J}_s(x,t) = n_s e \vec{v}_s(x,t) \tag{2.8}$$

where n_s is the superelectron number density and e is the charge on an electron. The equation of motion of a superelectron in the presence of an electric field is

$$m \frac{d\vec{v}_s(x,t)}{dt} = e \vec{E}(x,t) \tag{2.9}$$

which , when combined with Eq. (2.8) , yields

$$\frac{d\vec{J}_s(x,t)}{dt} = \frac{n_s e^2}{m} \vec{E}(x,t) \tag{2.10}$$

In the steady state , the current in a superconductor is constant. Therefore it follows from Eq. (2.10) that $\frac{d\vec{J}_s(x,t)}{dt} = 0$, or

$$\vec{E}(x,t) = 0 \tag{2.11}$$

This important conclusion asserts that , in the steady state , the electric field inside a superconductor vanishes. In other words ,the voltage drop across a superconductor is zero.

Eq. (2.11)leads immediately to another immediately important result. When this relation is combined with the Maxwell equation ,

$$\vec{\nabla} \times \vec{E} = - \frac{1}{c} \frac{d\vec{B}}{dt} \tag{2.12}$$

one finds that

$$\frac{d\vec{B}}{dt} = 0 \qquad (2.13)$$

This affirms that in the steady state the magnetic field is constant.

To proceed with this modification , let us substitute for $\vec{E}$ from Eq. (2.10) into (2.12) , which yields

$$\frac{d\vec{B}}{dt} = -\frac{mc}{n_s e^2} \vec{\nabla} \times \frac{d\vec{J}_s}{dt} \qquad (2.14)$$

This equation is invalid, as has just been seen ,because it predicts that $\frac{d\vec{B}}{dt} = 0$. To rectify this , London postulated the relation

$$\vec{B} = -\frac{mc}{n_s e^2} \vec{\nabla} \times \vec{J}_s \qquad (2.15)$$

or

$$\vec{J}_s = -\frac{n_s e^2}{mc} \vec{A} \qquad (2.16)$$

which has the same from as (2.14) , except that the time differentiations have been eliminated. We shall see presently that relation (2.16), know as the London equation , leads to results that are in agreement with experiment.

Eq. (2.15) is a relation between $\vec{B}$ and $\vec{J}_s$. These quantities are also related by the Maxwell equation

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J}_s \qquad (2.17)$$

We eliminate J_s between Eq. (2.15) and Eq. (2.17) [by laking the curl of (2.17) ,substituting for $\vec{\nabla} \times \vec{J}_s$ from Eq. (2.15), then using the identity of $\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla} (\vec{\nabla}.\vec{B}) - \nabla^2 B = - \nabla^2 B$, and $\vec{\nabla}.\vec{B} = 0$] , we find that

$$\nabla^2 \vec{B} = \frac{4\pi n_s e^2}{m c^2} \vec{B} \qquad (2.18)$$

Let us apply this field equation to a situation of simple geometry. The specimen is semi - infinite , with its surface lying in the yz plane (see Fig. 2.5) , and the field is applied in the y -direction. Since quantities vary only in the x -direction, Eq (2.18) reduced to

$$\frac{d^2 B_y}{d x^2} = \frac{4\pi n_s e^2}{mc^2} B_y \qquad (2.19)$$

Eq. (2.19) yields the solution

$$B_y (x) = B_y (0) e^{-x/\lambda_L} \qquad (2.20)$$

where $$\lambda_L = (mc^2/4\pi n_S e^2) \qquad (2.21)$$

is known as the London penetration depth.

Eq.(2.20) shows that the field decreases exponentially as one proceeds from the surface into the superconductor. Thus the field vanishes inside the bulk of the medium, in accord with the Meissner effect. As a matter of fact , this agreement was the primary motivation for postulating the London equation in the first place.

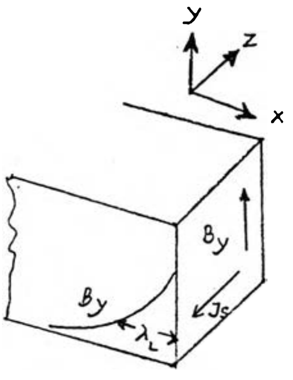


Figure 2.5 Solution of the London equation. The magnetic field decays exponentially within the superconductor.

Note , however , that Eq. (2.20) predicts that the field penetrates the sample to some extent , the distanec of penetration being roughly equal to λ_L. Thus the flux is not expelled entirely from the superconductor, as was once thought , but there is a small region near the surface in which there is an appreciable field , e.q. experimental work finds that λ_L= 500 Å , 1400 Å (37) for pure and high - T_c superconductors respectively.

Experiments indicate that the penetration depth increases with increasing temperature , and the function

$$\lambda_L(T) = \lambda_L(0)\,[1-(T/T_c)^4]^{-1/2} \qquad (2.22)$$

provides a good fit to the data for all temperatures (38) since n_s is the only variable quantity in Eq. (2.21) , We infer

$$n_s\,(T) = n_s\,(0)\,[\,1-(\,T/T_c)^4\,]^{-1/2} \qquad (2.23)$$

Eq. (2.23) is known as the Gorter - Casimir (5) formula.

2.2.2 Pippard's Generalized Equation

The London equation , which we have discussed in the previous section , was used to describe the electrodynamics of superconductors until about 1950. At that time , Pippard (7) started a series of measureements of the microwave surface impedance of superconductors ; his most important conclusion , as we have already mentioned , was that for pure superconductors the London equation should be replaced by a non - local equation.

Pippard introduced the coherence length while proposing a nonlocal generalization of the London equation (2.8).This was done in analogy to Chambers' nonlocal qeneralization (39) of Ohm's law from $\vec{J}(\vec{x}) = \sigma \vec{E}(\vec{x})$ to

$$\vec{J}(\vec{x}) = \frac{3\sigma}{4\pi\ell} \int \frac{\vec{R}[\vec{R}\vec{E}(\vec{x}')]}{R^4} e^{-R/\ell} d^3\vec{x}',$$

For this and other reasons , Pippard proprosed a nonlocal generalization of Eq. (2.16) in which $\vec{J}(\vec{x})$ is determined as a spatial average of $\vec{A}$ throughout some neighboring region of dimension $\xi_0 \sim 10^{-4}$ cm. For heavily doped alloys , ξ_0 is comparable with the electronic mean free path ℓ in the normal metal ; for pure metals , however, ξ_0 is not infinite , but instead tends to a characteristic length, known as Pippard coherence length.

The relation Pippard proposed to replace the London equation is

$$\vec{J}(\vec{x}) = -\frac{n_s e^2}{mc} \cdot \frac{3}{4\pi\xi_0} \int d^3\vec{x}' \frac{\vec{R}[\vec{R}.\vec{A}(\vec{x}')]e^{-R/\xi}}{R^4} \quad (2.24)$$

where $\vec{R} = \vec{X}-\vec{X}'$ and $R = |\vec{R}|$. The coherence length is given by

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell} \quad (2.25)$$

where ξ_0 is a constant of the pure material ; in particular $\xi = \xi_0$ in the pure material, and $\xi = \ell$ for $\ell \ll \xi_0$. The Pippard coherence length ξ , estimation by an uncertainty - principle , only electrons within $k_B T_c$ of the Fermi energy can play a major role in a phenomenon where sets in at T_c , and these electron have a momentum range $\Delta p \sim k_B T_c / v_F$, where V_F is the Fermi velocity. Thus $\Delta x \geq \hbar / \Delta p \simeq \hbar v_F / k_B T_c$,leading to the definition of a coherence length ξ_0.

$$\xi_0 = a \frac{\hbar v_F}{K_B T_c} \tag{2.26}$$

Using Eq. (2.22) , he computed the penetration depth for various values of ξ_0 and λ_L and compared the results with experimental data. He found (40) that he could fit the data on both tin and aluminum by the choice of a single parameter a = 0.15 in Eq. (2.26) For comparison , the BCS theory (11) of pure samples leads to a very similar nonlocal relation and identifies ξ_0 as

$$\xi_0 = \frac{0.18 \hbar V_F}{K_B T_c} = \frac{\hbar V_F}{\pi \Delta_0} \tag{2.27}$$

where Δ_0 is the energy gap at zero temperature.

The Pippard equation relates the induced supercurrent $\vec{J}$ to the vector potential $\vec{A}$. In the presence of an applied magnetic field , however , $\vec{A}$ contains contributions from the external currents as well as from $\vec{J}$ itself , thereby requiring a simultaneous solution of Eq. (2.24) and Maxwell's equation determining the magnetic field. Nevertheless , it is possible to extract the important physical features by noting the existence of two characteristic lengths. The vector potential varies with the self - consistent temperature - dependent penetration length , which need not be the same as the London penetration length [defined in Eq. (2.21)] , while the integral kernel has a temperature - independent range ξ, which is approximately the smaller of ξ and ℓ. If $\xi_0 \ll \lambda$, then the vector potential varies slowly and can be evaluated at $\vec{x} = \vec{x}'$. In this way , we obtain

$$J_k(\vec{x}) \qquad - \frac{n_s e^2 A_\ell(\vec{x})}{mc\xi_0} \cdot \frac{3}{4\pi} \int d^3 x \frac{R_k R_\ell e^{-R/\xi}}{R^4}$$

$$= - \frac{n_s e^2 A_\ell(\vec{x})}{mc} \delta_{k\ell} \int_0^\infty dR \quad e^{-R/\xi}$$

or

$$\vec{J}(\vec{x}) = - \frac{n_s e^2 \ell \vec{A}(\vec{x})}{mc(\ell + \xi_0)} , \quad \xi \ll \lambda \qquad (2.28)$$

Any sample that satisfies this local condition ($\xi \ll \lambda$) is known as a London superconductor , because Eq. (2.24) reproduces the form of London equation (2.16) but with the coefficient reduced by a factor $(1 + \xi/\ell)^{-1}$. Comparison with Eqs. (2.15) and (2.18) immediately gives the corresponding penetration depth at zero temperature

$$\lambda(0) = \lambda_L(0) \left(\frac{\ell + \xi_0}{\ell}\right)^{1/2} \text{ local limit } \xi \ll \lambda \qquad (2.29)$$

For a pure London superconductor ($\xi \ll \ell$) , Eq. (2.29) reduces to the previous London expression.

In practice , most superconducting elements at low temperature violate the condition for a pure London superconductor, which requires $\xi \simeq \xi_o << \lambda$. Hence London superconductors are usually heavily doped alloys, where the length ξ is determined by ℓ instead of ξ_o, and the following inequality holds : $\xi \simeq \ell << \lambda$ in this case Eq. (2.29) explains the observed increase in the penetration depth for dirty alloys where $\ell << \xi_o$, If a sample is a London superconductor, Eq. (2.22) shows that it remains one for all $T < T_c$ Since the penetration depth of any superconducting material increases rapidly as $T \rightarrow T_c$, however , all superconductors become London superconductor sufficiently close to T_c.

It is important to emphasize that eqs. (2.28) and (2.29) are only correct for $\xi << \lambda$, and typical nonlocal limit ($\xi >> \lambda$) when the material is konwn as a Pippard superconductor. We can calculate the penetration depth at zero temperature , and found that

$$\lambda(0) = \frac{8}{9} \left(\frac{\sqrt{3}}{2\pi} \right)^{1/3} [\lambda_L^2(0)\, \xi_0]^{1/3} \quad \text{nonlocal limit} = \xi >> \lambda \quad (2.30)$$

This expression is independent of the mean free path ℓ because the spatial integration in Eq. (2.24) is limited by the penetration depth λ and not by ξ . Eqs. (2.29) and (2.30) constitute a central result of the Pippard Theory.

2.2.3 Flux Quantization in a Superconducting Ring

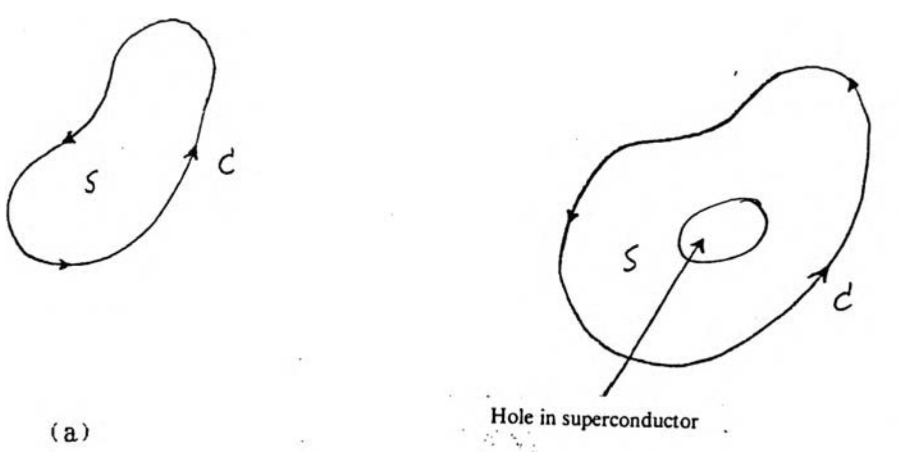
Flux quantization is an example of a long - range quantum effect which the coherence of the superconducting state extends over a ring or solenoid. Let us first consider the London equations imply a striking conservation law. For simplicity , we study only the linearized equations (2.8) , (2.9) and (2.10)

$$-\frac{e\vec{E}}{m} = \frac{d\vec{v}}{dt} = -\frac{1}{n_s c}\frac{d\vec{J}}{dt} \qquad (2.31)$$

Consider a surface S bounded by a fixed closed curve C that lies wholly in the superconducting material (See Fig 2.6). Independent of whether S also lies entirely in the superconductor (See Fig 2.6a or b), we may integrate Maxwell's equation (2.12) to obtain

$$\int d\vec{s}.\frac{d\vec{B}}{dt} = -c\int d\vec{s}\,(\vec{\nabla} \times \vec{E}) = -c\oint_C d\vec{l}.\vec{E} \qquad (2.32)$$

where the right side has been transformed with Stokes'theorem.



(a) (b)

Figure 2.6 Integration contour for evaluation of fluxoid (a) simply connected ; (b) multiply connected.

Since C lies in the superconductor , Eq. (2.31) is applied at every point , giving

$$\frac{d}{dt}\left[\int d\vec{S}.\vec{B} + \frac{mc}{n_s e^2}\oint_C d\vec{l}.\vec{J}\right] = 0 \qquad (2.33)$$

We see that the fluxoid [40] defined as

$$\phi = \int d\vec{S}.\vec{B} + \frac{mc}{n_s e^2}\oint_C d\vec{l}.\vec{J} \qquad (2.34)$$

remains constant for all time. It is clear that ϕ differs from the magnetic flux by an additional contribution arising from the induced supercurrent. With added assumptions , it is possible to derive more specific results.

1. If C is sufficiently far from the boundaries , then J is exponentially small , and ϕ reduces to the magnetic flux.
2. If the interior of C is wholly superconducting (See Fig 2.6a), then the other London equation (2.15) immediately implies that ϕ vanishes.
3. As a corollary of the previous conclusion, ϕ is the same for any path C' that can be deformed continuously into C , always remaining in the superconductor.

London also observed that Eqs. (2.15) and (2.8) can be written in terms of the vector potential as follows

$$\vec{\nabla} \times (m\vec{v}_s - \frac{e\vec{A}}{c}) = 0 \qquad (2.35)$$

where $\vec{B} = \vec{\nabla} \times \vec{A}$ (2.36)

The canonical momentum is given by

$$\vec{p} = m\vec{v} - \frac{e\vec{A}}{c} \qquad (2.37)$$

and Eq. (2.35) thus becomes

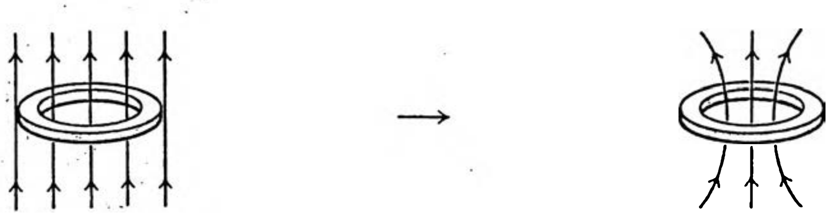
$$\vec{\nabla} \times \vec{p} = 0 \qquad (2.37)$$

which may be consider a generalized condition of irrotational flow. In addition , the fluxoid may be writhen as

$$\phi = \int (\vec{\nabla} \times \vec{A}) \cdot d\vec{S} - \frac{mc}{e} \oint d\vec{l} \cdot \vec{V}_s$$

$$= -\frac{c}{e} \oint_C (m\vec{v}_s - \frac{e\vec{A}}{c}) \cdot d\vec{l} \qquad (2.38)$$

or $$\phi = -\frac{c}{e} \oint \vec{P} \cdot d\vec{l} \qquad (2.39)$$



Normal ring in magnetic field

Cooled below T_c Superconducting ring in magnetic field ,magnetic field then removed

Figure 2.7 Flux trapping in a superconducting ring. Typical recent measurements are those of J. File and R.G. Mills , who find lifetimes of order 10^5 years.

This equation is reminiscent of the bohr - Sommerfeld quantization relation , and , indeed , London suggested that the fluxoid is quantized in units of hc /e (41). As shown in Fig. 2.7 , this prediction was subsequently confirmed ,but the observed quantum unit is hc / 2e (42,43) such persistent currents have been observed over long periods (44)

2.3 Thermodynamics of the Superconducting state

Like the electromagnetic properties , the thermal properties , Gibbs free energy , entropy and electronic specific heat of also change sharply as the temperature is lowered through the transition temperature for superconductivity. The Meissner effect shows that the transition in presence of a magnetic field through the normal and superconducting state boundary $H_c = H(0)[1-(T/T_c)^2]$ is reversible , and therefore that the laws of thermodynamies are applied to N-S phase transition. In this section , we show the discontinuity in these thermal properties and discuss the behavior of the electronic specific heat above and below T_c , which shows that entropy is carried by excitations in the superconducting state separated from the normal state by an energy gap.

2.3.1 Gibbs free energy

When the magnetic quantities are changed by small amounts , the work done on the system is given by $(4\pi)^{-1} \int d^3x\, \vec{H}.d\vec{B}$, so that the change in the Helmholtz free energy density becomes

$$dF = -SdT + (4\pi)^{-1}\vec{H}.d\vec{B} \qquad (2.40)$$

with the corresponding differential relation

$$S = -\left(\frac{\partial F}{\partial T}\right)_B , \quad H = 4\pi\left(\frac{\partial F}{\partial B}\right)_T \qquad (2.41)$$

Here we assume the volume is held constant , and S is the entropy density.

The flux expulsion associate with the Meissner effect indicates that a bulk superconductor in an external magnetic field H is uniquely characterized by the condition $\vec{B} = 0$, independent of the way the state is reached (see Fig 2.2). We therefore infer that the superconductor is in true thermodynamic equilibrium and accordingly apply the techinques of macroscopic thermodynamics. For most experiments , however , it is impossible to manipulate the flux density $\vec{B}$ directly ; instead , the external currents (in a solenoid , for example) control the magnetic field $\vec{H}$, and we prefer to make a Legendre transformation from the Helmholtz function F (T,B) to the Gibbs free energy density

$$G (T , H) = F - (4\pi)^{-1} \vec{B}.\vec{H} \qquad (2.42)$$

with the corresponding differential relations

$$dG = - SdT - (4\pi)^{-1} \vec{B}.d\vec{H} \qquad (2.43)$$

$$S = -\left(\frac{\partial G}{\partial T}\right)_H , \quad B = - 4\pi\left(\frac{\partial G}{\partial H}\right)_T \qquad (2.44)$$

Consider along superconducting cylinder in a parallel magnetic field. If the field $\vec{H} = H\hat{z}$ is increased at constant temperature , Eq. (2.44) gives

$$G_s(T , H) - G_n(T , 0) = - 4\pi \int_0^H B (H') d H' \quad (2.45)$$

To a good approximation , the normal state of most superconducting elementts is non magnetic (B = H) , and we find

$$G_s(T , H) - G_n(T , 0) = - (8\pi)^{-1} H^2 \quad (2.46)$$

In contrast , $\vec{B}$ vanishes in the superconductor , which yields

$$G_s (T , H) = G_s (T , 0) \quad (2.47)$$

The two phases are in thermodynamic equilibrium at the thermodynamic critical field H_c. This condition may be expressed by the equation

$$G_s (T , H_c) = G_n (T , H_c) \quad (2.48)$$

and a combination of Eqs. (2.46)-(2.48) immediatly gives

$$G_s (T , 0) = G_n (T , 0) - (8\pi)^{-1} H^2_c \quad (2.49)$$

$$F_s (T , 0) = F_n (T , 0) - (8\pi)^{-1} H^2_c \quad (2.50)$$

These equations show that a negative condensation energy $-H^2_c /8\pi$ per unit volume accompanies the formation of the superconducting state. We have , therefore , in an external magnetic field H a difference in the Gibbs free energy density between the superconducting and normal state. From Eqs. (2.46) - (2.49) , lead to the general result

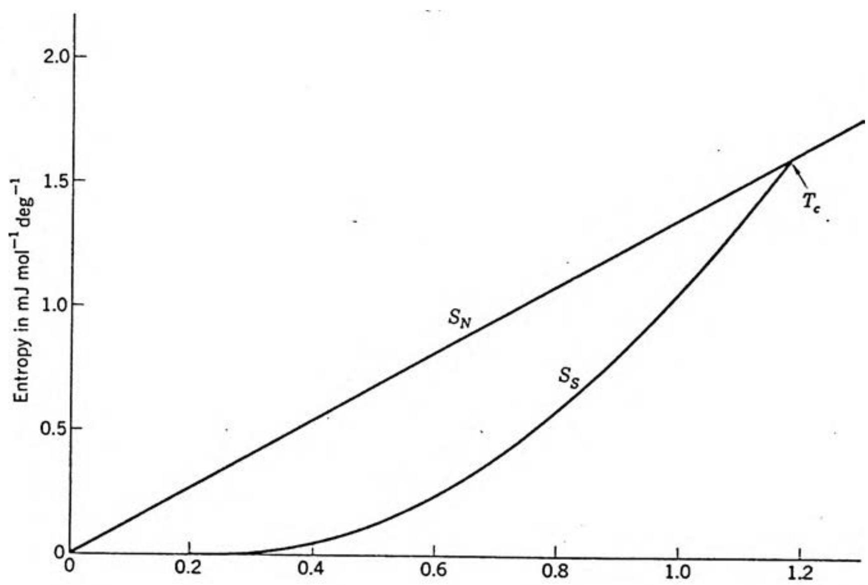
$$G_s (T , H) - G_n (T , H) = (8 \pi)^{-1} (H^2 - H^2_c) \quad (2.51)$$

2.3.2 Entropy and Specific heat

We now proceed to use the expressions above for the Gibbs free energy to compute the entropy, latent heat and the discontinuity in the electronic specific heat at the normal and superconducting state. The derivative of Eq. (2.51) with respect to temperature yield the entropy difference between the two state

$$S_s(T, H) - S_n(T, H) = (4\pi)^{-1} H_c \left(\frac{dH_c(T)}{dT}\right) \qquad (2.52a)$$

Figure 2.4 shows that the right side is negative , so that the superconducting phase has lower entropy than the normal phase. This is in agreement with experiment (45) shown in figure 2.8.



Temperature,K

Figure 2.8 Entropy S of alumimum in the normal and superconducting state as a function of the temperature.

Finally , from the thermodynamic identity , the specific heat

$$C_H = T\left(\frac{\partial S}{\partial T}\right)_H \qquad (2.53b)$$

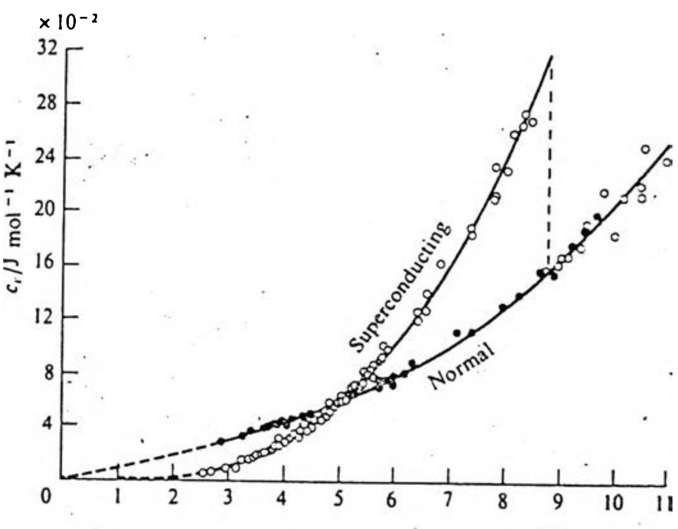
the difference in the electronic specific heats at constant field of the two phases , $C_{SH} - C_{NH}$, may be derived from Eqs. (2.52) as

$$C_{SH} - C_{ns} = \frac{T}{4\pi}\left[\left(\frac{dH_c}{dT}\right)^2 + H_c \frac{d^2H_c}{dT^2}\right] \qquad (2.53)$$

In particular, at the transition temperature T_c and $H_c = 0$, so we have for the transition in the absence of an applied magnetic field , so that the jump in the specific heat at T_c becomes

$$(C_s - C_n)_{Tc} = \frac{T_c}{4\pi}\left[\left(\frac{d\,Hc}{dT}\right)_{Tc}\right]^2 \qquad (2.54)$$

Eq. (2.54) has been well verified experimentally (46) shown in figure 2.9.



Temperature (K)

Figure 2.9 The heat capacity of Nb in the normal and superconducting state showing the sharp discontinuity at the critical temperature.

For zero applied field, at the transition temperature there is no latent heat (Eq. 2.52) , and there is a jump in the specific heat ; consequently the transition to the superconducting state is second - order phase transition in the absence of a magnetic field.

In practice , not only does one observe a discontinuity at T_c , one also obtains a deviation from the linear dependence of the electronic specific heat on T ($C_{en} = \gamma T$) , to an exponential dependence on T below T_c , proportional to $\exp(-\Delta/k_B T)$, Δ being the energy gap. To derive the exponential dependent, it is necessary to consider a system of elementary excitations or quasi - particles in the superconducting state that require a finite energy 2Δ to be excited above the ground state of the many - electron system. The existence of such quasi - pasticles is accounted for by the BCS theory (11)

2.4 Ginzburg - Landau Theory

This section will serve as the briefest of introductions to a theory whose ramifications reach extensitvely into the rest of this thesis. The theory is based on the pioneering work of Landau on second - order phase transition.

The phase transition at T_c singnal the appearance of an ordered state in which the electrons are partially condensed into a frictionless superfluid. Ginzburg and Landau (15) describe the condensate with a complex order parameter $\Psi(x)$. $|\Psi(x)|^2$ was to represent the local density of superconducting electrons , n_s. The observed second - order phase transition implies that $\Psi(x)$ vanishes for $T \geq T_c$, and that it increases in magnitude with increasing $T_c - T > 0$. Near T_c the quantity $|\Psi|^2$ is small , and the macroscopic free energy density F_s of the superconducting state in zero field is assumed to have and expansion of the from

$$F_s = F_{no} + \alpha|\Psi|^2 + {}^1/_2\, \beta|\Psi|^4 + \ldots. \qquad (2.55)$$

where F_{no} is the free energy density of the normal state in zero magnetic field and where α and β are real temperature dependent phenomenological constants. In equilibrium,

$$\frac{\partial F_s}{\partial |\Psi|^2} = 0 \, , \frac{\partial^2 F_s}{\partial |\Psi|^4} > 0 \, ,$$

and in addition we must have that $|\Psi|^2 = 0$ for $T \geqslant T_c$ and $|\Psi|^2 \geqslant 0$ for $T < T_c$. It follows that $\alpha_c = 0$, $\beta_c > 0$, and for $T < T_c$, $\alpha < 0$. Thus in equilibrium, for $T \leqslant T_c$,

$$|\Psi|^2 = |\Psi_\infty|^2 = -\frac{\alpha}{\beta} = \left(\frac{T_c - T}{\beta_c}\right)\left(\frac{d\alpha}{dT}\right)_{T=T_c} \tag{2.56}$$

and

$$F_s = F_{no} - \frac{\alpha^2}{2\beta} \tag{2.57}$$

where Ψ_∞ is the value in zero field (i.e., at an infinite distance from any boundary) , and we have assumed that $\alpha(T) = (T_c - T) \times \left(\frac{d\alpha}{dT}\right)_{T=T_c}$ and $\beta(T) = \beta(T_c)$. From Eq (2.57) one can get

$$H_c^2 = \frac{4\pi\alpha^2}{B} = \frac{4\pi(T_c - T)^2}{\beta_c}\left(\frac{d\alpha}{dT}\right)_{T=T_c} \tag{2.58}$$

The from of this expression is well known to be completely confirmed be experiment.

The second important step that Ginzburg and Landau look was to state that, if there was a spatial variation of Ψ, as well as a magnetic field $\vec{H}$ derived from a potential $\vec{A}$, the energy density in the superconducting phase F_{SH}, was given by

$$F_{SH} = F_{so} + \frac{H^2}{8\pi} + \frac{1}{2m}\left|\left(-i\hbar\hat{\nabla} - \frac{e\vec{A}}{c}\right)\Psi\right|^2 \tag{2.59}$$

The equation for Ψ many now be found from the requirement that the total free energy of the body $\int F_{SH}\, dV$, Thus , varying with respect to Ψ , we find that

$$\frac{1}{2m}\left(-i\hbar\nabla - \frac{e\vec{A}}{c}\right)^2\Psi + \frac{\partial F_{so}}{\partial \Psi^*} = 0 \qquad (2.60)$$

and moreover , at the boundary of the superconductor , in view of the arbitrariness of the variation Ψ^* , the following condition must hold :

$$\hat{n}\cdot\left(-i\hbar\nabla - \frac{e\vec{A}}{c}\right)\Psi = 0 \qquad (2.61)$$

Where $\hat{n}$ is the unit vector to the boundary.

So far as the equation for $\vec{A}$ is concerned , and using condition that $\vec{\nabla}\cdot\vec{A} = 0$ and vary the free energy with respect to $\vec{A}$, we obtain the usual expression ;

$$\nabla^2\vec{A} = \frac{-4\pi\vec{J}}{c} = \frac{2\pi e\hbar i}{mc}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) + \frac{4\pi e^2}{mc^2}|\Psi|^2\vec{A} \qquad (2.62)$$

in which the right - hand side constans the expression for the supercurrent

$$\vec{J} = -\frac{ie\hbar}{mc}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - \frac{e^2}{mc}|\Psi|^2\vec{A} \qquad (2.63)$$

The application of the Ginzburg - Landau equations , Eqs. (2.60) and (2.62) , to a planar boundary leads to the following results. One can define paramenter the penetration depth λ (of the dimension of length) , and K the Ginzburg - Landau parameter (dimensionless) as follows

$$\lambda^2 = \frac{mc^2\beta}{4\pi e^2|\alpha|} = \frac{mc^2}{4\pi e^2|\Psi|^2} = \frac{mc^2}{4\pi e^2 n_s} \qquad (2.64)$$

$$K^2 = \frac{1}{2\pi}\left(\frac{mc}{e\hbar}\right)^2 \beta = \left(\frac{2c^2}{\hbar c^2}\right) H_c \lambda^2 \qquad (2.65)$$

We can consider the dimensionless Ginzburg - Landau parameter K ,to be the ratio of penetration depth λ , to the coherence length ξ

$$K = \lambda / \xi \qquad (2.66)$$

where

$$\xi^2 = \frac{\hbar^2}{2m|\alpha|} \qquad (2.67)$$

Experiments (42,43, 47) indicate that , m replaced by an effective mass m^* , and e replaced by the charge of a pair of electrons e^* ,

$$m^* = 2m \quad , \quad e^* = 2e \qquad (2.68)$$

and n_s replaced by an effective superelectron density n_s^*

$$n^*_s = |\Psi|^2 = \frac{1}{2} n_s \qquad (2.69)$$

where n_s is the number of single electrons in the condensate.

With the identification $n_s^* = |\Psi|^2$, this agrees with the usual definition of the London penetration depth. In the original (15) formulation of the theory , it was thought that e^* and m^* would be the normal electronic value. However , experimental data has turned out to be fitted better if $e^* \simeq 2e$. The microscopic pairing theory of superconductivity makes it unambiguous that $e^* = 2e$ exactly , the charge of a pair of elections. In the free - electron approximation , it would then be natural to make $m^* = 2m$ and $n^*_s = (^1/_2)\ n_s$, where n_s is the number of single electrons in the condensate with these conventions , eq. (2.64) $\lambda^2 = m^* c^2 / 4\pi e^{*2}\ n^*_s = mc^2 / 4\pi e^2 n_s$ so the London penetration depth is unchanged by the pairing.

Eqs. (2.56) and (2.64) lead to an explicit form for temperature dependent of λ ,

$$\lambda \sim (1 - t)^{-1/2} \quad t = T/T_c \tag{2.70}$$

we have seen earlier , Eq. (2.22) , that the Gorter - Casimir temperature dependence $\lambda \sim (1 - t^4)^{-1/2}$ fits experimental data at all temperature. Near T_c , Eqs. (2.22) and (2.70) , the two forms of temperature dependence are in agreement. In fact , $(1 - t^4)^{-1/2} = (1+t^2)^{-1/2}\ (1+t)^{-1/2}\ (1-t)^{-1/2}$ and for t near 1 the first two terms are slowly varying , so that the dependence on t is dominated by singularity given by the last term , $(1 - t)^{-1/2}$, In similarity Eq. (2.7) , $H_c \sim (1 - t^2)$ reduces to Eq. (2.58) , $H_c \sim (1 - t)$, for t near 1 (or T near T_c).

A second important result that Ginzburg - Landau theory obtained was that, for K << 1 , the interphase surface energy σ_{ns}

between normal and superconducting phase is

$$\delta_{ns} = 1.89 \frac{\lambda}{K} \frac{H_c^2}{8\pi} \quad , \quad K \ll 1 \qquad (2.71)$$

thus explaning the very large positive energy ($\gg H_c^2 / 8\pi$) needed to explain the Meissner effect , and the structure in the intermediate state , physically , the significance of this result was that while the magnetic field decayed over a characteristic distance λ to a vanishingly small value in a the superconductor , decayed to zero toward the normal region over a much longer distance .

Ginzburg and Landau made further observation , without pursuing it , that for $K > 1/\sqrt{2}$, δ_{ns} becomes negative. This was subsequently recogonized as defining the difference between type I and type II superconductors (detail show in chapter 3). Solutions of the Ginzburg - Landau equations for special cases , we will discuss in chapter 3 .

2.5 Microscopic Theory of Superconductivity

Although the most remarkable properties of superconductor are those associated with electromagnetic fields , superconductors also exhibit striking thermodynamic effects, which played a central role in the development of the microscopic theory. The result , experimental by Maxwell (9) and Reynolds , et al.(10) discovered, what is now know as the isotope effect , that the transition temperature T_c of different isotopes of the same element varies with the isotopic mass , M , and it obeys the empirical law

$$T_c \sim M^{-1/2} \tag{2.72}$$

This result indicates that the dynamic of isotopic core effects superconducting state. The modern theory of superconductivity was promulgated by Bardeen , Cooper , and Schrieffer (11) in their classic paper in 1957. The BCS theory has now gained universal acceptance because it has proved capable of explaining all observed phenomena relating to superconductivity. Therefore , in interest of simplicity , let us instead give a brief qualitative , conceptual exposition of the BCS theory.

2.5.1 BCS Theory

The BCS theory has evolved from idea of Cooper (12) who first introduced the concept of an elementary superconductor (Cooper Pair). The basic idea that even a weak attraction can bind pairs of electrons into bound state was presented one year before the BCS theory. What he showed was that the Fermi sea of electrons is unstable against the formation of at least one bound pair, regardless of how weak the interaction is , so long as it is attractive. This is very important, because , in a bound state , electrons are paired their motion are correlated. The pairing can be broken only if an amount of energy equal to the binding energy is applied to the system.

Our two electrons pair are called a Cooper pair. The binding energy is strongest when the electrons forming the pair have equal and opposite momenta and spin (center of mass momentum is zero and opposite spins) , that is $k\uparrow$, $-k\downarrow$. It follows , therefore , that if there is any attraction between them , then all the electrons in the neighborhood of the Fermi surface condense into a system of Cooper pairs.

In technical literature, attractive interaction comes in only when one takes the motion of the ion cores into account. The physical idea is that the first electron polarizes the medium by attracting positive ions ; as illustrated in figure 2.10 , these excess positive ions , in trun , attract the second electron , giving an effective attractive interaction between the electrons.

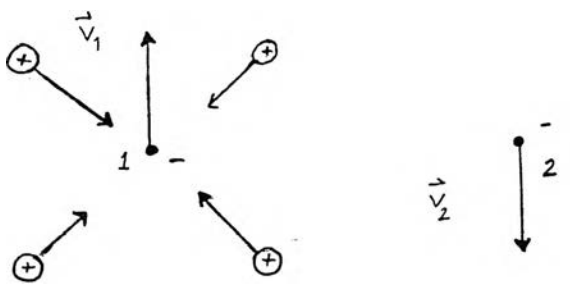


Figure 2.10 The screening of electron 1 by the positive ions of the lattice - solid circles represent the two electrons considered.

If this attraction is strong enough to overrid the repulsive screened Coulomb interaction , it gives rise to a net attractive interaction, and superconductivity results.

Since these lattice deformations are resisted by the same stiffness that makes a solid elastic , it is clear that the characteristic vibration , or phonon , frequencies will play a role, this coupling to the lattice means that an electron can emit a phonon, that is a set the ions into vibration , and it can also absorb a phonon. A possible intermediate process is one in which a phonon is emitted by one of the electrons and absorbed by the other ,as illustrated in figure 2.11 , inclusion of this intermediate process means that the energy of the two electrons is altered , so the phonon exchange has the same effect as a derect interaction between the electrons , reality the attraction between electrons is a second - order process.

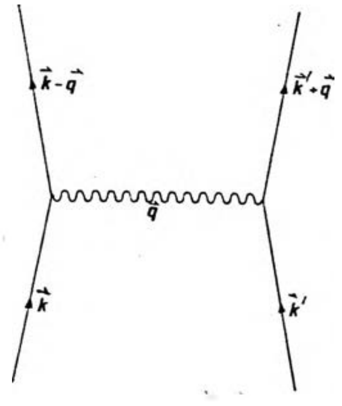


Figure 2.11 Phonon exchange between electrons. This is the fundamental process of superconductivity.

The equivalent direct interaction for the exchange of one phonon turns out to be where , following figure 2.13 , the momentum of the incoming electron are $\hbar\vec{k}$ and $\hbar\vec{k}'$, and the momentum of the exchanged phonon is $\hbar\vec{q}$.

From the suggestive results of Cooper , this line of reasoning led to a reduction of the problem of determining the ground state of Cooper pairs to the model BCS Hamiltonian :

$$H_{BCS} = \sum_{k\sigma} \epsilon_k c^+_{k\sigma} c_{k\sigma} + \Sigma \; V_{kk'} b^+_k b_k \qquad (2.73)$$

The first term is this expression gives the unperturbed energy of the electrons forming the pairs ; the second is the pairing interaction in which a pair of electrons in ($k\downarrow$, $-k\uparrow$) scatters to ($k\downarrow$, $-k\uparrow$),and

$$b^+_k = C^+_{k\uparrow} C^+_{-k\downarrow} \;, \quad b_k = C_{-k\downarrow} C_{k\uparrow} \qquad (2.74)$$

are , respectively , creation and annihilation operators for a pair of electrons in ($k\uparrow$, $- k\downarrow$). These operators obey the commutation relations of the so - called imperfect Bose gas :

$$[b_k \;, b^+_{k'}] = (1-n_{k\downarrow}-n_{-k\uparrow}) \delta_{kk'} \;, [b_k, b_{k'}] = [b^+_k, b^+_{k'}] = 0 \qquad (2.75)$$

The anticommutator of b_k and $b_{k'}$, is

$$\{b_k \;, b_k\}_+ = 2\, b_k\, b_{k'} \,, (1 - \delta_{kk'}) \qquad (2.76)$$

from which if follows that $(b_k)^2 = 0$, according to Schrieffer (48).

This point is essential to the theory and leads to the energy gap being present not only for dissociating a pair but also for making a pair move with a total momentum different from the common momentum of the rest of the pairs. It is this feature which enforce long - range order in the superfluid over macroscopic distance.

The determination of the ground state of the model BCS hamiltonian can now be accomplished by variational method. Since in the superconducting ground state we are interested only in completely paired states , we may retain only the part of the first term in Eq. (2.73) that connects pairs with zero net momentum ;

$$H_{BCS} = \sum_{k} 2 \epsilon_k b^{+}_k b_k + \sum_{kk'} V_{kk'} b^{+}_{k'} b_k \tag{2.77}$$

For the ground state , We take Schrieffer's wave function

$$|\Psi_0\rangle = \prod_{k} (u_k + v_k b^{+}_k) |0\rangle \tag{2.78}$$

Where $|0\rangle$ is the vacuum state and u_k , v_k are variational parameters that may be assumed to be real and , because of overall normalization of ,

$$|u_k|^2 + |v_k|^2 = 1 \tag{2.79}$$

The variational calculation , which is pretty standard (49) , yields the following results

$$v^2_k = (1/2)[1 - (E_k - \mu)/E_k] \tag{2.80}$$

where

$$E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2_k} \tag{2.81}$$

and μ (a Lagrange multiplier in the variational calculation) has the physical significance of the chemical potential (Fermi energy) ; while Δ_k , called the energy gap , satisfies gap equation ,

$$\Delta_k = \frac{-1}{2} \sum_{k'} \frac{V_{kk'} \Delta_{k'}}{E_k} \tag{2.82}$$

From this expression , one obtains in the simple model for which

$$V_{kk} = \begin{cases} -V , & |E_k - \mu| < \hbar\omega_D \text{ and } |\epsilon_K - \mu| < \hbar\omega_D \\ 0 & \text{Otherwise} , \end{cases} \tag{2.83}$$

the result that $\Delta_k = \Delta_o$ (for $|\epsilon_k - \mu| < \hbar\omega_D$) and zero otherwise where

$$\Delta_o = 2\,\hbar\omega \; \exp[\,-1/N(o)V\,] \tag{2.84}$$

where N(o) is the density of states in energy at the Fermi surface. Further , the condensation energy at absolute zero , i.e. the energy difference between the superconducting and the normal states, is found to be

$$\Delta G = (-{}^{1}/_{2})\; N(o)\; \Delta_o^2 \tag{2.85}$$

This is the condensation energy at T = 0 , which must by definition equal $H_c^{\,2}(o)/8\pi$, where $H_c(T)$ is the thermody - namic critical field.

Since we have identified E_k as the excitation energy of a fermion quasi - particle , the probability of its excitation in the thermal equilibrium is the usual Fermi function

$$f\,(E_k) = (e^{\beta E_k} + 1)^{-1} \tag{2.86}$$

It is fairly straightforward to generalize the gap equation to finite temperatures , Eq. (2.83) becomes

$$\Delta_k\,(T) = \frac{-1}{2} \sum_{k'} V_{kk'} \frac{\Delta_{k'}(T)}{E_{k'}(T)} \tanh\left[\frac{\beta E_{k'}(T)}{2}\right] \tag{2.87}$$

If $V_{kk'}$ is approximated by (2.83) , then Δ_k is again of form Δ_o (independent of k) and $\Delta_o\,(\beta)$ ($\beta = 1/k_BT$) satisfies

$$\frac{1}{N(o)V} = \int_0^{\hbar\omega} \frac{d\epsilon}{\sqrt{(\epsilon^2 + \Delta_o^2)}} \tanh\left[\frac{1}{2}\beta(\epsilon^2 + \Delta_o^2)^{1/2}\right] \quad (2.88)$$

This may be solved numerically. The gap $\Delta_o(T)$ decreases as tempesature increases , and vanishes entirely at $T = T_c$ as shown in Fig 2.12. Thus , as $T \rightarrow T_c$ and the gap vanishes, all the electron become normal.

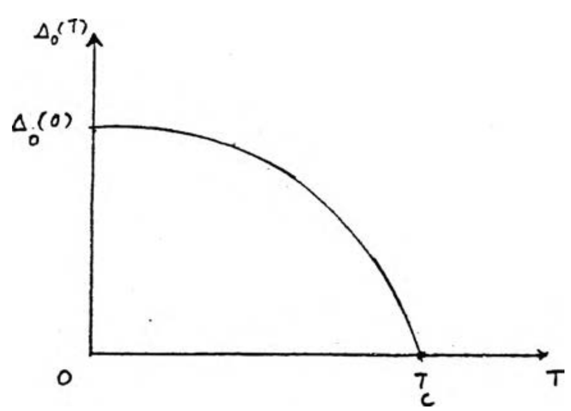


Figure 2.12 plot of the temperature dependence of the energy gap paramenter Δ_o (T).

Note that Δ_o vanishes with infinite slope as $T \rightarrow T_c$, leading to the second - order phasetransition The transition temperature is given by

$$\frac{1}{N(o)V} = \int_0^{\hbar\omega} \frac{dx}{x} \tanh\left(\frac{\beta_c}{2} x\right) \quad (2.89)$$

$$\frac{1}{N(o)V} = \ln(\hbar\omega\beta_c - \int_0^{\infty} dx \ln x \frac{d}{dx}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Hence

$$k_B\,T_c = \frac{2e^{\gamma}}{\pi}\,\hbar\omega\, e^{-1/N(o)V} \tag{2.90}$$

where γ is an Euler's constant , thus

$$k_BT_c = 1.14\,\hbar\omega\,\exp.\,[-1/N(o)V] \tag{2.91}$$

where $\hbar\omega$ is phonon energy for in the lattice , cutoff at the Debye energy $\hbar\,\omega_D$,

$$\hbar\omega = \hbar\omega_D = k_B\,\theta_D \tag{2.92}$$

with θ_D is the Debye temperature , $\theta_D = \hbar\omega_D / k_B$

Also at T = 0 K ,

$$\Delta_o(o) = \frac{\hbar\omega_D}{\sinh\,[1/N(o)V]} \tag{2.93}$$

which in the weak coupling limit , gives

$$\Delta_o(o) = 2\,\hbar\omega_D\,\exp\,[-1/N(o)V] \tag{2.94}$$

Comparing this with Eq. (2.91) , we see that

$$\frac{\Delta_o(o)}{kT_c} = \frac{2}{1.14} = 1.76$$

or

$$2\Delta_o(o) = 3.52\,k_B\,T_c \tag{2.95}$$

so that the gap at T = 0 is indeed comparable in energy to k_BT_c. The numerical factor 3.52 has been tested in many experiments and found to be resonable. That is , experimental value of $2\Delta_o(o)$ for different materials and different direction in k space generally fall in range 3.0 to 4.5 $k_B T_c$, with most clustered near the BCS value 3.50 $k_B T_c$.

Eq. (2.91) contains the isotope effect , dimensionally , ω_D must be given as $\omega_D \sim (K/M)^{1/2}$, where K is a force constant of the lattice , therefor $\omega_D \sim M^{-1/2}$, We already remarked that V should be independent of M , so Eq. (2.91) does give $T_c \sim M^{-1/2}$.

2.5.2 Strong - Coupling Theory

On the basics of the physical features of the BCS theory above , a wide variety of phenomena in superconductors has been worked out by numerous workers in the field. Deviations from the BCS theory occur , however , when , as point out in connection with a McMillan formula (14) , T_c is given by

$$T_c = \frac{\Theta}{1.45} \exp\left[-1.04\,(1+\lambda)/\ -\mu^*\,(1+0.62\lambda)\right] \quad (2.96)$$

where , for the strong - coupling superconductors , λ is the electron - phonon interaction strength , which is proportional to $(M\,\omega^2)^{-1}$, μ^* is the screened electron - electron Coulomb interaction and Θ is the Debye temperature , and for weak - coupling superconductors ($\lambda \ll 1$)

$$\frac{1.04\,(1+\lambda)}{\lambda - \mu^*(1+0.62\quad)} \simeq \frac{1}{\lambda - \mu^*} \equiv \frac{1}{N(o)V} \quad (2.97)$$

hence , McMillan formular reduces to Eq. (2.91) , original BCS theory , the electron - phonon coupling slrength λ is much greater than unity ; i.e., $\lambda \gg 1$: Superconductors for which this condition is true include Pb, Sn, Hg, Nb, and certain alloys like Nb_3 Sn and are called strong - coupling superconductors. These deviations can occur in three ways ,

1. The quasi - particle picture can become impedance if ,for example , the damping rate becomes comparable with the quasi - particle excitation energy.
2. The assumption of an effective two - body instantaneous interaction between quasi - particles may not provide an adequate representation of the retarded nature of the phonon - induced interaction
3. The pairing hypothesis may break down.

In most circumstance , only the first two possibilities are operative while the pairing hypothesis appears to be generally sounds for both weak and strong - coupling superconductors.

The theory of the strong - coupling superconductors was developed by Eliashberg (13) and others. It has the distinctive feature , however , that it provides a natural framework for correlating results from a number of different experiments that allows the many - body problem to be tackled in smaller , more tractable pieces. Most of these features arise from the fact that in the strong -coulping theory the energy gap parameter $\Delta(x,t)$ becomes a complex function of space and time , i.e., has real and imaginary part. The search for a superconductor is centered on now to control the parameters λ, and μ^* in the fabrication of a superconducting alloy , so that T_c will be as large as possible.