CHAPTER IV
RESULTS AND DISCUSSION

4.1 Example Process: Case Study

Consider the following process which was used by Bagajewicz (2004a):

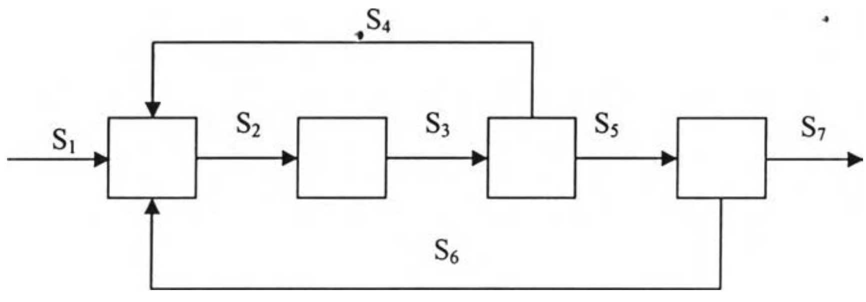

Figure 4.1 Example process

We assume that all variables are measured. The variance-covariance matrix of measurements is a diagonal matrix S = diag(1.0,0.2,1.0,1.0,1.0,1.0,1.0). The other assumptions are that biases follow normal distributions with zero means and standard deviations $\rho_1 = 3.0$, $\rho_2 = 4.0$, $\rho_3 = 4.0$, $\rho_4 = 5.0$, $\rho_5 = 6.0$, $\rho_6 = 5.0$. The stream S_7 is the output flowrate (or product flowrate). Therefore it is the flowrate for which economic value of accuracy is calculated. The two methods are to be compared based on two criteria: *accuracy of solution* and *time of computation*. The following results were obtained using an Intel 2.4 GHz processor and 1024 MB RAM memory. The solution, time of computation and maximum error are reported and interval sizes are indicated in the following tables.

4.1.1 The Financial Loss And The Probability In The Presence of Multiple Gross Errors

4.1.1 1 When Two Gross Errors Are Present

The results for the financial loss in the presence of two gross errors are shown in table 4.1 and table 4.2

Table 4.1 DEFL/ K_sT for two gross errors present in the system obtained by using the two methods

Locations of biases	Approximation method (interval size = 0.1)			Monte Carlo method				Relative error** (%)
	Solution	Time[a] (1/100th second)	Error*	Solution 1 N = 10^6	Solution 2 N = 10^7	Solution 3 N = 10^8	Time[b] (second)	
1, 2	0.2162	10	$0.6*10^{-5}$	0.2162	0.2162	0.2161	509	0.021
1, 3	0.2148	11	$0.5*10^{-5}$	0.2148	0.2148	0.2147	527	0.041
1, 4	0.2199	16	$0.4*10^{-5}$	0.2199	0.2198	0.2198	533	0.056
1, 5	0.2208	19	$0.8*10^{-5}$	0.2208	0.2208	0.2207	517	0.016
1, 6	0.2493	34	$1*10^{-5}$	0.2491	0.2491	0.249	545	0.101
2, 3	0.2017	69	$0.1*10^{-6}$	0.2018	0.2017	0.2017	553	0.002
2, 4	0.2005	34	$0.1*10^{-6}$	0.2005	0.2005	0.2005	493	0.008
2, 5	0.2024	17	$0.3*10^{-6}$	0.2023	0.2024	0.2024	485	0.026
2, 6	0.2056	14	$0.6*10^{-6}$	0.2056	0.2056	0.2056	497	0.026
3, 4	0.2006	13	$0.1*10^{-6}$	0.2006	0.2006	0.2006	491	0.004
3, 5	0.2007	12	$0.1*10^{-6}$	0.2007	0.2007	0.2007	577	0.005
3, 6	0.2057	12	$0.1*10^{-6}$	0.2057	0.2057	0.2057	502	0.004
4, 5	0.2098	52	$0.5*10^{-6}$	0.2098	0.2098	0.2098	560	0.011
4, 6	0.2054	34	$1*10^{-6}$	0.2054	0.2054	0.2054	542	0.001
5, 6	0.2051	38	$0.3*10^{-6}$	0.2051	0.2052	0.2051	512	0.002

where

Solution: average of the overestimate and the underestimate = $(J_U^* - J_L^*)/2$

N: number of trials in the Monte Carlo method.

Time[a] : sum of computation time in calculating the overestimate and the underestimate. Time[b]: computation time when number of trials N = 10^8; when N increase 10 times, computation time increase roughly 10 times (i.e., computation time when N = 10^6 is about Time[b]/100 second).

Error*: is defined as $(J_U^* - J_L^*)$, all values smaller than $0.1*10^{-6}$ are rounded to $0.1*10^{-6}$.

Relative error**: relative error between solutions obtained by the two methods.

Relative error = (solution – solution 3)*100/solution 3

The above notations are also applied to other tables showing calculation results.

Table 4.2 DEFL/ K_sT for two gross errors present in the system obtained by using the approximation method at different interval sizes

Locations of biases	Interval size = 0.1			Interval size = 0.2		
	Solution	Time (1/100th second)	Error*	Solution	Time (1/100th second)	Error*
1, 2	0.2162	10	$0.6*10^{-5}$	0.2162	3	$2.3*10^{-5}$
1, 3	0.2148	11	$0.5*10^{-5}$	0.2148	2	$1.9*10^{-5}$
1, 4	0.2199	16	$0.4*10^{-5}$	0.2197	4	$3.4*10^{-5}$
1, 5	0.2208	19	$0.8*10^{-5}$	0.2204	6	$3*10^{-5}$
1, 6	0.2493	34	$1*10^{-5}$	0.2487	10	$6.2*10^{-5}$
2, 3	0.2017	69	$0.1*10^{-6}$	0.2017	17	$0.25*10^{-6}$
2, 4	0.2005	34	$0.1*10^{-6}$	0.2005	10	$0.1*10^{-6}$
2, 5	0.2024	17	$0.3*10^{-6}$	0.2024	3	$1.0*10^{-6}$
2, 6	0.2056	14	$0.6*10^{-6}$	0.2056	3	$2.3*10^{-6}$
3, 4	0.2006	13	$0.1*10^{-6}$	0.2006	3	$0.1*10^{-6}$
3, 5	0.2007	12	$0.1*10^{-6}$	0.2007	4	$0.3*10^{-6}$
3, 6	0.2057	12	$0.8*10^{-6}$	0.2057	4	$3.2*10^{-6}$
4, 5	0.2098	52	$0.5*10^{-6}$	0.2098	13	$1.9*10^{-6}$
4, 6	0.2054	34	$1*10^{-6}$	0.2054	8	$4.4*10^{-6}$
5, 6	0.2051	38	$0.3*10^{-6}$	0.2052	9	$1.3*10^{-6}$

The accuracy of solution and computation time of the approximation method depend mainly on two factors: interval size chosen and parameters W_{ij} (to be used in the MT test statistics). When interval size (size of subinterval $[a_i,b_i]$ divided) decreases, accuracy increases (the error decreases) but computation time also increases. Table 4.2 shows that when interval size decreases 2 times (from 0.2 to 0.1), the error decreases about 3-5 times but computation time increases about 3 – 5 times. The parameters W_{ij} will determine the size of the regions divided (e.g. P_1, P_2, P_3, P_4) which will affect the computation time. The larger the size of the rectangular regions (e.g. P_4), the more the amount of computation, hence the longer the computation time. The results show that at the same number of gross errors, computation time varies significantly. Moreover, approximation method is also subjected to round-off problem, for example, the number of subintervals divided (e.g., the range [$-K_1$, K_1] divided by interval size) may be rounded off, which will somehow affect accuracy of solution. Clearly, the results show that at the same number of gross errors, the error also varies significantly. For financial loss calculation in the presence of two biases, the computation time of approximation method is not more than one second and the error is not more than $1*10^{-5}$. The relative error between solutions obtained by the two methods is not more than 0.1%.

Regarding the Monte Carlo method, the accuracy of solution and the computation time of the Monte Carlo method depend on the number of trials N. It is well known that the higher the number of trials N, the better its solution. When we increase N, we obtain more accurate solution at the expense of longer computation time. When number of trials N increases 10 times, computation time increases roughly 10 times (based on the author's calculation results not shown in the tables). It is obvious that we have convergence of solution when number of trials N $\geq 10^6$. Really, the relative error between solution 1 (at $N = 10^6$) & solution 3 (at $N = 10^8$) is usually not more than 0.1% and that between solution 2 (at $N = 10^7$) & solution 3 (at $N = 10^8$) is usually not more than 0.05%. Therefore, we can say that the Monte Carlo method at number of trials $N = 10^6$ has satisfactory accuracy of solution and relatively fast computation speed (a few seconds, not more than 9 seconds when four gross errors are present). At the same number of gross errors, the computation time of Monte Carlo method is virtually unchanged. The same arguments as stated above are also observed for other cases (more than two gross errors are present) and also for the probability calculation as shown in the following tables.

Comparing between the two methods, Monte Carlo method can give us the best solution at the expense of long computation time. When two gross errors are present (the number of variables is two), the approximation method is superior to Monte Carlo method because its solution accuracy is satisfactory (the relative error between its solution and Monte Carlo method's solution at $N = 10^8$ is not more than 0.1%) and computation time is shorter (not more than one second while that of Monte Carlo method ranges from about 6 seconds at number of trials N $= 10^6$ to about 600 seconds at number of trials $N = 10^8$).

The results show clearly that the financial loss depends on the number and the location of gross errors or more specifically, the financial loss is the function of the location of measurement sensors. In other words, the financial loss is the function of existing instrumentation. Table 4.1 show that, when two gross errors are present, the two largest financial losses are incurred at biases' locations of 1,6 and 1,5. This can be explained by the recognition that undetected gross error in stream S1 causes the largest induced bias in stream S7 (output stream or product stream) then follow stream S6 and S5. Really, we see that streams S1 & S7 are part of an equivalent set (or input stream S1 = output stream S7) and streams S5 & S6 are connected directly to stream S7 through a node, therefore undetected gross errors in these streams cause the worst effect on measurement accuracy of stream S7 (through data reconciliation treatment). This argument can be checked by looking at the value of the coefficient α_i in the expression $\hat{\delta}_p^i(\theta_i) = \alpha_i\theta_i$ for induced bias in stream S7 caused by undetected gross error anywhere in the system: stream S1 has the largest value α_i (0.3774), then follow stream S6 and S5 (-0.2453 and 0.1321 respectively). Note that because streams S1 & S7 are part of an equivalent set (Bagajewicz and Jiang, 1998), calculation results for biases at streams 1, 5 and 1, 6 and 1, 2, 3 are the same as calculation results for biases at streams 7, 5 and 7, 6 and 7, 2, 3. In other words, if stream S1 is replaced by stream S7, the calculation results are unchanged.

The results also confirm our expectation that that financial loss in the presence of biases is larger than the financial loss without biases. Really, we see that the financial loss without biases is $DEFL^0 = 0.19947 K_s T \hat{\sigma}_p = 0.19947 K_s T$ and all the results shown above (which are financial losses $DEFL/K_sT$ for the presence of two biases) are larger than $0.19947 = DEFL^0/ K_sT$. However, when a gross errors detection strategy such as MIMT is used to detected gross errors, the increase in financial loss is not too much (not more than 130% based on the calculation results obtained so far at the given assumptions).

The results for the probability in the presence of two gross errors are shown in table 4.3 and table 4.4

Table 4.3 The probability for two gross errors present in the system obtained by using the two methods

$P\{\hat{m}_p \geq m_p^{*} | i1,i2\} = \frac{\Phi^2_{i1,i2}}{2}\frac{1}{2}S$, S: the solutions given below

Loca-tions of biases	Approximation method (interval size = 0.1)			Monte Carlo method				Relative Error** (%)
	Solution	Time^a (1/100th second)	Error*	Solution 1 N = 10^6	Solution 2 N = 10^7	Solution 3 N = 10^8	Time^b (second)	
1, 2	1	8	$4*10^{-5}$	0.9998	0.9998	1	482	0.0007
1, 3	1	8	$3.2*10^{-5}$	0.9998	0.9999	1	479	0.0009
1, 4	1	14	$4.6*10^{-5}$	0.9997	0.9998	1	463	0.0026
1, 5	1	13	$4.8*10^{-5}$	0.9998	0.9999	1	458	0.0009
1, 6	1	27	$9.6*10^{-5}$	0.9996	0.9997	1	476	0.0022
2, 3	1	51	$0.1*10^{-5}$	0.9999	1	1	471	0.0009
2, 4	1	27	$0.2*10^{-5}$	1	1	1	447	0.0005
2, 5	1	12	$0.4*10^{-5}$	1	1	1	425	0.001
2, 6	1	12	$1*10^{-5}$	1	0.9999	1	435	0.0008
3, 4	1	9	$0.1*10^{-5}$	1	1	1	432	0.0001
3, 5	1	9	$0.2*10^{-5}$	1.0001	1	1	436	0.0007
3, 6	1	8	$0.8*10^{-5}$	0.9999	0.9999	1	443	0.001
4, 5	1	40	$1*10^{-5}$	0.9999	1.0001	1	445	0.0019
4, 6	1	27	$1.1*10^{-5}$	1	0.9999	1	542	0.0001
5, 6	1	27	$0.9*10^{-5}$	1	0.9999	1	512	0.0008

The same observations as stated above (i.e. how interval size affects computation time and solution accuracy of the approximation method and how the number of trials N affects those of the Monte Carlo method) can be verified from table 4.3 and the following table 4.4. For the probability calculation in the presence of two biases, the error of approximation method is not more than $1*10^{-4}$ and the relative error between solutions obtained by the two methods is not more than 0.01%. In general, at the same number and the same location of biases, computation times of both methods in probability calculation are less than those in financial loss calculation because financial loss calculation requires more computation.

Table 4.4 Probability for two gross errors present in the system obtained by using the approximation method at different interval sizes

$P\{\hat{m}_p \geq m_p^* | i1,i2\} = \frac{\Phi_{i1,i2}^2}{2}\frac{1}{2}S$, S: the solutions given below

Locations of biases	Interval size = 0.1			Interval size = 0.2		
	Solution	Time (1/100th second)	Error	Solution	Time (1/100th second)	Error
1, 2	1	8	$4*10^{-5}$	1	2	$20*10^{-5}$
1, 3	1	8	$3.2*10^{-5}$	1	2	$10*10^{-5}$
1, 4	1	14	$4.6*10^{-5}$	1	1	$20*10^{-5}$
1, 5	1	13	$4.8*10^{-5}$	1	1	$20*10^{-5}$
1, 6	1	27	$9.6*10^{-5}$	1	3	$40*10^{-5}$
2, 3	1	51	$0.1*10^{-5}$	1	6	$0.5*10^{-5}$
2, 4	1	27	$0.2*10^{-5}$	1	3	$0.6*10^{-5}$
2, 5	1	12	$0.4*10^{-5}$	1	2	$1*10^{-5}$
2, 6	1	12	$1*10^{-5}$	1	1	$4*10^{-5}$
3, 4	1	9	$0.1*10^{-5}$	1	1	$0.4*10^{-5}$
3, 5	1	9	$0.2*10^{-5}$	1	1	$0.4*10^{-5}$
3, 6	1	8	$0.8*10^{-5}$	1	1	$3*10^{-5}$
4, 5	1	40	$1*10^{-5}$	1	5	$3*10^{-5}$
4, 6	1	27	$1.1*10^{-5}$	1	4	$4*10^{-5}$
5, 6	1	27	$0.9*10^{-5}$	1	3	$4*10^{-5}$

If biases are assumed to follow normal distribution with zero means, it is obvious that the probability P is the function of $\Phi_{i1,i2}^2$, which is the probability of such set of gross errors at a particular location to develop, only. Really, the results show that the probability P takes value of $\frac{\Phi_{i1,i2}^2}{4} x1$ regardless of the location of gross errors. This probability is viewed as the confidence with which the financial loss as shown above is known. Recall that, under assumptions of negligible process variations and normal distributions, the probability without biases has been shown to be 0.25 under assumptions o (Bagajewicz et al., 2003). Therefore the probability in the presence of biases is less than the probability without biases (under simplified assumptions of negligible process variations and normal distributions with zero means) since $\Phi_{i1,i2}^2$ is less than one.

4.1.1.2 When Three Gross Errors Are Present

The results for the financial loss in the presence of three biases calculated by approximation method and Monte Carlo method are shown in table 4.5

Table 4.5 DEFL/ K_sT for three gross errors present in the system obtained by using the two methods

Location of biases	Approximation method (interval size = 0.2)			Monte Carlo method				Relative Error** (%)
	Solution	Time[a] (second)	Error*	Solution 1 $N = 10^6$	Solution 2 $N = 10^7$	Solution 3 $N = 10^8$	Time[b] (second)	
1, 2, 3	0.2218	10	$11*10^{-5}$	0.222	0.222	0.222	719	0.099
1, 2, 4	0.2192	4	$3.1*10^{-5}$	0.2193	0.2194	0.2195	662	0.128
1, 2, 5	0.2279	3	$4.8*10^{-5}$	0.2277	0.2278	0.2279	641	0.009
1, 2, 6	0.2489	3	$11*10^{-5}$	0.2484	0.2486	0.2487	649	0.104
1, 3, 4	0.2194	1	$2.9*10^{-5}$	0.2196	0.2197	0.2197	654	0.133
1, 3, 5	0.2213	1	$3.3*10^{-5}$	0.2209	0.2211	0.2211	658	0.071
1, 3, 6	0.2489	3	$11*10^{-5}$	0.2487	0.249	0.249	644	0.065
1, 4, 5	0.2577	19	$8.4*10^{-5}$	0.2577	0.2581	0.2582	641	0.183
1, 4, 6	0.2482	6	$13*10^{-5}$	0.2477	0.2479	0.2479	637	0.114
1, 5, 6	0.247	8	$12*10^{-5}$	0.2471	0.2474	0.2475	652	0.208
2, 3, 5	0.2128	54	$0.5*10^{-5}$	0.2132	0.2131	0.2131	652	0.125
2, 3, 6	0.2056	26	$0.6*10^{-5}$	0.2056	0.2056	0.2056	648	0.016
2, 4, 5	0.2073	9	$0.5*10^{-5}$	0.2072	0.2072	0.2072	628	0.047
2, 4, 6	0.2053	6	$0.5*10^{-5}$	0.2053	0.2053	0.2053	612	0.012
2, 5, 6	0.2048	17	$0.6*10^{-5}$	0.2048	0.2048	0.2048	613	0.001
3, 4, 5	0.2097	4	$0.4*10^{-5}$	0.2097	0.2098	0.2098	623	0.025
3, 4, 6	0.2054	3	$0.5*10^{-5}$	0.2053	0.2054	0.2054	610	0.003
3, 5, 6	0.2052	4	$0.5*10^{-5}$	0.2052	0.2052	0.2052	609	0.022

Tables 4.1, 4.2 and table 4.5 show that when the number of gross errors is increased by one (from two to three), computation time of the approximation method increases significantly. On the same step size basis, computation time can increase 50-200 times, which can be seen from table 4.2 and table 4.5 (the increase in computation time once again depends on the parameter W_{ij} or the size of the rectangular regions P_1, P_2, P_3, P_4...). The results also show that when the number of gross errors increases, the error of the approximation method also increases (solution accuracy decreases) significantly but this thing is partly due to the increase in interval size used. These observations are also applied to the probability calculation. For financial loss calculation in the presence of three biases,

the error of the approximation method is not more than $1.3*10^{-5}$ and the relative error between solutions obtained by the two methods is not more than 0.2%.

Regarding the Monte Carlo method, when the number of gross errors is increased by one (from two to three), computation time increases a little bit (roughly 1.2-1.3 times). The relative error between Monte Carlo method's solution at $N = 10^6$ and that at $N = 10^8$ is usually not more than 0.1% except some values as large as 0.2%.

Comparing between the two methods, when three gross errors are present (the number of variables is three), computation time of approximation method is comparable to that of Monte Carlo method at number of trials $N = 10^6$ (about a few seconds) but it is still significantly less than that of Monte Carlo method at number of trials $N = 10^8$.

From the results shown in table 4.1 (two biases are present) and table 4.5 (three biases are present), it is also obvious that the financial loss increases when the number of gross errors increases. For example, financial losses incurred due to biases at location 1, 2 and 1, 3 are 0.2162 and 0.2148 respectively, while those at location 1, 2, 3 and 1, 2, 3, 5 are 0.2218 and 0.2591, respectively.

We know that undetected gross error in stream S1 causes the largest induced bias in stream S7 (output stream or product stream) then follow streams S6 and S5 and when two gross errors are present, the two largest financial losses are incurred at biases' locations of 1,6 and 1,5. However, table 4.3 show that when three gross errors are present, the three largest financial losses are incurred at biases' locations of 1, 4, 5 and 1, 3, 6 and 1, 4, 6 rather than the location 1, 5, 6 (the fourth largest financial loss incurred). The reason is that the financial loss depends strongly on the magnitude of the induced bias which in turn depends not only on the coefficients α_i but also the on the power of the gross error detection strategy to detect sets of gross errors at specific locations. Compared with the set of gross errors at location 1, 4, 5; the set of gross errors at location 1, 5, 6 render larger coefficients α_i ($\alpha_6 > \alpha_4$) but smaller magnitudes of maximum undetected gross errors (that is, the set of gross errors at location 1, 4, 5 is more resistant to the gross errors detection than that at location 1, 5, 6). Thus the financial loss incurred due to a set of gross errors at a particular location (i.e. at particular streams) is a function of the two factors: (i) the coefficients α_i that indicate the effect of undetected gross errors in these streams to measurement accuracy of product stream and (ii): the power of the gross error detection strategy to detect this set of gross errors at this specific location. A set of gross errors at specific locations causes large financial loss when it possesses: (i): large coefficients α_i and/or (ii): more resistance to gross error detection strategy.

The results for the probability in the presence of three biases calculated by approximation method and Monte Carlo method are shown in table 4.6

Table 4.6 The probability for three gross errors present in the system obtained by using the two methods

$P\{\hat{m}_p \geq m_p^* | i1,i2,i3\} = \frac{\Phi^3_{i1,i2,i3}}{2} - \frac{1}{2}S$, S: the solutions given below

Locations of biases	Approximation method (interval size = 0.2)			Monte Carlo method				Relative Error** (%)
	Solution	Time[a] (second)	Error*	Solution 1 $N = 10^6$	Solution 2 $N = 10^7$	Solution 3 $N = 10^8$	Time[b] (second)	
1, 2, 3	1.0006	10	$20*10^{-5}$	0.9999	1	1	611	0.059
1, 2, 4	1	4	$16*10^{-5}$	1.0005	1	0.9999	583	0.003
1, 2, 5	1	3	$28*10^{-5}$	1.0004	1	0.9999	554	0.007
1, 2, 6	0.9999	3	$32*10^{-5}$	1.0007	1.0001	0.9999	575	0.001
1, 3, 4	1	1	$15*10^{-5}$	1.0005	1	0.9999	564	0.004
1, 3, 5	1	1	$20*10^{-5}$	1.0006	1.0001	0.9999	556	0.006
1, 3, 6	1	3	$30*10^{-5}$	1.0007	1	0.9999	563	0.002
1, 4, 5	0.9999	19	$31*10^{-5}$	1.0011	1.0002	1	563	0.004
1, 4, 6	0.9999	6	$35*10^{-5}$	1.0006	0.9999	0.9999	564	0.001
1, 5, 6	0.9999	5	$30*10^{-5}$	1.0008	1	0.9999	585	0.003
2, 3, 5	0.9984	53	$4.1*10^{-5}$	0.9995	1.0001	1	562	0.158
2, 3, 6	0.9998	26	$3.2*10^{-5}$	1.0002	1	1	549	0.018
2, 4, 5	1	9	$3.1*10^{-5}$	1	1	1	551	0.001
2, 4, 6	1	6	$4.1*10^{-5}$	1.0002	1	1	544	0.001
2, 5, 6	1	17	$3.3*10^{-5}$	1.0001	1	1	562	0.002
3, 4, 5	1	4	$3*10^{-5}$	1.0002	1	1	559	0.003
3, 4, 6	1	3	$4.3*10^{-5}$	1.0002	1	1	550	0.001
3, 5, 6	1	4	$3*10^{-5}$	1	1	1	538	0.001

The same arguments (i.e. how and what factors affect computation time and solution accuracy of the approximation method and the Monte

Carlo method) as stated above can be verified from tables 4.3, 4.4 and table 4.6. For probability calculation in the presence of three biases, the error of approximation method is not more than $4*10^{-4}$ and the relative error between solutions obtained by the two methods is not more than 0.2%. Concerning the results, if biases are assumed to follow normal distributions with zero means, the probability P is the function of $\Phi^{3}_{i1,i2,i3}$ only regardless of the location of gross errors (all the results above are $\Phi^{3}_{i1,i2,i3}*1/4$).

4.1.1.3 When Four Gross Errors Are Present

The results for the financial loss in the presence of four biases calculated by approximation method and Monte Carlo method are shown in table 4.7

Table 4.7 DEFL/ K_sT for four gross errors present in the system obtained by using the two methods

Location of biases	Approximation method (interval size = 0.4)			Monte Carlo method				Relative Error (%)
	Solution	Time[a] (second)	Error[a]	Solution 1 $N = 10^6$	Solution 2 $N = 10^7$	Solution 3 $N = 10^8$	Time[b] (second)	
1, 2, 3, 5[c]	0.2591	146	$13*10^{-4}$	0.2598	0.2592	0.2592	798	0.027
1, 2, 3, 6	0.2441	112	$3.3*10^{-4}$	0.2443	0.2443	0.2442	783	0.058
1, 2, 4, 6	0.2474	33	$3.7*10^{-4}$	0.2481	0.2479	0.2479	789	0.191
1, 3, 4, 5	0.2563	38	$4.2*10^{-4}$	0.2561	0.256	0.256	810	0.117

1, 2, 3, 5[c]: interval size = 0.5 was used

The same arguments as stated above for the approximation method and Monte Carlo method can be verified from tables 4.1, 4.5 and table 4.7. For financial loss calculation in the presence of four biases, the error of the approximation method is not more than $1.3*10^{-3}$ and the relative error between solutions obtained by the two methods is not more than 0.2%.

Comparing between the two methods, when four gross errors are present (number of variables is four), computation time of approximation method is comparable to that of Monte Carlo method at number of trials $N = 10^7$ (about 80 seconds) but it is still significantly less than that of Monte Carlo method at number of trials N = 10^8. Therefore it can be said that at high number of gross errors (≥ 5), the approximation method is not superior to Monte Carlo method because computation time of approximation increases significantly while that of Monte Carlo method increases a little bit with the number of gross errors. At high number of gross errors, we should use large interval size in approximation method to reduce computation time, which will increase the errors or reduce accuracy of solutions. An alternative choice at high number of gross errors is the Monte Carlo method at low number of trials (N = 10^6 or N = 10^7) whose solution is satisfactorily accurate and computation time is acceptable. Short computation time is important because the financial loss calculation can be used in sensor network design which needs to explore many alternatives combinatorially.

Concerning the results, from tables 4.5 and 4.7, it is obvious that the financial loss increases when more gross errors are present.

The results for the probability in the presence of four biases calculated by approximation method and Monte Carlo method are shown in table 4.8

Table 4.8 The probability for four gross errors present in the system obtained by using the two methods

$$P\{\hat{m}_p \geq m_p^* | i1,i2,i3,i4\} = \frac{\Phi^4_{i1,i2,i3,i4}}{2} \frac{1}{2} S,$$ S: the solutions given below

Location of biases	Approximation method (interval size = 0.4)			Monte Carlo method				Relative Error (%)
	Solution	Time[a] (second)	Error*	Solution 1 N = 10^6	Solution 2 N = 10^7	Solution 3 N = 10^8	Time[b] (second)	
1, 2, 3, 5[c]	0.9981	83	$55*10^{-4}$	0.9988	0.9998	1	710	0.189
1, 2, 3, 6	0.999	77	$6.8*10^{-4}$	0.9999	0.9999	1	714	0.1
1, 2, 4, 6	1	22	$8.5*10^{-4}$	0.9995	0.9998	1	726	0.003
1, 3, 4, 5	0.9992	26	$9.1*10^{-4}$	1	1	1	735	0.08

1, 2, 3, 5[c]: interval size = 0.5 was used

Again the same arguments as stated above for the approximation method and Monte Carlo can be verified from tables 4.3, 4.6 and table 4.8. For probability calculation in the presence of four biases, the error of the approximation method is not more than $6*10^{-3}$ and the relative error between solutions obtained by the two methods is not more than 0.2%. Once again, if biases are assumed to follow normal distribution with zero means, the probability P is the function of $\Phi^4_{i1,i2,i3,i4}$ only regardless of the location of gross errors.

4.1.2 Effect of Changing Parameters

The effect of changing parameters the effectiveness of approximation method is investigated. The means of the biases' probability distribution functions are changed to nonzero values. In other words, if biases are assumed to follow normal distributions with nonzero means (and same standard deviations as shown above), we obtain the following results shown in two tables: table 4.9 for positive means (all means = +1) and table 4.10 for negative means (all means = -1):

Table 4.9 DEFL/ K_sT for mutiple gross errors present in the system obtained by using the approximation method when parameters are changed

Locations of biases	Zero means		Positive means (+1)		Negative means (-1)	
	Solution	Error	Solution	Error	Solution	Error
1, 2	0.2162	$0.6*10^{-5}$	0.2033	$0.6*10^{-5}$	0.228	$0.6*10^{-5}$
1, 3	0.2148	$0.5*10^{-5}$	0.2029	$0.5*10^{-5}$	0.2258	$0.4*10^{-5}$
1, 5	0.2208	$0.8*10^{-5}$	0.2064	$0.7*10^{-5}$	0.2341	$0.7*10^{-5}$
1, 6	0.2493	$1*10^{-5}$	0.2333	$1.5*10^{-5}$	0.263	$1.4*10^{-5}$
2, 6	0.2056	$0.06*10^{-5}$	0.2082	$0.07*10^{-5}$	0.2028	$0.09*10^{-5}$
4, 5	0.2098	$0.05*10^{-5}$	0.211	$0.07*10^{-5}$	0.2083	$0.07*10^{-5}$
5, 6	0.2051	$0.03*10^{-5}$	0.2081	$0.06*10^{-5}$	0.202	$0.07*10^{-5}$
1, 2, 3	0.2218	$11*10^{-5}$	0.2016	$10.1*10^{-5}$	0.2418	$10.2*10^{-5}$
1, 2, 4	0.2192	$3.1*10^{-5}$	0.2086	$6.14*10^{-5}$	0.2285	$6.1*10^{-5}$
1, 3, 6	0.2489	$11*10^{-5}$	0.2336	$10.4*10^{-5}$	0.2619	$10.4*10^{-5}$
1, 4, 5	0.2577	$8.4*10^{-5}$	0.2389	$12.3*10^{-5}$	0.2752	$12.2*10^{-5}$
3, 4, 5	0.2097	$0.4*10^{-5}$	0.211	$1.0*10^{-5}$	0.2081	$1.0*10^{-5}$
3, 5, 6	0.2052	$0.5*10^{-5}$	0.2083	$0.84*10^{-5}$	0.202	$0.84*10^{-5}$
1, 2, 3, 5[a]	0.2591	$13*10^{-4}$	0.2112	$12.6*10^{-4}$	0.3094	$12.6*10^{-4}$
1, 2, 3, 6	0.2441	$3.3*10^{-4}$	0.2324	$3.15*10^{-4}$	0.2534	$3.14*10^{-4}$
1, 2, 4, 6	0.2474	$3.7*10^{-4}$	0.232	$3.7*10^{-4}$	0.2604	$3.7*10^{-4}$
1, 3, 4, 5	0.2563	$4.2*10^{-4}$	0.2381	$4.13*10^{-4}$	0.2734	$4.13*10^{-4}$

Interval size = 0.1, 0.2, 0.4 were used for the financial loss calculation in the presence of two, three, four gross errors respectively.

1, 2, 3, 5[a]: interval size = 0.5 was used

From table 4.9 and the following table 4.10, we can see that when parameters such as the means of the biases' probability distributions change, the computation time and the errors (solution accuracy) of the approximation method are virtually unchanged at the same number of gross errors and the same location. If the means of biases' probability distributions are changed to nonzero values, the financial loss and the probability also change; the change may be an increase or decrease depending on the signs of coefficients α_i and the means.

Table 4.10 Probability for mutiple gross errors present in the system obtained by using the approximation method when parameters are changed

$$P\{\hat{m}_p \geq m_p^* | i1,i2\} = \frac{\Phi^2_{i1,i2}}{2}\frac{1}{2}S$$

or $$P\{\hat{m}_p \geq m_p^* | i1,i2,i3\} = \frac{\Phi^3_{i1,i2,i3}}{2}\frac{1}{2}S$$

or $$P\{\hat{m}_p \geq m_p^* | i1,i2,i3,i4\} = \frac{\Phi^4_{i1,i2,i3,i4}}{2}\frac{1}{2}S;$$ S: the solutions given below

Location	Zero means		Positive means (+1)		Negative means (-1)	
of biases	Solution	Error	Solution		Solution	Error
1, 2	1	$4*10^{-5}$	1.036	$3.8*10^{-5}$	0.964	$3.73*10^{-5}$
1, 3	1	$3.2*10^{-5}$	1.0337	$3.2*10^{-5}$	0.9663	$3.14*10^{-5}$
1, 5	1	$4.8*10^{-5}$	1.0395	$4.6*10^{-5}$	0.9605	$4.62*10^{-5}$
1, 6	1	$9.6*10^{-5}$	1.0369	$9.3*10^{-5}$	0.9631	$9.12*10^{-5}$
2, 6	1	$1*10^{-5}$	0.9918	$0.95*10^{-5}$	1.0082	$1*10^{-5}$
4, 5	1	$1*10^{-5}$	0.996	$0.86*10^{-5}$	1.004	$0.9*10^{-5}$
5, 6	1	$0.9*10^{-5}$	1.036	$0.98*10^{-5}$	0.964	$1*10^{-5}$
1, 2, 3	1.0006	$20*10^{-5}$	1.0574	$20.1*10^{-5}$	0.9438	$20.4*10^{-5}$
1, 2, 4	1	$16*10^{-5}$	1.0286	$16.2*10^{-5}$	0.9714	$15.4*10^{-5}$
1, 3, 6	1	$30*10^{-5}$	1.0352	$29.6*10^{-5}$	0.9647	$28.1*10^{-5}$
1, 4, 5	0.9999	$31*10^{-5}$	1.0436	$30.6*10^{-5}$	0.9563	$29.8*10^{-5}$
3, 4, 5	1	$3*10^{-5}$	0.9956	$2.8*10^{-5}$	1.0044	$2.9*10^{-5}$
3, 5, 6	1	$3*10^{-5}$	0.9904	$2.61*10^{-5}$	1.0096	$3*10^{-5}$
1, 2, 3, 5[a]	0.9981	$55*10^{-4}$	1.1121	$24.2*10^{-4}$	0.8789	$24.5*10^{-4}$
1, 2, 3, 6	0.999	$6.8*10^{-4}$	1.025	$6.5*10^{-4}$	0.9723	$6.7*10^{-4}$
1, 2, 4, 6	1	$8.5*10^{-4}$	1.0356	$8.6*10^{-4}$	0.9646	$8.6*10^{-4}$
1, 3, 4, 5	0.9992	$9.1*10^{-4}$	1.0419	$9*10^{-4}$	0.9563	$9*10^{-4}$

Interval size = 0.1, 0.2, 0.4 were used for the probability calculation in the presence of two, three, four gross errors respectively.

1, 2, 3, 5[a]: interval size = 0.5 was used

4.2 Summary

- Approximation method

  - Computation time and solution accuracy (the error) depend on interval size used. When interval size increases, computation time decreases but the error increases.

  - When the number of gross errors is increased by one, the computation time and the error increase significantly.

- Monte Carlo method

  - The higher the number of trials N, the more accurate the solution but the longer the computation time.

  - When number of trials $N \geq 10^6$, convergence of solution is attained (the relative error between Monte Carlo method's solution at $N = 10^6$ and that at $N = 10^8$ is usually not more than 0.1% except some values as large as 0.2%).

  - When the number of biases is increased by one, computation time increases a little bit (about 120 – 130%).

- Approximation method vs. Monte Carlo method

  - At low number of biases (≤ 4), the approximation method is superior to Monte Carlo method because it provides satisfactorily accurate solutions and needs shorter computation time.

  - At high number of biases (> 4), we should use large interval size in approximation method to reduce computation time, which will increase the errors or reduce accuracy of solutions. An alternative choice at high number of gross errors is the Monte Carlo method at low number of trials ($N = 10^6$ or $N = 10^7$)

- Financial loss as a function of plant instrumentation

  - Financial loss in the presence of biases is larger than financial loss without biases. At the same number of biases, the financial loss is the function of location of biases.

  - When more biases are present, financial loss increases.