## CHAPTER V

## WEATHER FORECASTING BY TWO-LEVEL MODEL

In this chapter, we will apply the two-level model which is the basic model of more levels model to forecast the weather. In this study, Krishnamurti's theories and techniques (Krishnamurti,1986) are used as a basis for numerical solutions.

## The Model Equations

The physical model consists of two layers bounded by surfaces as shown in Fig.5.1


Figure 5.1 Schematic diagram of the vertical level structure.

We can derive the model equation by applying the quasi-geostrophic vorticity equation (4.15) at level 1 and level 2 . To do this we must evaluate the divergence term at each level by using finite difference approximations to the vertical derivatives as follows:

$$
\left(\frac{\partial \omega}{\partial \mathrm{p}}\right)_{1} \cong \frac{\bar{\omega}-\omega_{0}}{\overline{\mathrm{p}}-\mathrm{p}_{0}} \quad, \quad\left(\frac{\partial \omega}{\partial \mathrm{p}}\right)_{2} \cong \frac{\omega_{\mathrm{g}}-\bar{\omega}}{\mathrm{p}_{\mathrm{g}}-\overline{\mathrm{p}}}
$$

let $\quad \Delta \mathrm{p}=\overline{\mathrm{p}}-\mathrm{p}_{0}=\mathrm{p}_{\mathrm{g}}-\overline{\mathrm{p}}=500 \mathrm{hPa}$
So, the vorticity equations at level 1 and level 2 can be written as

$$
\begin{align*}
& \frac{\partial \nabla^{2} \psi_{1}}{\partial \mathrm{t}}=-\left(\hat{\mathrm{k}} \times \nabla \psi_{1}\right) \cdot \nabla\left(\nabla^{2} \psi_{1}+\mathrm{f}\right)+\frac{\mathrm{f}_{0} \bar{\omega}}{\Delta \mathrm{p}}  \tag{5.1}\\
& \frac{\partial \nabla^{2} \psi_{2}}{\partial \mathrm{t}}=-\left(\hat{\mathrm{k}} \times \nabla \psi_{2}\right) \cdot \nabla\left(\nabla^{2} \psi_{2}+\mathrm{f}\right)-\frac{\mathrm{f}_{0} \bar{\omega}}{\Delta \mathrm{p}} \tag{5.2}
\end{align*}
$$

We next evaluate $\left(\frac{\partial \psi}{\partial \mathrm{p}}\right)$ using the difference formula

$$
\left(\frac{\partial \psi}{\partial p}\right)=\frac{\psi_{2}-\psi_{1}}{\Delta p} \quad \text { at } 500 \mathrm{hPa}
$$

then, we can write the thermodynamic energy equation (4.22) at 500 hPa as follow:

$$
\begin{equation*}
\frac{\partial\left(\psi_{1}-\psi_{2}\right)}{\partial \mathrm{t} C}=-\left[\widehat{\mathrm{k}} \times \nabla \frac{\left(\psi_{1}+\psi_{2}\right)}{2}\right] \cdot \nabla\left(\psi_{1}-\psi_{2}\right)+\frac{\sigma \Delta \mathrm{p} \bar{\omega}}{\mathrm{f}_{0}} \tag{5.3}
\end{equation*}
$$

Now eq.(5.1), eq.(5.2) and eq.(5.3) become a closed set of prediction equations in the variables $\psi_{1}$ and $\psi_{2}$. We next add eq.(5.1) and eq.(5.2) to obtain

$$
\begin{equation*}
\frac{\partial \nabla^{2}\left(\psi_{1}+\psi_{2}\right)}{\partial \mathrm{t}}=-\left(\hat{\mathrm{k}} \times \nabla \psi_{1}\right) \cdot \nabla\left(\nabla^{2} \psi_{1}+\mathrm{f}\right)-\left(\hat{\mathrm{k}} \times \nabla \psi_{2}\right) \cdot \nabla\left(\nabla^{2} \psi_{2}+\mathrm{f}\right) \tag{5.4}
\end{equation*}
$$

and then subtract eq.(5.2) from eq.(5.1) and add the result to $-2 \lambda^{2}$ time eq.(5.3) to get

$$
\begin{align*}
\frac{\partial\left[\left(\nabla^{2}-2 \lambda^{2}\right)\left(\psi_{1}-\psi_{2}\right)\right]}{\partial \mathrm{t}}= & -\left(\hat{\mathrm{k}} \times \nabla \psi_{1}\right) \cdot \nabla\left(\nabla^{2} \psi_{1}+\mathrm{f}\right)+\left(\hat{\mathrm{k}} \times \nabla \psi_{2}\right) \cdot \nabla\left(\nabla^{2} \psi_{2}+\mathrm{f}\right) \\
& +\lambda^{2}\left[\hat{\mathrm{k}} \times \nabla\left(\psi_{1}+\psi_{2}\right)\right] \cdot \nabla\left(\psi_{1}-\psi_{2}\right) \tag{5.5}
\end{align*}
$$

where $\quad \lambda^{2}=\frac{\mathrm{f}_{0}^{2}}{\sigma(\Delta \mathrm{p})^{2}}$

Eq.(5.4) and eq.(5.5) are the model equations for two-level model. Eq.(5.4) states that the local rate of change of the vertically averaged vorticity (that is, the average of the 250 and 750 hPa vorticities) is equal to the average of the 250 and 750 hPa vorticity advections. And eq.(5.5) states that the local rate of change of the $250-750 \mathrm{hPa}$ thickness is proportional to the difference between the vorticity advections at 250 and 750 hPa plus the thermal advection. These two equations can be used to forecast the streamfunction and wind fields at level 250 and 750 hPa . Eq.(5.4) and eq.(5.5) can be rewritten in Jacobian form as

$$
\begin{align*}
\frac{\partial \nabla^{2}\left(\psi_{1}+\psi_{2}\right)}{\partial t}= & -J\left(\psi_{1}, \zeta_{a 1}\right)-J\left(\psi_{2}, \zeta_{\mathrm{a} 2}\right)  \tag{5.6}\\
\frac{\partial\left[\left(\nabla^{2}-2 \lambda^{2}\right)\left(\psi_{1}-\psi_{2}\right)\right]}{\partial \mathrm{t}}= & -\mathrm{J}\left(\psi_{1}, \zeta_{\mathrm{a} 1}\right)+\mathrm{J}\left(\psi_{2}, \zeta_{\mathrm{a} 2}\right) \\
& +\lambda^{2}\left[\widehat{\mathrm{k}} \times \nabla\left(\psi_{1}+\psi_{2}\right)\right] . \nabla\left(\psi_{1}-\psi_{2}\right) \tag{5.7}
\end{align*}
$$

where $\mathrm{J}\left(\psi, \zeta_{\mathrm{a}}\right)$ is a Jacobian which can be written as,

$$
\mathrm{J}\left(\psi, \zeta_{\mathrm{a}}\right)=\frac{\partial \psi}{\partial \mathrm{x}} \frac{\partial \zeta_{\mathrm{a}}}{\partial \mathrm{y}}-\frac{\partial \psi}{\partial \mathrm{y}} \frac{\partial \zeta_{\mathrm{a}}}{\partial \mathrm{x}}
$$

and

$$
\zeta_{\mathrm{a} 1}=\nabla^{2} \psi_{1}+\mathrm{f}
$$

$$
\begin{aligned}
\zeta_{\mathrm{a} 2} & =\nabla^{2} \psi_{2}+\mathrm{f} \\
\lambda^{2} & =\frac{\mathrm{f}_{0}^{2}}{\sigma(\Delta \mathrm{p})^{2}} \\
\sigma & =\frac{\mathrm{R}^{2} \overline{\mathrm{~T}}}{(\Delta \mathrm{p})^{2}}\left[\frac{1}{\mathrm{~g}}\left(\frac{\partial \mathrm{~T}}{\partial \mathrm{z}}\right)+\frac{1}{\mathrm{c}_{\mathrm{p}}}\right]
\end{aligned}
$$

The values of $\lambda$ and $\sigma$ can be calculated by using standard values (in Appendix A).

## Region Covered by the Model and Horizontal Grid Structure

The forecasting area is shown in Fig.5.2, which covered the area between 90E180E longitudes and $0-45 \mathrm{~N}$ latitudes. The total number of the grid-points is $38 \times 19$ points with the grid-space of $2.5^{\circ}$ latitudes $/ 2.5^{\circ}$ longitudes and time space of 1800 seconds. The input data are 200 hPa and 850 hPa wind data analyzed by ECMWF (European Centre for Medium Range Weather Forecast) at 12 UTC ( $30 / 8 / 1993$ ).


Figure 5.2 The forecasting area ( $0-45 \mathrm{~N}, 90 \mathrm{E}-180 \mathrm{E}$ ) and horizontal grid structure

## Boundary Conditions

In the zonal direction, we extended the domain by adding two grid points and using cyclic continuity condition. The extended grid-point values in the east can be calculated by linear interpolating.

$$
\begin{align*}
\psi(\mathrm{L}, \mathrm{j}, \mathrm{k}) & =\psi(1, \mathrm{j}, \mathrm{k}))  \tag{5.8}\\
\psi(\mathrm{L}-1, \mathrm{j}, \mathrm{k}) & =(\psi(\mathrm{L}, \mathrm{j}, \mathrm{k}))+\psi(\mathrm{L}-2, \mathrm{j}, \mathrm{k})) / 2
\end{align*}
$$

But in the north and south domain boundaries, we assumed that the streamfunctions are constants which are equal to the initial streamfunctions.

$$
\begin{equation*}
\psi(t)=\psi(t=0) \tag{5.9}
\end{equation*}
$$

## Numerical Techniques

The finite difference scheme is used to approximate the differential equation on a grid of points in space and time. In order that a finite difference scheme be computationally stable in the sense that a solution of the difference equations will approximate a solution of the original system, it turns out that the ratio of the time and space increments must satisfy certain conditions. However, for finite difference solutions stability alone does not guarantee accurate solutions because all such solutions are subject to truncation error due to the approximate nature of the finite difference estimates of space and time derivatives.

## A. Space Differencing Scheme

The center space differencing scheme which is in the second order of accuracy is used as followed:

$$
\begin{aligned}
\left(\frac{\partial \psi}{\partial x}\right)_{i, j, k} & =\frac{\psi(\mathrm{i}+1, \mathrm{j}, \mathrm{k})-\psi(\mathrm{i}-1, \mathrm{j}, \mathrm{k})}{2 \Delta \mathrm{x}} \\
\left(\frac{\partial \psi}{\partial y}\right)_{\mathrm{i}, \mathrm{j}, \mathrm{k}} & =\frac{\psi(\mathrm{i}, \mathrm{j}+1, \mathrm{k})-\psi(\mathrm{i}, \mathrm{j}-1, \mathrm{k})}{2 \Delta \mathrm{y}} \\
\left(\frac{\partial^{2} \psi}{\partial \mathrm{x}^{2}}\right)_{\mathrm{i}, \mathrm{j}, \mathrm{k}} & =\frac{\psi(\mathrm{i}+1, \mathrm{j}, \mathrm{k})-2 \psi(\mathrm{i}, \mathrm{j}, \mathrm{k})+\psi(\mathrm{i}-1, \mathrm{j}, \mathrm{k})}{(\Delta \mathrm{x})^{2}} \\
\left(\frac{\partial^{2} \psi}{\partial y^{2}}\right)_{\mathrm{i}, \mathrm{j}, \mathrm{k}} & =\frac{\psi(\mathrm{i}, \mathrm{j}+1, \mathrm{k})-2 \psi(\mathrm{i}, \mathrm{j}, \mathrm{k})+\psi(\mathrm{i}, \mathrm{j}-1, \mathrm{k})}{(\Delta \mathrm{y})^{2}}
\end{aligned}
$$

And the Laplacian become

$$
\left(\nabla^{2} \psi\right)_{i, j, k}=\frac{\psi(\mathrm{i}+1, \mathrm{j}, \mathrm{k})-2 \psi(\mathrm{i}, \mathrm{j}, \mathrm{k})+\psi(\mathrm{i}-1, \mathrm{j}, \mathrm{k})}{(\Delta \mathrm{x})^{2}}+\frac{\psi(\mathrm{i}, \mathrm{j}+1, \mathrm{k})-2 \psi(\mathrm{i}, \mathrm{j}, \mathrm{k})+\psi(\mathrm{i}, \mathrm{j}-1, \mathrm{k})}{(\Delta \mathrm{y})^{2}}(5.10)
$$

And the differential form of Jacobian (Mesinger and Arakawa, 1976) can be written as,

$$
\begin{aligned}
\mathrm{J}(\psi, \zeta) & =\frac{\partial \psi}{\partial \mathrm{x}} \frac{\partial \zeta}{\partial y}-\frac{\partial \psi}{\partial y} \frac{\partial \zeta}{\partial \mathrm{x}} \\
& =\frac{1}{3}\left\{\mathrm{~J}_{1}(\psi, \zeta)+\mathrm{J}_{2}(\psi, \zeta)+\mathrm{J}_{3}(\psi, \zeta)\right\}
\end{aligned}
$$

where
$\mathrm{J}_{1}(\psi, \zeta)$ is a finite difference analog of the term $\frac{\partial \psi}{\partial \mathrm{x}} \frac{\partial \zeta}{\partial y}-\frac{\partial \psi}{\partial y} \frac{\partial \zeta}{\partial x}$
$\mathrm{J}_{2}(\psi, \zeta)$ is a finite difference analog of the term $\frac{\partial\left[\zeta \frac{\partial \psi}{\partial \mathrm{x}}\right]}{\partial \mathrm{y}}-\frac{\partial\left[\zeta \frac{\partial \psi}{\partial \mathrm{y}}\right]}{\partial \mathrm{x}}$ $\mathrm{J}_{3}(\psi, \zeta)$ is a finite difference analog of the term $\frac{\partial\left[\psi \frac{\partial \zeta}{\partial y}\right]}{\partial \mathrm{x}}-\frac{\partial\left[\psi \frac{\partial \zeta}{\partial \mathrm{x}}\right]}{\partial \mathrm{y}}$

The following is a final form of the Jacobian for a rectangular mesh of grid points at distance, $\Delta \mathrm{x}$ and $\Delta \mathrm{y}$, apart (Arakawa, 1966)

$$
\begin{align*}
\mathrm{J}_{\mathrm{i}, \mathrm{j}, \mathrm{k}}(\psi, \zeta)= & \frac{1}{12 \Delta \mathrm{x}(\mathrm{j}) \Delta \mathrm{y}}[(\psi(\mathrm{i}, \mathrm{j}-1, \mathrm{k})+\psi(\mathrm{i}+1, \mathrm{j}-1, \mathrm{k})-\psi(\mathrm{i}, \mathrm{j}+1, \mathrm{k})-\psi(\mathrm{i}+1, \mathrm{j}+1, \mathrm{k})) \\
& (\zeta(\mathrm{i}+1, \mathrm{j}, \mathrm{k})-\zeta(\mathrm{i}, \mathrm{j}, \mathrm{k})) \\
& +(\psi(\mathrm{i}-1, \mathrm{j}-1, \mathrm{k})+\psi(\mathrm{i}, \mathrm{j}-1, \mathrm{k})-\psi(\mathrm{i}-1, \mathrm{j}+1, \mathrm{k})-\psi(\mathrm{i}, \mathrm{j}+1, \mathrm{k}))(\zeta(\mathrm{i}, \mathrm{j}, \mathrm{k})-\zeta(\mathrm{i}-1, \mathrm{j}, \mathrm{k})) \\
& +(\psi(\mathrm{i}+1, \mathrm{j}, \mathrm{k})+\psi(\mathrm{i}+1, \mathrm{j}+1, \mathrm{k})-\psi(\mathrm{i}-1, \mathrm{j}, \mathrm{k})-\psi(\mathrm{i}-1, \mathrm{j}+1, \mathrm{k}))(\zeta(\mathrm{i}, \mathrm{j}+1, \mathrm{k})-\zeta(\mathrm{i}, \mathrm{j}, \mathrm{k})) \\
& +(\psi(\mathrm{i}+1, \mathrm{j}-1, \mathrm{k})+\psi(\mathrm{i}+1, \mathrm{j}, \mathrm{k})-\psi(\mathrm{i}-1, \mathrm{j}-1, \mathrm{k})-\psi(\mathrm{i}-1, \mathrm{j}, \mathrm{k}))(\zeta(\mathrm{i}, \mathrm{j}, \mathrm{k})-\zeta(\mathrm{i}, \mathrm{j}-1, \mathrm{k})) \\
& +(\psi(\mathrm{i}+1, \mathrm{j}, \mathrm{k})-\psi(\mathrm{i}, \mathrm{j}+1, \mathrm{k}))(\zeta(\mathrm{i}+1, \mathrm{j}+1, \mathrm{k})-\zeta(\mathrm{i}, \mathrm{j}, \mathrm{k})) \\
& +(\psi(\mathrm{i}, \mathrm{j}-1, \mathrm{k})-\psi(\mathrm{i}-1, \mathrm{j}, \mathrm{k}))(\zeta(\mathrm{i}, \mathrm{j}, \mathrm{k})-\zeta(\mathrm{i}-1, \mathrm{j}-1, \mathrm{k})) \\
& +(\psi(\mathrm{i}, \mathrm{j}+1, \mathrm{k})-\psi(\mathrm{i}-1, \mathrm{j}, \mathrm{k}))(\zeta(\mathrm{i}-1, \mathrm{j}+1, \mathrm{k})-\zeta(\mathrm{i}, \mathrm{j}, \mathrm{k})) \\
& +(\psi(\mathrm{i}+1, \mathrm{j}, \mathrm{k})-\psi(\mathrm{i}, \mathrm{j}-1, \mathrm{k}))(\zeta(\mathrm{i}, \mathrm{j}, \mathrm{k})-\zeta(\mathrm{i}+1, \mathrm{j}-1, \mathrm{k}))] \tag{5.11}
\end{align*}
$$

## B. Time Integration Scheme

In order to forecast the future circulation we must extrapolate ahead in time using a finite difference approximations. In this study we use an explicit time differencing scheme. This scheme is a slight modification of the Euler backward predictor-corrector technique, or Matsuno scheme (Matsuno,1966). Given the equation:

$$
\frac{\partial \mathrm{F}}{\partial \mathrm{t}}=\mathrm{G}
$$

the predictor and corrector are defined by the relations

$$
\begin{array}{ll}
F_{2}^{(1)}=F_{1}+G_{1} \Delta t & \text { (predictor) } \\
F_{2}=F_{1}+G_{2}^{(1)} \Delta t & \text { (corrector) } \tag{5.12b}
\end{array}
$$

Because of the finite differences are only approximations to the actual derivatives, so they have errors called truncation errors. And this phenomena is called computational instability. For the Euler backward scheme it was required the stability condition which
can be computed by applying it to the one-dimensional advection equation (Krishnamurti, 1986) :

$$
\left.\left.\begin{array}{rl}
\frac{\partial F}{\partial t}=c \frac{\partial F}{\partial x} \\
F_{m^{\prime}, n+1}= & F_{m, n}+\frac{c \Delta t}{2 \Delta x}\left(F_{m+1, n}-F_{m-1, n}\right) \\
F_{m, n+1}= & F_{m, n}+\frac{c \Delta t}{2 \Delta x}\left(F_{m^{\prime}+1, n}-F_{m^{\prime}-1, n+1}\right) \\
= & F_{m, n}+\frac{c \Delta t}{2 \Delta x}\left[\left\{F_{m+1, n}+\frac{c \Delta t}{2 \Delta x}\left(F_{m+2, n}-F_{m, n}\right)\right\}\right. \\
& -\left\{F_{m-1, n}+\frac{c \Delta t}{2 \Delta x}\left(F_{m, n}-F_{m-2, n}\right)\right.
\end{array}\right\}\right]
$$

Trial solution of the form $\quad F_{m, n}=A^{(n)} e^{i \alpha m \Delta x}$
Here $A^{(n)}$ is the amplitude at time level $n$. We define an amplification factor $|\lambda|$ by $A^{(n+1)}=\lambda \cdot A^{(n)}$. If $|\lambda| \leq 1$ we call the solution $F_{m, n}$ stable or else the solution is unstable. Now we get

$$
\begin{aligned}
\lambda & =1+\frac{\mathrm{c} \Delta \mathrm{t}}{2 \Delta \mathrm{x}}\left[\mathrm{e}^{\mathrm{i} \alpha \Delta \mathrm{x}}+\left(\frac{\mathrm{c} \Delta \mathrm{t}}{2 \Delta \mathrm{x}}\right)\left(\mathrm{e}^{2 \mathrm{i} \alpha \Delta \mathrm{x}}-1\right)-\mathrm{e}^{-\mathrm{i} \alpha \Delta \mathrm{x}}-\left(\frac{\mathrm{c} \Delta \mathrm{t}}{2 \Delta \mathrm{x}}\right)\left(1-\mathrm{e}^{-2 \mathrm{i} \alpha \Delta \mathrm{x}}\right)\right] \\
& =1+\frac{\mathrm{c} \Delta \mathrm{t}}{2 \Delta \mathrm{x}}\left(\mathrm{e}^{\mathrm{i} \alpha \Delta \Delta \mathrm{x}}-\mathrm{e}^{-\mathrm{i} \alpha \Delta \mathrm{x}}\right)+\left(\frac{\mathrm{c} \Delta \mathrm{t}}{2 \Delta \mathrm{x}}\right)^{2}\left(\mathrm{e}^{2 \mathrm{i} \alpha \Delta \mathrm{x}}+\mathrm{e}^{-2 i \alpha \Delta \mathrm{x}}-2\right) \\
& =1+\frac{\mathrm{c} \Delta \mathrm{t}}{\Delta \mathrm{x}} \mathrm{i} \sin \alpha \Delta \mathrm{x}-\left(\frac{\mathrm{c} \Delta \mathrm{t}}{\Delta \mathrm{x}}\right)^{2}(\sin \alpha \Delta \mathrm{x})^{2}
\end{aligned}
$$

so

$$
\begin{aligned}
|\lambda|^{2} & =\left(\frac{c \Delta t}{\Delta x}\right)^{2}(\sin \alpha \Delta x)^{2}+1-2\left(\frac{c \Delta t}{\Delta x}\right)^{2}(\sin \alpha \Delta x)^{2}+\left(\frac{c \Delta t}{\Delta x}\right)^{4}(\sin \alpha \Delta x)^{4} \\
& =1-\left(\frac{c \Delta t}{\Delta x}\right)^{2}(\sin \alpha \Delta x)^{2}+\left(\frac{c \Delta t}{\Delta x}\right)^{4}(\sin \alpha \Delta x)^{4} \\
|\lambda| & =\sqrt{1-\left(\frac{c \Delta t}{\Delta x}\right)^{2}(\sin \alpha \Delta x)^{2}+\left(\frac{c \Delta t}{\Delta x}\right)^{4}(\sin \alpha \Delta x)^{4}}
\end{aligned}
$$

The condition for stability requires that $|\lambda| \leq 1$ for every $\alpha$, therefore

$$
\left|\frac{c \Delta t}{\Delta x}\right| \leq 1
$$

For a two-dimensional grid with uniform grid space d , in the x and y directions it can be shown that $\quad \Delta t$ and d must satisfy

$$
\left|\frac{\mathrm{c} \Delta t}{\mathrm{~d}}\right| \leq \frac{1}{\sqrt{2}}
$$

The speed c is just the maximum wind speed, typically, $\mathrm{c}<50 \mathrm{~ms}^{-1}$ (Lindzen, 1990). In this study the minimum grid interval is 196 km , so the time increment must be less than 2,772 seconds.

## C. Over-Relaxation Method

The most practical scheme for solving Poisson eq.(5.6) and Helmholtz eq.(5.7) on a large grid mesh is a subsequent iteration technique known as relaxation. To illustrate this method ( Haltiner and Williams, 1979), consider the Poisson and Helmholtz equations

$$
\begin{aligned}
\nabla^{2} \mathrm{G} & =\mathrm{F} & (\delta=0) \\
\nabla^{2} \mathrm{G}-\mathrm{HG} & =\mathrm{F} & (\delta=1)
\end{aligned}
$$

or

$$
\begin{equation*}
\nabla^{2} \mathrm{G}-\delta \mathrm{HG}=\mathrm{F} \tag{5.13}
\end{equation*}
$$

Where F is a known " Forcing " function, H is a known positive coefficient and $\delta=0$ or 1 , eq.(5.13) can be rewritten in finite difference as

$$
\begin{equation*}
\nabla^{2} G(i, j)-\delta H G(i, j)=F(i, j) \tag{5.14}
\end{equation*}
$$

Now assume an initial estimate and let $G^{v}(i, j)$ represent the $v$ th estimate. Then the residual $R^{v}(i, j)$ for the $v$ th estimate is defined as follows :

$$
\begin{equation*}
R^{v}(i, j)=\nabla^{2} G^{v}(i, j)-\delta H G^{v}(i, j)-F(i, j) \tag{5.15}
\end{equation*}
$$

The objective of the subsequent iterations is to reduce the residuals to some acceptably small value although the exact solution with the $\mathrm{R}(\mathrm{i}, \mathrm{j})=0$ everywhere will not be reached. Given the $v$ th estimate $G^{v}(i, j)$, an improved value $G^{v+1}(i, j)$, which will temporarily reduce the residual $\mathrm{R}^{\mathrm{V}}(\mathrm{i}, \mathrm{j})$ to zero, may be obtained by giving

$$
\begin{equation*}
G^{v+1}(i, j)=G^{v}(i, j)+\alpha R^{v}(i, j) \Delta x(j) \Delta y \tag{5.16}
\end{equation*}
$$

Where $\alpha$ is an over-relaxation coefficient. Eq.(5.15) and eq.(5.16) are the two steps of over-relaxation method. If the method is convergent, $\mathrm{G}^{v}(\mathrm{i}, \mathrm{j})$ should approach the true solution $\mathrm{G}(\mathrm{i}, \mathrm{j})$ at all grid points as $v \rightarrow \infty$.

## Data Initialization

In this subsection we propose the method for construction of streamfunction from the analyzed wind field by applying Krishnamurti's technique (Krishnamurti,1986).

Given the horizontal wind components $u$ and $v$, the streamfunction ( $\psi$ ) can be computed using a program (in Appendix C). The various steps in this method are:
1). Compute the relative vorticity $(\zeta)$ over an array $i=1,2,3, \ldots \mathrm{~L}$ (in the westeast direction) and $\mathrm{J}=1,2,3, \ldots \mathrm{M}$ (in the south-north direction). Here one calculates

$$
\begin{gather*}
\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \\
\zeta(i, j, k)=\frac{v(i+1, j, k)-v(i-1, j, k)}{2 \Delta x(j)}-\frac{u(i, j+1, k)-u(i, j-1, k)}{2 \Delta y} \tag{5.17}
\end{gather*}
$$

Since $u=-\frac{\partial \psi}{\partial y}$ and $v=\frac{\partial \psi}{\partial x}$, then $\zeta=\nabla^{2} \psi$
2). Define appropriate boundary conditions for the streamfunction. The value of streamfunction at the boundaries can be computed by using the continuity equation. The net mass flux out of the domain may be expressed by

$$
\mathrm{M}_{\mathrm{F}}=\oint \mathrm{V}_{\mathrm{n}} \mathrm{ds}
$$

Where

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{n}}=-\mathrm{V} \text { at the southern boundary } \\
& \mathrm{V}_{\mathrm{n}}=-\mathrm{U} \text { at the western boundary } \\
& \mathrm{V}_{\mathrm{n}}=+\mathrm{V} \text { at the northern boundary } \\
& \mathrm{V}_{\mathrm{n}}=+\mathrm{U} \text { at the easthern boundary }
\end{aligned}
$$

We assume that the outward normal velocity at the boundary can be corrected to yield a net zero outward mass flux by the relation

$$
\begin{equation*}
\oint_{s} V_{n}^{c} d s=0 \tag{5.18}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{V}_{\mathrm{n}}^{\mathrm{c}}=\mathrm{V}_{\mathrm{n}}+\varepsilon\left|\mathrm{V}_{\mathrm{n}}\right| \tag{5.19}
\end{equation*}
$$

the correction, is proportional to the magnitude of the outward normal velocity. Thus, we obtain the correction coefficient, $\varepsilon$, by the relation

$$
\begin{equation*}
\varepsilon=-\frac{\oint_{:} \mathrm{V}_{\mathrm{n}} \mathrm{ds}}{\oint_{:}\left|\mathrm{V}_{\mathrm{n}}\right| \mathrm{ds}} \tag{5.20}
\end{equation*}
$$

$$
\begin{equation*}
\varepsilon=-\frac{\sum \mathrm{V}_{\mathrm{n}} \Delta \mathrm{~s}}{\sum\left|\mathrm{~V}_{\mathrm{n}}\right| \Delta \mathrm{s}} \tag{5.21}
\end{equation*}
$$

Here $s$ is a length element along the boundary, of the domain. The boundary wind $u$ and $v$ are next corrected at each point of the boundary by these relations. The streamfunction at one point $i=1, j=M$ the northwestern corner, is assumed to be known, ( $\psi=0$ ) its value at the remaining points of the boundary is calculated using the corrected normal velocity $\mathrm{V}_{\mathrm{n}}$.
or

$$
\begin{gather*}
V_{n}^{c}=\frac{\partial \psi}{\partial s} \\
\psi_{2}=\psi_{1}+\frac{\left(V_{n 1}^{c}+V_{n 2}^{c}\right)}{2} \Delta s \tag{5.22}
\end{gather*}
$$

Where $\psi_{1}$ denotes a know value and $\psi_{2}$ is the adjacent neighbor where the value of $\psi$ is being defined. $\Delta \mathrm{s}$ is the grid size $\Delta \mathrm{x}$ or $\Delta \mathrm{y}$ depending on the boundary.
3). The next step requires the solution of a Poisson equation subject to the above boundary conditions. This can be carried out using the relaxation method eq.(5.15) and eq.(5.16) described below for a rectangular domain. There are two steps in an iteration manner over all grid points. These are:

$$
\mathrm{R}=\nabla^{2} \psi-\zeta
$$

and

$$
\psi(v+1)=\psi(v)+\alpha R \Delta x \Delta y
$$

Which can be written in finite difference as :

$$
\begin{align*}
R^{v}(i, j, k) & =\frac{\left(\psi^{v}(i+1, j, k)+\psi^{v+1}(i-1, j, k)-2 \psi^{v}(i, j, k)\right)}{(\Delta x(j))^{2}} \\
& -\frac{\left(\psi^{v}(i, j+1, k)+\psi^{v+1}(i, j-1, k)-2 \psi^{v}(i, j, k)\right)}{(\Delta y)^{2}}-\zeta(i, j, k) \tag{5.23}
\end{align*}
$$

and

$$
\begin{equation*}
\psi^{v+1}(i, j, k)=\psi^{v}(i, j, k)+\alpha R^{v}(i, j) \Delta x(j) \Delta y \tag{5.24}
\end{equation*}
$$

Where $\alpha$ is an over relaxation coefficient, and $v$ is the order of approximation

## Summary of the Procedure for Forecasting

The procedure for forecasting with a two-level model can now be summarized in two parts as follows :

## Part 1. Initialize data (Program NWPI in Appendix C )

1). Use the input velocity wind field at time $t=0$ to compute the relative vorticity $(\zeta)$ at all grid points by eq.(5.17)
2). Define appropriate boundary conditions and compute the velocity correction coefficient $(\varepsilon)$ by eq.(5.21) and the correction velocity $\left(v_{n}^{\mathrm{c}}\right)$ by eq.(5.19) and then compute the streamfunctions $(\psi)$ at the boundary by eq. (5.22)
3). Use relaxation method by eq.(5.23) and eq.(5.24) to compute the streamfunctions $(\psi)$ at all grid points.

## Part 2. Forecast weather ( Program NWPF in Appendix D )

1). Use initial streamfunctions $(\psi)$ form part 1 to compute the relative vorticity $(\zeta)$ by eq.(5.10) and the absolute vorticity $\left(\zeta_{a}\right)$
2). Compute the Jacobian $\mathrm{J}\left(\psi_{1}, \zeta_{\mathrm{a} 1}\right)$ and $\mathrm{J}\left(\psi_{2}, \zeta_{\mathrm{a} 2}\right)$ by eq.(5.11) and compute the last term on the right-side of eq.(5.7)
3). Compute the forcing function ( the right-side of eq.(5.6) and eq.(5.7) )
4). Integrate eq.(5.6) and eq.(5.7) by extrapolating ahead with a time increment $\Delta t$ using eq.(5.12)
5). Use the new values form step 4 to compute the summation of streamfunctions $\left(\psi_{1}+\psi_{2}\right)$ and the difference of streamfunctions $\left(\psi_{1}-\psi_{2}\right)$ by relaxation method eq.(5.15) and eq.(5.16)
6). Use the new values form step 5 to compute the streamfunctions $\psi_{1}$ and $\psi_{2}$
7). Repeat step 2-6 until the desired forecast time reached.

## Forecasting Results



In this study, the proto-type model has been developed for the IBM (PC/AT) 80386-25 compatible computer. The 850 hPa and 200 hPa grid point values of the wind fields (1200 UTC, 30 / 8 / 1993) are used as initial data, which are results of objectively analyzed fields of the original First GARP Global Experiment (FGGE) observations by the European Centre for Medium Range Weather Forecast (ECMWF). The data initialization ( by Program NWPI in Appendix C ) take 2 minutes to complete and the results of initial streamfunctions are shown in Fig. (5.3a) and Fig.(5.3b). These streamfunctions are the input data for forecasting the future circulation ( by Program NWPF in Appendix D ) and the results of $24-\mathrm{hr}, 48-\mathrm{hr}$, $72-\mathrm{hr}$, and $96-\mathrm{hr}$ forecast are shown in Fig.(5.4), Fig.(5.5), Fig.(5.6), and Fig.(5.7) respectively.

Streamfunction at 250 hPa


Figure 5.3a The result of initialized data at level 250 hPa

## Streamfunction at 750 hPa



Figure 5.3b The result of initialized data at level 750 hPa

## Streamfunction at 250 hPa



Figure 5.4a The result of $24-\mathrm{hr}$ forecast at level 250 hPa

Streamfunction at 750 hPa


Figure 5.4 b The result of $24-\mathrm{hr}$ forecast at level 750 hPa

Streamfunction at 250 hPa


Figure 5.5a The result of $48-\mathrm{hr}$ forecast at level 250 hPa

Streamfunction at 750 hPa


Figure 5.5 b The result of $48-\mathrm{hr}$ forecast at level 750 hPa

Streamfunction at 250 hPa


Figure 5.6a The result of 72 -hr forecast at level 250 hPa

Streamfunction at 750 hPa


Figure 5.6b The result of $72-\mathrm{hr}$ forecast at level 750 hPa

## Streamfunction at 250 hPa



Figure 5.7a The result of $96-\mathrm{hr}$ forecast at level 250 hPa

Streamfunction at 750 hPa


Figure 5.7b The result of $96-\mathrm{hr}$ forecast at level 750 hPa

