RETROFIT WITH/WITHOUT RELOCATION OF HEAT EXCHANGER NETWORKS FOR A MULTI- CRUDE REFINERY

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ABSTRACT

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An effective way to reduce energy usage in a refinery is to design efficient heat exchanger networks (HENs) by using process optimization on the Mixed Integer Liner Programming (MILP) method. In this work, retrofit designs with/without relocation of HENs are done by GAMS (General Algebraic Modeling System) software. This methodology can generate networks where utility cost, heat exchanger areas and selection of matches are optimized simultaneously. In addition, the simplicity in model assumption of non-isothermal mixing with constraints such as stream splitting and allowed/forbidden matches make the model structure more convenient to use. This MILP model can be successfully applied to the crude refinery, providing both retrofit designs with/without relocation. In a special scenario, relocation topology can be used for further reduction in total cost, which also gives the highest annual cost saving for retrofitting HENs. This research also determines the best HENs for light, intermediate and heavy crude refinery.

บทคัดย่อ

วรพล ศรีพยัพ: แบบจำลองเพื่อการปรับปรุงแบบมีและไม่มี การเปลี่ยนตำแหน่ง เครือข่ายแลกเปลี่ยนความร้อนของหอกลั่นน้ำมันดิบ อ. ที่ปรึกษา: ผศ. ดร. กิติพัฒน์ สีมานนท์ และ ศ. ดร. มิเกล บากาเฮวิซ 222 หน้า

ทางเลือกหนึ่งที่มีประสิทธิภาพ ในการลดปริมาณาการใช้พลังงานใน โรงกลั่นน้ำมัน คือ การออกแบบระบบแลกเปลี่ยนความร้อนที่มีประสิทธิภาพ โดยอาศัยแบบจำลองทางคณิตศาสตร์ มิกส์อินทีเจอร์ ลีเนียโปรแกรมมิง วิทยานิวิพนธ์นี้ เป็น การออกแบบ ปรับปรุง เครือข่าย แลกเปลี่ยนความร้อนเดิม โดย วิธี เปลี่ยนต่ำแหน่งและไม่เปลี่ยนตำแหน่ง ของ เครื่องแลกเปลี่ยน ความร้อนเดิม โดย ใช้ โปรแกรม จี เอ เอม เอส ด้วยวิธีนี้ เราสามารถปรับปรุง เครือข่ายแลกเปลี่ยน ความร้อน ซึ่งมีประสิทธิภาพสูงสุดของ การใช้พลังงานของระบบ การเพิ่มพื้นที่ของเครื่อง แลกเปลี่ยนความร้อนและการจับคู่ของเครือข่าย โดย เงื่อนไขเหล่านี้ จะ ถูกพิจารณาโดยพร้อมกัน นอกจากนี้ เงื่อนไขการจับคู่ของเครือ ข่าย แบบ นอนไอโซเทอร์มอล มิกส์ซิง เช่น การ แยกสาย และการจับคู่แบบอิสระ ยังสามารถ นำมา ประยุกต์ ใช้กับสถานการณ์จริง โดย แบบจำลองทาง คณิตศาสตร์ มิกส์อินทีเจอร์ ลีเนียโปรแกรมมิง โดย วิธี เปลี่ยนต่ำแหน่งและไม่เปลี่ยนตำแหน่ง ของ เครื่องแลกเปลี่ยนความร้อนเดิม นั้น ประสพผลสำเร็จในการนำมาประยุกต์ ใช้กับ โรงกลั่น น้ำมัน โดย เฉพาะ การออกแบบ ปรับปรุง เครือข่ายแลกเปลี่ยนความร้อนเดิม โดย วิธี เปลี่ยนต่ำ แหน่ง เดิม สามารถ ปรับปรุง เครือข่ายแลกเปลี่ยนความร้อน ให้ ลดค่าใช้จ่ายรายปีได้สูงที่สุด วิทยานิพนธ์ ยัง ศึกษาเพื่อหา เครือข่ายแลกเปลี่ยนความร้อนที่ดีที่สุด สำหรับ ชนิดของน้ำมันดิบ แบบเบา กลาง และ หนัก สำหรับ โรงกลั่นน้ำมัน

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ABBREVIATIONS

Sets

B = { (i,j) | more than one heat exchanger unit is permitted between hot stream i and cold stream j }

C^z = { j | j is a cold stream present in zone z }

C_n^z = { j | j is a cold stream present in temperature interval n in zone z }

CU^z = { j | j is a heating utility present in zone z } $(CU^z \subset C^z)$

H^z = { i | i is a hot stream present in zone z }

H_m^z = { i | i is a hot stream present in temperature interval m in zone z }

HU^z = { i | i is a heating utility present in zone z } $(HU^z \subset H^z)$

M^z = { m | m is a temperature interval in zone z }

M_i^z = { m | m is a temperature interval belonging to zone z, in which hot stream i is presented }

m_i^0 = { m | m is the starting temperature interval for hot stream i }

m_i^f = { m | m is the final temperature interval for hot stream i }

N_j^z = { n | n is a temperature interval belonging to zone z, in which cold stream j is presented }

NI^H = { i | non-isothermal mixing is permitted for hot stream i }

NI^C = { j | non-isothermal mixing is permitted for cold stream j }

n_j^0 = { n | n is the starting temperature interval for cold stream j }

n_i^f = { n | n is the final temperature interval for cold stream j }

P = { (i,j) | heat exchange match between hot stream i and cold stream j is permitted }

P_{im}^H = { j | heat transfer from hot stream i at interval m to cold stream j is permitted }

P_{jm}^{C} = { i | heat transfer from hot stream i to cold stream j at interval n is permitted }

PA^{z} ={ i | I is a hot stream present in zone z and is a pump around stream)

S^{H} = { i | splits are allowed for hot stream i }

S^{C} = { j | splits are allowed for cold stream j }

Z = {z| z is a heat transfer zone}

Parameters

$A_{ij}^{a^0}$	Area of an existing exchanger between streams i and j in zone z prior to retrofit
A_{ij}^{z,k^0}	Area of the k-th existing exchanger between streams i and j in zone z prior to retrofit
$A_{ij\max}^{z}$	Maximum shell area for an exchanger matching hot stream i and cold stream j in zone z
$A_{ij\max}^{z^N}$	Maximum area for a new heat exchanger matching hot stream i and cold stream j in zone z .
$\Delta A_{ij\max}^{z^0}$	Maximum area addition for an existing heat exchanger matching hot stream i and cold stream j in zone z
$\Delta A_{ij\max}^{z,k^0}$	Maximum area addition for the k-th existing heat exchanger matching hot stream i and cold stream j in zone z
$\hat{C}_{im}$	Heat capacity of hot stream i at temperature interval m
$\hat{C}_{jn}$	Heat capacity of cold stream j at temperature interval n
c_{i}^{H}	Cost of heating utility i
c_{j}^{C}	Cost of cooling utility j
c_{ij}^{F}	Fixed charge cost for a heat exchanger matching hot stream i and cold stream j

$c_{ij}^{A^N}$	Variable cost for a new heat exchanger matching hot stream *i* and cold stream *j*
$c_{ij}^{A^O}$	Area addition cost for an existing heat exchanger matching hot stream *i* and cold stream *j*
$CHEAD_{im,jn}$	Temperature difference between interval *m* of hot stream *i* and interval *n* of cold stream *j* at cold end
F_i	Flow rate of hot process stream *i*
F_j	Flow rate of cold process stream *j*
F_i^U	Upper bound for the flow rate of heating utility *i*
F_j^U	Upper bound for the flow rate of cooling utility *j*
$FPR_{i,r}^H$	Candidate values for pump around flowrate *i*
h_{im}	Film heat transfer coefficient for hot stream *i* in interval *m*
h_{jn}	Film heat transfer coefficient for cold stream *j* in interval *n*
$\Delta H_{im}^{z,H}$	Enthalpy change for hot stream *i* at interval *m* of zone *z*
$\Delta H_{jn}^{z,C}$	Enthalpy change for cold stream *j* at interval *n* of zone *z*
$HHEAD_{im,jn}$	Temperature difference between interval *m* of hot stream *i* and interval *n* of cold stream *j* at hot end
$k_{\max}$	Maximum number of heat exchangers allowed between hot stream *i* and cold stream *j* in zone *z* when $(i,j)\in B$
k_e	Number of existing heat exchangers between hot stream *i* and cold stream *j* in zone *z* when $(i,j)\in B$
k_e'	Number of existing heat exchangers in the original network
q_{ijm}^L	Lower bound for heat transfer from hot stream *i* at interval *m* to cold stream *j*

q^{L}_{ijn}	Lower bound for heat transfer from hot stream *i* to cold stream *j* at interval *n*
Q_{PA}	Total PA load
T^{U}_{m}	Upper temperature of interval *m*
T^{L}_{m}	Lower temperature of interval *m*
T^{U}_{n}	Upper temperature of interval *n*
T^{L}_{n}	Lower temperature of interval *n*
ΔT_{i}	Temperature range of stream *i*
ΔT_{j}	Temperature range of stream *j*
ΔT^{ML}_{mn}	Mean logarithmic temperature difference between intervals *m* and *n*
$U^{z^0}_{ij}$	Number of existing heat exchangers between hot stream *i* and cold stream *j* in zone *z*
$U^{z^N}_{ij}$	Maximum number of new heat exchangers allowed for the retrofit design
$\Gamma^{z,H}_{im,jn}$	Upper bound
λ_{k}	Weight factor

Variables

A^{z}_{ij}	Total required area for a match between hot stream *i* and cold stream *j* in zone *z*
A^{z,k^0}_{ij}	Area of the *k*-th existing heat exchanger between hot stream *i* and cold stream *j* in zone *z* after retrofit
$A^{z^N}_{ij}$	Area of a new heat exchanger between hot stream *i* and cold stream *j* in zone *z*
$\Delta A^{z^0}_{ij}$	Area addition for an existing heat exchanger between hot stream *i* and cold stream *j* in zone *z*

$\Delta A_{ij}^{z,k^0}$	Area addition for the *k*-th existing heat exchanger between hot stream *i* and cold stream *j* in zone *z*
FP_i^H	Flowrate of pumparound stream
$K_{ijm}^{z,H}$	Determines the beginning of a heat exchanger at interval *m* of zone *z* for hot stream *I* with cold stream *j*. Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$
$K_{ijn}^{z,C}$	Determines the beginning of a heat exchanger at interval *n* of zone *z* for cold stream *j* with hot stream *i*. Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$
$\hat{K}_{ijm}^{z,H}$	Determines the end of a heat exchanger at interval *m* of zone *z* for hot stream *i* with cold stream *j*. Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$
$\hat{K}_{ijn}^{z,C}$	Determines the end of a heat exchanger at interval *n* of zone *z* for cold stream *j* with hot stream *i*. Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$
k_e	Number of existing heat exchangers between hot stream *i* and cold stream *j* in zone *z* when $(i,j) \in B$
$q_{im,jn}^{z}$	Heat transfer from hot stream *i* at interval *m* to cold stream *j* at interval *n* in zone *z*
$\bar{q}_{inm}^{z,H}$	Non-isothermal mixing heat transfer for hot stream *i* between intervals *m* and *n* in zone *z*
$\bar{q}_{jmn}^{z,C}$	Non-isothermal mixing heat transfer for hot stream *i* between intervals *m* and *n* in zone *z*
$\hat{q}_{ijm}^{z,H}$	Heat transfer from hot stream *i* at interval *m* to cold stream *j* in zone *z*
$\hat{q}_{ijn}^{z,C}$	Heat transfer to cold stream *j* at interval ***n*** from hot stream *j* in zone *z*
$\tilde{q}_{ijm}^{z,H}$	Auxiliary continuous variable utilized to compute the hot side heat load of each heat exchanger when several exchangers exist between hot stream *i* and cold stream *j* in zone *z*

$\tilde{q}_{ijn}^{z,C}$	Auxiliary continuous variable utilized to compute the cold side heat load of each heat exchanger when several exchangers exist between hot stream i and cold stream j in zone z
$\tilde{q}_{im,jn}^{z,H}$	Auxiliary continuous variable utilized to compute the area of individual heat exchangers between hot stream i with cold stream j in zone z when $(i,j) \in B$
QP_i^z	Pump around load
U_{ij}^z	Number of heat exchangers between hot stream i and cold stream j in zone z
$X_{im,jn}^z$	Auxiliary continuous variable equals to zero when an exchanger ends at interval m for hot stream i and at interval n for cold stream j. A value of one corresponds to all other cases
$\hat{X}_{ijm}^{z,k}$	Auxiliary binary variable that determines whether the k-th between hot stream i with cold stream j in zone z exists at interval m of when $(i,j) \in B$
$Y_{ijm}^{z,H}$	Determines whether heat is being transferred from hot stream i at interval m to cold stream j. Defined as binary when $(i,j) \notin B$ and as continuous when $(i,j) \in B$
$Y_{ijn}^{z,C}$	Determines whether heat is being transferred from hot stream i to cold stream j at interval n. Defined as binary when $(i,j) \notin B$ and as continuous when $(i,j) \in B$
$\delta_{ij}^{z,k}$	Auxiliary binary variable used for heat exchanger relocation. Auxiliary binary variable that determines whether the k-th original heat exchanger of zone z has is serving the match between hot stream i and cold stream j, when $(i,j) \notin B$
$\delta_{ij}^{z,hk}$	Auxiliary binary variable used for heat exchanger relocation. This variable determines whether the k-th original heat exchanger of zone z has is serving the h-th exchanger streams i and j, when $(i,j) \in B$

$\alpha_{ijm}^{z,H}$	Auxiliary continuous variable equal to one when heat transfer from interval *m* of hot stream *i* to cold stream *j* occurs in zone *z* and it does not correspond to the beginning nor the ending of a heat exchanger. A value of zero corresponds to all other cases
$\alpha_{ijn}^{z,C}$	Auxiliary continuous variable equal to one when heat transfer from hot stream *i* to interval *n* of cold stream *j* occurs in zone *z* and it does not correspond to the beginning nor the ending of a heat exchanger. A value of zero corresponds to all other cases