CHAPTER III
MATHEMATICAL MODEL FORMULATION

3.1 Problem Definition

This work addresses the planning of crude oil purchasing to satisfy the product specification and demand with the highest profit. Planning activities involve the manipulation of crude oil purchase decisions, processing and management over time periods. The model represents a scheme of a refinery including product paths, capacities and yields of several units.

A unit model consists of blending relations and production yields. Blending relations represent intermediate blending possibilities for producing each product. The production yield is assumed to be a simple yield relation for simplification of solving purpose. It is based on average value obtained from plant data. Processing of a unit must satisfy bound constraints for both maximum and minimum unit feed.

Pricing decision is integrated with the planning model. The starting point is the development of the price-demand relation using microeconomic method and "Utility function" concept. The relation obtained is integrated into the planning model. Product demand and price are now considered as the decision variables. They are simultaneously determined with the optimal crude oil processing.

The goal is to maximize the gross refinery margin (GRM) function which is the different between the product revenue minus the cost of raw materials, inventory and the operating costs.

First, the planning model was developed without pricing decision and was made deterministic. Then uncertainty of demands and prices were incorporated into the model. Next, the pricing decision was integrated. Finally, financial risk management is discussed.

3.2 General Mathematical Formulation

3.2.1 Assumption

The following assumptions are proposed to the model:

a) Perfect mixing and linear blending is assumed in the process.

b) Material losses are neglected.

c) The property state of each crude oil or products is decided only by specific key components such as sulfur and aromatics contents.

d) Utility function concept used to develop the price-demand model is based on the Constant Elasticity of Substitution (CES) type.

3.2.2 General Mathematical Model

A set-up of input-output balancing is based on the network structure proposed by Pinto *et al.* (2000). Figure 3.1 shows the general representation of balancing a production unit.

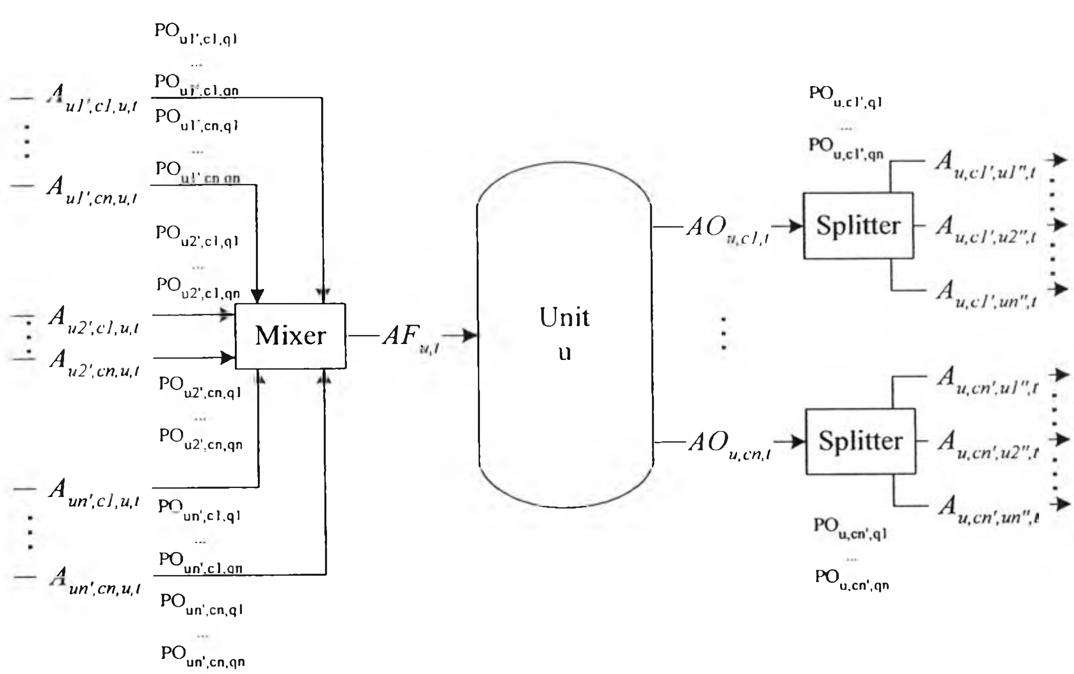

Figure 3.1 Balancing of a typical unit.

From Figure 3.1, commodity c_1 from unit u_1' is sent to unit u at flow rate $A_{u1',c1,u,t}$ in period t. The same unit u_1' may send different commodities c $(c_2, c_3, \ldots, c_n)$ to unit u. In addition, u' $(u_2', u_3', \ldots, u_n')$ can feed commodities c $(c_1, c_2, \ldots, c_n)$ to unit u. The summation of feed for unit u is represented by $AF_{u,t}$. Parameters $PO_{u1',c1,q}$

denote properties q of commodity c, flow from u_l'. Variables $AO_{u,c,t}$ represents the outlet flow rate of commodity c from unit u in time period t. A splitter is represented at every outlet stream because a product stream can be sent to more than one unit for further processing or storage.

The model of a typical unit u in Figure 3.1 is represented by two sets of equations. The first set involves balance equations and the other involves stream property equations.

Stream Balance equations include:

1. Balance of feeds to unit u which is represented

$$AF_{u,t} = \sum_{c}\sum_{u'} A_{u',c,u,t} \tag{3.1}$$

2. Balance of products from splitter which is represented by

$$AO_{u,c,t} = \sum_{u'} A_{u,c,u',t} \tag{3.2}$$

3. Balance of products from unit u which is represented in two ways:

- For percent yields that do not depend on the feed properties, the amount of products is equal to the total inlet flow multiply by a constant, the percent yield of that unit.

$$AO_{u,c,t} = AF_{u,t} \times yield_{u,c} \tag{3.3}$$

- For percent yields that depend on the feed properties, the amount of products is equal to the sum of each inlet flow times percent yield of each inlet flow.

$$AO_{u,c,t} = \sum_{u' \in ctank} \sum_{c' \in C_O} (A_{u',c',u,t} \times cyield_{c',c}) \tag{3.4}$$

Stream property equations:

The calculation of product properties that can be accomplished in two ways:

1. Product properties leaving unit u calculated by the sum of the flow fraction times the properties of each flow as in the following equation. These are called blending equations.

$$PO_{u,c,q,t} = \frac{\sum_{u' \in U'} \sum_{c' \in CO_{u'}} (A_{u',c',u,t} \times pro_{u',c,q})}{\sum_{u' \in U'} \sum_{c' \in CO_{u'}} A_{u',c',u,t}} \tag{3.5}$$

The equation is non-linear. However, this is not an equation we use in the model. We use bounds on this property. This is further discussed below.

2. Product properties from unit u that can be determined over average values obtained from plant data, e.g. isomerate from isomerization unit and reformate from reformer unit:

$$PO_{u,c,q,t} = pro_{u,c,q} \tag{3.6}$$

Capacity constraint:

The stream flowing to each unit should be within established minimum and maximum values

$$ux_u \geq AF_{u,t} \geq un_u \tag{3.7}$$

The allowable quantity of crude oil refined in each time period is shown in the following equation:

$$ox_c \geq AC_{c,t} \geq on_c \tag{3.8}$$

The allowable quantity of finish product stored in each time period is limited:

$$stox_c \geq AS_{c,t} \tag{3.9}$$

Quality constraint:

The product quality must be greater or equal to its minimum specifications and must not be over its maximum specifications. The set of product (C_p) must satisfy the following equation:

$$px_{c,q} \geq PO_{u,c,q,t} \geq pn_{c,q} \tag{3.10}$$

Demand constraint:

The amount of each product sold plus the volume of lost demand of that product in each period of time must be equal to the amount of product demand in that time period.

$$dem_{c,t} = sales_{c,t} \tag{3.11}$$

These balance equations and stream property equations are applied to a set of different operation units to develop a general planning model for refinery. These units are include crude oil charging tanks, distillation units, naptha pretreating unit, catalytic reforming units, hydrodesulfurization unit, intermediate tanks such as isomerate, reformate or naptha tanks, and product units. Equations that are set to these units are as follow:

Charging tank model

Charging tank is the unit to which all crude oils are assumed to be unloaded and then delivered to the other units. It is assumed that the charging tanks have no capacity limit in order to find the exact amount of each crude oil refined to satisfy demand in each time period. Equation (3.12) describes the outlet stream from crude charging tanks

$$AO_{ctank,o,t} = \sum_{u'} A_{c\tan k,o,u',t} \tag{3.12}$$

Crude distillation unit (CDU) model

Total feeds flow to CDUs are represented by the following equation:

$$AF_{CDU,t} = \sum_{u'} A_{u',c,CDU,t} \tag{3.13}$$

The amount of product yield depends on feed flow and feed properties:

$$AO_{CDU,c,t} = \sum_{u'} (A_{u,c',CDU,t} \times cyield_{c',c}) \tag{3.14}$$

Total outlet flow of CDUs is the summation of all CDUs product flowing to other units:

$$AO_{CDU,c,t} = \sum_{u'} A_{CDU,c,u',t} \tag{3.15}$$

Properties of product streams are determined from properties of each fraction from each feed. The properties based on volume basis can be calculated from:

$$PO_{CDU,c,q,t} = \frac{\sum_{u'} (A_{u',c',CDU,t} \times cyield_{c',c} \times pro_{u',c,q})}{\sum_{u'} (A_{u',c,CDU,t} \times cyield_{c',c})} \qquad q \in \mathrm{AV}_q \tag{3.16}$$

Properties which are based on weight basis can be calculated from:

$$PO_{CDU,c,q,t} = \frac{\sum_{u'} (A_{u',c',CDU,t} \times cyield_{c',c} \times pro_{u',c,q} \times pro_{u',c,SG})}{\sum_{u'} (A_{u',c,CDU,t} \times cyield_{c',c} \times pro_{u',c,SG})} \qquad q \in \mathrm{AW}_q \tag{3.17}$$

Naptha pretreating unit (NPU) model

Feed flow of NPU is determined by Equation (3.18):

$$AF_{NPU,t} = \sum_{u'} A_{u',nap,NPU,t} \tag{3.18}$$

Products from NPUs can be calculated from Equation (3.19):

$$AO_{NPU,nap,t} = \left(\sum_{u'} A_{u',nap,NPU,t} \right) \times yield_{NPU,nap} \tag{3.19}$$

Total outlet flow of NPUs is the summation of all NPUs product flowing to other units:

$$AO_{NPU,c,t} = \sum_{u} A_{NPU,c,u,t} \tag{3.20}$$

Catalytic reformer unit (CRU) model

Feed flow of CRUs are considered by Equation (3.21):

$$AF_{CRU,t} = \sum_{u'} A_{u',c,CRU,t} \tag{3.21}$$

Amount of product yield depends on feed flow and feed properties:

$$AO_{CRU,c,t} = AF_{CRU,t} \times yield_{CRU,c} \tag{3.22}$$

Balance of CRUs outlet flow is as follows:

$$AO_{CRU,c,t} = \sum_{u} A_{CRU,c,u,t} \tag{3.23}$$

Intermediate tanks model

- *Isomerization unit (ISOU) model*

Feed flow of ISOU is calculated from Equation (3.24):

$$AF_{ISOU,t} = \sum_{u'} A_{u',c,ISOU,t} \tag{3.24}$$

Production yield can be estimated from the following equation:

$$AO_{ISOU,c,t} = AF_{ISOU,t} \times yield_{ISOU,c} \tag{3.25}$$

Outlet balance flow of ISOU can be calculated from Equation (3.26)

$$AO_{ISOU,c,t} = \sum_{u} A_{ISOU,c,u,t} \tag{3.26}$$

- *Kerosine treating unit (KTU) model*

Total feed flow of KTU is shown in the Equation (3.27)

$$AF_{KTU,t} = \sum_{u'} A_{u',c,KTU,t} \tag{3.27}$$

Product from KTU is estimated to equal to its feed.

$$AO_{KTU,c,t} = AF_{KTU,t} \tag{3.28}$$

Balance of outlet flow from KTU is as follow:

$$AO_{KTU,c,t} = \sum_{u} A_{KTU,c,u,t} \tag{3.29}$$

- Reformate, Isomerate, Light naptha and Heavy naptha tank

Feed flow of these intermediate tanks can be estimated from Equation (3.30):

$$AF_{u,t} = \sum_{u'} A_{u',c,u,t}$$
$$u \in REFT,\ ISOT,\ LNT\ and\ HNT \tag{3.30}$$

Products of these units are estimated to equal to their feeds as in the following equation:

$$AO_{u,c,t} = AF_{u,t}$$
$$u \in REFT,\ ISOT,\ LNT\ and\ HNT \tag{3.31}$$

Outlet flow balance equation of the units is as follow:

$$AO_{u',c,t} = \sum_{u} A_{u',c,u,t}$$
$$u' \in REFT,\ ISOT,\ LNT\ and\ HNT \tag{3.32}$$

- Purchased intermediate tank

For the purchased intermediate such as MTBE, the outlet flow of its tank can be calculated from Equation (3.33):

$$AO_{u',c,t} = \sum_{u} A_{u',c,u,t}$$
$$u' \in Purchased\ intermediate\ tank \tag{3.33}$$

Hydrodesulfurization (HDS) unit

Feed flow of HDS can be estimated from the following equation:

$$AF_{HDS,t} = \sum_{u'} A_{u',c,HDS,t} \tag{3.34}$$

Production yield from HDS is calculated from Equation (3.34)

$$AO_{HDS,c,t} = AF_{HDS,t} \times yield_{HDS,c} \tag{3.35}$$

Outlet flow balance equation is as follow:

$$AO_{HDS,c,t} = \sum_{u} A_{HDS,c,u,t} \tag{3.36}$$

Product pool model

- Fuel gas (FG) tank

Total production of FG tank can be found from the following equation:

$$AF_{FGT,t} = \sum_{u'} A_{u',c,FGT,t} \tag{3.37}$$

The amount of product flow out from FG tank is represented by:

$$AO_{FGT,c,t} = AF_{FGT,t} \tag{3.38}$$

- Liquefied petroleum gas (LPG) tank

The total production of LPG tank can be calculated from Equation (3.39)

$$AF_{LPGT,t} = \sum_{u'} A_{u',c,LPGT,t} \tag{3.39}$$

The amount of product flow out from LPG tank is represented by:

$$AO_{LPGT,c,t} = AF_{LPGT,t} \tag{3.40}$$

- Gasoline pool (GSP)

Gasoline is produced by blending the intermediate streams. Equation (3.41) represents the feed flow of GSPs:

$$AF_{GSP,t} = \sum_{u'} A_{u',c,GSP,t} \tag{3.41}$$

The amount of product flow out from GSPs are determined by Equation (3.42)

$$AO_{GSP,c,t} = AF_{GSP,t} \tag{3.42}$$

The properties of products from GSPs can be calculated from the following equation:

$$PO_{GSP,c,q,t} = \frac{\sum_{u'} \sum_{c} (A_{u',c',GSP,t} \times pro_{u',c,q})}{\sum_{u'} \sum_{c} A_{u',c,GSP,t}} \tag{3.43}$$

- Jet fuel (JP-1) tank

The total production of JP-1 can be calculated from the following equation:

$$AF_{JPT,t} = \sum_{u'} A_{u',c,JPT,t} \tag{3.44}$$

The amount of product flow out from JP-1 tank is represented by:

$$AO_{JPT,c,t} = AF_{JPT,t} \tag{3.45}$$

- *Diesel Oil pool (DSP)*

DSP is the unit that produces the diesel oil. The amount of DSP feed can be calculated as follow:

$$AF_{DSP,t} = \sum_{u'} A_{u',c,DSP,t} \tag{3.46}$$

The amount of product flow out from DSP is determined by Equation (3.47):

$$AO_{DSP,c,t} = AF_{DSP,t} \tag{3.47}$$

- *Fuel oil (FO) pool*

Different types of fuel oils may be produced with different viscosity, or other properties. All are blended in their fuel oil pools. The following equation represents the feed flow to each fuel oil pool.

$$AF_{FOP,t} = \sum_{u'} A_{u',c,FOP,t} \tag{3.48}$$

The amount of product flow out from fuel oil pools are determined by Eqaution (3.49):

$$AO_{FOP,c,t} = AF_{FOP,t} \tag{3.49}$$

Quality constraints for product units

The properties concerned in this work are different upon the products. For FG and LPG, the properties of these two products are not taken into account since FG is burned as an energy source in the plant whereas LPG properties are mostly in the range of its specification. Other properties for each product can be described as follows:

Gasoline products

The properties involving with these products are octane number (RON), aromatic content (ARO), and Reid vapor pressure (RVP). These properties are of importance for production of gasoline products.

Diesel oil (DO)

Only two properties are used in the DO production that is CI and S. These properties are the specification for the Diesel products.

Fuel oil (FO)

There are four properties (S, V50, V100, PPI) used in the fuel oil production. S, V50, V100, and PPI are required for the fuel oils.

- *Gasoline blending*

Property constraints of product from gasoline pool are RON, ARO, and RVPI and can be calculated from the following equations:

For RON;
$$pn_{Gasoline,RON} \leq \frac{\sum_{u'}\sum_{c}(A_{u',c',GSP,t} \times pro_{u',c,RON})}{\sum_{u'}\sum_{c} A_{u',c,GSP,t}} \tag{3.50}$$

For ARO and RVPI
$$\frac{\sum_{u'}\sum_{c}(A_{u',c',GSP,t} \times pro_{u',c,q})}{\sum_{u'}\sum_{c} A_{u',c,GSP,t}} \leq px_{Gasoline,q}$$

$$q \in ARO\ and\ RVPI \tag{3.51}$$

-Fuel oil bleding

Property constraints for fuel oil product are calculated from the equations as follow:

$$pn_{fueloil,q} \leq PO_{FOP,fueloil,q,t} \leq px_{fueloil,q} \tag{3.52}$$

Fuel oil products are produced by blending of FO intermediates which typically are sent from CDUs. Properties on volume basis of fuel oil product can be calculated as follow:

$$PO_{FOP,fueloil,q,t} = \frac{\sum_{u'}\sum_{c}(A_{u',c,CDU,t} \times cyield_{c',FO} \times pro_{c,FO,q})}{\sum_{u'}\sum_{c}(A_{u',c,CDU,t} \times cyield_{c',FO})}$$

$$q \in \mathrm{AV}_q \qquad (3.53)$$

Properties on weight basis of fuel oil product can be calculated as follow:

$$PO_{FOP,fueloil,q,t} = \frac{\sum_{u'}\sum_{c}(A_{u',c,CDU,t} \times cyield_{c,FO} \times pro_{c,FO,q} \times pro_{c,FO,SG})}{\sum_{u'}\sum_{c}(A_{u',c,CDU,t} \times cyield_{c,FO} \times pro_{c,FO,SG})}$$

$$q \in \mathrm{AW}_q \qquad (3.54)$$

- Diesel blending

Property of diesel oil intermediate from CDU must satisfy the property constraints of diesel oil. Property constraint is as follow:

$$pn_{diesel,q} \leq PO_{DSP,diesel,q,t} \leq px_{diesel,q} \qquad (3.55)$$

Diesel oil products are produced by blending of DO intermediates which typically are sent from CDUs. Properties on volume basis of fuel oil product can be calculated as follow:

$$PO_{DSP,dieseloil,q,t} = \frac{\sum_{u'}\sum_{c}(A_{u',c,CDU,t} \times cyield_{c',DO} \times pro_{c,DO,q})}{\sum_{u'}\sum_{c}(A_{u',c,CDU,t} \times cyield_{c',DO})} \qquad (3.56)$$

Refinery Fuel Balance

Some outlet commodities might be burnt in the refinery to provide the energy required for operation of the different units and to provide utilities (steam, electricity, etc.).

The commodity that are used might be sold as a product and burnt as an energy source for the plant. The balance equation of this commodity is as follow:

$$AO_{u,c,t} - Burnt_{c,t} = MANU_{c,t} \tag{3.57}$$

where $AO_{u,c,t}$ is equal to the amount of commodity leaving from the process.

The refinery fuel balance is calculated based on weight basis. The refinery fuel balance is shown by the following equation:

$$Used_t = (\sum_c Burnt_{c,t} \times SG_c) \tag{3.58}$$

where SG_c is the specific gravity of commodity used as energy source. $Used_t$ is energy consumption for operating the process expressed in ton of oil equivalence and can be calculated from the following equation:

$$Used_t = \sum_u^{UN} (AF_{u,t} \times density_u \times fuel_u) \tag{3.59}$$

where $AF_{u,t}$ is volume of feed and $density_u$ is density of feed to each unit. This density is an average value for each unit except for CDUs that the density is different with crude oil types. Energy consumption for each unit is calculated by using $fuel_u$ which is percent of energy consumption for each unit.

3.3 Objective Function

The objective function in this model is profit that is obtained by the product sales minus crude oil cost, intermediate cost, storage cost, expense from lost demand, and expense from discounted product. This is shown by the following equation:

$$\text{Max Profit} = \sum_t \sum_p TP_{p,t} - \sum_t \sum_o TO_{o,t} - \sum_t \sum_i TI_{i,t} - \sum_t \sum_p TS_{p,t} - \sum_t \sum_p TL_{p,t} - \sum_t \sum_p TD_{p,t} \tag{3.60}$$

where:

$TP_{p,t}$ comes from

$$TP_{p,t} = MANU_{p,t} \times cp_{p,t} \qquad \forall p \in C_p \tag{3.61}$$

$MANU_{p,t}$ is equal to the amount of product produced in that time period.

$$MANU_{p,t} = \sum_u AO_{u,p,t} \qquad \forall u \in UP_u;\ p \in C_p \tag{3.62}$$

$AO_{u,p,t}$ is the amount of product flow out from production unit in each time period.

$TO_{o,t}$ comes from

$$TO_{o,t} = AC_{o,t} \times co_{o,t} \qquad \forall o \in C_o \tag{3.63}$$

$AC_{o,t}$ is equal to the amount of crude oil refined in that time period.

$$AC_{o,t} = \sum_u AO_{u,o,t} \qquad \forall u \in UC_u,\ o \in C_o \tag{3.64}$$

Note that $AO_{u,o,t}$ is amount of crude oil flow out from crude oil storage tank in each time period.

$TI_{i,t}$ comes from

$$TI_{i,t} = AI_{i,t} \times ci_{i,t} \qquad \forall i \in C_{ia} \qquad (3.65)$$

where $AI_{i,t}$ is equal to the amount of purchased intermediate added in that time period.

$$AI_{i,t} = \sum_{u} AO_{u,i,t} \qquad \forall u \in UI_{u}, i \in C_{ia} \qquad (3.66)$$

$AO_{u,ia,t}$ is amount of purchased intermediate flow out from their storage tank in each time period.

$TS_{p,t}$ comes from

$$TS_{p,t} = \left(\frac{AS_{p,t} + AS_{p,t-1}}{2} \right) \times cp_{p,t} \times int \qquad \forall p \in C_{p} \qquad (3.67)$$

where cost of storage is the cost of financing the investment in the working capital that it represents. The financial cost incurred relates to the average stock level over the period. Unless the stock levels are known, they are assumed that the average stock level is equal to the arithmetic mean of the opening and closing stock (Favennec, 2001). $AS_{p,t}$ represents closing stock and $AS_{p,t-1}$ represents opening stock. Int represents the average rate of interest payable in that period.

The balance of product storage can be found in the following equation:

$$AS_{p,t} = AS_{p,t-1} + MANU_{p,t} - Sales_{p,t} - AD_{p,t} \qquad (3.68)$$

where $sales_{p,t}$ is equal to the amount of product demand in that time period (capacity constraint in Equation (3.11)).

$TL_{p,t}$ comes from

$$TL_{p,t} = AL_{p,t} \times cl_{p,t} \qquad \forall p \in C_p \tag{3.69}$$

Alp,t is the product volume that cannot satisfy its demand. The demand constraint in Equation (3.11) is modified to the following equation:

$$dem_{c,t} = sales_{c,t} + AL_{c,t} \tag{3.70}$$

and $cl_{p,t}$ is assigned to equal $cp_{p,t}$ as follows:

$$cl_{p,t} = cp_{p,t} \qquad \forall p \in C_p \tag{3.71}$$

In Equation (3.70), the demands of each product must be equal to the volume of that product sale plus the volume of lost demand of that product. The volume of the lost demand is taken into account as the opportunity cost if that production cannot satisfy the demand.

$TD_{p,t}$ comes from

$$TD_{p,t} = AD_{p,t} \times cp_{p,t} \times disc \qquad \forall p \in C_p \tag{3.72}$$

where $AD_{p,t}$ is the product volume that over demand and sold as discount. The balance of discount volume can be found in Equation (3.68).

3.4 Pricing Decision

In pricing decision, the price-demand model is first developed by using microeconomic and mathematical method. In developing the price-demand model, we solve the consumer problem starting with "Utility Function". In microeconomics, utility function is a measure of the happiness or satisfaction gained by consuming goods and services.

Considering the two products, with demands d1 (for our products) and d_2 (for the competition products), we maximize the consumer utility (satisfaction), $u(d_1,d_2)$ subject to his budget limitation, that is:

$$\text{Max } u\ (d_1,d_2)$$

$$\text{s.t.} \quad P_1 d_1 + P_2 d_2 = Y \tag{3.73}$$

where p_1 is our product's selling price, p_2 the competitor's product price and Y the consumer budget. From this, we obtained the demand function. Then it would be applied to the planning model for integrating pricing decision with the planning and scheduling model.

A typical utility function has constant elasticity of substitution which we focus on in this work. The constant elasticity of substitution (CES) utility has the form of:

$$u(d_1, d_2) = (x_1^{\rho} + x_2^{\rho})^{1/\rho} \tag{3.74}$$

Where x is the function of demand, $x_i = x_i(d_i)$, which we could call "satisfaction functions". The satisfaction functions are determined by considering the different in quality between products and the reaction of consumers to prices. We propose the satisfaction functions as follows:

$$x_1 = d_1 \qquad \text{and} \qquad x_2 = \frac{\beta}{\alpha} d_2 \tag{3.75}$$

where β is a measure of how much a consumer prefers product 1 to product 2 and α a measure of how much the consumer population is aware of the quality of product 1, that is our product.

Then, we could write the utility function as follows:

$$u\ (d_1, d_2) = \left[(d_1)^{\rho} + (\frac{\beta}{\alpha} d_2)^{\rho} \right]^{1/\rho} \tag{3.76}$$

s.t. the budget constraint:

$$P_1\,(d_1) + P_2\,(d_2) \;\leq\; Y \tag{3.77}$$

Now we maximize the utility function with the budget constraint by using the Lagrange's method.

$$L\,(d_1,d_2,\lambda) = f\,(d_1,d_2) + \lambda \cdot g(d_1,d_2) \tag{3.78}$$

So,

$$L = \left[(d_1)^{\rho} + (\frac{\beta}{\alpha}\,d_2)^{\rho}\right]^{1/\rho} - \lambda\,[\,P_1(d_1) + P_2(d_2) - Y] \tag{3.79}$$

Then,
$$\frac{\partial L}{\partial d_1} = \frac{1}{\rho}[(d_1)^{\rho} + (\frac{\beta}{\alpha}\,d_2)^{\rho}]^{\frac{1}{\rho}-1} \times \rho\,(d_1)^{\rho-1} - \lambda P_1 = 0 \tag{3.80}$$

$$\therefore\quad [(d_1)^{\rho} + (\frac{\beta}{\alpha}\,d_2)^{\rho}]^{\frac{1}{\rho}-1} = \lambda P_1 (d_1)^{1-\rho} \tag{3.81}$$

And
$$\frac{\partial L}{\partial d_2} = \frac{1}{\rho}[(d_1)^{\rho} + (\frac{\beta}{\alpha}\,d_2)^{\rho}]^{\frac{1}{\rho}-1} \times \rho\,(d_2)^{\rho-1} - \lambda P_2 = 0 \tag{3.82}$$

$$[(d_1)^{\rho} + (\frac{\beta}{\alpha}\,d_2)^{\rho}]^{\frac{1}{\rho}-1} \times (\frac{\beta}{\alpha})^{\rho} = \lambda P_2 (d_2)^{1-\rho} \tag{3.83}$$

$$\therefore\quad \lambda \mathrm{P}_1 (\mathrm{d}_1)^{1-\rho} \times (\frac{\beta}{\alpha})^{\rho} = \lambda \mathrm{P}_2 (\mathrm{d}_2)^{1-\rho} \tag{3.84}$$

Then
$$P_1 (d_1)^{1-\rho} = (\frac{\alpha}{\beta})^{\rho} P_2 (d_2)^{1-\rho} \tag{3.85}$$

And from the budget constraint in Equation (3.77)

$$d_2 = \frac{Y - P_1(d_1)}{P_2} \tag{3.86}$$

We can rearrange. $(\frac{\alpha}{\beta})^\rho P_2 \left[\frac{Y - P_1(d_1)}{P_2}\right]^{1-\rho}$

$$P_1(d_1)^{1-\rho} = (\frac{\alpha}{\beta})^\rho P_2 \left[\frac{Y - P_1(d_1)}{P_2}\right]^{1-\rho} \tag{3.87}$$

Then the price-demand relation we get is as follows:

$$P_1(d_1)^{1-\rho} = (\frac{\alpha}{\beta})^\rho P_2^{\rho}\left[Y - P_1(d_1)\right]^{1-\rho} \tag{3.88}$$

Anyway, another issue that we need to concern is the demand constraint. Typically the customer would not purchase more product than what they want, so the total demand is limited by their real need.

Let D represents the total demand of product. then we could write

$$d_1 + d_2 \leq D \tag{3.89}$$

And this is the "demand constraint".

Then if we have the demand of product, d1 and d2, violate the demand constraint, the utility function would become

$$u(d_1,d_2) = [(d_1)^\rho + \{\frac{\beta}{\alpha}(D - d_1)\}^\rho]^{1/\rho} \tag{3.90}$$

With $$d_1 + d_2 = D \tag{3.91}$$

Maximum $u(d_1,d_2)$ is obtained by differentiating $u\ (d_1,d_2)$ with respect to d_1, that is:

$$\frac{\partial u(d_1,d_2)}{\partial d_1} = (\frac{1}{\rho})[(d_1)^\rho +$$

$$[\frac{\beta}{\alpha}(D-d_1)^\rho]^{\frac{1}{\rho}-1}[\rho(d_1)^{\rho-1} - \rho(\frac{\beta}{\alpha})^\rho(D-d_1)^{\rho-1}] = 0 \qquad (3.92)$$

Solve the above equation and rearrange; we get the price-demand relation under the demand constraint case as follows:

$$(d_1)^{1-\rho} = (\frac{\beta}{\alpha})^\rho (D-d_1)^{1-\rho} \qquad (3.93)$$

Linearization

The price-demand relation we obtained for both cases need linearization before we implement them in GAMS.

The budget case,

From $$P_1(d_1)^{1-\rho} = (\frac{\alpha}{\beta})^\rho P_2^\rho\,[Y - P_1(d_1)]^{1-\rho} \qquad (3.94)$$

Rearrange, $$P_{1c,t} = \left(\frac{\alpha}{\beta}\right)^\rho P_{2c,t}^\rho \left(\frac{Y - P_{1c,t}(d_{1c,t}^{(Y)})}{d_{1c,t}^{(Y)}}\right)^{1-\rho} \qquad (3.95)$$

Then elevate the two sides of the equation to 1/(1-ρ),

$$(P_{1c,t})^{\frac{1}{1-\rho}} = \left(\frac{\alpha}{\beta}\right)^{\frac{\rho}{1-\rho}} P_{2c,t}^{\frac{\rho}{1-\rho}} \left(\frac{Y - P_{1c,t}(d_{1c,t}^{(Y)})}{d_{1c,t}^{(Y)}}\right) \qquad (3.96)$$

Rearrange,

$$d_{1c,t}^{(Y)}\left(P_{1c,t}\right)^{\frac{1}{1-\rho}}\left(\frac{\beta}{\alpha P_{2c,t}}\right)^{\frac{\rho}{1-\rho}} + P_{1c,t}d_{1c,t}^{(Y)} = Y \tag{3.97}$$

Now the product of demand and price is linearized by discretizing the price term.

Let $$\mathrm{P}_{1c,t} = \sum z_{1_c}^{k}\mathrm{P}_{1_c}^{k} \quad \text{and} \quad \sum z_{1_c}^{k} = 1 \tag{3.98}$$

Where $z_{1_c}^{k}$ are binary variable and then substitute into eq.,

$$d_{1c,t}^{(Y)} \sum z_{1_c}^{k}\left[\left(\frac{\beta}{\alpha P_{2c,t}}\right)^{\frac{\rho}{1-\rho}}\left(P_{1_c}^{k}\right)^{\frac{1}{1-\rho}} + P_{1_c}^{k}\right] = Y \tag{3.99}$$

Let $$A_{k,c,t} = \left[\left(\frac{\beta}{\alpha P_{2c,t}}\right)^{\rho/1-\rho}\left(P_{1c}^{k}\right)^{1/1-\rho} + P_{1c,t}^{k}\right] \tag{3.100}$$

Substitute in to equation, we get

$$\sum z_{1_c}^{k} d_{1c,t}^{(Y)} A_{k,c,t} = Y \tag{3.101}$$

Let $$v_{1c,t}^{k} = z_{1c}^{k}\mathrm{d}_{1c,t}^{(Y)} \tag{3.102}$$

Then, $$\sum v_{1_c}^{k} A_{k,c,t} = Y \tag{3.103}$$

With $$\sum z_{1_c}^{k} = 1 \tag{3.104}$$

And

$$v_{1_c}^{k} - z_{1_c}^{k}\Omega \le 0$$
$$v_{1_c}^{k} \ge 0$$
$$(d_{1c,t}{}^{(Y)} - v_{1_c}^{k}) - (1 - z_{1_c}^{k})\Omega \le 0 \qquad (3.105\text{-}3.108)$$
$$(d_{1c,t}{}^{(Y)} - v_{1_c}^{k}) \ge 0$$

Where Ω = large number , z = binary variable

The demand case

If the demand of product, (d_1+d_2), violates the total demand, D*, the price-demand relation in the demand case is used instead:

From
$$(d_1)^{1-\rho} = (\frac{\beta}{\alpha})^{\rho}(D - d_1)^{1-\rho} \qquad (3.109)$$

This need to be linearized by elevating the two sides of the equation to $1/(\rho - 1)$ and rearrange,

$$d_{1c,t}^{(D)}\left[1 + \left(\frac{\beta}{\alpha}\right)^{\frac{\rho}{\rho-1}}\right] = \left(\frac{\beta}{\alpha}\right)^{\frac{\rho}{\rho-1}} D \qquad (3.110)$$

So, from this equation there is only one variable, $d_{1c,t}^{(D)}$ and it can be calculated as a function of parameters.

Now, we need to determine that which case of demand would be selected by using the following method,

First, we rise up the following equation

$$d_{1c,t}^{(Y)} + \left(\frac{Y - P_{1c,t}d_{1c,t}^{(Y)}}{P_{2c,t}}\right) - D + (Gamma \cdot w) > 0 \qquad (3.111)$$

Where $Gamma$ = large number, w = binary variable

in the 1st case, if $d^{(Y)}_{1c,t} + \left(\dfrac{Y - P_{1c,t} d^{(Y)}_{1c,t}}{P_{2c,t}} \right) < D$ then we obtain $w = 1$

and in the 2nd case, if $d^{(Y)}_{1c,t} + \left(\dfrac{Y - P_{1c,t} d^{(Y)}_{1c,t}}{P_{2c,t}} \right) > D$ then we obtain $w = 1$ or 0

For 2nd case, the constraint to force the right value of w is as follows:

Let
$$d_{1c,t} = w \cdot d^{(Y)}_{1c,t} + (1 - w) \cdot d^{(D)}_{1c,t} \tag{3.112}$$

$$d_{2c,t} = w \cdot \left(\frac{Y - P_{1c,t} d^{(Y)}_{1c,t}}{P_{2c,t}} \right) + (1 - w) \cdot (1 - d^{(D)}_{1c,t}) \tag{3.113}$$

With
$$d_{1c,t} + d_{2c,t} \leq D \tag{3.114}$$

Then we linearize the above equations as follows:

From eq. (3.112) $\quad \mathrm{d}_{1c,t} = w \cdot \mathrm{d}^{(Y)}_{1c,t} + (1 - w) \cdot \mathrm{d}^{(D)}_{1c,t}$

Let $\quad m_{c,t} = w \cdot \mathrm{d}^{(Y)}_{1c,t}$ and Ω = large number

So,
$$d_{1c,t} = m_{c,t} + (1 - w) \cdot d^{(D)}_{1c,t} \tag{3.115}$$

Now we introduce these equations,

$$\begin{aligned} m_{c,t} - w \cdot \Omega &\le 0 \\ m_{c,t} &\ge 0 \\ (d_{1c,t}^{(Y)} - m_{c,t}) - (1 - w) \cdot \Omega &\le 0 \\ d_{1c,t}^{(Y)} - m_{c,t} &\ge 0 \end{aligned} \qquad (3.116\text{-}3.119)$$

And from eq. (3.113), $d_{2c,t} = w \cdot \left(\dfrac{Y - P_{1c,t} d_{1c,t}^{(Y)}}{P_{2c,t}} \right) + (1 - w) \cdot (1 - d_{1c,t}^{(D)})$

Let $n_{c,t} = w \cdot \left(\dfrac{Y - P_{1c,t} d_{1c,t}^{(Y)}}{P_{2c,t}} \right)$ and Ω = large number

So, $$d_{2c,t} = n_{c,t} + (1 - w) \cdot (1 - d_{1c,t}^{(D)}) \qquad (3.120)$$

Now we introduce these equations,

$$\begin{aligned} n_{c,t} - w \cdot \Omega &\le 0 \\ n_{c,t} &\ge 0 \\ \left[\left(\frac{Y - P_{1c,t} d_{1c,t}^{(Y)}}{P_{2c,t}} \right) - n_{c,t} \right] - (1 - w) \cdot \Omega &\le 0 \\ \left(\frac{Y - P_{1c,t} d_{1c,t}^{(Y)}}{P_{2c,t}} \right) - n_{c,t} &\ge 0 \end{aligned} \qquad (3.121\text{-}3.124)$$

With $$d_{1c,t} + d_{2c,t} \le D \qquad (3.125)$$

So, in the 1st case "w" is one and in the 2nd case "w" must be zero.

Equation (3.103)-(3.108), (3.111), (3.116)-(3.119), (3.121)-(3.125) were implemented in GAMS and added into the deterministic model for integrating pricing decision with the planning model. A case study was applied and the deterministic model with integrated pricing decision was run and tested again. Then it was adjusted to the stochastic model to handle uncertainty issue.

3.5 Stochastic Formulation

The stochastic formulation technique used in this thesis is the two-stage stochastic program with fixed recourse. The uncertainty is introduced through the demand and product price parameter in the general planning model without pricing. For the planning model with pricing decision, the uncertainty is introduced through the consumer budget and the total demand of products. First-stage decisions are the amount of crude oil purchased, $AO_{p,t}$, for every planning period. Second-stage decisions are the amount of product production, $MANU^{s}_{p,t}$, amount of product stock, $AS^{s}_{p,t}$, amount of intermediate purchased, AIsp,t, amount of product that cannot satisfy demand, $AL^{s}_{p,t}$, and amount of discount sales, $Ad^{s}_{p,t}$. These second-stage scenarios are denoted by the index s and assumed to occur with individual probabilities p_s. It is assumed that the random events which occur at the second-stage are finite and independent from the first-staged decisions.

The stochastic results are obtained by using sampling algorithm method which was discussed by Aseeri and Bagajewicz (2003). In this method, a full deterministic model is run for each scenario and then, after that scenario is solved, the first staged variables (commitment to buy a certain sets of crudes) are fixed and rerun the same model for all the rest of the scenarios. After that, the highest expected GRM risk curves and non-dominated curve with this one are selected and discussed.

3.6 Model Interfaces

The model interfaces are generated. Mathematical equations in this model are developed not specific with unit names or number of units. Input data such as set of products, set of production units, or product paths of different units can be input through the model interfaces instead of directly fixing them in GAMS. Therefore, this model can be applied to different refineries with a set of typical units and it is more convenient for the model user.

The model interfaces are created by using the Microsoft Visual Basic. Input data from model users are sent from the model interfaces to the Excel application and then to GAMS model by applying the GAMS Data Exchange (GDX) facilities as shown in Figure 3.2. The results of the model are sent backward through the model interfaces and then to the model user.

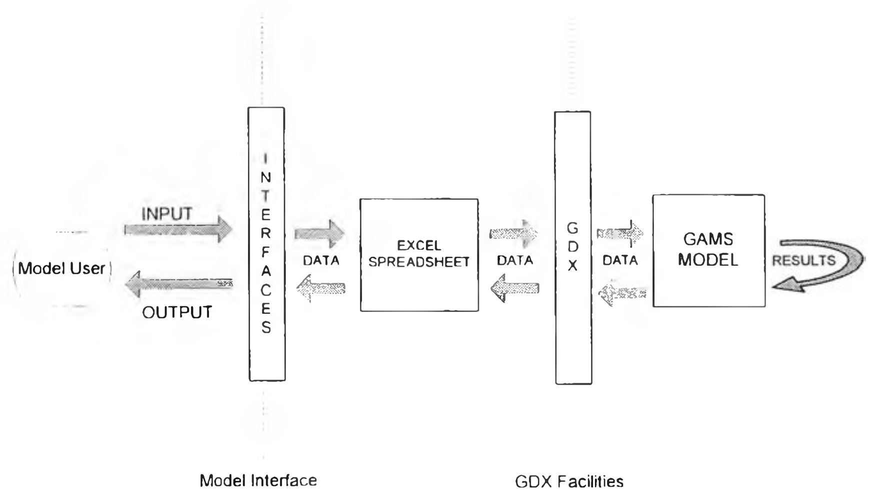

Figure 3.2 Data flow diagram of the planning and scheduling mode.

3.7 Case Study

The model was applied and tested by using the data from the Bangchak Refinery. Figure 3.3 shows a simplified scheme of the refinery. It has two atmospheric distillation units (CDU2 and CDU3), two naphtha pretreating units (NPU2 and NPU3), one light naphtha isomerization unit (ISOU), two catalytic reforming units (CRU2 and CRU3), one kerosene treating unit (KTU), one gas oil hydrodesulphurization (GO-HDS), and one deep gas oil hydrodesulphurization (DGO-HDS). The commercial products from the refinery are liquefied petroleum gas (LPG), gasoline RON 91 (SUPG), gasoline RON 95 (ISOG), jet fuel (JP-1), high speed diesel (HSD), fuel oil 1 (FO1), fuel oil 2 (FO2), and low sulfur fuel oil (FOVS). Fuel gas (FG) and some amount of FOVS produced from the process are used as an energy source for the plant. The summary of feeds and products that processed in each unit can be found in the Table 3.1. All units, feeds, products, unit interconnections, feed and product paths of each unit are input in the planning model. The intermediate streams from each unit are blended in product pools to satisfy product specification. There are three product pools for blending products: gasoline pool (GSP), diesel pool (DSP), and fuel oil pool (FOP). The streams that used for blending each product are shown in Table 3.2.

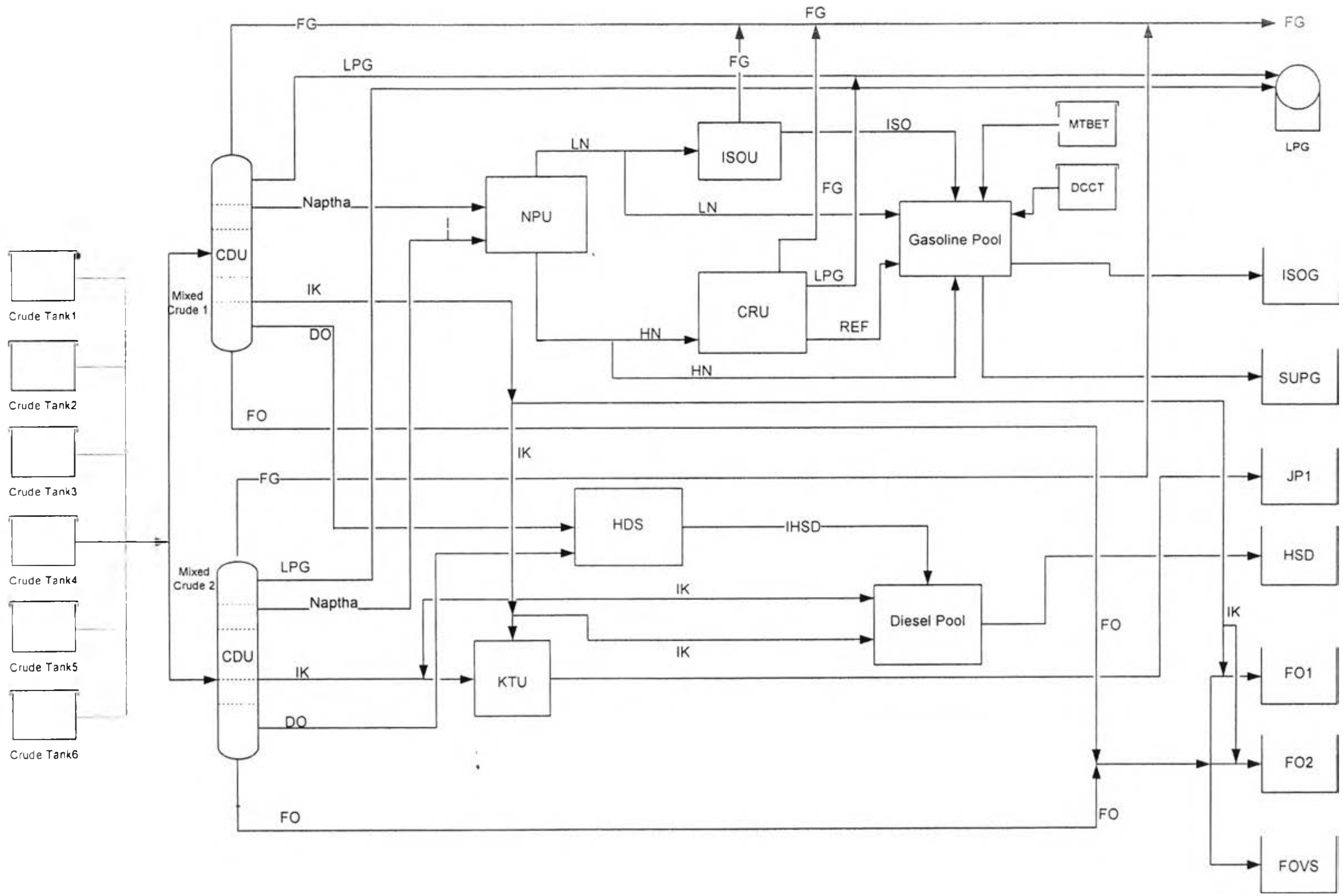

Figure 3.3 Simplified scheme of Bangchak Petroleum Public Company Limited.

Table 3.1 Summary of feeds and products for each unit

Unit	Feed	Product
CDU	- Crude mixture	- FG - LPG - LN - MN - HN - IK - DO - FO
NPU	- LN - MN - HN	- LN - MN - HN
ISOU	- LN	- FG - ISO
CRU	- MN - HN	- FG - LPG - REF
KTU	- IK	- JP-1
HDS	- DO	- IHSD
ISOT	- ISO	- ISO
REFT	- REF	- REF
LNT	- LN	- LN
HNT	- HN	- HN
MTBET	- MTBE (purchased)	- MTBE
DCCT	- DCC (purchased)	- DCC

Table 3.2 Intermediate streams for product blending in each pool

Pool	Intermediate	Product
GSP91	- ISO - REF - LN - HN - MTBE - DCC	- SUPG
GSP95	- ISO - REF - LN - HN - MTBE - DCC	- ISOG
DSP	- IK - DO - IHSD	- HSD
FO1P	- IK - FO	- FO1
FO2P	- IK - FO	- FO2
FOVSP	- FO	- FOVS

There are six crude oil types for feeding the refinery: Oman, Tapis, Labuan, Seria light, Phet, and Murban. Data of all units and commodities (crude oils, intermediates, products) can be found in Appendix B.

In applying the case study, there are some specific restrictions of the refinery. They need to be input in the model and are as follow:

▪ PHET crude has to be fed to CDU2 only due to the limitation of unit. This operation rule is represented in Equation (3.124).

$$A_{PHETT,PHET,CDU3,t} = 0 \quad (3.124)$$

▪ There is an operating rule in blending gasoline product with MTBE. The amount of MTBE in gasoline must not be over 10%. This rule is shown in the following equation:

$$A_{MTBET,MTBE,GSP,t} \leq AF_{GSP,t} \times 0.1 \quad (3.125)$$

▪ The recipe used in blending FO1 and FO2 with IK is 7 and 2.5% of the FO1 and FO2 volume, respectively. This is shown in the following equations:

$$\sum_{CDU} A_{CDU,IK,FO1P,t} = AF_{FO1P,t} \times 0.07 \quad (3.126)$$

$$\sum_{CDU} A_{CDU,IK,FO2P,t} = AF_{FO2P,t} \times 0.025 \quad (3.127)$$

3.8 Model Testing

Data from Bangchak Refinery was used in testing the model. All properties are input to the planning model.

The refinery produces eight commercial products (LPG, SUPG, ISOG, JP-1, HSD, FO1, FO2, and FOVS) using two crude distillation units (CDU2 and CDU3) and six productive units (NPU2, NPU3, CRU2, CRU3, ISOU, GO-HDS, DGO-HDS, KTU). The maximum plant production capacity is 120 kbd. The production yields and unit capacity can be found in Appendix B.

Demand in each period was considered to be satisfied by the production. Uncertainty was introduced in market demand and price for the general stochastic model without pricing. For the model with pricing decision added, uncertainty was taken into account in consumer budget and total demand of product.

The model was tested in three-time-period planning. First the general deterministic linear programming model were solved to obtain the results including amount of type of crude oil used, amount and property of product refined, amount of product stored, and profit. The stochastic programming model was then formulated with uncertainty in the product demands and prices by using a deterministic linear programming model as a basis. The stochastic solution was found by using sampling algorithm method. The results are compared with the deterministic model.

After that, the price-demand relation was developed and added into the general deterministic and stochastic planning model. The model with pricing decision is now changed from Linear Programming model to the Mixed Integer Programming (MIP) one. The deterministic model with pricing decision was solved first, and then it was adjusted to the stochastic programming model and solved again. The results of these two types of pricing and planning model were compared to those of the general planning model without pricing. Moreover, risk curves form these solutions are analyzed.

The proposed model was implemented it the modeling system GAMS. The linear and mixed integer linear model was solved by CPLEX 9.0 solver. GAMS was run on a Pentium IV / 2.4 GHz PC platform.