CHAPTER IV

DISCUSSION OF NUMERICAL RESULTS

4.1 Convergence and Accuracy of Numerical Solutions

A computer program based on the numerical solution schemes described in the pervious chapter was developed to study load transfer from an axially loaded elastic bar to a multilayered poroelastic half-space. First of all, the convergence of solutions with respect to the following parameters is investigated:

1. the number of alpha terms, K, used in the assumed displacement function, $w(z,t)$ in eqn (2.46)

2. the total number of elements, N_t, used in the discretization of the volume of the bar as shown in Fig. 4

3. the upper limit of Hankel transform parameter, ξ_L, used in the numerical integration of eqn (2.17) to determine the flexibility matrix of the multilayered half-space.

Table 1 shows the convergence of nondimensionalized axial displacement at the top of the bar, $\Delta_0 E_b A / V_0 a$, with respect to the number of alpha terms. Δ_0 is the axial displacement of the bar at $z=0$ due to the axial load V_0 applied at the top of the bar as shown in Fig. 5. In addition, A is the cross-sectional area of the bar at $z=0$. It can be seen that the solutions have good convergence. Note that, when the ratio of E_b/E_h is greater than 1000, the solutions converge with a few terms of alpha. This is due to the fact that the elastic bar will behave as a rigid bar for the higher value E_b/E_h and the assumed displacement function of the rigid bar requires that K is equal to one.

Table 2 and Table 3 present the convergence of nondimensionalized displacement, $\Delta_0 E_h A / V_0 a$, with respect to N_t and ξ_L, respectively. The solutions in both Table 2 and Table 3 show good convergence for increasing values of N_t and ξ_L. It should be noted that numerical solutions presented hereafter will be evaluated with K, N_t and ξ_L being equal to 12, 14 and 25, respectively.

Numerical solutions obtained from the two numerical Laplace inversion schemes, namely, Stehfest, given by eqn (3.1), and Schapery, given by eqn (3.3), are compared in Table 4. It is evident that the solutions from both schemes agree very closely. However, Stehfest's scheme requires more computational effort than Schapery's scheme. In this thesis, the Laplace inversion is carried out by using Schapery's scheme in order to determine quasi-static behavior of the bar.

The literature review indicates that there are a number of researchers in the past who studied the problem of load transfer from a bar to a homogeneous medium. Consider a bar-homogeneous half-space system as shown in Fig. 5. The comparison between solutions from existing studies and the present study is presented in Tables 5 and 6. Table 5 shows the comparison of numerical solutions for nondimensionalized axial displacement of an elastic bar embedded in homogeneous ideal elastic medium obtained from present procedure and Selvadurai and Rajapakse[3]. As evident from the table, the numerical results from both schemes are in good agreement with only about 7% maximum difference. The difference is due to the fact that the flexibility matrix in the present study was evaluated by using numerical methods (due to the complexity of the integrals involved in the global stiffness matrix of the multilayered medium). By contrast, the flexibility matrix obtained from Selvadurai and Rajapakse's approach[3] is in the form of Lipschitz-Hankel integrals which can be expressed in terms of complete elliptic integrals and computed accurately by using a high-precision software library for special mathematical functions.

Table 6 presents final solutions for nondimensionalized axial displacement of the bar-homogeneous half-space system of Fig. 6. using the present scheme and the scheme proposed by Niumpradit and Karasudhi[4]. It is evident that the trend of both solutions is in good agreement, i.e. nondimensionalized axial displacement decreases with increasing values of the ratio E_b/E_h. In addition, the maximum difference between both solutions is about 8%.

It should be noted that as time approaches infinity pore pressure tends to zero and the medium becomes an ideal elastic one. Compared with the solution obtained by Selvadurai and Rajapakse[3] in Table 5, the numerical solutions obtained from the present study show smaller difference than those obtained from Niumpradit and Karasudhi[4]. Therefore, it can be concluded that the present solution scheme gives a better approximation than the scheme proposed by Niumpradit and Karasudhi[4].

4.2 Numerical Results for Bar-Multilayered System

The quasi-static behavior of an elastic bar embedded in a multilayered poroelastic half-space is investigated in this section. A layered system consisting of two poroelastic layers bonded to an underlying poroelastic half-space, as shown in Fig. 6, is considered in the parametric study.

The properties of the first layer are $B^{(1)} = 1.0$, $\nu^{(1)} = 0.25$ and $\nu_u^{(1)} = 0.50$; for the second layer, $B^{(2)} = 0.80$, $\nu^{(2)} = 0.25$ and for the underlying half-space, $B^{(3)} = 0.60$, $\nu^{(3)} = 0.20$, $\nu_u^{(3)} = 0.30$. In addition, $\kappa^{(3)} / \kappa^{(2)} = 0.50$. These properties are kept as constants throughout this section while other properties are varied in order to investigate their effects on time histories of the axial displacement of the bar. The total length of the bar , unless otherwise specified, is set to be 10a. A nondimensional time, $t^* = c^{(2)} t / a^2$, in the range $10^{-5} \le t^* \le 10^5$ is considered in the numerical study. Note that $c^{(2)}$ is the consolidation coefficient of the second layer given by eqn (2.12). The time histories of nondimensionalized axial displacements of the bar, $\Delta_0 E_h^{(2)} A / V_0 a$ (in which Δ_0, A and V_0 are as defined in section 4.1 and $E_h^{(2)}$ denotes the modulus of elasticity of the second layer), are shown from Fig. 7 to Fig. 12.

The influence of permeability on the nondimensionalized displacement is demonstrated in Fig. 7 by setting the ratio of $\kappa^{(1)} / \kappa^{(2)}$ equal to 0.001, 0.01, 0.1, 1.0, 10 and 100, respectively. The thickness of the first and second layer is equal to 5a. It is found that the ratio

$\kappa^{(1)}/\kappa^{(2)}$ has a significant influence on nondimensionalized axial displacement rate. The variation of $\kappa^{(1)}/\kappa^{(2)}$ results in the shift of nondimensionalized axial displacement profile in the time scale. The earliest final displacement is reached for the maximum value of $\kappa^{(1)}/\kappa^{(2)}$ and the latest for the minimum ratio of $\kappa^{(1)}/\kappa^{(2)}$. It should be noted that the numerical results in Fig. 7 show identical initial and final nondimensionalized axial displacement of the bar. This is due to the fact that the material properties, ν, ν_u and μ and the thickness of the two layers are the same for all values of $\kappa^{(1)}/\kappa^{(2)}$.

The influence of layer thickness on nondimensionalized axial displacement of the bar is presented in Fig. 8 in view of the ratio $h^{(1)}/h^{(2)}$. Note that the total thickness of the two layers is equal to $10a$ and the ratio of $\kappa^{(1)}/\kappa^{(2)}$ is 0.001. It is evident that the initial displacements of the bar for different values of $h^{(1)}/h^{(2)}$ are different and their order of magnitude is identical to that of $h^{(1)}/h^{(2)}$. This is due to the fact that the undrained behavior of the bar-multilayered medium system is mainly governed by undrained Poisson's ratio. The higher value of ratio $h^{(1)}/h^{(2)}$ means the lesser undrained compressibility of the multilayered medium since $\nu_u^{(1)} > \nu_u^{(2)}$. It is also found that the time to reach the final displacement increases with increasing the value of $h^{(1)}/h^{(2)}$. These features are due to the fact that the multilayered medium becomes more impermeable for the higher ratio of $h^{(1)}/h^{(2)}$, since $\kappa^{(1)}/\kappa^{(2)} = 0.001$. The final nondimensionalized axial displacements of the bar are also identical for all values of $h^{(1)}/h^{(2)}$ since the elastic properties (drained) for different layers are identical.

Fig. 9 shows the effect of drained Poisson's ratio on nondimensionalized axial displacement of the bar. The values of drained Poisson's ratio for the second layer are set at $\nu^{(2)} = 0.20$, 0.25 and 0.30. It is found that the final displacement depends on drained Poisson's ratio because at the final condition, i.e. time approaches infinity, the water has already been drained and excess pore pressure becomes zero. The multilayered poroelastic half-space tends to behave like a multilayered elastic one. The lower value of drained Poisson's ratio means the higher compressibility of that layer since its thickness is kept constant. Note that the initial displacements are identical since the undrained property of different layers is identical for each value of $\nu^{(2)}$.

Fig. 10 shows the effect of shear modulus on nondimensionalized axial displacement of the bar. The different values of shear modulus are shown in the figure. It is found that the higher value of shear modulus causes the reduction in the bar displacement. This feature can be explained by the fact that the layered medium becomes more rigid when shear moduli of one and/or more layers are increased.

The influence of the bar length is presented in Fig. 11. The length of the bar is varied from $10a$ to $30a$. The thickness of the first and second layer is the same and the total thickness of these two layers are equal to the total length of the bar. It is found that the bar displacement is decreased for increasing the bar length. This is due to the fact that longer bar means the contact surface between the bar and the medium is increased. This implies that the longer bar will have more resisting force than the shorter one.

Fig. 12 demonstrates the relationship between the ratio $E_b/E_h^{(2)}$ and nondimensionalized axial displacement of the bar. It is evident that the displacement decreases with increasing $E_b/E_h^{(2)}$ increases. This is due to the fact that the strain of the bar depends on E_b and the bar which has higher ratio $E_b/E_h^{(2)}$ will have less strain than the lower one.

Fig. 13 shows the bar-multilayered system modelled from the real system of a pile embedded in surrounding soil. The properties and profiles of soil are given in Fig.13. The radius and length of the pile is 0.25 m. and 31.50 m., respectively. The time-histories behavior of the pile is shown in Fig. 14 and 15.

Fig. 14 and 15 demonstrate the nondimensionalized displacement and nondimensionalized axial stress of the pile, respectively. The final displacement is occured when the nondimansional time, t^*, reaches 10^3. It can be seen that the maximum and minimum stress are occured at z = 0 and z = h_b, respectively. In addition, It is difficult to identify the influence of individual parameters (layer thickness, material properties, etc.) separately on the displacement and axial stress of the pile.

It should be noted that since the final behavior of the pile occurs when the nondimensional time, t^*, reaches 10^3, the time, t^*, can be converted to the real time, t, from the relation given in section 4.2. The consolidation coefficient, $c^{(2)}$, can be determined from eqn (2.12) by using the properties of the second layer ($\nu^{(2)} = 0.25$, $\nu_u^{(2)} = 0.35$, $B^{(2)} = 1.0$, $\kappa^{(2)} = 1x10^{-11}$ m^4/kg-sec, $\mu = 5.6x10^5$

kg/m^2). It can be shown that, for the pile-soil system as shown in Fig. 13, the final displacement occurs when the real time reaches 18 years and the final displacement of the pile which is subjected to service load about 130 tons is about 0.007 meter.