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TABLES AND FIGURES

TABLE 1. Convergence of nondimensionalized displacement, $\Delta_0 E_h A / V_0 a$, of an elastic bar embedded in a homogeneous poroelastic half-space with respect to K . ($h_b = 10a$, $\xi_L = 25$, $N_t = 10$)

K	E_b / E_h				
	10	100	1000	10000	100000
4	0.6388	0.3731	0.3342	0.3301	0.3296
6	0.6392	0.3732	0.3342	0.3301	0.3296
8	0.6399	0.3733	0.3342	0.3301	0.3296
10	0.6401	0.3733	0.3342	0.3301	0.3296
12	0.6401	0.3733	0.3342	0.3301	0.3296
14	0.6401	0.3733	0.3342	0.3301	0.3296

TABLE 2. Convergence of nondimensionalized displacement, $\Delta_0 E_h A / V_0 a$, of an elastic bar embedded in a homogeneous poroelastic medium with respect to N_t . ($h_b = 10a$, $\xi_L = 25$, $K = 14$)

N_t	E_b / E_h				
	10	100	1000	10000	100000
10	0.6468	0.3754	0.3364	0.3323	0.3319
12	0.6439	0.3745	0.3354	0.3313	0.3309
14	0.6418	0.3738	0.3347	0.3306	0.3302
16	0.6401	0.3733	0.3342	0.3301	0.3297

TABLE 3. Convergence of nondimensionalized axial displacement, $\Delta_0 E_h^{(2)} A / V_0 a$, with respect to ξ_L ($h_b = 10a$, $h^{(1)} = 6a$, $h^{(2)} = 4a$, $\kappa^{(1)} / \kappa^{(2)} = 0.001$, $N_t = 14$, $K = 10$, $t^* = 10^5$)

ξ_L	$E_b / E_h^{(2)}$				
	10	100	1000	10000	100000
15	0.6136	0.3579	0.3199	0.3159	0.3155
20	0.6153	0.3578	0.3199	0.3159	0.3155
25	0.6136	0.3578	0.3199	0.3159	0.3155
30	0.6137	0.3578	0.3199	0.3159	0.3155

TABLE 4. Comparison between solutions from Stehfest and Schapery scheme ($h_b = 10a$, $h^{(1)} = h^{(2)} = 5a$, $G^{(1)}/G^{(3)} = 0.5$, $G^{(2)}/G^{(3)} = 0.5$, $v^{(2)} = 0.25$, $\kappa^{(1)}/\kappa^{(2)} = 0.001$, $N_t = 14$, $K = 10$, $E_b/E^{(2)} = 1000$ and $\xi_L = 25$)

t^*	$\Delta_0 E_h^{(2)} A / V_0 a$	
	Stehfest scheme	Schapery scheme
10^{-4}	0.2306	0.2306
10^{-3}	0.2307	0.2308
10^{-2}	0.2311	0.2311
10^{-1}	0.2319	0.2321
1	0.2336	0.2338
10	0.2359	0.2362
10^2	0.2397	0.2403
10^3	0.2458	0.2462
10^4	0.2506	0.2450
10^5	0.2508	0.2507

Table 5. Comparison of nondimensionalized axial displacement obtained from present study and Selvadurai & Rajapakse⁽³⁾

E_b / E_h	$\Delta_0 E_h A / V_0 a$	
	Present Study	Selvadurai & Rajapakse ⁽³⁾
10	0.6468	0.6043
100	0.3754	0.3431
1000	0.3364	0.3137
10000	0.3323	0.3096

Table 6. Comparison of final solutions for nondimensionalized axial displacement obtained from present study and Niumpradit & Karasudhi⁽⁴⁾

E_b / E_h	$\Delta_0 E_h A / V_0 a$	
	Present study	Niumpradit & Karasudhi ⁽⁴⁾
10	0.6468	0.6944
100	0.3754	0.4017
1000	0.3364	0.3657
10000	0.3323	0.3612

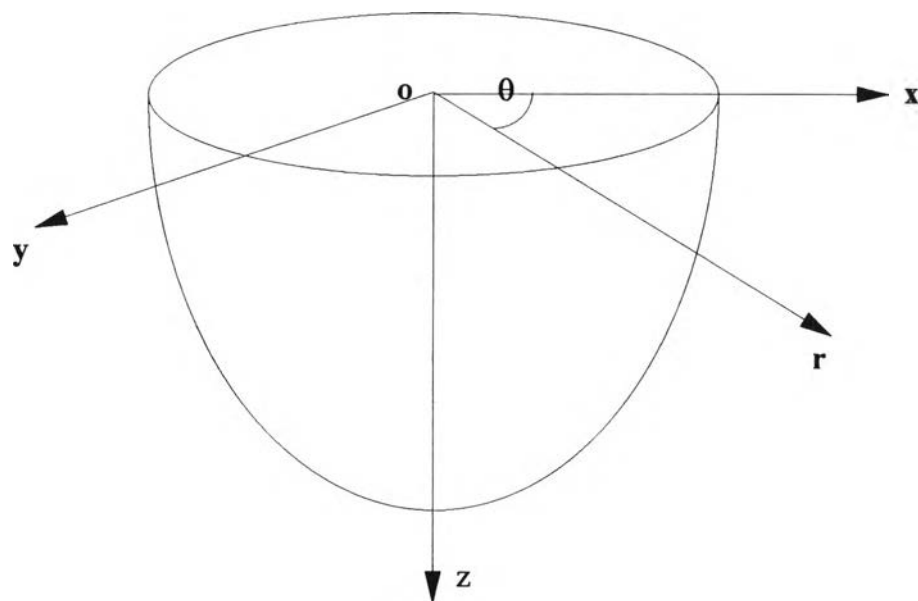


Figure 1 A homogeneous poroelastic half-space

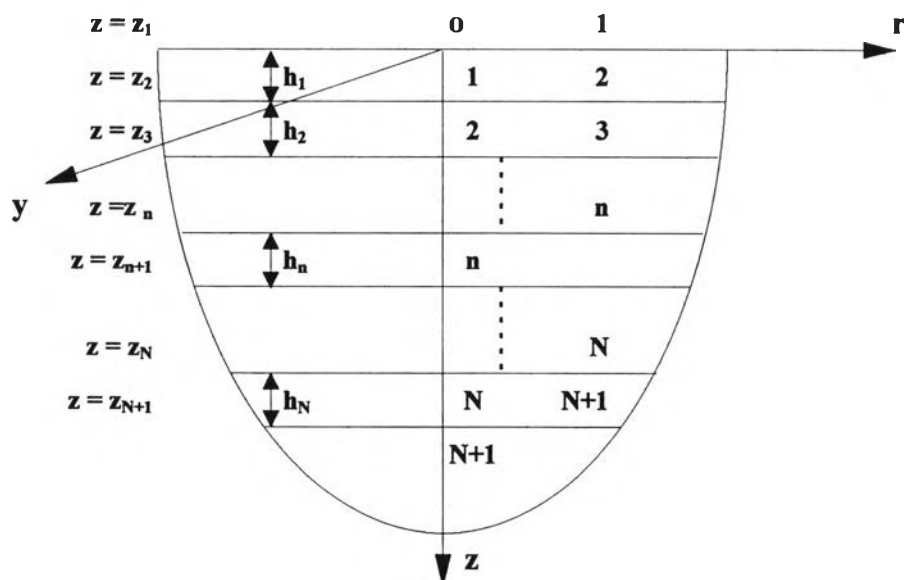


Figure 2 Geometry of multilayered half-space

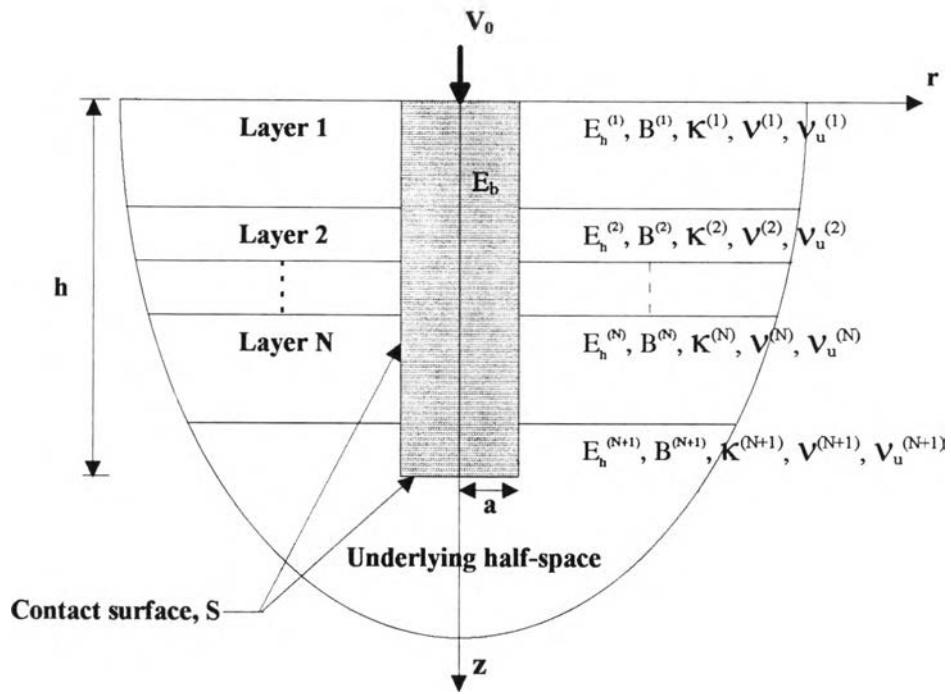


Figure 3 Geometry of bar-multilayered medium system

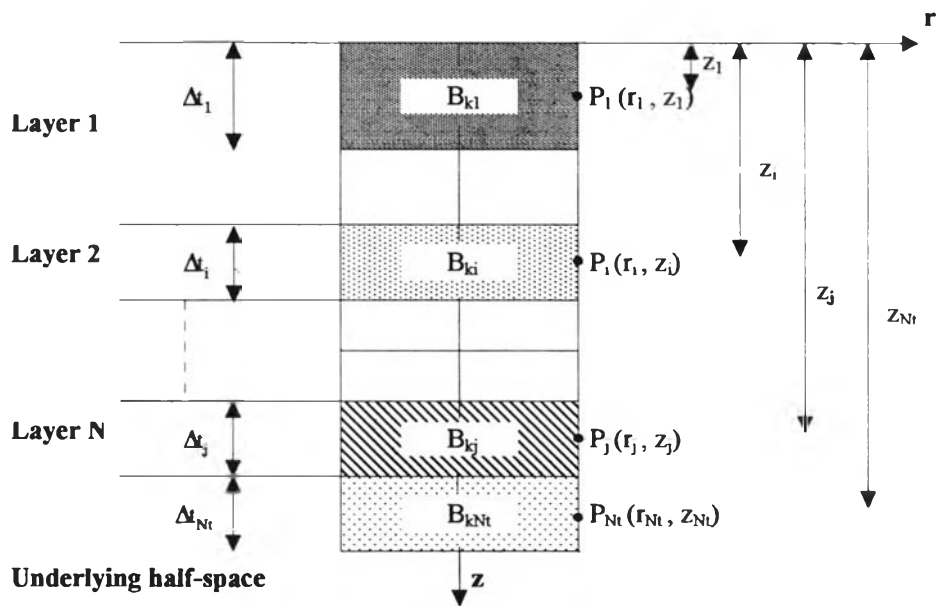


Figure 4 Basic elements used in discretizing multilayered medium

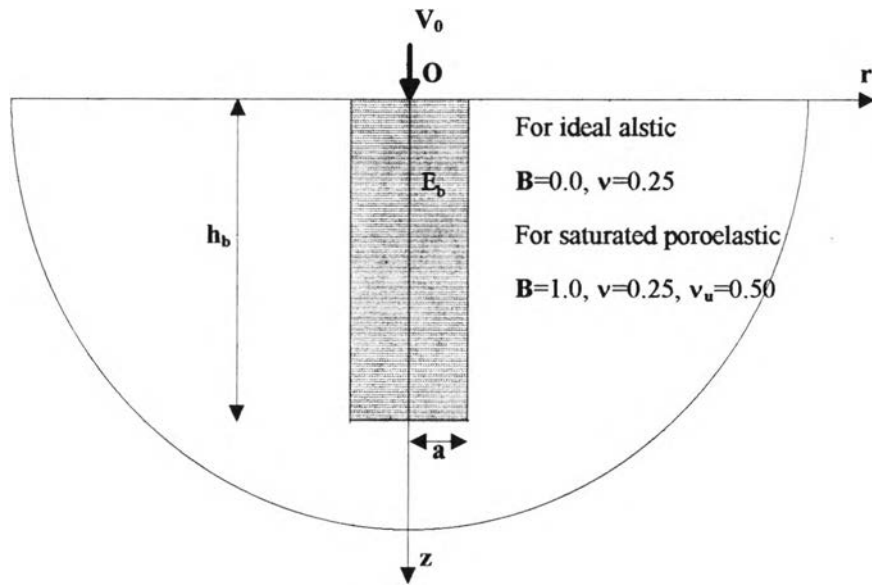


Figure 5 Bar-homogeneous medium system considered in this study

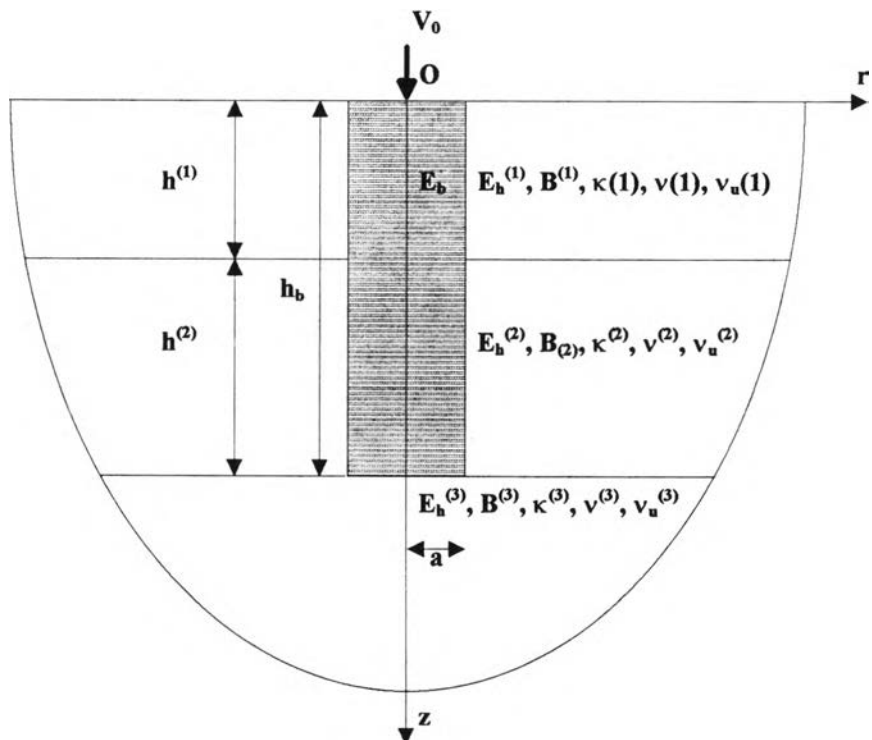


Figure 6 Numerical representation of bar-multilayered system

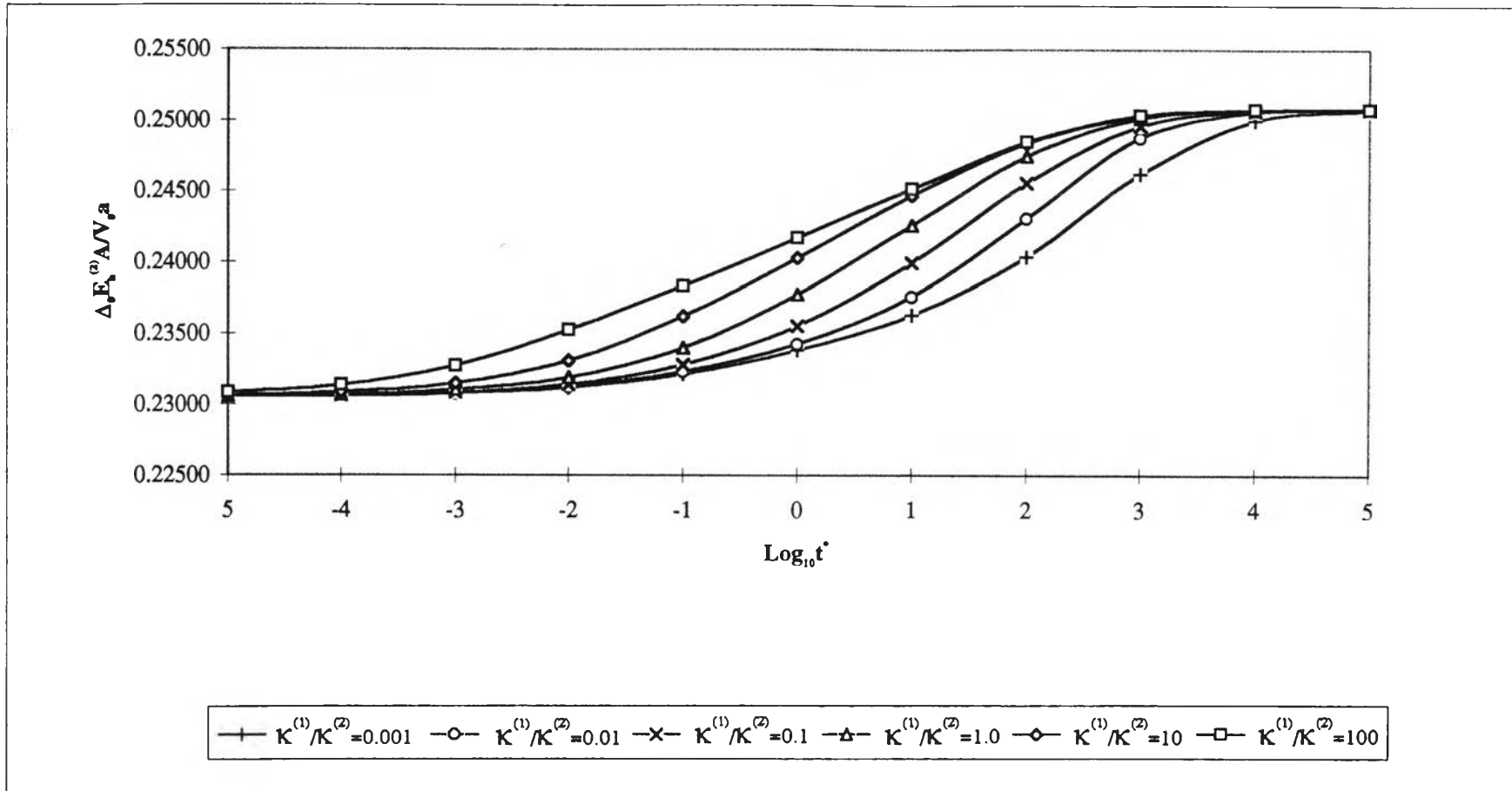


FIGURE 7 Nondimensionalized bar displacement for different $\kappa^{(1)}/\kappa^{(2)}$

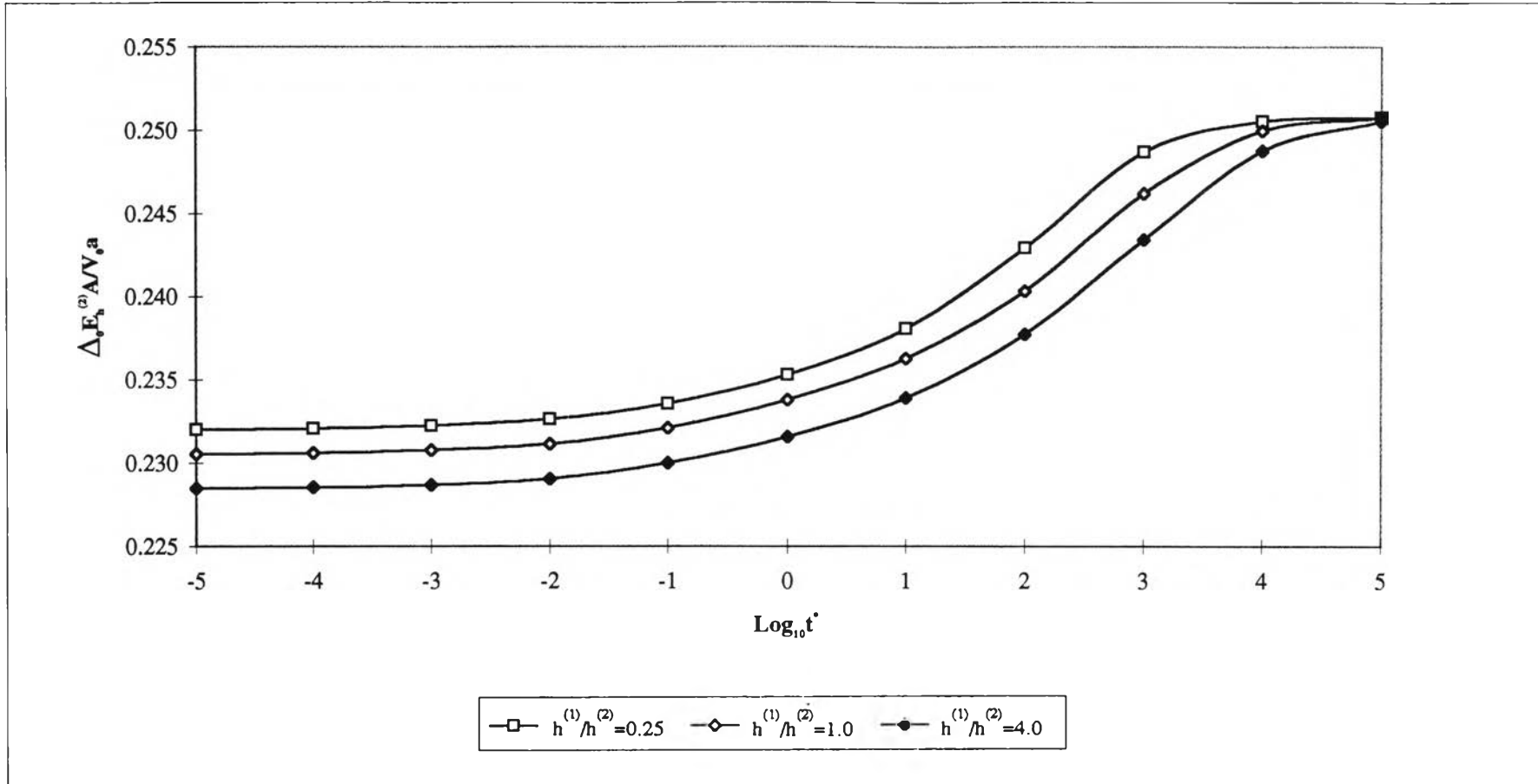


FIGURE 8 Nondimensionalized bar displacement for different $h^{(1)}/h^{(2)}$

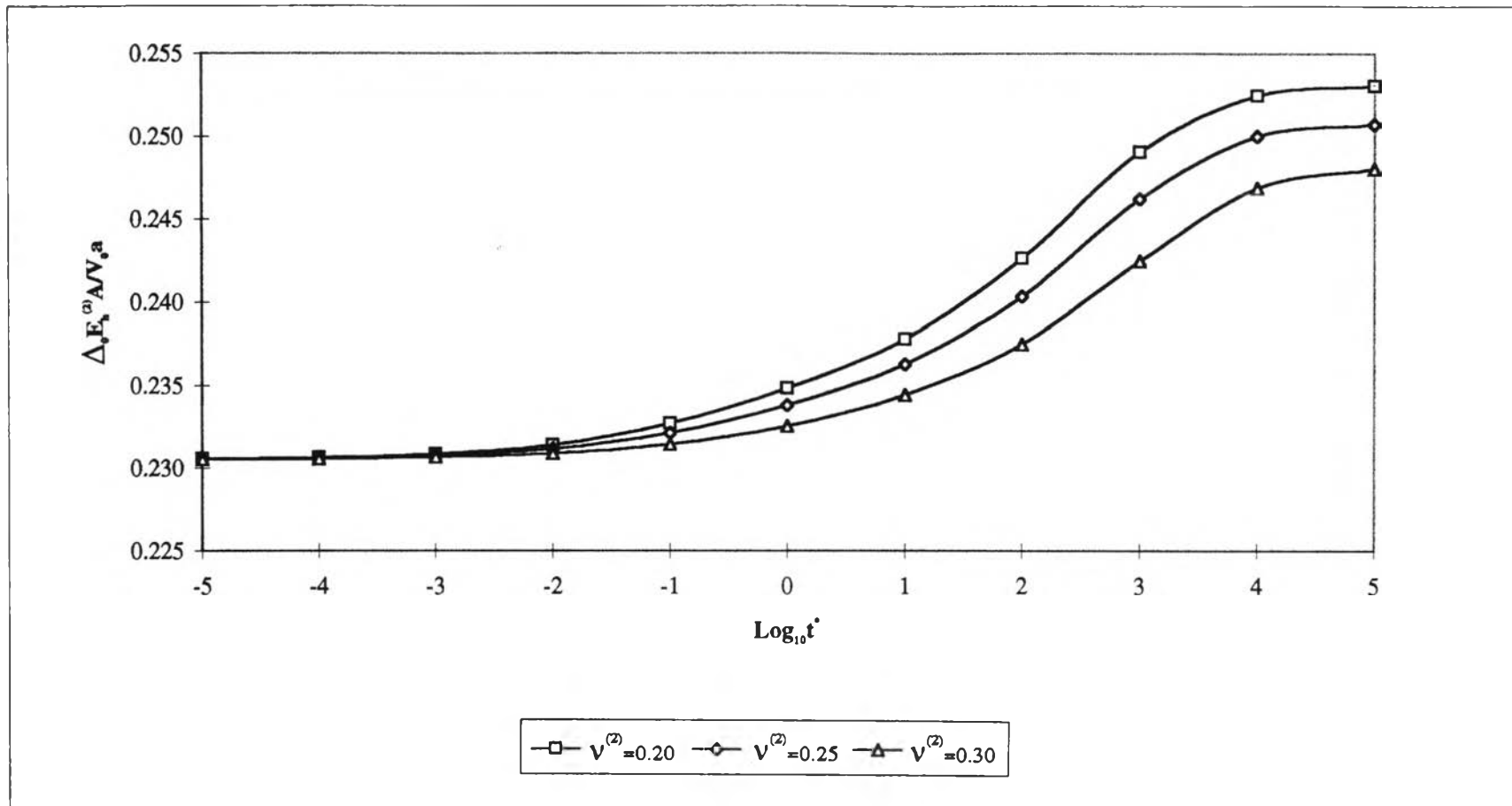


FIGURE 9 Nondimensionalized bar displacement for different $\nu^{(2)}$

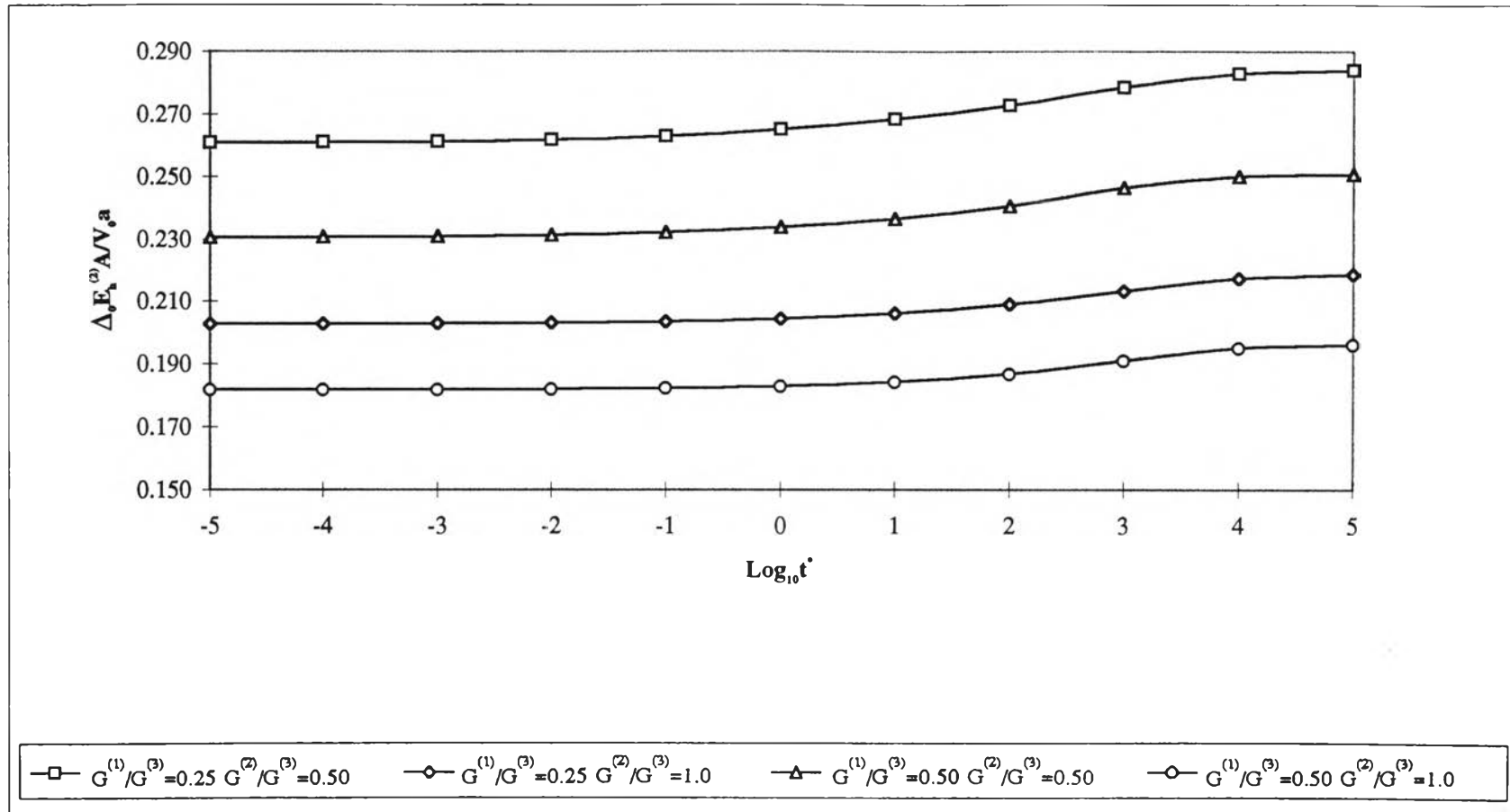


FIGURE 10 Nondimensionalized bar displacement for different shear moduli

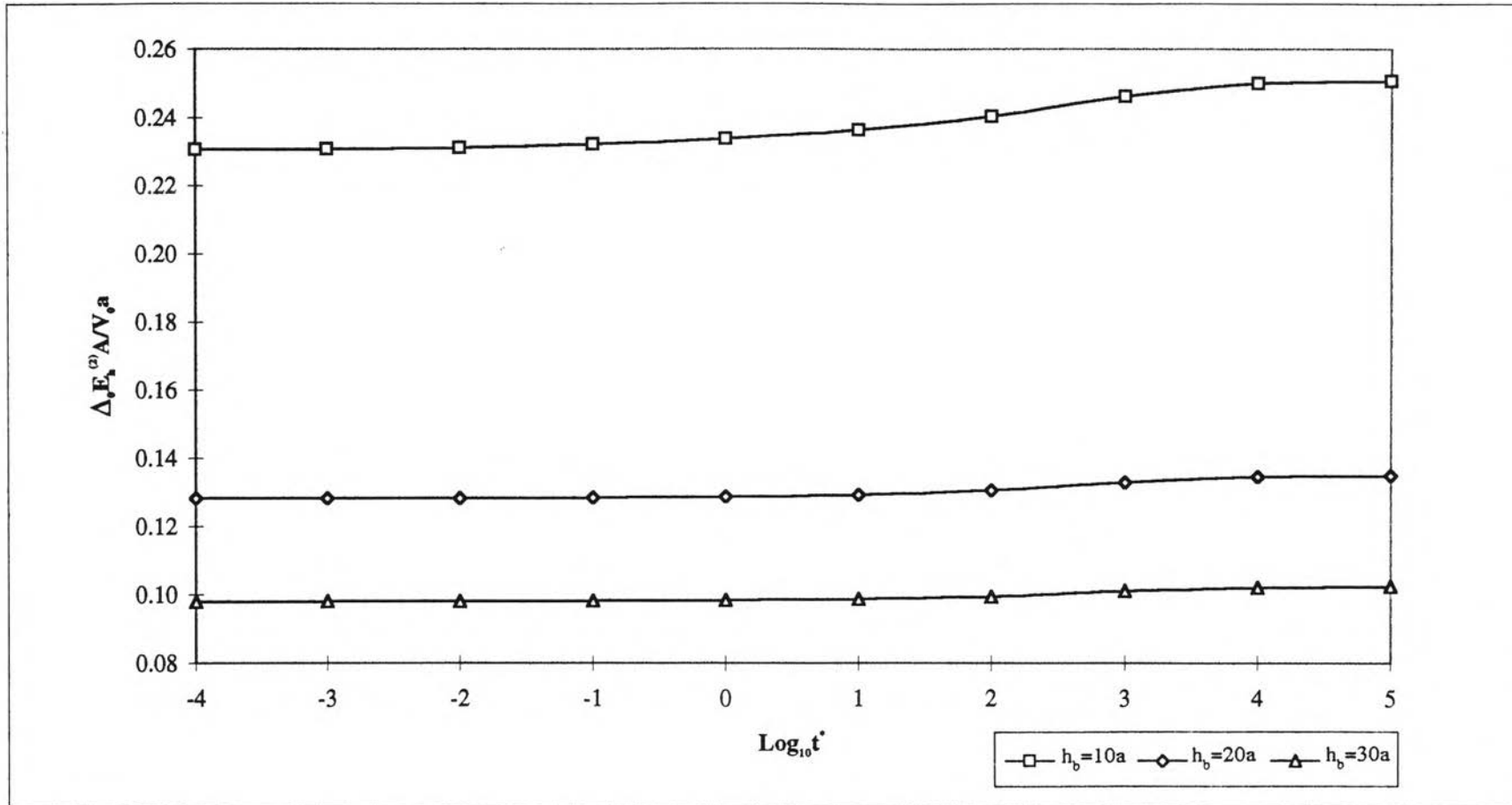


FIGURE 11 Nondimensionalized bar displacement for different bar length

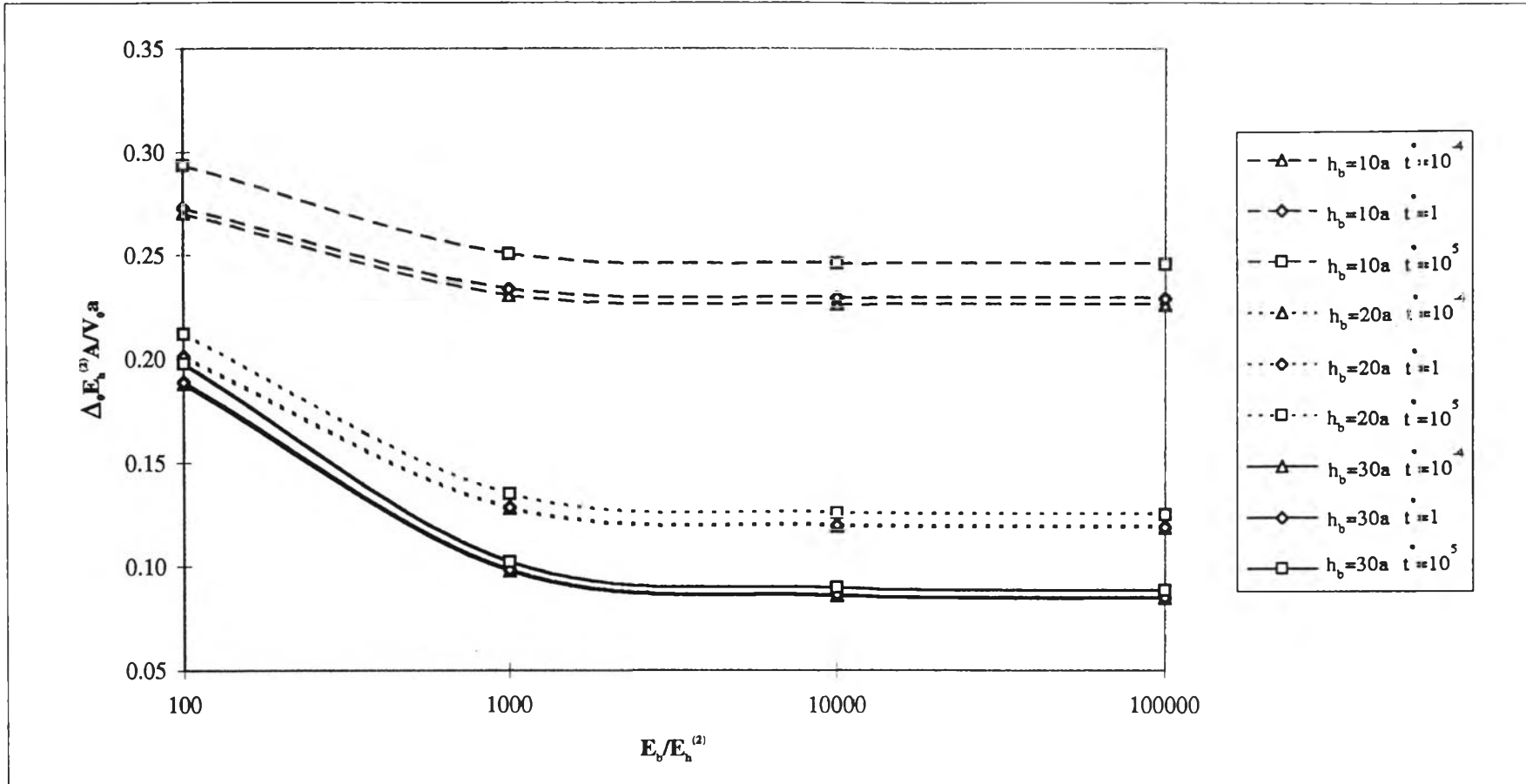
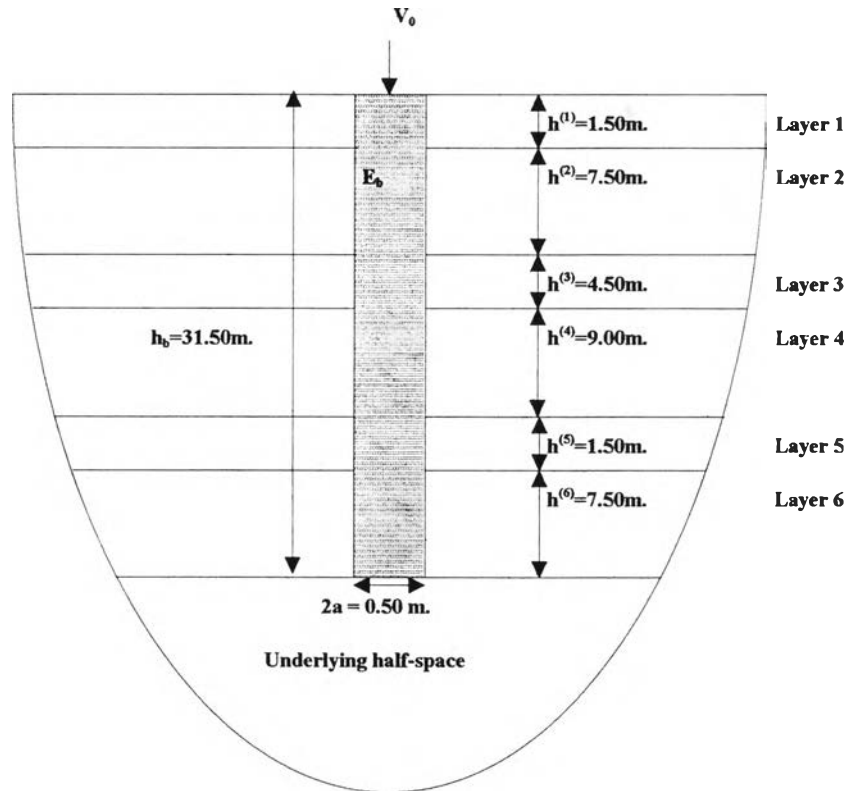


FIGURE 12 Nondimensionalized bar displacement for different $E_b/E_h^{(2)}$



Layer properties

- Layer 1 : $\nu^{(1)} = 0.15$, $\nu_u^{(1)} = 0.25$, $\mu^{(1)}/\mu^{(2)} = 0.54$, $\kappa^{(1)}/\kappa^{(2)} = 100$, $B^{(1)} = 0.50$
- Layer 2 : $\nu^{(2)} = 0.25$, $\nu_u^{(2)} = 0.45$, $\mu^{(2)} = 58 \text{ ksc.}$, $\kappa^{(2)} = 1 \times 10^{-11} \text{ m}^4/\text{kg-sec}$, $B^{(2)} = 1.00$
- Layer 3 : $\nu^{(3)} = 0.30$, $\nu_u^{(3)} = 0.45$, $\mu^{(3)}/\mu^{(2)} = 2$, $\kappa^{(3)}/\kappa^{(2)} = 1$, $B^{(3)} = 1.00$
- Layer 4 : $\nu^{(4)} = 0.35$, $\nu_u^{(4)} = 0.45$, $\mu^{(4)}/\mu^{(2)} = 4.63$, $\kappa^{(4)}/\kappa^{(2)} = 1$, $B^{(4)} = 1.00$
- Layer 5 : $\nu^{(5)} = 0.20$, $\nu_u^{(5)} = 0.45$, $\mu^{(5)}/\mu^{(2)} = 6.94$, $\kappa^{(5)}/\kappa^{(2)} = 50$, $B^{(5)} = 1.00$
- Layer 6 : $\nu^{(6)} = 0.35$, $\nu_u^{(6)} = 0.45$, $\mu^{(6)}/\mu^{(2)} = 7.71$, $\kappa^{(6)}/\kappa^{(2)} = 1$, $B^{(6)} = 1.00$
- Underlying half-space : $\nu^{(7)} = 0.20$, $\nu_u^{(7)} = 0.45$, $\mu^{(7)}/\mu^{(2)} = 10.42$, $\kappa^{(7)}/\kappa^{(2)} = 50$, $B^{(6)} = 1.00$
- $E_p/E_h^{(2)} = 1000$

FIGURE 13 Geometry of pile-soil system

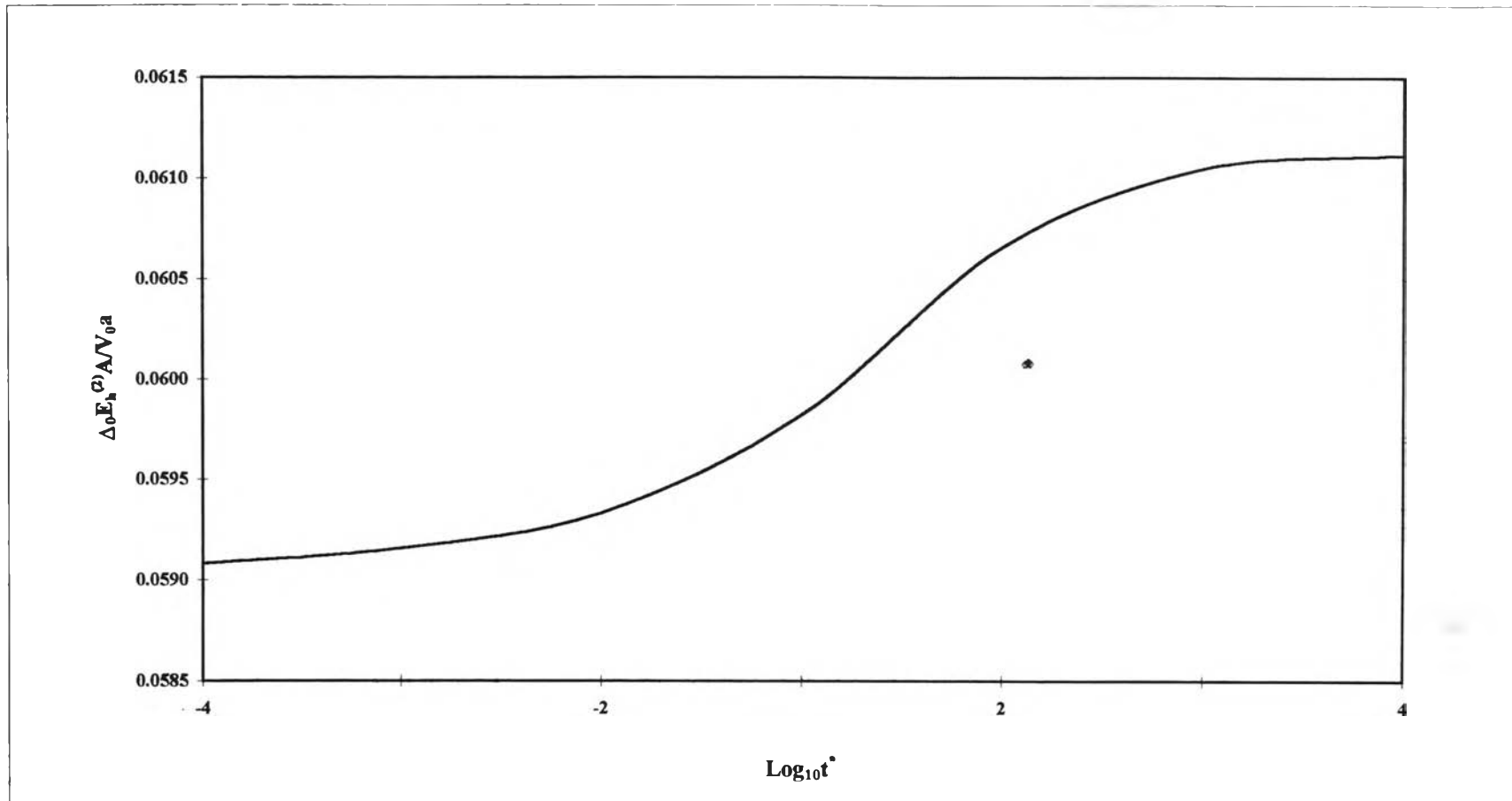


FIGURE 14 Nondimensionalized pile displacement of system in FIGURE 13

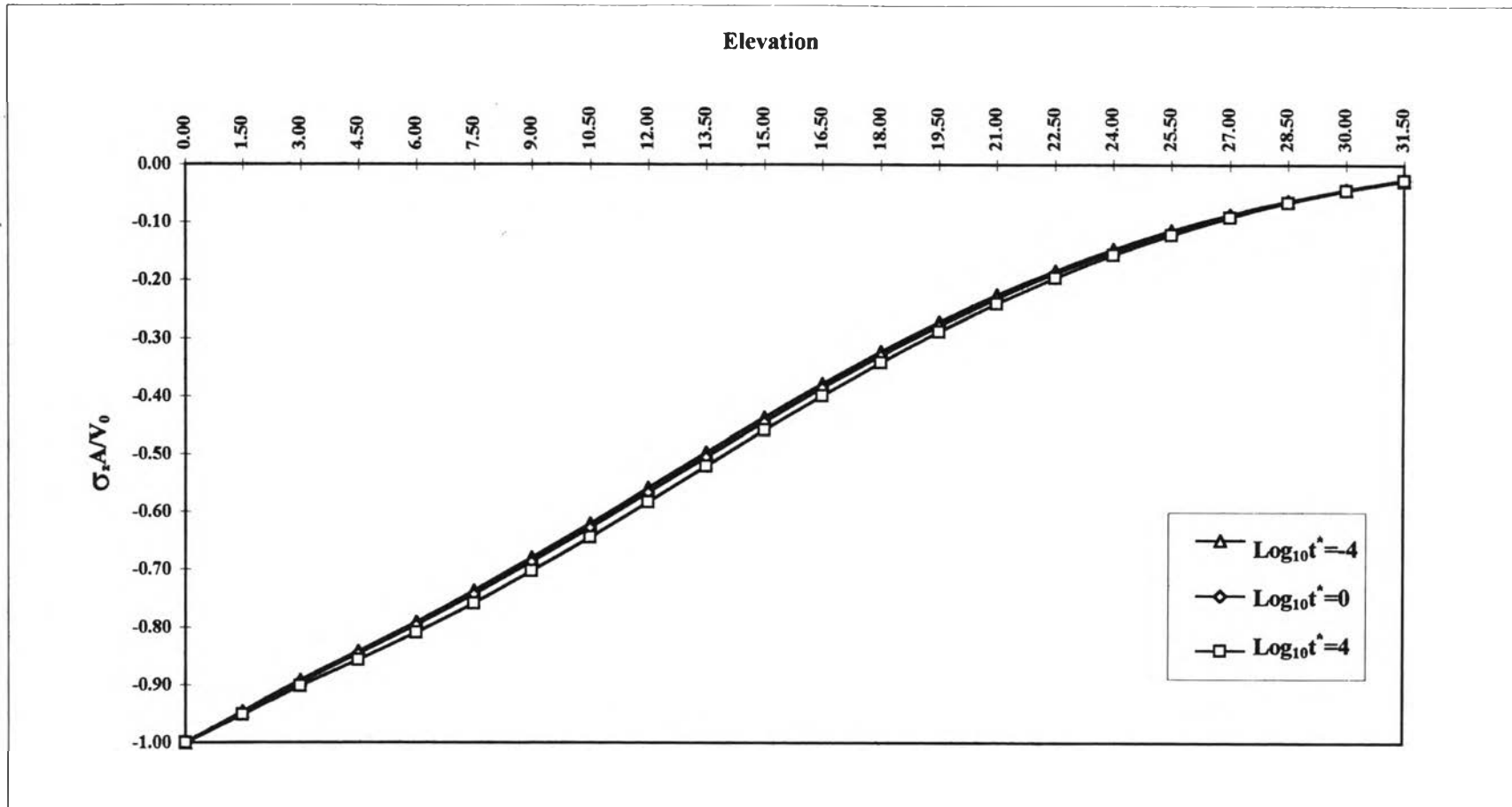


FIGURE 15 Nondimensionalized bar axial stress of system in FIGURE 13

APPENDICES

APPENDIX A

MATRICES $\mathbf{R}(\xi, z, s)$ AND $\mathbf{S}(\xi, z, s)$

The matrices $\mathbf{R}(\xi, z, s)$ and $\mathbf{S}(\xi, z, s)$ in eqns (2.18) and (2.19), respectively, are given by⁽⁹⁾.

$$\mathbf{R}(\xi, z, s) = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_1 \end{bmatrix} \quad (\text{A-1})$$

$$\mathbf{S}(\xi, z, s) = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_1 \end{bmatrix} \quad (\text{A-2})$$

where

$$\mathbf{R}_1 = \begin{bmatrix} -2\mu a_3 \eta \delta_2 e^{\gamma z} & -2\mu a_3 \eta \delta_2 e^{-\gamma z} & a_1 z e^{\xi z} & -a_1 z e^{-\xi z} \\ 0 & 0 & 0 & 0 \\ 2\mu a_3 \eta \delta_1 e^{\gamma z} & -2\mu a_3 \eta \delta_1 e^{-\gamma z} & -(a_1 z - \frac{a_2}{\xi}) e^{\xi z} & -(a_1 z + \frac{a_2}{\xi}) e^{-\xi z} \\ 2\mu a_3 \eta e^{\gamma z} & 2\mu a_3 \eta e^{-\gamma z} & -2\mu a_4 \eta e^{\xi z} & -2\mu a_4 \eta e^{-\xi z} \end{bmatrix} \quad (\text{A-3})$$

$$\mathbf{R}_2 = \frac{1}{2} \begin{bmatrix} e^{\xi z} & e^{-\xi z} & -e^{\xi z} & -e^{-\xi z} \\ e^{\xi z} & e^{-\xi z} & e^{\xi z} & e^{-\xi z} \\ -e^{\xi z} & e^{-\xi z} & e^{\xi z} & -e^{-\xi z} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A-4})$$

$$\mathbf{S}_1 = \mu \begin{bmatrix} -4\mu a_3 \xi \eta \delta_1 e^{\gamma z} & 4\mu a_3 \xi \eta \delta_1 e^{-\gamma z} & (2a_1 \xi z - 1) e^{\xi z} & (2a_1 \xi z + 1) e^{-\xi z} \\ 0 & 0 & 0 & 0 \\ 4\mu a_3 \xi \eta \delta_2 e^{\gamma z} & 4\mu a_3 \xi \eta \delta_2 e^{-\gamma z} & -2(a_1 \xi z - a_4) e^{\xi z} & 2(a_1 \xi z + a_4) e^{-\xi z} \\ -2a_3 \delta_1 e^{\gamma z} & 2a_3 \delta_1 e^{-\gamma z} & 2a_4 \delta_2 e^{\xi z} & -2a_4 \delta_2 e^{-\xi z} \end{bmatrix} \quad (\text{A-5})$$

$$S_2 = \frac{\mu}{2} \begin{bmatrix} 2\xi e^{\xi z} & -2\xi e^{-\xi z} & -2\xi e^{\xi z} & 2\xi e^{-\xi z} \\ \xi e^{\xi z} & -\xi e^{-\xi z} & \xi e^{\xi z} & -\xi e^{-\xi z} \\ -2\xi e^{\xi z} & -2\xi e^{-\xi z} & 2\xi e^{\xi z} & 2\xi e^{-\xi z} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (A-6)$$

where

$$a_1 = \frac{1}{2(1-2\nu_u)} \quad (A-7)$$

$$a_2 = \frac{(3-4\nu_u)}{2(1-2\nu_u)} \quad (A-8)$$

$$a_3 = \frac{B(1+\nu_u)(1-\nu)}{3(\nu_u-\nu)} \quad (A-9)$$

$$a_4 = \frac{(1-\nu_u)}{(1-2\nu_u)} \quad (A-10)$$

$$c = 2\mu a_3 \kappa \eta \quad (A-11)$$

$$\eta = \frac{B(1+\nu_u)}{3(1-\nu_u)} \quad (A-12)$$

$$\delta_1 = \frac{\kappa \gamma \eta}{s} \quad (A-13)$$

$$\delta_2 = \frac{\kappa \xi \eta}{s} \quad (A-14)$$

APPENDIX B

DERIVATION OF STRAIN ENERGY OF ELASTIC BAR
EMBEDDED IN MULTILAYERED POROELASTIC HALF-SPACE

The strain energy of an axially loaded elastic bar embedded in a multilayered poroelastic half-space in Laplace domain, as shown in Fig. 3, can be determined as follows:

The axial strain corresponding to eqn (2.47) can be expressed as

$$\bar{\varepsilon}(z,s) = \sum_{k=1}^K \frac{-(k-1)}{h_b} \bar{\alpha}_k e^{-(k-1)z/h_b} \quad (\text{B-1})$$

In view of the conventional constitutive relation the strain energy of an elastic bar can be written as

$$U_b = \int_{h_b} \int_{\lambda} \int_{\varepsilon} \bar{\sigma} \bar{\varepsilon} d\lambda dz = \int_0^{h_b} \frac{\pi a^2 E^* \bar{\varepsilon}^2}{2} dz \quad (\text{B-2})$$

Note that $E^* = E_b - E_h$. Since the half-space is multilayered, eqn (B-3) can be rewritten in view of eqn (B-2) as

$$\begin{aligned} U_b &= \frac{\pi a^2}{2h_b^2} \int_0^{h^{(1)}} E^{(1)*} \sum_{m=1}^K \sum_{n=1}^K (m-1)(n-1) \bar{\alpha}_m \bar{\alpha}_n e^{-(m+n-2)z/h_b} dz \quad (\text{B-3}) \\ &+ \frac{\pi a^2}{2h_b^2} \int_{h^{(1)}}^{h^{(1)}+h^{(2)}} E^{(2)*} \sum_{m=1}^K \sum_{n=1}^K (m-1)(n-1) \bar{\alpha}_m \bar{\alpha}_n e^{-(m+n-2)z/h_b} dz \\ &+ \dots \dots \dots \\ &+ \frac{\pi a^2}{2h_b^2} \int_{h^{(1)}+h^{(2)}+\dots+h^{(N-1)}}^{h^{(1)}+h^{(2)}+\dots+h^{(N)}} E^{(N)*} \sum_{m=1}^K \sum_{n=1}^K (m-1)(n-1) \bar{\alpha}_m \bar{\alpha}_n e^{-(m+n-2)z/h_b} dz \end{aligned}$$

By integrating eqn (B-4), the strain energy of an elastic bar embedded in a multilayered poroelastic half-space can be obtained in the following form

$$U_b = \sum_{m=1}^K \sum_{n=1}^K \bar{\alpha}_m \bar{\alpha}_n D_{mn} \quad (\text{B-4})$$

in which

$$D_{mn} = \frac{\pi a^2 (m-1)(n-1)}{2h_b (m+n-2)} \sum_{j=1}^{N_i} E^{(i)*} (e^{-(m+n-2)(z_i - \Delta t_i/2)} - e^{-(m+n-2)(z_i + \Delta t_i/2)}) \quad (\text{B-5})$$

where z_i and Δt_i denote the distance from the top of the bar to the middle of the i^{th} element and the thickness of the i^{th} element, respectively.



VITA

Mr. Jaruek Teerawong was born in Bangkok in 1968. He graduated from Faculty of Engineering, Chiang Mai University in 1990. He pursued his education for Master Degree in Civil Engineering in Chulalongkorn University in 1994.