Chapter 3 Introduction to Axiomatic Design Theory

3.1 Introduction

Axiomatic Design is a theory of the conceptual aspect of design process developed by Nam P. Suh [38], the Ralph E. & Eloise F. Cross Professor of Manufacturing in the Department of Mechanical Engineering at Massachusetts Institute of Technology, USA.

Suh began to develop this theory with two design axioms and the framework for design in the mid 1970's. However, his Axiomatic Design Theory was rapidly grown in 1985 while he was serving in the U.S. Government as assistant director for engineering of the National Science Foundation (NSF). It was later published in a book, *The Principle of Design* [38], in 1990.

For a long time, the science of designing has been traditionally thought that it can only be taught through experience and required much creativity. Axiomatic Design was developed to provide the scientific basis for design. It is based on the abstraction of the good design decisions and processes which categorized into two axioms, "Independence Axiom" and "Information Axiom".

This chapter will introduce the basic concepts of Axiomatic Design which will be used as a basis for developing the classification methodology of the intelligent manufacturing systems.

3.2 Design and Design Processes

The design world of the axiomatic approach is made up of four domains as illustrated in Figure 3.1 : Customer Domain, Functional Domain, Physical Domain, and Process Domain.



Figure 3.1 : Domains of Design World

	Customer Domain {CAs}	Functional Domain {FRs}	Physical Domain {DPs}	Process Domain {PVs}
Manufacturing	Attributes which consumers desire	Functional requirements specified for the product	Physical variables which can satisfy the functional requirements	Process variables that can control design parameters (DPs)
Materials	Desired performance	Required properties	Microstructure	Processes
Software	Attributes desired in the software	Output	Input variables or algorithms	Subroutines
Organization	Customer satisfaction	Functions of the organization	Programs or offices	People and other resources that can support the programs
Systems	Attributes desired of the overall system	Functional requirements of the system	Machines or components, sub- components	Resources (human, financial, materials, etc.)

Table 3.1 Four Domains of the Design World of Various Fields

The axiomatic design defines the design as the creation of synthesized solutions in the form of products, processes or systems that satisfy perceived needs through the *mapping* between the **Functional Requirements (FRs)** in the functional domain and the **Design Parameters (DPs)** of the physical domain. Table 3.1 shows four domains of the design world of various fields.

In the case of process design, the DPs in the physical domain are mapped into the process domain in terms of the **Process Variables (PVs)** which defined as the parameters and quantities controlling the manufacturing process.



Figure 3.2 : The Design Process according to Wilson

Figure 3.2 illustrates the design process simplified by Wilson [7], [38]. It shows that the design process begins with the recognition of a societal need. The need is formalized, resulting in a set of FRs. The selection of FRs, which defines the design problem, is left to the designer. Once the need is formalized, ideas are generated to

create a product (process, large system, concurrent engineering, or any other applications). This product is then analyzed and compared with the original set of FRs through a feedback loop. When the product does not fully satisfy the specified FRs, then one must either come up with a new idea, or change the FRs to reflect the original need more accurately. This iterative process continues until the designer produces an acceptable result.

3.3 Zigzagging

The design process progresses from a system level to levels of more detail which may be represented in terms of a design hierarchy (see Figure 3.3). The decisions which are made at higher levels affect the statement of the problem at lower levels by zigzagging. This process enables designers to decompose a given design problem into different domains : functional, physical, and process domain.



Figure 3.3 : Design Hierarchy and Zigzagging

At a given level of the design objects, there exists a set of functional requirements. Before these FRs can be decomposed, the corresponding design parameters must be selected. Once a functional requirement is satisfied by the corresponding design parameters, that FR can be decomposed into a set of subrequirements, and the process is repeated.

3.4 Hierarchy of FRs and DPs

There are two very important facts about design and design process, which should be recognized by all designers :

- 1. FRs and DPs have hierarchy, and they can be decomposed.
- 2. FRs at the ith level cannot be decomposed into the next level of the FR hierarchy without first going over to the physical domain and developing a solution that satisfies the ith level FRs with all the corresponding DPs. That is, we have to travel back and forth between the functional domain and the physical domain in developing the FR and DP hierarchies.

Figure 3.4 shows an example of functional and physical hierarchy of a lathe [38]. With this figure, it is clear that the entire FR hierarchy can not be constructed without referring to the DP hierarchy at each corresponding level. For example, without having decided to use a tailstock, these FRs should not be mentioned : tool holder, positioner, and support structure.

Remark : FRs are defined to be the minimum set of independent requirements that completely characterize the design objective for a specific need. FRs must be independent of other FRs, and thus can be stated without considering other FRs.



Lathe Funtional Hierarchy

Lathe System Physical Hierarchy

Figure 3.4 Lathe Functional and Physical Hierarchy

3.5 Design Axioms

The design axioms [38] are basic principles that can be used to develop many specific methodologies and problem solving techniques. They provide a tool for analysis, particularly during conceptual design. The two Design Axioms are stated as follows :

aintain the independence of FRs.
ne Information Axiom
nimize the information content of the design.

The first axiom, the Independence Axiom, states that the independence of functional requirements (FRs) must always be maintained. It can be restated as the following alternate statement :

- Alternate Statement 1 An optimal design always maintains the independence of FRs.
- Alternate Statement 2 In an acceptable design, the DPs and the FRs are related in such a way that specific DP can be adjusted to satisfy its corresponding FR without affecting other functional requirement.

The second axiom, The Information Axiom, states that, among those designs that satisfy the independence axiom, the design with the highest probability of functional success is the best design. This axiom can be restates as follows : Alternate StatementThe best design is a functionally uncoupled design that hasminimum information content.

3.6 Mathematical Representation

In the Axiomatic Design approach, "Design" is defined as the mapping process between the FRs in the functional domain and the DPs in the physical domain. At a given level of design hierarchy, the set of independence FRs that define the specific design goals constitutes a vector $\{FRs\}$ with *m* components. Similarly, the set of DPs in the physical domain also constitutes a vector $\{DPs\}$ with *n* components. The relationship between these two vectors can be written as follows :

$${FR}_{m\times 1} = [A]_{m\times n} {DP}_{n\times 1}$$
 (3.1)

where **{FR}** is the functional requirement vector, **{DP}** is the design parameter vector, and **[A]** is the design matrix. Equation 3.1 is called "**The Design Equation**" for the product and can be written as

$$FR_{i} = \sum_{j} A_{ij} DP_{j}$$
 (3.1a)

The left-handed side of the design equation represents "What we want to achieve ?", and the right-handed side of the equation represents "How we propose to satisfy a requirement specified in the left-handed side ?"

The design matrix **[A]** is of the form

$$[\mathbf{A}] = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & & \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix}$$
(3.2)

The element A_{ij} can be expressed as

$$A_{ij} = \frac{\partial FR_i}{\partial DP_j}$$
(3.3)

 A_{ij} must be evaluated at a specific design point in the physical space unless A_{ij} is a constant. In a nonlinear case, A_{ij} varies with both FR_i and DP_j.

In process design, a similar relationship exists between the design parameter vector, {DPs}, of the physical domain and process variable vector, {PVs}, of the process domain. The design equation for a process may be written as

$$\{DP\} = [B]\{PV\}$$
 (3.4)

where [B] is the design matrix that characterize the process design.

When m = n, [A] is a square matrix. For example, when m = n = 3; {FR}, {DP}, and [A] can be written as

$$\{FR\} = \begin{cases} FR_1 \\ FR_2 \\ FR_3 \end{bmatrix}$$

$$\{DP\} = \begin{cases} DP_1 \\ DP_2 \\ DP_3 \end{cases}$$

$$(3.5)$$

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

and the Design Equation 3.1 can be written as

$$\begin{bmatrix} FR_1 \\ FR_2 \\ FR_3 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{bmatrix}$$
(3.6)

The most simplest case of design occurs when [A] becomes a diagonal matrix, that is

$$A_{ij} \neq 0$$
 when $i = j$
 $A_{ij} = 0$ when $i \neq j$

or $A_{12} = A_{13} = A_{21} = A_{23} = A_{31} = A_{32} = 0$ and A_{11} , A_{22} , $A_{33} \neq 0$. That is :

$$[\mathbf{A}] = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix} \text{ where } X \neq 0 \quad (3.7)$$

Then, Equation 3.6 can be written as

$$FR_1 = A_{11}DP_1$$

 $FR_2 = A_{22}DP_2$ (3.8)
 $FR_3 = A_{33}DP_3$

A design that can be represented by Equation 3.8 satisfies Axiom 1, since the independence of FRs is assured when each DP is change. That is, FR₁ can be satisfied by simply changing DP₁, and similarly FR₂ and FR₃ can be changed independently without affecting any other FRs by varying DP₂ and DP₃, respectively. This kind of design is defined as an *Uncoupled Design*.

The opposite design of an uncoupled design is the *Coupled Design*, whose design matrix consists of mostly nonzero elements. For example, the 3×3 design matrix can be written as

$$FR_{1} = A_{11}DP_{1} + A_{12}DP_{2} + A_{13}DP_{3}$$

$$FR_{2} = A_{21}DP_{1} + A_{22}DP_{2} + A_{23}DP_{3}$$

$$FR_{3} = A_{31}DP_{1} + A_{32}DP_{2} + A_{33}DP_{3}$$
(3.9)

Equation 3.9 states that a change in FR_1 cannot be accomplished by simply changing DP_1 , since this will also affect FR_2 and FR_3 . Such a design clearly violates Axiom 1.

A special case of Equation 3.6 where the design matrix is triangular can be represented as

$$\begin{cases} FR_1 \\ FR_2 \\ FR_3 \end{cases} = \begin{bmatrix} A_{11} & 0 & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{bmatrix}$$
(3.10)

The relationship between FRs and DPs is

$$FR_{1} = A_{11}DP_{1}$$

$$FR_{2} = A_{21}DP_{1} + A_{22}DP_{2}$$

$$FR_{3} = A_{31}DP_{1} + A_{32}DP_{2} + A_{33}DP_{3}$$
(3.11)

In this case, the independence of the FRs can be assured if we adjust the DPs in a particular order. If we vary DP₁ first, then the value of FR₁ can be set. Although it also affects FR₂ and FR₃, we can then change DP₂ to set the value of FR₂, without affecting FR₁. Finally, DP₃ can be changed to control DP₃ without affecting FR₁ and FR₂. If we had reversed the order and change DP₃ first to set FR₃, and then DP₂ to set the value of FR₂, the value of FR₃ would have changed while changing DP₂. This kind of system is called a *Decoupled or Quasi-Coupled Design*.

3.7 Quantitative Measure for Functional Independence

Consider the two-dimensional design equation which can be written as

$$\begin{cases} FR_1 \\ FR_2 \end{cases} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{cases} DP_1 \\ DP_2 \end{cases} = \begin{bmatrix} A_{11} \\ A_{21} \end{bmatrix} DP_1 + \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} DP_2$$
 (3.12)

$$\mathbf{FR} = \mathbf{C}_1 \mathbf{DP}_1 + \mathbf{C}_2 \mathbf{DP}_2 = \mathbf{FR}_1 \mathbf{i} + \mathbf{FR}_2 \mathbf{j}$$
(3.13)

where the vectors $\mathbf{C}_{\mathbf{k}}$ are

$$\mathbf{C_1} = \begin{cases} \mathbf{A_{11}} \\ \mathbf{A_{21}} \end{cases} \quad \text{and} \quad \mathbf{C_2} = \begin{cases} \mathbf{A_{12}} \\ \mathbf{A_{22}} \end{cases}$$

Equation 3.13 can be graphically represented in the functional space as in Figure 3.5 which is called **"DP isogram"**.



Figure 3.5 Graphical representation of Equation 3.13

Taking the dot product, the angular relationship, θ , between C_1 and C_2 can be obtained as

$$\cos \theta = \frac{C_1 \cdot C_2}{|C_1| \cdot |C_2|}$$
(3.14)

There are two important qualitative measures for functional independence, "Reangularity, R" and "Semangularity, S". Reangularity is defined as $R = \sin \theta$ which can be expressed as

$$R = \sin \theta = \sqrt{1 - \cos^2 \theta} \qquad (3.15)$$

*** For the two-dimensional case which has the design equation as equation 3.12,

$$\begin{cases} FR_1 \\ FR_2 \end{cases} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{cases} DP_1 \\ DP_2 \end{cases} = \begin{bmatrix} A_{11} \\ A_{21} \end{bmatrix} DP_1 + \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} DP_2$$

Equation 3.15 can be written as

R =
$$\sqrt{1 - \frac{(A_{11}A_{12} + A_{21}A_{22})^2}{(A_{11}^2 + A_{21}^2)(A_{12}^2 + A_{22}^2)}}$$
 (3.16)

*** For the three-dimensional case which has the design equation as

FR ₁		A ₁₁	A ₁₂	A ₁₃	$\left[DP_{l} \right]$	
FR ₂	=	A ₂₁	A ₂₂	A ₂₃	DP ₂	ł
FR ₃		A_{31}	A ₃₂	A 33	DP ₃	

Equation 3.15 can be written as

$$R = \sqrt{1 - \frac{(A_{11}A_{12} + A_{21}A_{22} + A_{31}A_{32})^2}{(A_{11}^2 + A_{21}^2 + A_{31}^2)(A_{12}^2 + A_{22}^2 + A_{32}^2)}} \cdot \sqrt{1 - \frac{(A_{11}A_{13} + A_{21}A_{23} + A_{31}A_{33})^2}{(A_{11}^2 + A_{21}^2 + A_{31}^2)(A_{13}^2 + A_{23}^2 + A_{33}^2)}} \cdot \sqrt{1 - \frac{(A_{12}A_{13} + A_{22}A_{23} + A_{32}A_{33})^2}{(A_{12}^2 + A_{22}^2 + A_{32}^2)(A_{13}^2 + A_{23}^2 + A_{33}^2)}} \cdot \sqrt{1 - \frac{(A_{12}A_{13} + A_{22}A_{23} + A_{32}A_{33})^2}{(A_{12}^2 + A_{22}^2 + A_{32}^2)(A_{13}^2 + A_{23}^2 + A_{33}^2)}} \cdot (3.17)}$$

Notice that Reangularity, R, in Equation (3.17) is the product of the sines of all

the angles between pairs of DP isogram :
$$\begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix} DP_1, \begin{bmatrix} A_{12} \\ A_{22} \\ A_{32} \end{bmatrix} DP_2, \text{ and } \begin{bmatrix} A_{13} \\ A_{23} \\ A_{33} \end{bmatrix} DP_3$$

From Equation 3.16 and 3.17, it can be deducted that, for the n-dimensional case, Equation 3.15 can be written as

$$R = \prod_{\substack{i=1,n-1\\j=1+i,n}} \sqrt{1 - \frac{(\sum_{k=1}^{n} A_{ki} A_{kj})^2}{(\sum_{k=1}^{n} A_{ki}^2)(\sum_{k=1}^{n} A_{kj}^2)}}$$
(3.18)

which is the product of the sines of all the angles between each pairs of DP isogram in the functional space.

The maximum value of Reangularity, R, occurs when the isograms are mutually orthogonal which indicates that each DP is independent on each DPs. The limit of R is zero happened when two or more of the isograms are parallel which, in turn, is the coupled design. When the degree of coupling increases, the value of R decreases. It is clear that Reangularity, R, is used for measuring the orthogonality between the DPs.

To characterize the functional independence, not only the value of R, which measures only the orthogonality between the DPs, can definitely measured since it is very important to know the angular relationship between the corresponding axes of DPs and FRs. Therefore, Semangularity (S) which is required to characterize the functional independence among pairs of DP and FR was defined as

S =
$$\prod_{j=1}^{n} (\frac{|A_{jj}|}{\sqrt{\sum_{k=1}^{n} A_{kj}^{2}}})$$
 (3.19)

The value of Semangularity (derived from Latin words meaning "Same angle quality"), S, converge to unity when all off-diagonal elements are zero. It means that the DP isograms are parallel to the coordinates in the functional space, and the FR

isograms are parallel to the coordinates of the design space. Such design is an Uncoupled Design.

At this state, it is obvious that Semangularity and Reangularity are useful measures for determining the degree of coulping between FRs due to the particular set of DPs chosen as a result of the design process.

3.8 The Information Axiom

The information axiom [38], Axiom 2, is the axiom which deal with the minimization of a parameter called "Information Content". It states that among all designs that satisfy Axiom 1, the one that possesses the least information is the best.

According to the Axiomatic Design concept, the information can be in the form of drawings, equations, specifications, operational instructions, software, etc. It is the measure of knowledge required to satisfy a given FR at a given level of the FR hierarchy. The idea of information is related closely to the probability of achieving the FR. However, Suh[38] stated that the exact form of the definition of information content is not important, as long as it is an accurate predictor of relative complexity, and it is defined with a consistent definition.

In order to develop a proper quantitative measure for information content of a design, there are some important viewpoints as follows.

 All values of information content, whether they are associated with the same or different attributes (length, hardness, cost, etc.) are comparable as long as the underlying probabilities are the same. Hence, they can be added or substracted without regard to the original units that define the probabilities of success, since all of these probabilities are directly related to the success of achieving a given design task.

- 2. The information content is equal to the complexity of the task involved. As the complexity of a task increases, so the probability of success decreases.
- 3. The information content of a message is the minimum information required to satisfy an FR within specified tolerances.
- 4. The reduction in uncertainty due to a message is related to the ratio of prior and posterior probabilities due to the message.

From the information theory developed by Shannon and Weaver[35], in terms of probability, p, Information Content is defined as

$$I = \log_2(\frac{1}{p})$$
 (3.20)

where p is the probability of success of a given task as expressed in form of FRs. The base of the logarithm is taken to be 2 so that the information content has the unit of "bits". However, when the natural logarithm is used, the unit for information is "nats". These units are equivalent and 1 bit = 1.443 nats. That is

I =
$$K \ln(\frac{1}{p})$$
 = $\log_2(\frac{1}{p})$ (3.21)

where K = 1.443.

In a design situation, Information Content can also be defined to be

$$I = \log_2(\frac{\text{range}}{\text{tolerance}})$$
 (3.22)

where range is the design range and tolerance is the design tolerance.

3.9 Additional Theorems and Corollaries

The followings are some of the Theorems and Corollaries developed and implied from the Axiomatic Design Theory described in previous sections.

3.9.1 Theorems

Theorem 1 (Coupling Due to Insufficient Number of DPs)

When the number of DPs is less than the number of FRs, either a coupled design results or the FRs cannot be satisfied.

Theorem 2 (Decoupling of Coupled Design)

When a design is coupled due to the greater number of FRs than DPs (i.e., m > n), it may be decoupled by the addition of new DPs so as to make the number of FRs and DPs equal to each other, if a subset of the design matrix containing n×n elements constitutes a triangular matrix.

Theorem 3 (Redundant Design)
 When there are more DPs than FRs, the design is either a redundant design or a coupled design.
 Theorem 4 (Ideal Design)

In an ideal design, the number of DPs is equal to the number of FRs.

Theorem 5 (Need for New Design)

When a given set of FRs is changed by the addition of a new FR, or substitution of one of the FRs with a new one, or by selection of a completely different set of FRs, the design solution given by the original DPs cannot satisfy the new set of FRs. Consequently, a new design solution must be sought.

Theorem 6 (Path Independence of Uncoupled Design)

The information content of an uncoupled design is independent of the sequence by which the DPs are changed to satisfy the given set of FRs.

Theorem 7 (Path Dependence of Coupled and Uncoupled Designs)

The information contents of coupled and decoupled designs depend on the sequence by which the DPs are changed and on the specific paths of the changes of these DPs.

Theorem 8 (Independence and Tolerance)

A design is an uncoupled design when the designer-specified tolerance is greater than

$$\sum_{\substack{j\neq i\\j=1}}^{n} \frac{\partial FR_{i}}{\partial DP_{j}} \Delta DP_{i}$$

so that the nondiagonal elements of the design matrix can be neglected from design consideration.

Theorem 9 (Design for Manufacturability)

For a product to be manufacturable, the design matrix for the product, **[A]** (which relates the **FR** vector for the product to the **DP** vector of the product) times the design matrix for the manufacturing process, **[B]** (which relates the **DP** vector to the **PV** vector of the manufacturing process) must yield either a diagonal or triangular matrix. Consequently, when any one of these design matrices, that is, either **[A]** or **[B]**, represents a coupled design, the product cannot be manufactured.

Theorem 10(Modularity of Independence Measures)

Suppose that a design matrix **[DM]** can be partitioned into square submatrices that are nonzero only along the main diagonal. Then the reangularity and semangularity for **[DM]** are equal to the products of the corresponding measures for each of the submatrices.

Theorem 11 (Invariance)

Reangularity and Semangularity for a design matrix are invariant under alternative orderings of the FR and DP variables, as long as the orderings preserve the association of each FR with its corresponding DP.

Theorem 12 (Sum of Information)

The sum of information for a set of events is also information, provided that proper conditional probabilities are used when the events are not statistically independent.

Theorem 13 (Information Content of the Total System)

If each DP is probabilistically independent of other DPs, the information content of the total system is the sum of the information of all individual events associated with the set of FRs that must be satisfied.

Theorem 14 (Information Content of Coupled vs. Uncoupled Designs)

When the state of FRs is changed from one state to another in the functional domain, the information required for the change is greater for a coupled process than for an uncoupled process.

3.9.2 Corollaries

Corollary 1 (Decoupling of Coupled Designs)

Decouple or separate parts or aspects of a solution if FRs are coupled or become interdependent in the designs proposed.

Corollary 2 (Minimization of FRs)

Minimize the number of FRs and constraints.

(Integration of Physical Parts) **Corollary 3** Integrate design features in a single physical part if FRs can be independently satisfied in the proposed solution. **Corollary 4** (Use of Standardization) Use standardized or interchangeable parts if the use of these parts is consistent with FRs and constraints. **Corollary 5** (Use of Symmetry) Use symmetrical shapes and/or components if they are consistent with the FRs and constraints. **Corollary 6** (Largest Tolerance) Specify the largest allowable tolerance in stating FRs. **Corollary** 7 (Uncoupled Design with Less Information) Seek an uncoupled design that requires less information than coupled designs in satisfying a set of FRs.

Corollary 8 (Effective Reangularity for a Scalar) The effective reangularity R for a scalar design "matrix" or element is unity.

3.10 Conclusion

Axiomatic Design is a theory of the conceptual aspect of design process proposed by Suh [38]. It was developed to provide the scientific basis for the design process based on the two axioms, "*Independence Axiom*" and "*Information Axiom*".

The axiomatic design defines the design as the creation of synthesized solutions in the form of products, processes or systems that satisfy perceived needs through the *mapping* between the Functional Requirements (FRs) in the functional domain and the Design Parameters (DPs) of the physical domain. In the case of process design, the DPs in the physical domain are mapped into the process domain in terms of the Process Variables (PVs) which defined as the parameters and quantities controlling the manufacturing process.

At a given level of design hierarchy, the set of independence FRs that define the specific design goals constitutes a vector $\{FRs\}$ with *m* components whereas the set of DPs in the physical domain also constitutes a vector $\{DPs\}$ with *n* components. The relationship between these two vectors can be written as

$${FR}_{m\times 1} = [A]_{m\times n} {DP}_{n\times 1}$$
 or
 $FR_i = \sum_j A_{ij} DP_j$

while [A] is the design matrix of the form

$$[A] = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & & \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix}$$

In order to classify how good a design is, a design can be considered to be an Uncouple, Couple, or Decouple Design by its design matrix. The most preferable design is the Uncouple Design which its FRs can be changed independently without affecting any other FRs by varying DPs.

There are also the additional theorems and corollaries related to the Axiomatic Design theory presented in this chapter.