

Portfolio construction under group risk parity strategy
in the Stock Exchange of Thailand



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การจัดพอร์ตโฟลิโอภายใต้กลยุทธ์ความเสี่ยงสมดุลแบบกลุ่ม
ในตลาดหลักทรัพย์แห่งประเทศไทย



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โดยทั่วไปแล้วการจัดพอร์ตโฟลิโอภายใต้กลยุทธ์ความเสี่ยงสมดุลมักถูกใช้กับหลายกลุ่มสินทรัพย์ สำหรับพอร์ตโฟลิโอที่มีสินทรัพย์ประเภทเดียวนั้น กลยุทธ์ความเสี่ยงสมดุลแบบกลุ่มได้ถูกนำมาใช้ โดยความเสี่ยงที่เกิดจากแต่ละกลุ่มของสินทรัพย์นั้นถูกทำให้เท่ากัน การศึกษานี้แสดงประสิทธิภาพของการจัดพอร์ตโฟลิโอภายใต้กลยุทธ์ความเสี่ยงสมดุลแบบกลุ่มในตลาดหลักทรัพย์แห่งประเทศไทย โดยจำแนกการจัดกลุ่มเป็นสองรูปแบบได้แก่ประเภทของอุตสาหกรรมและขนาดของหลักทรัพย์ และเปรียบเทียบประสิทธิภาพที่ได้จากการจัดพอร์ตโฟลิโอโดยวิธีการอื่น ซึ่งได้แก่กลยุทธ์ความเสี่ยงสมดุลทั่วไป, ความผันผวนต่ำสุดและการกระจายน้ำหนักอย่างเท่าเทียม การศึกษาถูกแบ่งออกเป็นสองช่วงคือตั้งแต่ปีพ.ศ.๒๕๕๙ จนถึงปีพ.ศ.๒๕๖๓ และช่วงที่ตลาดหลักทรัพย์ปรับตัวลดลงพร้อมความผันผวนที่สูง ผลการศึกษาพบว่าไม่มีข้อบ่งชี้ที่ชัดเจนว่ากลยุทธ์ความเสี่ยงสมดุลแบบกลุ่มนั้นให้ผลลัพธ์ที่ดีกว่าแบบอื่นในช่วงปีพ.ศ.๒๕๕๙ จนถึงปีพ.ศ.๒๕๖๓ อย่างไรก็ตาม เป็นที่สังเกตได้ว่าในช่วงที่ตลาดปรับตัวลดลงพร้อมกับความผันผวนที่สูงนั้น กลยุทธ์ความเสี่ยงสมดุลแบบกลุ่มโดยประเภทของอุตสาหกรรมให้ผลลัพธ์ที่ดีกว่าในแง่ของผลตอบแทนรายปีและผลตอบแทนต่อหน่วยความเสี่ยงโดยสรุป เราเชื่อว่าการจัดพอร์ตโฟลิโอภายใต้กลยุทธ์ความเสี่ยงสมดุลแบบกลุ่มโดยประเภทอุตสาหกรรมให้ข้อได้เปรียบในแง่ของผลตอบแทนและความเสี่ยงเมื่อเปรียบเทียบกับกลยุทธ์อื่นภายใต้เงื่อนไขความผันผวนของดัชนีหลักทรัพย์ที่สูง

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Risk parity strategy has been commonly applied to construct a portfolio with multiple asset classes. For single asset class, group risk parity is introduced such that risks from each group are equally contributed. This study investigates and compares performances of group risk parity strategy in Thai market under two different grouping methods which are sector and size with non-group risk parity, minimum-variance, and equal-weight strategies. The study mainly focus on two periods which are during 2016-2020 and in years with highly volatile down market. The analysis indicates that there is no clear evidence that group risk parity strategy outperforms others during 2016-2020. However, in the period with high volatility and large negative movement of the market, it is noticeable that group risk parity strategy with grouping stocks by sector outperforms others in terms of annualized return and Sharpe ratio. In conclusion, we believe that grouping stocks by sector under group risk parity strategy provides an advantage in risk return compromisation among other strategies during the volatile market.

Field of Study:	Financial Engineering	Student's Signature
	
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1. Introduction

Quantitative strategy in portfolio management has been widely used and studied by investors and researchers for many decades. By using volatility as a measure of risk, the main objective of the strategy is to manage the tradeoff between risk and reward from different choices of investment.

In 1952, Harry Markowitz introduced his dissertation on "Portfolio Selection" or also known as "Modern Portfolio Theory" which has been considered to be a notable theory in portfolio selection and become a fundamental framework in portfolio construction. The main concept of this theory is to create a diversified portfolio that maximizes expected return with a given level of risk or, in another way, minimizes risk with given level of expected return. However, since inputs of the model incorporate mainly with expected return and risk, there is a major criticism about its sensitivity to inputs. It has been shown that a minor change in expected return can lead to major change in the performances of optimal portfolio (Chopra & Ziemba, 2013). (Chopra & Ziemba, 2013)

In order to reduce the complication of parameter estimation, investors have adopted simpler methods of portfolio construction. One strategy that is widely used by many investors is "60/40" strategy where investors simply invest 60% in equity and 40% in bond. Although bond reduces overall risk of portfolio, there is still a concentration of risk since over 90% of portfolio's risk is originated from equity (Qian, 2011). Another method is called "equal-weight" or "1/n" portfolio construction which weights of each asset are equally portioned in the portfolio. Lastly, in the case of single asset class, one might simply invest in what is called "index portfolio" which simply replicate the weight of each constituent in the same amount with the weight of each stock in the index. Since most of indices are cap-weight index (weights of each constituent are determined using the market value such that stocks with higher market capitalization will have more weight in the portfolio), risk contributions to portfolio are mainly from large cap stocks.

Considering these methods of portfolio construction, there are studies about the performances between different methods and cap-weight index portfolio construction. The most general case is to compare between equal-weight portfolio and cap-weight index portfolio which most of the results suggest that equal-weight portfolio outperforms cap-weight index portfolio in the long-term. The main reasons are from rebalancing and size effects. The rebalancing takes a positive return from reversal, volatility, and lead-lag characteristic of stock returns. Moreover, smaller companies tend to

outperform larger companies and equal-weight portfolio will participate more gain from smaller companies than cap-weight index portfolio (Plyakha et al., 2012), (Malladi & Fabozzi, 2017).

However, since there are different risk profiles among each asset class or underlying within one asset class, contributions of risk from each constituent to portfolio are also different. As a result, portfolio's risk and weight may concentrate in one asset class or a group of underlying. The study suggests that equal-weight and minimum variance strategies gives high concentration in risk contribution and weight of specific asset respectively (Maillard et al., 2010).

Unlike other strategies, risk parity approach mainly focuses on risk contributions from each asset class and constructs a portfolio such that risk contributions from every assets are equal. In order to achieve this, there are many optimization techniques which subject to different variable constraints. For example, one may consider a long-only portfolio which requires a simpler optimization model, comparing with long-short portfolio. Generally, risk parity strategy is applied to a portfolio construction that contains different asset classes (e.g. equity, bond, and commodity) since risk profiles are different and may lead to a better result from diversification. For multi-asset portfolio construction, there is a comparison between risk parity, 60/40, and equal-weight portfolios which the result suggest that risk parity strategy provides a better compromise between risk and return than other strategies (Bai et al., 2016).

However, there are only a few discussion on the topic when it comes to single asset class (such as equity) and the results are still ambiguous. One example of applying risk parity strategy with single asset class is alternative-weighted equity indexation in European stock market. The study compares cap-weight index of Eurostoxx 50 with risk parity portfolio under same set of underlying from 1993 to 2011. The results suggest that risk parity portfolio outperforms Eurostoxx 50 index in both risk and return aspects. It achieves higher yearly return (10.7% versus 7.1%) with lower volatility (21.2% versus 22.9%) and drawdown (55.1% versus 66.6%) over testing period which results a better balance in terms of risk-return contribution. In contrast, Eurostoxx 50 index shows high concentration in some specific stocks and this causes a betting in risk-return contribution (Bruder & Roncalli, 2012).

In addition, more complicated method is required in order to achieve an efficient risk parity portfolio of a single asset class. The reason is that risk parity tends to give more weight in low volatility stocks which generally clustered in the same sector or group and this results in a group or sector concentration of stocks in portfolio which gives a poorer diversification of the portfolio. For example,

the problem with sector bias comes as each sector reacts differently to the different market driven factors. For example, higher energy price might benefit the energy sector but impact the transportation sector. Thus, if the majority of constituents are clustered in one specific group, portfolio's risk may not be well-diversified. In order to improve overall diversification in single asset risk parity portfolio, group risk parity is introduced. The optimization technique is applied to groups of one asset class (e.g. groups of equity) such that risk contributions are equal from each group, not individual assets. Bai, Scheinberg, et al. (2016) suggests that applying group risk parity with different group of stocks in S&P 500 provides a better diversification in terms of risk contribution from each group than other portfolio construction strategies. The results indicates that highest group risk contribution of group risk parity, equal-weight, and minimum variance portfolios are 11.13%, 23.81%, and 65.10% respectively.

In this paper, we introduce group risk parity strategy as an alternative way of risk-based approach to single asset portfolio construction with stocks in the Stock Exchange of Thailand (SET). Two ways of grouping which are grouping by sector and market capitalization are considered. We examine in many aspects whether results of group risk parity portfolios outperform non-group risk parity, equal-weight, minimum variance, and cap-weight index portfolios or not. Moreover, we investigate the effect of change in market volatility to risk contribution profile of group risk parity portfolio and observe the relationship between market volatility and performances of group risk parity, non-group risk parity, equal-weight, minimum variance, and cap-weight index portfolios.



2. Research Questions and Hypotheses

2.1 How does group risk parity portfolio compare with non-group risk parity, equal-weight, minimum variance, and cap weight index portfolios in terms of risk-adjusted return, volatility, maximum drawdown, value-at-risk, and risk diversification?

To start with, the result of sector or group concentration may exist under non-group risk parity since it uses volatility of stocks as a criteria. In order to solve this issue, group risk parity strategy is applied such that risks are equally contributed from each group. We consider two grouping methods which we believe that these would give satisfied results.

Firstly, Bai, Scheinberg, et al. (2016) studies about non-group risk parity portfolio and risk parity portfolio with grouping by sector in the US market comparing with equal-weight and minimum variance portfolios. The results are compared in many aspects e.g. excess return, volatility, and risk contribution. Under risk parity strategy, it indicates that the volatility of portfolio lies between equal-weight and minimum variance portfolios where the realized excess return is higher than the minimum variance portfolio. As a result, it shows better compromise between risk and return than the other two. For group risk parity, stocks in S&P 500 index are grouped according with Global Industry Classification Standard (GICS Sector). The results suggest that equal-weight portfolio gives lowest excess return and highest volatility. For minimum variance portfolio, it shows a poorer result from high risk concentration in both group and stock levels where major risk is contributed from one sector and about 40% of total risk is contributed from a single stock. Unlike others, group risk parity gives a highest excess return and better risk diversification in single asset class portfolio since risks contributed from each group are almost equal.

Secondly, for the case of market capitalization, from what we have known, there is no result of group risk parity under value weighted criteria. However, there is a study which indicates that groups of stocks with different market capitalization have different risk-return characteristics. Considering 10 developed countries from 1988 to 1999, it can be found that groups of small-cap and large-cap stocks tend to generally have high and low in both volatility and return respectively. The differences between average annualized volatility and return of small-cap and large-cap stocks are 3.1% and 4.5% where the highest differences are around 7.4% and 13.7% respectively (Eun et al., 2008). Given that the effect of size still exists, grouping stocks under market capitalization criteria would avoid concentration of large-cap stocks in group risk parity portfolio.

Moreover, from what we have found,

- There is no comparison between risk parity with and without grouping in single asset class.
- There is no comparison between group risk parity portfolio and index portfolio.
- The result of single asset class risk parity portfolio in Thai market has never been mentioned.

We propose our hypotheses that, firstly, group risk parity portfolio would give highest risk-adjusted return comparing with non-group risk parity equal-weight and minimum variance portfolios. Secondly, the volatility of group risk parity portfolio would be lower than non-group risk parity, equal-weight, and cap-weight index portfolios but higher than minimum variance portfolio. Moreover group risk parity

would give lowest maximum drawdown and value-at-risk among others. Lastly, group risk parity would give the best performance in terms of risk diversification from results of risk contribution profile and Herfindahl index. In addition, for obtain a better picture of group risk parity strategy, we will also look at other performance measure such as total and excess returns.

2.2 Does group risk parity outperform others portfolio construction strategies in the down market in terms of risk-adjusted return, volatility, maximum drawdown, value-at-risk, and risk diversification?

It is commonly known that the decrease in asset price is accompanied with increase in volatility. Risk parity portfolio is constructed based on the risk of each constituent such that the stock with high volatility would have low weight in the portfolio. In this case, the portfolio under risk parity strategy may avoid a significant losses in the down market. Also, there is a suggestion in US market that risk parity strategy clearly outperforms other strategies in the down market.

Moreover, there exists an idea about sector rotation in asset allocation of the portfolio. The idea indicates that different sectors of stock have advantages over others in different phases of the market or business cycle and therefore tend to outperform of others. One reason behind this is that stocks are grouped into sectors based on companies' core activities which react differently to economic cycles and results in a different performances and returns over time (e.g. utilities sector is typically considered to be safest sector in the down market due to its core business model which deliver the basic necessities) (Sassetti & Tani, 2006).

For group risk parity portfolio, the strategy tries to balance the risk contribution from each group not individual stocks where non-group risk parity portfolio attempts to equalize risk contribution from each constituent which may lead to a concentration in group with low volatility. Thus, group risk parity would reduce a group concentration and give a better result in risk diversification. In this topic, we examine whether group risk parity strategy outperforms other strategies (including non-group risk parity) or not. We believe that, during the down market, group risk parity would give the same results as we proposed in 2.1.

2.3 Does the change in index volatility affect the risk contribution profile of group risk parity portfolio? What is the correlation between index volatility and performances of group risk parity, equal-weight, minimum variance, and cap-weight index portfolios?

In general, there is a period of stock market which has higher or lower volatility than the average throughout the years. Index volatility generally contributed by the volatility of individual constituents where the weights are put mostly at large cap stocks.

For group risk parity, the strategy mainly focuses on equalizing the risk contribution from each group of underlying. Where the main factor of strategy is asset's volatility, there should be a correlation between index's volatility and portfolio performances. To begin with, the uncertain and frequent change in the states of index volatility (high and low) would affect the performance in terms of risk diversification of group risk parity strategy since it would be difficult for one to apply the strategy. Moreover, higher index volatility would result in lower performance in terms of risk-adjusted return of every portfolios as it generally occurs during the down market. However, since group risk parity gives high weight to low volatility group, the performance in terms of risk-adjusted return of group risk parity portfolio should perform better than other strategies during the high volatility period.

3. Methodology

3.1 Overview of Risk Parity

Risk Parity is portfolio construction strategy such that risk contributions from each asset or constituents are equal. Considering a portfolio that has $x = (x_1, x_2, \dots, x_n)$ as the weight matrix of n risky assets, σ_i^2 is the variance of asset i , σ_{ij} is the covariance between asset i and j , and let Σ be the covariance matrix. We have that risk of portfolio is

$$\sigma(x) = \sqrt{x^T \Sigma x} = \sqrt{\sum_i x_i^2 \sigma_i^2 + \sum_i \sum_{i \neq j} x_i x_j \sigma_{ij}}. \quad (1)$$

The marginal risk contribution, which is the quantity that measures change of risk contribution of each asset to portfolio's overall risk given change in asset weight, can be written as:

$$\partial_{x_i} \sigma(x) = \frac{\partial \sigma(x)}{\partial x_i} = \frac{x_i \sigma_i^2 + \sum_{i \neq j} x_j \sigma_{ij}}{\sigma(x)} \quad (2)$$

or

$$\partial_{x_i} \sigma(x) = \frac{\Sigma x}{\sqrt{x^\top \Sigma x}}. \quad (3)$$

The risk contribution of asset i , which represents the contribution of asset i to the total risk of portfolio, is

$$\sigma_i(x) = x_i \cdot \partial_{x_i} \sigma(x) = x_i \cdot \frac{\partial(x)}{\partial x_i} = RC_i, \quad (4)$$

where the risk of portfolio can be considered as a summation of risk contributions from individual assets:

$$\sigma(x) = \sum_{i=1}^n \sigma_i(x). \quad (5)$$

Since the risk parity portfolio is the portfolio that risk contributions from every assets are equal, it satisfies

$$x_i \cdot \frac{\partial \sigma(x)}{\partial x_i} = x_j \cdot \frac{\partial \sigma(x)}{\partial x_j}, \forall i, j. \quad (6)$$

In general, the total weight from each asset is restricted and we have the normalized risk parity problem which can be written as an optimization problem:

$$\begin{aligned} x_i \cdot \frac{\partial \sigma(x)}{\partial x_i} &= x_j \cdot \frac{\partial \sigma(x)}{\partial x_j}, \forall i, j \\ \text{s.t. } \sum_{i=1}^n x_i &= 1. \end{aligned} \quad (7)$$

3.2 General RP Optimization Technique (Least-squares model with general bounds)

Following Bai, Scheinberg, and Tutuncu (2016), least-squares optimization for solving risk parity problem with general bounds is introduced as follows:

$$\begin{aligned}
& \min_x \sum_{i=1, j=1}^n (x_i(\Sigma x)_i - x_j(\Sigma x)_j)^2 \\
& \text{s.t.} \quad a_i \leq x_i \leq b_i \\
& \quad \sum_{i=1}^n x_i = 1.
\end{aligned} \tag{8}$$

where a_i and b_i are constants that represents the bounds of weight of asset i . If short sales is allowed, a_i will be less than zero. The optimization technique in (8) minimizes the difference of risk contributions between each asset which leads to a risk parity portfolio. On the other hand, one may consider using the average risk contribution (θ) which can be presented as

$$\frac{\sum_{j=1}^n x_j(\Sigma x)_j}{n} = \theta. \tag{9}$$

We can also minimize the difference of risk contribution of asset i and the average value such that (8) can be written as

$$\begin{aligned}
& \min_{x, \theta} \sum_{i=1, j=1}^n (x_i(\Sigma x)_i - \theta)^2 \\
& \text{s.t.} \quad a_i \leq x_i \leq b_i \\
& \quad \sum_{i=1}^n x_i = 1.
\end{aligned} \tag{10}$$

3.3 Group RP Optimization Technique

Unlike non-group risk parity, for group risk parity, the risk contributions are equal from each group instead of individual assets. The optimization for group risk parity can be written as a nonconvex problem:

$$\begin{aligned}
& \min_{x, \theta} \sum_{j=1}^l \left(\sum_{i \in \mathcal{G}_j} x_i(\Sigma x)_i - \theta \right)^2 \\
& \text{s.t.} \quad a_i \leq x_i \leq b_i \\
& \quad \sum_{i=1}^n x_i = 1,
\end{aligned} \tag{11}$$

where \mathcal{G}_j stands for the j th group and l is the total number of groups. We can also write (10) and (11) in another form of optimization problem as

$$\min_{x \in X, \theta} F(x, \theta) = \sum_i ((A_i x)^\top (B_i x) - \theta)^2, \quad (12)$$

where $x \in \mathbb{R}^n$, $A_i, B_i \in \mathbb{R}^{m \times n}$ and X is defined by a linear constraints. In case of non-group risk parity (10), $A_i = \Sigma_i \in \mathbb{R}^{1 \times n}$ as i th row of covariance matrix, and $B_i = e_i \in \mathbb{R}^{1 \times n}$ as the i th column of the identity matrix. For the case of group risk parity (11), $A_j \in \mathbb{R}^{m_j \times n}$ is defined by a submatrix of Σ which correspond to rows with indices from set \mathcal{G}_j , and $B_j \in \mathbb{R}^{m_j \times n}$ is defined as:

$$(B_j)_{i,k} = \begin{cases} 1, & k = k_i \\ 0, & \text{otherwise.} \end{cases}$$

To reduce the parameters in the optimization problem, one might consider using $M_i = A_i^\top B_i \in \mathbb{R}^{n \times n}$ such that (12) can be written as a nonconvex optimization problem:

$$\min_{x \in X, \theta} F(x, \theta) = \sum_{i=1}^n F_i(x) = \sum_{i=1}^n (x^\top M_i x - \theta)^2 \quad (13)$$

However, it has been shown that the optimization problems (10), (11), and (13) can lead to a local solutions that is not global solution. The algorithm for solving this issue is introduced in the next section.

3.4 Grouping Selections

3.4.1 Group by Sector

Stock is normally grouped according with its core business activity where it can be divided into sector and industry groups. In general, every member in the sector tends to move toward the same direction and incorporates the similar risk profile. For Stock Exchange of Thailand (SET), stocks are grouped into 8 sectors. However, it can be noticed from the list of members that some of the group has only few members and this could result a non-diversified portfolio for our group risk parity problem. In order to increase a diversification among groups, we introduce another standard of grouping stocks by using The Global Industry Classification Standard (GICS). Following GICS, stocks can be grouped into 11 sectors as follows:

Group	Sector
1	Energy
2	Financials
3	Utilities
4	Consumer Staples
5	Materials
6	Industrials
7	Communication Services
8	Consumer Discretionary
9	Information Technology
10	Health Care
11	Real Estate

Table 1. List of groups by sector under Global Industry Classification Standard (GICS)

3.4.2 Group by Size

There has been studies about the nature of stock return and volatility of small and large market capitalization companies. Some of the findings suggest that small cap stock give more return and also higher volatility due to its size, growth opportunity, etc. Consequently, a group of underlying with different sizes might result in a different risk profile. Using Small-Mid-Large Cap as a criterion, however, there is no standard of how to set the level of market cap for each category. In this study, stocks will be ranked by its market value and sorted from highest to lowest value. We group the first 30% of the list into group1 which is considered to have high market value. The last 30% from the list will be listed into low market capitalization group which is group3. Lastly, the less which is 40% with moderate market value will be grouped into group2.

3.5 Up and Down Market Identification

In general, there are many ways to identify the state of market depending on different strategies of market participants. For a longer term, we consider a maximum drawdown of the index and use 20% and -20% as a signal. The down market is identified when maximum drawdown is lower than 20% and opposite to the maximum drawdown for the bull market.

3.6 Volatility States Classification

We construct the chart of annualized volatility of SET100 index using 20-day rolling volatility throughout the testing period. Then we apply the arithmetic average to classify states of volatility into high and low states. The period where the volatility is above the mean will be classified as high volatility period and otherwise for low volatility period.

3.7 Portfolio Performance Measurements

In this study, testing periods are divided according to our research questions. For the first question, we consider the testing period from the beginning of 2016 to the end of 2020 with 1-year rolling window and 6-month rebalancing. For second and last question, beginning and ending points of each cycle of down market and volatility states are considered to be starting and ending points of each testing period respectively. Performances of each portfolio with different strategies will be reviewed and compared using these following measurements:

a) Annualized Portfolio Return

We use the cumulative return of each portfolio to calculate portfolio's annualized total return.

The formulation is as follows:

$$\text{Annualized Portfolio Return} = (1 + \text{Cumulative Return})^{\frac{365}{\text{Day Held}}} - 1 \quad (20)$$

where cumulative return and holding days are calculated from the beginning to the end of each testing period.

b) Annualized Excess Return

We calculate the excess return using the geometric difference of annualized returns:

$$\text{Annualized Excess Return} = \frac{(1 + \text{Annualized Portfolio Return})}{(1 + \text{Annualized Risk-free Rate})} - 1 \quad (21)$$

where annualized return and risk-free rate are calculated from each testing period.

c) Portfolio Annualized Volatility

Since the testing period is between 2016 and 2020, 5-year standard deviation of daily return (σ) is used to calculate annualized volatility of portfolio which is

$$\text{Portfolio Annualized Volatility} = \sqrt{252} \times \sigma. \quad (22)$$

For the case where we look into some specific period which is less than 5 years, we simply apply the range between starting and ending points of observation period to calculate the standard deviation.

d) Maximum Drawdown

We measure the largest historical loss of the portfolio from a peak to a trough of each testing period and compute the maximum drawdown as:

$$\text{Maximum Drawdown} = \frac{(\text{Trough Value} - \text{Peak Value})}{\text{Peak Value}}. \quad (23)$$

e) Sharpe Ratio

We evaluate the performance of portfolios with different returns and risks by using Sharpe ratio which can be calculated as:

$$\text{Sharpe Ratio} = \frac{\text{Annualized Portfolio Return} - \text{Annualized Risk-free Rate}}{\text{Annualized Portfolio Standard Deviation}} \quad (24)$$

where annualized portfolio return, risk-free rate, and standard deviation are calculated from the beginning to the end of each testing period.

f) Value-at-Risk

We apply non-parametric value-at-risk approach where the historical returns of each testing period are ranked from worst to best and the value-at-risk is determined at 95th percentile.

g) Risk Contribution and Highest Risk Contribution

We measure risk contribution from each group or individual stocks during different testing

periods where the highest risk contribution can be defined as $\max_j \frac{\sum_{i \in G_j} x_i (\Sigma x)_i}{x^T \Sigma x}$.

h) Herfindahl Index

We compare Herfindahl Index which is used to measure the risk concentration in each portfolio during different testing periods. It can be defined as $h(x) = \sum_{i=1}^n \left[\frac{x_i (\Sigma x)_i}{x^T \Sigma x} \right]^2$. For a perfect risk parity portfolio that consisted of n groups, Herfindahl Index is equal to $\frac{1}{n}$.

3.8 Rebalancing Method

In order to investigate performances and effects of using different rebalancing and in-sample data range, we apply multiple rebalancing methods which are 12-month, 6-month, 3-month, and 1-month rebalancing periods together with 12-month, 6-month, 3-month, and 1-month training data range before each rebalancing date.

4. Data

In this study, we focus on the Thai market with stocks in SET100 universe. Most of the data will be provided by the Stock Exchange of Thailand (SET), Bloomberg, and Datastream. Details are as follows:

- i. Given the longest testing period is from 2008 to 2020 and the strategy of group risk parity requires 1-year historical returns as an input, we consider historical data of daily adjusted prices, returns, and market capitalizations of constituents in SET100 index from 2007 to 2020. The underlying which has less than 1-year historical data prior beginning of each testing or rebalancing period will not be included in the list. Source of data is from SET, Bloomberg, and Datastream.
- ii. The Global Industry Classification Standard codes of each stock are provided by Bloomberg.

For excess return and Sharpe ratio, we consider annualized risk-free rate using average 10-year bond yield during the testing period. Data of bond yield is retrieved from Bloomberg.

5. Results

5.1 Risk Contributions

Firstly, we perform simulations for each year according to different training data sets and rebalancing periods. We have 12 portfolios a year for each strategy (except equal-weight strategy and cap weight strategy). For example, portfolio simulations in 2020 consist with 99 stocks and have 4

periods of training samples with 4 rebalancing frequencies. We use sample data in 2019 which have 244 daily returns to estimate expected returns, variances and covariance for the simulation in the beginning of 2020.

For a perfect risk contribution in sector level, the Herfindahl index should be equal to 0.0909 (given that we have 11 sectors). As a comparison in table 2, group risk parity by sector gives the best diversification among others in terms of sector risk contribution and Herfindahl index with moderate concentration in stock level. Similarly for risk contribution in size level, group risk parity by size gives the optimal numbers with Herfindahl index equals to 0.3333 and highest group risk contribution is 33.41%.

For non-group risk parity and equal-weight strategies, there are moderate risk concentration in group levels, where non-group risk parity gives lowest risk contribution in the level of individual stock since it equalizes the risk individually.

Moreover, minimum variance portfolio gives highest stock and sector risk concentration especially in financial and utilities sectors with 15% of risk contributed from a single stock and 27.39% from utilities sector. The number of risk contribution from single underlying is increased gradually to 36.35% given 12-month rolling window without rebalance in 2020.

Lastly, for cap weight portfolio, it shows high risk concentration in stock and sector levels with extremely high risk concentration in size level where 22.29% and 76.72% of risk is contributed from energy sector and large cap stocks respectively. This characteristic generally occurs in cap weight portfolio in every rebalancing point since it weights in accordance with market capitalization of each stock.

As it can be observed from Herfindahl index, group risk parity strategy and our numerical optimization provide almost perfect results in terms of risk equalization. However, the risk contribution profile is subjected to change over time with movement of stocks, different rebalancing frequencies, and different training data sets.

Strategy	Highest Stock RC (%)	By Sector		By Size	
		Highest Group by Sector RC (%)	Herfindahl Index	Highest Group by Size RC (%)	Herfindahl Index
GRP by Sector	5.19%	9.82%	0.0912	40.48%	0.3467
GRP by Size	2.86%	14.59%	0.1023	33.41%	0.3333
Risk Parity	1.57%	13.79%	0.1012	39.04%	0.3410
Minimum Variance	15.00%	27.39%	0.1814	51.32%	0.4124
Equal-Weight	2.32%	14.36%	0.1019	38.34%	0.3435
Cap Weight	10.88%	22.29%	0.1173	76.72%	0.6248

Table 2. Highest risk contribution and Herfindahl index in the beginning of 2020 from individual stock, group of sector and size

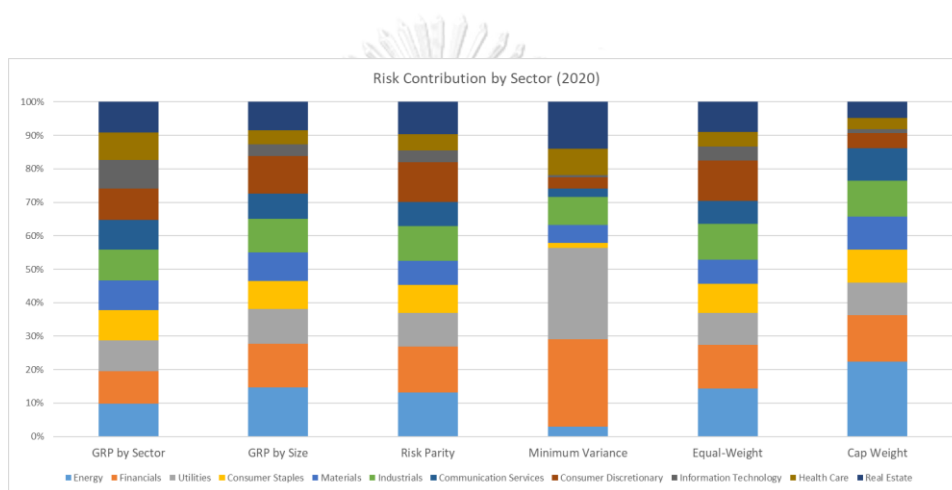


Figure 1. Sector Risk Contribution of different portfolios in 2020.

It can be observed that group risk parity by sector portfolio gives almost equal risk contribution from every sector. For minimum variance portfolio, it shows lowest risk diversification since there are high risk concentrations in financial and utilities sectors.

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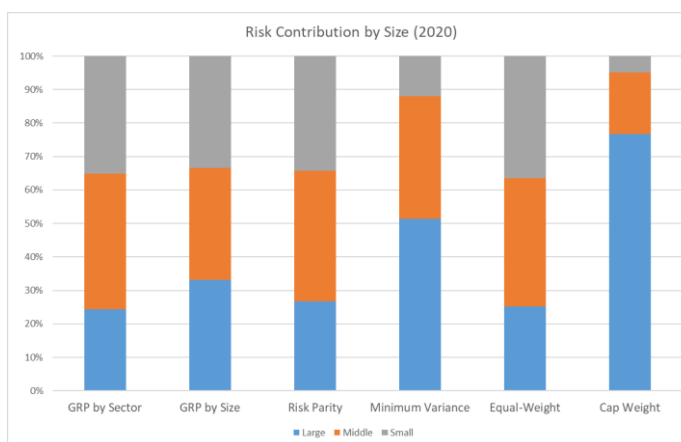


Figure 2. Size Risk Contribution of different portfolios in 2020.

Cap weight portfolio has highest risk contributed from stocks with large market capitalization.

5.2 Effect of inputs and rebalancing to risk contribution profile

Changes in input parameters, such as stock prices, training data sets, and rebalancing frequencies, affect the risk contribution profile of portfolios. To measure level of changes in stock prices, we use volatility as measurement. For comparison, we select year 2017 and 2020 which have lowest and highest annualized volatility (6.36% and 29.45% respectively). We use 12-month training data with no rebalancing to investigate the time series of risk contribution. From figure 3, it can be clearly seen that risk contribution profile in 2017 is more stable than in 2020 with highest Herfindahl indices equal to 0.0997 and 0.1589 respectively. Thus, it can be concluded that less volatile market gives better result in terms of stability of risk equalization than the volatile market.

Moreover, we investigate the effect of rebalancing frequency in 2020 with 12-month rolling window size. As it can be noticed in figure 4 that monthly rebalancing helps equalize the risk, especially in volatile period around the end of the year. The highest Herfindahl indices of portfolio without and with monthly rebalancing are 0.1589 and 0.0961 respectively.

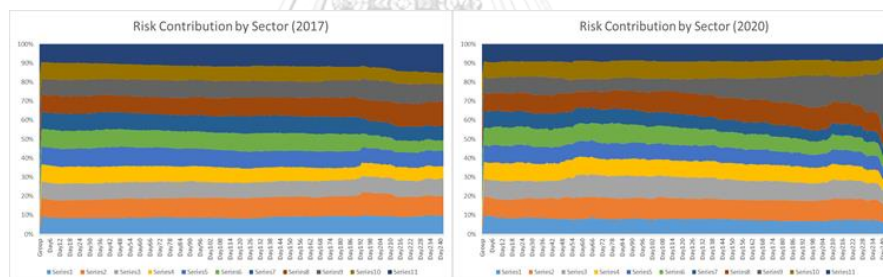


Figure 3. Risk contribution profiles in 2017 and 2020 (12-month training data without rebalancing)

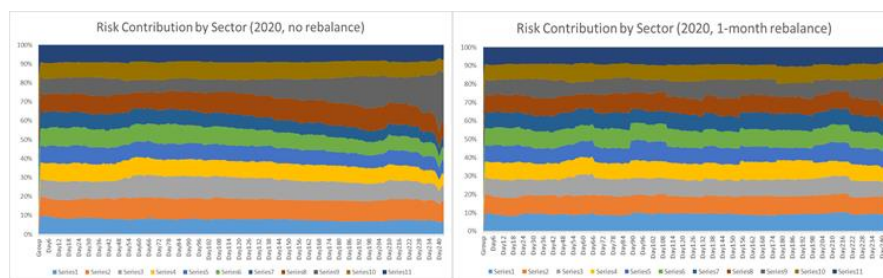


Figure 4. Risk contribution profiles in 2020 without and with 1-month rebalancing frequency (12-month rolling window)

5.3 Portfolio performances from 2016 to 2020

We measure performances of portfolios under different strategies which are group risk parity by sector, group risk parity by size, non-group risk parity, minimum variance, equal-weight, and cap weight. We ignore using 1-month rolling window size since we find that risk parity cannot be constantly achieved during the testing period.

For cap weight portfolio, the performance tends to outperform others from 2016 to 2018. The reason is that energy and financial sectors which have highest weight and risk contribution in portfolios outperform most of other sectors during these 3 years.

Measures	2017										
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real Estate
Weight Contribution	17.61%	18.46%	3.86%	9.80%	10.90%	10.50%	9.29%	6.29%	2.01%	5.52%	5.78%
Risk Contribution	23.31%	19.22%	2.22%	8.42%	11.31%	8.79%	11.07%	5.77%	1.32%	3.68%	4.90%
Cumulative Return	32.48%	35.08%	8.89%	-5.65%	10.20%	-8.32%	10.03%	21.82%	-4.87%	-5.03%	30.79%

Table 3. Weight contribution, risk contribution, and cumulative return by sector in 2017.

Cap weight portfolio gives more weight to energy and financial sectors which contribute majority of risks to portfolio. Performance of the portfolio tends to follow the movement of energy and financial sectors. Please refer to appendix for similar results in 2016 and 2018.

From the result in table 4 and figure 5, it can be observed that minimum variance portfolios, apart from high risk contribution in stock and group levels, it has lowest annualized variance. But standard deviations of performances are also very high comparing with others which confirms the sensitivity to inputs that has been mentioned earlier. Moreover, there is no clear evidence that group risk parity by sector and size outperform others. Moreover group risk parity portfolios does not evidently have lower annualized volatility than equal-weight and non-group risk parity portfolios.

However, it can also be observed that group risk parity by sector clearly outperforms others in terms of risk and return compromisation in 2020 where market is in the down trend with very high volatility since it gives highest average annualized return and moderate annualized volatility.

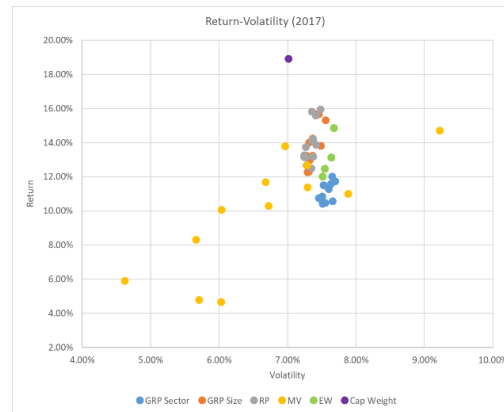


Figure 5. Return and Volatility profile of portfolios under different strategy in non-volatile year (2017).

There is no clear evidence that group risk parity by sector outperforms other portfolios. Minimum variance portfolio has very high sensitivity to inputs as it can be notice from dispersion in performances. In years with the same conclusion please refer to appendix.

Year	Portfolio	Annualized Return		Annualized Volatility		Sharpe Ratio		95% VaR		Maximum Drawdown	
		Average	SD	Average	SD	Average	SD	Average	SD	Average	SD
2016	GRP Sector	25.62%	0.88%	14.82%	0.10%	1.58	0.06	-1.46%	0.04%	-21.57%	0.54%
	GRP Size	25.95%	0.80%	15.06%	0.14%	1.58	0.04	-1.49%	0.01%	-22.49%	0.26%
	RP	24.93%	0.96%	14.51%	0.17%	1.57	0.06	-1.40%	0.01%	-21.90%	0.43%
	MV	18.55%	3.42%	11.34%	0.75%	1.45	0.34	-0.98%	0.05%	-18.70%	0.68%
	EW	25.85%	0.91%	14.99%	0.18%	1.58	0.04	-1.45%	0.02%	-22.23%	0.38%
	SET100 TRI	26.72%	-	15.90%	-	1.54	-	-1.84%	-	-24.73%	-
2017	GRP Sector	11.06%	0.55%	7.57%	0.08%	1.13	0.07	-0.75%	0.03%	-13.25%	0.23%
	GRP Size	13.79%	1.23%	7.37%	0.09%	1.53	0.15	-0.75%	0.01%	-14.26%	0.61%
	RP	14.04%	1.16%	7.34%	0.08%	1.57	0.15	-0.70%	0.02%	-14.18%	0.57%
	MV	9.94%	3.37%	6.68%	1.20%	1.07	0.39	-0.62%	0.10%	-11.61%	0.91%
	EW	13.12%	1.12%	7.59%	0.07%	1.39	0.14	-0.78%	0.02%	-14.33%	0.54%
	SET100 TRI	18.91%	-	7.01%	-	2.34	-	-0.67%	-	-16.59%	-
2018	GRP Sector	-20.52%	0.91%	14.30%	0.20%	-1.61	0.04	-1.69%	0.06%	-22.58%	0.64%
	GRP Size	-18.91%	0.46%	14.08%	0.20%	-1.53	0.03	-1.53%	0.03%	-21.08%	0.44%
	RP	-19.79%	0.26%	13.86%	0.05%	-1.61	0.02	-1.54%	0.06%	-21.75%	0.27%
	MV	-13.49%	4.10%	11.26%	0.67%	-1.42	0.31	-1.22%	0.08%	-12.94%	1.27%
	EW	-20.49%	0.30%	14.26%	0.16%	-1.62	0.02	-1.61%	0.01%	-22.48%	0.29%
	SET100 TRI	-8.85%	-	13.18%	-	-0.87	-	-1.35%	-	-12.83%	-
2019	GRP Sector	7.57%	0.54%	12.07%	0.12%	0.46	0.04	-1.26%	0.03%	-15.00%	0.23%
	GRP Size	6.84%	0.56%	11.67%	0.06%	0.41	0.05	-1.27%	0.03%	-15.81%	0.33%
	RP	6.77%	0.42%	11.21%	0.07%	0.42	0.04	-1.20%	0.02%	-15.65%	0.23%
	MV	7.78%	4.76%	10.55%	2.00%	0.62	0.52	-0.92%	0.21%	-17.94%	0.77%
	EW	7.68%	0.51%	11.86%	0.05%	0.48	0.04	-1.27%	0.03%	-16.44%	0.31%
	SET100 TRI	5.08%	-	10.50%	-	0.29	-	-0.99%	-	-12.43%	-
2020	GRP Sector	10.65%	1.89%	32.13%	0.33%	0.29	0.06	-2.91%	0.22%	-46.00%	1.21%
	GRP Size	5.21%	0.39%	32.50%	0.20%	0.12	0.01	-2.80%	0.20%	-43.87%	0.28%
	RP	4.45%	0.97%	31.76%	0.11%	0.10	0.03	-2.69%	0.27%	-43.23%	1.07%
	MV	-13.20%	7.35%	26.81%	2.00%	-0.55	0.27	-2.13%	0.29%	-33.36%	0.18%
	EW	5.98%	1.19%	32.42%	0.17%	0.14	0.04	-2.83%	0.27%	-44.58%	0.77%
	SET100 TRI	-11.42%	-	33.70%	-	-0.38	-	-2.41%	-	-36.70%	-

Table 4. Portfolios' performances from 2016 to 2020.

There is no clear evidence that group risk parity strategies outperform others. Except in 2020 where group risk parity by sector clearly gives a better risk-return result.

5.4 Portfolio performances during the down market and high volatility

From the result in 2020, we further explore the result in the down market and high volatility periods. Consider year 2008, 2011, 2013, 2015, and 2020 where there are high volatilities and large negative movement in stock market comparing with 2016-2019 as indicated in table 5.

Year	Annualized Volatility												Max. Drawdown
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real-Estate	SET index	
2016	31.88%	29.47%	23.85%	30.15%	33.51%	34.82%	39.70%	34.71%	31.58%	29.08%	28.06%	13.98%	-21.11%
2017	25.52%	21.42%	26.81%	22.37%	21.11%	24.85%	22.71%	28.15%	30.00%	26.88%	27.00%	6.36%	-12.44%
2018	33.27%	29.05%	33.77%	31.28%	27.73%	28.32%	29.74%	36.44%	46.26%	28.12%	26.98%	11.89%	-15.80%
2019	30.77%	23.90%	26.05%	29.61%	29.82%	29.36%	28.52%	32.49%	41.16%	25.81%	22.43%	9.26%	-11.04%
Average	30.36%	25.96%	27.62%	28.35%	28.04%	29.34%	30.17%	32.95%	37.25%	27.47%	26.11%	10.37%	-15.10%
2008	54.75%	48.86%	38.22%	36.48%	51.97%	55.41%	64.76%	50.38%	33.33%	37.29%	52.74%	32.81%	-56.55%
2011	36.99%	35.31%	19.18%	32.18%	36.03%	37.60%	44.32%	37.45%	36.88%	31.58%	37.21%	22.18%	-25.23%
2013	31.81%	35.96%	30.72%	38.50%	39.26%	50.99%	43.69%	43.21%	42.99%	33.98%	46.09%	20.58%	-22.37%
2015	36.53%	27.50%	18.66%	27.73%	34.66%	34.37%	38.83%	31.97%	38.25%	28.43%	28.93%	13.56%	-21.92%
2020	57.64%	52.00%	44.74%	44.50%	51.62%	54.58%	44.17%	56.87%	73.40%	38.82%	45.72%	29.45%	-35.99%
Average	43.54%	39.92%	30.30%	35.88%	42.71%	46.59%	47.15%	43.98%	44.97%	34.02%	42.14%	23.72%	-32.41%

Table 5. Annualized Sector Volatility of SET index Volatility

Unlike less volatile period, cap weight portfolio gives poorer result in volatile periods since it has comparatively high risk and weight concentration in energy and financial sectors which do not outperform during the period.

Measures	2020											
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real Estate	
Weight Contribution	18.15%	16.31%	9.60%	10.49%	8.24%	11.85%	9.68%	4.76%	0.95%	4.93%	5.03%	
Risk Contribution	22.29%	13.91%	9.81%	9.85%	9.81%	10.85%	9.68%	4.46%	1.21%	3.32%	4.80%	
Cumulative Return	-0.42%	7.00%	-1.52%	7.54%	4.60%	-14.68%	-13.18%	19.90%	196.81%	-0.66%	-3.01%	

Table 6. Weight contribution, risk contribution, and cumulative return by sector in 2020

Cap weight portfolio gives more weight to energy and financial sectors which contribute majority of risks to portfolio. Performance of the portfolio tends to follow the movement of energy and financial sectors. Please refer to appendix for similar results in other years.

From the result in figure 6, we observe the same characteristic of sensitivity to inputs in minimum variance portfolios. There are high uncertainties in performances given different training data sets and rebalancing frequencies where performances of other strategies are more stable.

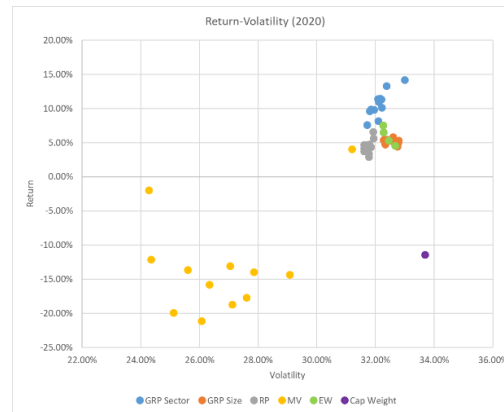


Figure 6. Return and Volatility profile of portfolios under different strategy in highly volatile year (2020).

It can be noticed that minimum variance strategy has a very high dispersion in risk-return results. For group risk parity strategy, it shows better risk-return compromise since it has highest returns with moderate volatility levels. In years with the same conclusion please refer to appendix.

By comparing table 5 and 7, we notice that group risk parity strategy by sector gives a better diversification in sector level since it reduces sectors' annualized volatilities for every testing year.

As a result of better risk diversification in volatile periods, it can also be seen in table 8 that for group risk parity by sector, unlike in less volatile periods, volatilities of portfolios lie between equal-weight and minimum variance portfolios in every year. In terms of maximum drawdown and 95% Value-at-Risk, there is a clear result that grouping stocks under sector or size with group risk parity strategy do not provide lowest numbers in both terms. This means that group risk parity strategy by both sector and size cannot avoid significant losses in the down market. However, comparing with others, grouping stocks by sector under group risk parity strategy provides better compromise between risk and return during volatile periods since it gives highest annualized return with moderate annualized volatility.

Year	Annualized Volatility of Group Risk Parity by Sector										
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real-Estate
2016	19.87%	17.86%	16.88%	19.28%	18.54%	21.09%	24.64%	20.50%	17.66%	22.99%	15.85%
2017	10.92%	9.97%	11.76%	11.92%	11.45%	9.15%	13.91%	14.86%	17.17%	13.80%	12.00%
2018	20.26%	19.42%	20.83%	16.54%	17.82%	16.08%	19.50%	16.48%	33.66%	17.89%	14.41%
2019	20.03%	13.11%	13.87%	17.05%	16.59%	14.85%	20.76%	15.24%	36.64%	16.08%	11.89%
Average	17.77%	15.09%	15.84%	16.20%	16.10%	15.30%	19.70%	16.77%	26.29%	17.69%	13.54%
2008	46.50%	36.79%	32.26%	21.51%	39.36%	35.47%	43.20%	29.15%	20.90%	32.84%	31.50%
2011	29.06%	26.69%	15.21%	21.49%	21.79%	24.58%	31.58%	23.84%	25.31%	26.54%	24.81%
2013	21.34%	23.39%	34.16%	21.18%	23.00%	34.61%	33.05%	29.62%	31.70%	25.95%	31.19%
2015	23.26%	17.02%	15.06%	15.47%	27.09%	18.75%	23.87%	17.01%	21.93%	17.86%	17.76%
2020	43.98%	36.18%	33.96%	31.16%	36.09%	38.67%	31.46%	37.86%	69.45%	28.91%	34.92%
Average	32.83%	28.01%	26.13%	22.16%	29.47%	30.42%	32.63%	27.49%	33.86%	26.42%	28.04%

Table 7. Annualized Sector Volatility under Group Risk Parity by Sector Strategy.

Group risk parity by sector strategy reduces volatilities of every sector.

Year	Portfolio	Annualized Return		Annualized Volatility		Sharpe Ratio		95% VaR		Maximum Drawdown	
		Average	SD	Average	SD	Average	SD	Average	SD	Average	SD
2008	GRP Sector	-44.17%	0.62%	25.76%	0.29%	-1.89	0.02	-2.61%	0.03%	-53.74%	0.51%
	GRP Size	-50.51%	0.24%	30.60%	0.49%	-1.80	0.03	-2.94%	0.07%	-59.25%	0.31%
	RP	-48.92%	0.69%	27.01%	0.35%	-1.98	0.02	-2.62%	0.06%	-56.66%	0.45%
	MV	-39.73%	4.18%	18.90%	1.36%	-2.35	0.23	-1.97%	0.14%	-44.29%	1.32%
	EW	-50.42%	0.20%	30.63%	0.51%	-1.80	0.03	-2.92%	0.05%	-59.47%	0.32%
SET100 TRI	-47.20%	-	37.44%	-	-1.38	-	-3.57%	-	-58.94%	-	
2011	GRP Sector	4.73%	0.94%	19.70%	0.24%	0.05	0.05	-1.88%	0.05%	-22.93%	0.45%
	GRP Size	-0.37%	0.99%	21.77%	0.11%	-0.19	0.05	-2.30%	0.03%	-26.52%	0.44%
	RP	0.86%	1.33%	20.82%	0.08%	-0.14	0.06	-2.19%	0.03%	-25.43%	0.41%
	MV	4.45%	4.91%	16.19%	1.04%	0.04	0.30	-1.46%	0.15%	-18.24%	1.12%
	EW	-0.23%	0.84%	21.77%	0.09%	-0.18	0.04	-2.33%	0.03%	-26.53%	0.43%
SET100 TRI	2.16%	-	24.14%	-	-0.06	-	-2.10%	-	-25.45%	-	
2013	GRP Sector	0.76%	0.97%	23.47%	0.39%	-0.13	0.04	-2.98%	0.11%	-28.55%	0.45%
	GRP Size	-2.24%	1.00%	24.06%	0.37%	-0.25	0.04	-2.99%	0.06%	-28.97%	0.66%
	RP	-0.69%	1.29%	23.16%	0.74%	-0.19	0.05	-2.88%	0.14%	-28.02%	0.76%
	MV	-1.41%	8.68%	19.62%	3.10%	-0.22	0.36	-2.16%	0.21%	-21.89%	1.74%
	EW	-2.60%	0.39%	24.24%	0.26%	-0.26	0.01	-3.02%	0.06%	-29.18%	0.33%
SET100 TRI	-2.39%	-	21.61%	-	-0.29	-	-2.54%	-	-21.01%	-	
2015	GRP Sector	-6.16%	0.86%	14.14%	0.10%	-0.63	0.06	-1.47%	0.02%	-17.96%	0.17%
	GRP Size	-7.76%	1.02%	14.73%	0.05%	-0.71	0.07	-1.47%	0.02%	-19.43%	0.38%
	RP	-6.92%	1.06%	14.13%	0.06%	-0.68	0.07	-1.45%	0.03%	-18.52%	0.38%
	MV	-10.02%	2.56%	11.78%	0.64%	-1.08	0.21	-1.18%	0.09%	-17.11%	1.95%
	EW	-6.97%	0.80%	14.90%	0.03%	-0.65	0.05	-1.51%	0.03%	-19.52%	0.26%
SET100 TRI	-14.09%	-	15.27%	-	-1.10	-	-1.41%	-	-22.46%	-	
2020	GRP Sector	10.65%	1.89%	32.13%	0.33%	0.29	0.06	-2.91%	0.22%	-46.00%	1.21%
	GRP Size	5.21%	0.39%	32.50%	0.20%	0.12	0.01	-2.80%	0.20%	-43.87%	0.28%
	RP	4.45%	0.97%	31.76%	0.11%	0.10	0.03	-2.69%	0.27%	-43.23%	1.07%
	MV	-13.20%	7.35%	26.81%	2.00%	-0.55	0.27	-2.13%	0.29%	-33.36%	0.18%
	EW	5.98%	1.19%	32.42%	0.17%	0.14	0.04	-2.83%	0.27%	-44.58%	0.77%
SET100 TRI	-11.42%	-	33.70%	-	-0.38	-	-2.41%	-	-36.70%	-	

Table 8. Portfolios' performances in 2008, 2011, 2013, 2015 and 2020.

Group risk parity by sector portfolios indicate better performance in terms of risk-return compromise with highest annualized return with moderate annualized volatility.

6. Conclusion

In conclusion, risk parity is a portfolio construction strategy such that risk contributions are equal from every assets. It shows a better compromise between risk and return among other portfolio construction strategies such as equal-weight and minimum variance portfolios. For a single asset class, group risk parity is applied such that risk contributions from each group are equal. In this study, we examine the performances between group risk parity with grouping by sector and size, non-group risk parity, minimum variance, and equal-weight strategies in Thai market.

Firstly, we show that equal group risk contribution can be achieved. From the result in the beginning of 2020, it is shown that the optimization technique provides almost perfect risk equalization for both grouping by sector and size. In addition, moderate concentrations of risk are found in non-group risk parity and equal-weight portfolios. Unlike others, minimum variance and cap weight portfolios have very high risk concentrations in both single stock and group levels.

Secondly, the effects of input parameters and rebalancing frequency to risk contribution profile are analyzed. It is shown as expected that risk contribution is more difficult to be managed during the high volatility period. For rebalancing frequency, it indicates that frequently rebalance the portfolio helps maintaining equal risk contribution profile.

Moreover, we examine portfolios' performances from 2016 to 2020 which we find no clear evidence that group risk parity strategy by sector or size provides better performance than other strategies. For minimum variance portfolio, there is high sensitivity to inputs and rebalancing frequency since standard deviations of portfolio performances are significantly higher than others in every measure.

Lastly, it is noticeable that grouping stocks by sector under group risk parity strategy outperforms other portfolio construction strategies in terms of risk-return compromise in highly volatile down market. It also indicates that the volatilities of group risk parity by sector lie between minimum variance and equal-weight portfolios. Moreover, we find that group risk parity by sector minimizes the overall sectors' volatilities, and thus reduces the negative effect of volatility on portfolio performances. As a result, we believe that applying group risk parity by sector strategy under high volatility period is the most preferable strategy to gain higher risk-return compromise.

7. Appendix

A. Weight contribution by sector from cap weight portfolio (index portfolio)

Year	Weight by sector in Cap Weight Portfolio										
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real Estate
2008	41.16%	19.99%	2.56%	3.07%	7.99%	6.83%	6.61%	3.47%	1.72%	1.43%	5.18%
2011	32.84%	24.62%	2.17%	8.13%	8.99%	5.71%	6.87%	3.77%	1.73%	1.30%	3.87%
2013	21.34%	23.42%	2.23%	10.73%	11.89%	4.53%	12.54%	4.59%	0.76%	2.60%	5.37%
2015	16.03%	23.23%	2.38%	8.21%	9.75%	8.40%	16.04%	4.79%	1.82%	3.93%	5.42%
2016	14.23%	19.46%	2.99%	7.67%	12.61%	11.69%	10.56%	5.83%	2.26%	5.96%	6.73%
2017	17.61%	18.46%	3.86%	9.80%	10.90%	10.50%	9.29%	6.29%	2.01%	5.52%	5.78%
2018	19.46%	17.44%	3.16%	10.88%	10.44%	11.47%	8.38%	6.66%	0.75%	4.35%	7.00%
2019	19.33%	17.12%	7.51%	9.56%	10.13%	10.74%	7.76%	5.58%	1.16%	4.93%	6.19%
2020	18.15%	16.31%	9.60%	10.49%	8.24%	11.85%	9.68%	4.76%	0.95%	4.93%	5.03%

B. Risk contribution by sector from cap weight portfolio (index portfolio)

Year	Risk Contribution by Sector in Cap Weight Portfolio										
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real Estate
2008	51.69%	20.25%	1.27%	1.60%	5.38%	5.35%	6.06%	2.48%	0.82%	0.87%	4.24%
2011	36.73%	28.36%	0.53%	6.07%	9.43%	5.78%	4.79%	2.94%	0.96%	0.48%	3.92%
2013	24.93%	26.81%	0.65%	8.39%	13.48%	3.10%	11.95%	4.23%	0.47%	1.40%	4.58%
2015	19.47%	22.33%	1.06%	6.00%	7.91%	9.32%	18.36%	4.58%	1.36%	3.00%	6.62%
2016	21.09%	18.06%	1.97%	5.68%	13.81%	10.57%	10.30%	5.20%	1.89%	4.67%	6.76%
2017	23.31%	19.22%	2.22%	8.42%	11.31%	8.79%	11.07%	5.77%	1.32%	3.68%	4.90%
2018	23.30%	15.57%	3.10%	10.30%	10.87%	11.64%	9.06%	6.72%	0.75%	2.12%	6.55%
2019	28.68%	15.65%	6.58%	8.18%	11.45%	9.16%	6.23%	5.41%	0.44%	3.36%	4.87%
2020	22.29%	13.91%	9.81%	9.85%	9.81%	10.85%	9.68%	4.46%	1.21%	3.32%	4.80%

C. Weight contribution by size from cap weight portfolio (index portfolio)

Year	Weight Contribution by Size in Cap Weight Portfolio		
	Large	Mid	Small
2008	85.06%	12.38%	2.56%
2011	82.69%	13.79%	3.51%
2013	81.43%	15.40%	3.17%
2015	80.10%	16.28%	3.63%
2016	76.23%	18.38%	5.40%
2017	76.80%	17.95%	5.24%
2018	77.84%	17.35%	4.81%
2019	78.07%	17.55%	4.38%
2020	77.37%	18.30%	4.33%

D. Risk contribution by size from cap weight portfolio (index portfolio)

Year	Risk Contribution by Size in Cap Weight Portfolio		
	Large	Mid	Small
2008	89.35%	8.78%	1.87%
2011	86.41%	11.08%	2.51%
2013	85.82%	11.81%	2.37%
2015	81.54%	15.01%	3.45%
2016	78.09%	16.24%	5.67%
2017	80.81%	14.80%	4.38%
2018	79.96%	15.25%	4.79%
2019	78.55%	16.71%	4.74%
2020	76.72%	18.42%	4.86%

E. Weight contribution by sector from group risk parity by sector portfolio

Year	Weight by Sector in Group Risk Parity by Sector Portfolio										
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real Estate
2008	5.84%	7.52%	11.27%	10.90%	9.74%	8.77%	5.41%	9.04%	12.42%	9.08%	10.00%
2011	8.64%	7.54%	12.88%	10.30%	8.73%	8.89%	6.92%	9.09%	9.90%	9.93%	7.18%
2013	7.90%	9.21%	8.90%	9.75%	8.82%	11.09%	7.34%	8.26%	9.00%	9.61%	10.13%
2015	7.86%	10.73%	9.59%	11.24%	8.14%	8.56%	8.38%	9.58%	8.71%	9.38%	7.82%
2016	7.84%	10.94%	10.47%	10.63%	7.78%	9.10%	8.58%	9.24%	8.04%	8.75%	8.64%
2017	8.86%	10.25%	9.97%	8.51%	8.19%	8.49%	7.78%	8.32%	10.33%	8.29%	11.00%
2018	9.71%	11.85%	8.29%	9.18%	8.87%	12.18%	7.96%	9.21%	5.39%	7.88%	9.48%
2019	8.15%	9.86%	10.59%	8.79%	8.19%	9.86%	8.85%	8.22%	7.76%	9.06%	10.68%
2020	7.98%	11.96%	10.80%	8.35%	8.44%	9.06%	8.62%	9.05%	5.27%	10.16%	10.31%

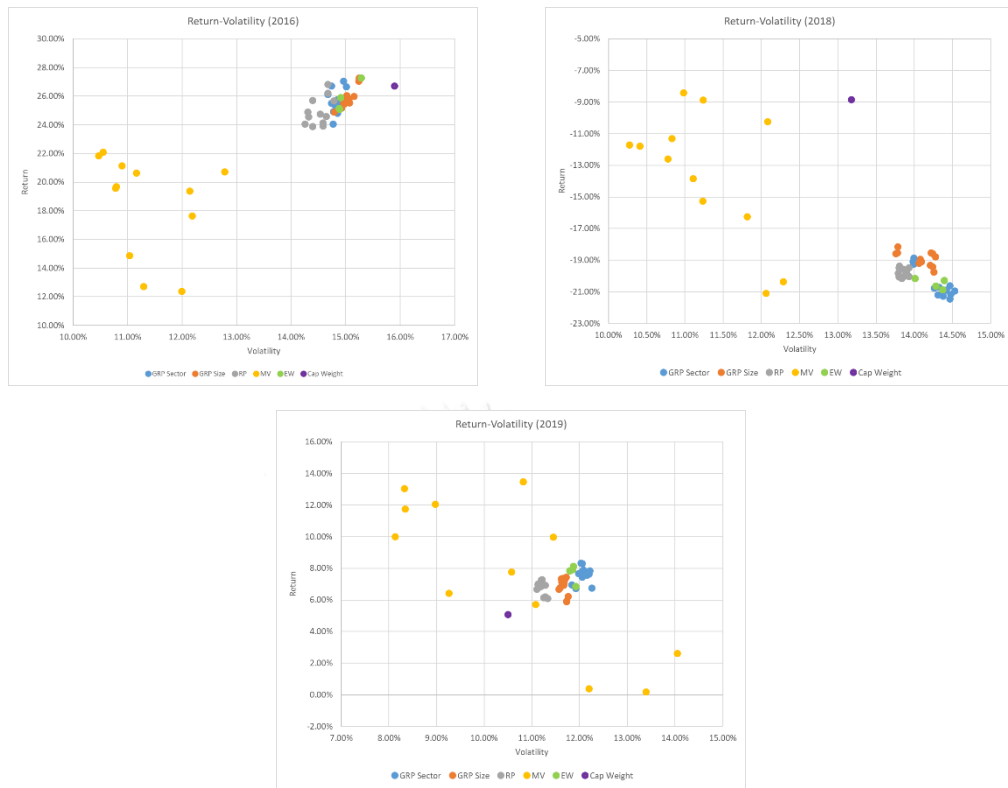
F. Risk contribution by sector from group risk parity by sector portfolio

Year	Risk Contribution by Sector in Group Risk Parity by Sector Portfolio										
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real Estate
2008	8.58%	9.75%	9.01%	8.48%	8.89%	9.50%	8.91%	9.00%	9.51%	9.08%	9.30%
2011	9.69%	9.14%	8.22%	9.07%	8.95%	9.57%	8.80%	9.55%	8.65%	9.02%	9.33%
2013	9.49%	9.92%	8.07%	9.09%	9.60%	9.71%	8.89%	9.49%	7.68%	7.65%	10.42%
2015	9.25%	9.40%	8.34%	8.41%	8.66%	10.26%	9.18%	9.54%	8.68%	8.25%	10.03%
2016	8.99%	9.44%	8.61%	8.77%	9.25%	9.63%	9.30%	9.21%	8.92%	8.33%	9.55%
2017	9.20%	9.54%	8.83%	9.06%	9.05%	9.35%	9.12%	9.33%	8.28%	8.89%	9.34%
2018	9.34%	9.03%	8.65%	9.01%	8.35%	9.65%	8.72%	10.41%	8.48%	8.35%	10.01%
2019	9.47%	9.56%	9.47%	8.87%	8.77%	9.04%	8.45%	9.61%	8.53%	8.58%	9.64%
2020	9.82%	9.77%	9.18%	9.02%	8.80%	9.33%	8.77%	9.46%	8.47%	8.22%	9.16%

G. Average sector cumulative return

Year	Average Sector Cumulative Return										
	Energy	Financials	Utilities	Consumer Staples	Materials	Industrials	Communication Services	Consumer Discretionary	Information Technology	Health Care	Real Estate
2008	-53.54%	-58.84%	-17.97%	-21.31%	-45.95%	-62.33%	-40.14%	-45.04%	-44.67%	-45.04%	-57.07%
2011	-3.87%	-2.13%	2.17%	17.27%	-18.11%	-18.76%	42.50%	19.84%	-19.10%	64.98%	4.93%
2013	-9.31%	-3.94%	1.08%	0.81%	-12.99%	10.36%	36.29%	-2.89%	21.96%	6.92%	-9.62%
2015	-7.13%	-9.00%	-10.15%	-18.82%	6.40%	-16.06%	-39.28%	5.47%	8.11%	26.29%	-7.65%
2016	33.04%	30.66%	26.77%	42.34%	7.07%	19.95%	30.27%	22.45%	9.86%	2.12%	15.86%
2017	32.48%	35.08%	8.89%	-5.65%	10.20%	-8.32%	10.03%	21.82%	-4.87%	-5.03%	30.79%
2018	-16.60%	0.96%	-13.23%	-24.04%	-13.63%	-20.04%	-16.33%	-33.90%	-45.17%	-2.17%	-23.64%
2019	3.59%	10.86%	23.48%	22.41%	1.77%	-7.53%	20.95%	-5.21%	-1.49%	4.58%	3.75%
2020	-0.42%	7.00%	-1.52%	7.54%	4.60%	-14.68%	-13.18%	19.90%	196.81%	-0.66%	-3.01%

H. Return-Volatility profiles in non-volatile year (2016, 2018, and 2019)



I. Return-Volatility profiles in highly volatile year (2008, 2011, 2013, and 2015)



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