STRUCTURAL OPTIMIZATION UNDER LIMITED NATURAL FREQUENCY CONSTRAINTS USING COMPREHENSIVE LEARNING PHASOR PARTICLE SWARM OPTIMIZATION

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การออกแบบโครงสร้างที่เหมาะสมที่สุดภายใต้ข้อจำกัดความถี่ธรรมชาติด้วยวิธีการของกลุ่มอนุภาคเฟสเซอร์แบบเบ็ดเสร็จ

น.ส.เอะ โซ โพเน

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิศวกรรมศาสตรมหาบัณฑิต
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Field of Study	Civil Engineering
Thesis Advisor	Associate Professor Dr. SAWEKCHAI TANGARAMVONG, Ph.D.

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เอะ โซ โพเน : การออกแบบโครงสร้างที่เหมาะสมที่สุดภายใต้ข้อจำกัดความถี่ธรรมชาติด้วยวิธีการของกลุ่มอนุภาคเฟสเซอร์แบบเบ็ดเสร็จ. (STRUCTURAL OPTIMIZATION UNDER LIMITED NATURAL FREQUENCY CONSTRAINTS USING COMPREHENSIVE LEARNING PHASOR PARTICLE SWARM OPTIMIZATION) อ.ที่ปรึกษาหลัก : เสวกชัย ตั้งอร่ามวงศ์

ในงานวิจัยนี้ได้มีการเสนอการเพิ่มประสิทธิภาพของกลุ่มอนุภาคเฟสเซอร์ด้วยกลยุทธ์การเรียนรู้แบบครอบคลุม(CLPPSO)สำหรับการออกแบบที่เหมาะสมที่สุดของโครงสร้างข้อหมุนแบบโดมภายใต้ข้อจำกัดความถี่ที่จำกัดรูปแบบที่เสนอเป็นเทคนิคPSOแบบใหม่ที่มีการผสมผสานโดยตรงตามทฤษฎีเฟสเซอร์ในวิชาคณิตศาสตร์และกลยุทธ์การเรียนรู้ที่ครอบคลุมเพื่อการเพิ่มประสิทธิภาพกลุ่มอนุภาคในการสร้างแบบจำลองพารามิเตอร์การควบคุมอนุภาคมุมเฟสที่รวมฟังก์ชันไซน์และโคไซน์ตามคาบจะถูกนำไปใช้ตามหลักพื้นฐานโดยจะใช้เฉพาะตำแหน่งที่ดีที่สุดก่อนหน้าของอนุภาคทั้งหมดเพื่อปรับปรุงความเร็วของอนุภาคตัวอย่างในระหว่างกระบวนการจะปรับให้เหมาะสมที่สุดสิ่งนี้ทำให้อัลกอริธึมสามารถป้องกันความแปรปรวนของฝูงเพื่อเลี่ยงการลู่เข้ากันก่อนเวลาอันควรเพื่อแสดงประสิทธิภาพและความทนทานของอัลกอริธึมCLPPSOที่เสนอเป็นโครงสร้างข้อหมุนแบบโดม3มิติมี120บาร์600บาร์1410บาร์ตามลำดับได้รับการทดสอบเรียบร้อยแล้วและเปรียบเทียบผลลัพธ์อื่นที่รายงานโดยใช้อัลกอริธึมเมตาฮิวริสติกที่แตกต่างกันในเอกสารเกี่ยวข้องกับวิธีแก้ปัญหาที่เหมาะสมที่สุด

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Ei Cho Pyone : STRUCTURAL OPTIMIZATION UNDER LIMITED NATURAL FREQUENCY CONSTRAINTS USING COMPREHENSIVE LEARNING PHASOR PARTICLE SWARM OPTIMIZATION. Advisor: Assoc. Prof. Dr. SAWEKCHAI TANGARAMVONG, Ph.D.

In this research, a phasor particle swarm optimization with comprehensive learning strategy (CLPPSO) is proposed for the optimal design of dome-like truss structures under the limited frequency-constraints. The proposed scheme is a new variant of PSO techniques with the direct combination of both the phasor theory in mathematics and comprehensive learning strategy to the particle swarm optimization. In order to model particle control parameters, a phase angle incorporating the periodic sine and cosine functions is essentially applied through which only the previous best positions of all particles are used to update the exemplar particle’s velocity during the optimization process. This empowers the algorithm to keep the swarm's variability from eschewing premature convergence. To demonstrate the effectiveness and robustness of the proposed CLPPSO algorithm, three benchmarks of 120- bar, 600-bar and 1410-bar of dome truss structures are successfully tested, and the results are compared with those reported using different metaheuristic in the literature regarding their optimum solutions.

Field of Study:	Civil Engineering	Student's Signature ....
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CHAPTER 1
INTRODUCTION

1.1 Background

In the engineering industry, structural optimization can administrate the design solutions for specific structural components or materials to be creative. For achieving the best performance and safety for a structure, the crucial part of design process is to maximize the stiffness or minimize the stresses, and weight or compliance under constraints of crisp condition, for instance, displacement constraints, stress constraints, buckling constraints and natural frequency constraints. Starting from 1960's, the mathematical programming techniques that impose limitations to only a few of many design variables are introduced in structural optimization. Afterward, the optimality criteria methods of deterministic techniques that usually provided an approximate optimum design with a few structural analyses without depending on the problem size are presented in the late 1960's (Tushaj, Lak, & Research, 2017). However, deterministic methods are difficult to be applied to the non-smooth and non-convex optimization problems and have time consuming effects.

Figure 1. The collapse of Tacoma Narrow Bridge.

To address this problem, other optimization techniques called meta-heuristics approaches, which generate a combination of random populations with less computational effort than other optimization algorithms, iterative methods, or simple heuristics by searching over a large set of feasible solutions in a pseudo-random way following their inspiring principle without requiring gradient information, have transpired (Blum & Roli, 2003). During the last two decades, many meta-heuristic methods have been developed in structural optimization field owing to a consequence of growing computational power. Meta-heuristic techniques, being mostly inspired by concepts found in nature, are used for combinatorial optimization. Meta-heuristic algorithms are mainly based on a wide variety of evolution, swarm intelligence and colony, social sciences and human activities, and physical sciences (Degertekin, Yalcin Bayar, & Lamberti, 2021). Even though many meta-heuristic algorithms have been introduced in the field of structural optimization, just a few of them have been published on the structural optimization under multiple frequency constraints. In addition, there has no algorithm that is well suited to solve all optimization problems in the structural optimization research area due to the relationship between how well an algorithm performs and the optimization problem on which it is run (Wolpert & Macready, 1997). Thus, there has a potential that a novel developed algorithm can perform in the structural optimization problem better than the current ones (Millan-Paramo & Abdalla Filho, 2020).

Since the structural optimization under natural frequency constraints is highly implicit nonlinear and/or non-convex, the efficient and robust optimization techniques are developed to provide the best performance and safety of a structure with reasonable computing efforts. The natural frequencies and mode shapes are the inherent properties of a structure to find its dynamic response of the unloaded structure. In real structures, there have a large number of natural frequencies depending on the mass and stiffness of the structure. For each natural frequency, a vibration shape which is named mode shape (e.g., horizontal, vertical, torsional, bending etc.) is obtained. The higher the frequencies are, the more complex the mode shapes become. It is important that the frequencies of the structure have to be isolated from to the natural frequencies to avoid undesirable vibrations and resonance under external excitations (Bellagamba & Yang, 1981; Grandhi, 1993). Therefore, the natural frequencies are taken into account as one of the constraints in the structural optimization for the sake of generating not only the complexity in the optimal design of structures but also the difficulty in the transposing of vibration modes, which may face the convergence problems to the optimizer through structural size and shape modifications.

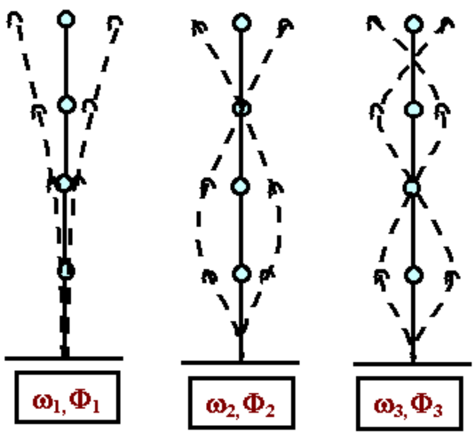


Figure 2. Example mode shapes of the first three frequencies.

Based on the abovementioned background, this study proposes a new variant of PSO techniques so-called a phasor particle swarm optimization with comprehensive learning strategy (CLPPSO) for the sizing optimization of dome structures under the limited frequency constraints. The proposed scheme is a new variant of PSO techniques with the direct combination of both the phasor theory in mathematics and comprehensive learning strategy to the particle swarm optimization. In order to model particle control parameters, a phase angle incorporating the periodic sine and cosine functions is essentially applied through which only the previous best positions of all particles are used to update the exemplar particle’s velocity during the optimization process. The aim of this work is to minimize the total weight of dome-like truss structures under limited frequency constraints with the proposed technique.

1.2 Research Objective

The objectives of this research are:

(1) To propose the comprehensive learning phasor particle swarm optimization (CLPPSO) algorithm in the field of structural optimization.

(2) To implement the natural frequency and intrinsic structural responses in the sizing optimization process.

(3) To evaluate, analyze and compare the results of the proposed scheme with those obtained from other developed optimization methods.

1.3 Scope

(1) The analyses of three prominent dome-like truss structures will be performed by the eigenvalue method.

(2) All benchmarks will be optimized to demonstrate the robustness and efficiency of the proposed CLPPSO.

(3) The MATLAB program is applied to optimize and analyze with an iterative manner to minimize the total weight of the structure subjected to the natural frequency constraints.

(4) The violations of these constraints are considered as penalty functions and will have accounted them for the total weight.

1.4 Methodology

In this study, a phasor particle swarm optimization with comprehensive learning strategy (CLPPSO) is proposed. The generic idea is based on the direct integration with the phasor theory in mathematics and comprehensive learning strategy to the particle swarm optimization to empower the algorithm to keep the swarm's variability from eschewing premature convergence. The particle control parameters of PSO are endorsed by the periodic functions of phase angle (θ) and the comprehensive learning strategy helps to be dispirited from the premature convergence of the PSO. The algorithm is tested on several well-known benchmark truss problems for sizing optimization under stress, buckling and displacement constraints. The obtained results were compared to those of some well-known meta-heuristics which successfully solved benchmarks highlighted efficiency and accuracy of the discrete optimization problems.

To investigate the efficiency and viability of the proposed CLPPSO method, three prominent benchmarks of three-dimensional dome-like trusses will be explored for sizing optimization with the natural frequency constraints. The goal is to find the optimum cross-sections of the members by minimizing the weight of the structure under limited natural frequency constraints. The first example is mid-scale dome-like truss of 120-bar which has 49 nodes and 7 design groups. The large-scale dome-like truss of 600-bar which is composed of 216 nodes and 25 design groups, is presented as the second benchmark. The third one is also the 1410-bar large-scale dome-like truss, and it has 390 nodes and 47 design groups. The nonstructural additional masses are attached to the free nodes of the design examples. The eigenvalue method is conducted to analyze the natural frequencies and distinctive mode shapes of an undamped free vibration dome-like truss system. In addition, the direct stiffness method is applied to analyze the mass and stiffness of all benchmark structures. The coding procedure is implemented with MATLAB program. The optimization procedure terminates when the minimum cross-sections are obtained without violating its given constraints at the maximum no. of iterations. The penalty function method will be applied to the weight minimization process if the allowable natural frequency constraints are violated. The results obtained from the proposed method will be compared with those of other meta-heuristic optimization algorithms recently presented in literature.

CHAPTER 2
LITERATURE REVIEW

In order to obtain the optimal design, the size, shape, and topology of a structure can be optimized individually or by combination of each other. In sizing optimization, the goal is to find the optimal weight corresponding to the minimum cross-sectional areas under the limited design constraints. In shape optimization, the aim is to find the most suitable shape of the structure corresponding to its geometric layout under the limited design constraints whereas the target of topology optimization is to find the optimal structure corresponding to internal member configuration of the structure. Most of the limited design constraints are stress, displacement, buckling and natural frequency.

2.1 Overview on structural optimization with natural frequency constraints

The optimization problem minimizes the total weight (W) of the structure, which is considered as the objective function. The member cross-sectional areas, namely A_d for each d-th member, are considered as design variables. In practical design, the discrete values of the rolled steel sections are considered as cross-sectional variables and the algorithm is permitted to choose only one of a discrete set of available values. Due to the increase in the computational time of discrete value variables, the cross-sectional design variables are usually assumed to be continuous. Generally, there has many typical constraints such as mass, stress, displacements, natural frequencies, velocities, and accelerations which can be subdivided as equality and inequality constraints. To manipulate the dynamic characteristics of the structure, its inherent structural responses and the required natural frequency are appraised as the constraints in this study.

The optimization problem can be mathematically described as follows:

$$\begin{aligned} &\text{Find} && A_d \quad \text{for} \quad \forall d \in \{1, \ldots, n_d\} \\ &\text{Minimize} && W = \sum_{d=1}^{n_d} \rho_d A_d L_d \\ &\text{Subjected to} && \omega_j \le \omega_j^*, \quad \omega_k \le \omega_k^* \\ & && A_{\min} \le A_d \le A_{\max} \end{aligned} \tag{1}$$

where n_d as the total number of (pin-jointed) truss members, ρ_d as the material density, L_d as the length of a generic d-th member, ω_j and ω_k as the response natural frequencies (i.e., the j-th and k-th modes, respectively), ω_j^* and ω_k^* as the natural frequency limits, A_{min} and A_{max} as the lower and upper limits on the available sectional areas, respectively.

2.2 Constraints Handling

As constraints handling process, the penalty function f is applied to the problem in Eq. (1) for updating the objective function (total design weight) W:

$$\left.\begin{aligned} f &= W(1+\varepsilon_1 . C)^{\varepsilon_2} \\ C &= c_\omega^j + c_\omega^k \end{aligned}\right\}, \tag{2}$$

where $c_\omega^j = \begin{cases} \dfrac{|\omega_j|}{|\omega_j^*|} - 1, & \text{if} \quad \omega_j > \omega_j^* \\ 0, & \text{if} \quad \omega_j \le \omega_j^* \end{cases}$, $c_\omega^k = \begin{cases} \dfrac{|\omega_k|}{|\omega_k^*|} - 1, & \text{if} \quad \omega_k > \omega_k^* \\ 0, & \text{if} \quad \omega_k \le \omega_k^* \end{cases}$,

The penalty factor C is associated with the violation of natural frequency constraints, c_ω^j and c_ω^k stand for the parameters that indicate the natural frequency conditions are satisfied or violated. The parameters ε_1 and ε_2 impinge on the exploration and the exploitation of the search owing to the effect of settling on how much a violated solution is penalized (Joines & Houck, 1994). The more the optimization process proceeds, the more the intensification level of penalizing infeasible solutions increase. This strategy permits more diversification in the search space during the early stages of optimization and more intensification in the final stages to balance the exploration and exploitation of the search process (A Kaveh & Zolghadr, 2018). In this study, the parameter ε_1 is set to 1 and ε_2 is subjected to monotonic increase from 1.5 (subsequently increase to 3), respectively.

2.3 Large-span steel domes

Large-span steel domes are used as the resourceful and profitable structures to span a large space utilizing minimum surfaces with the economical consumption of building materials. In these days, they can mostly be seen over the structures which are required to have very large open spaces as well as iconic aesthetic standards such as airports, jurisdictive buildings, and sports arenas (Chen & Lui, 2005).

Figure 3. Science world at TELUS world of science in Vancouver.

The major advantages of large-span steel domes are as follows:

(1) They are relatively lightweight in comparison with reinforced concrete buildings.
(2) The site spaces will be increased from 5% to 8% as they can be prefabricated in the supplier and then assembled on-site construction.
(3) They can reduce the construction period about over 40% than that of the reinforced concrete structures as the prefabricated components offer high precision, and the construction hoisting speed is fast.
(4) Due to the prefabrication process, they can also reduce the labor cost as the smaller number of workers are required for on-site construction.

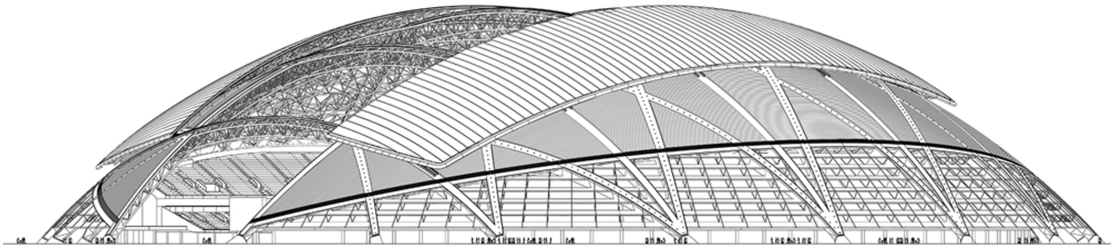


Figure 4. World largest free-spanning dome of National Sport Stadium in Singapore.

Figure 5. Aerial view-of Jewel Changi Airport in Singapore.

2.4 Direct stiffness method

The direct stiffness method is performed to construct the element stiffness of the dome truss depending upon their material properties, section properties, and member configurations assuming that the truss is loaded at the joints as the concentrated loads and the member of the truss is subjected to only axial forces which remain constant along the length of the member. In addition, the joints are also assumed as frictionless pins (or internal hinges). The element local stiffness matrix can be expressed as:

$$[k_e'] = \frac{EA_d}{L_d}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

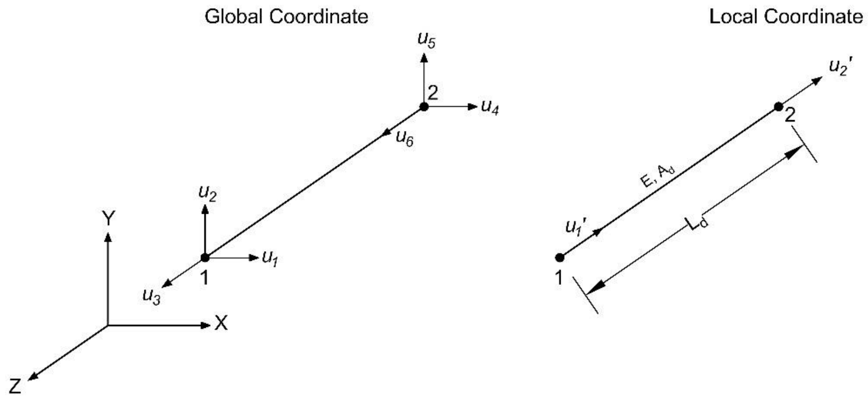
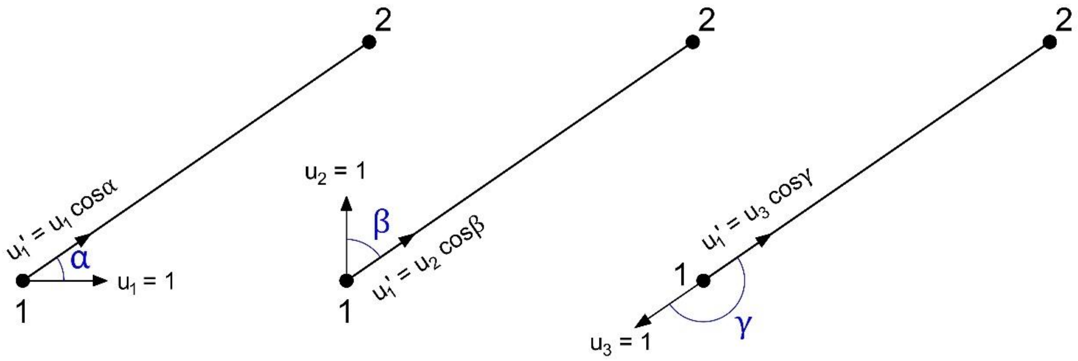


Figure 6. Typical Two-node Truss element in Global and Local coordinates.

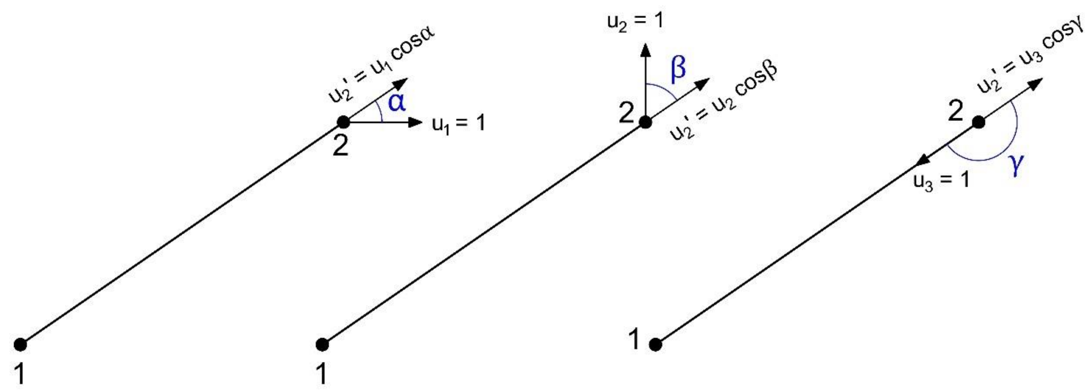
A unit displacement is applied in the global coordinate in order to determine the corresponding displacement in local coordinate. The relationship between the displacements of local and global coordinates is derived as follows:

$$\begin{Bmatrix} u_1' \\ u_2' \end{Bmatrix} = \begin{bmatrix} C_x & C_y & C_z & 0 & 0 & 0 \\ 0 & 0 & 0 & C_x & C_y & C_z \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix}$$

$$C_x = \cos\alpha, C_y = \cos\beta, C_z = \cos\gamma$$



(i)



(ii)

Figure 7. 1 unit displacement applying in global coordinate to determine corresponding displacement in local coordinate system, (i) at node 1, (ii) at node 2.

Then, the global stiffness matrix for a truss element can be stated as follows:

$$[k_e] = \frac{EA_d}{L_d}\begin{bmatrix} C_x & 0 \\ C_y & 0 \\ C_z & 0 \\ 0 & C_x \\ 0 & C_y \\ 0 & C_z \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} C_x & C_y & C_z & 0 & 0 & 0 \\ 0 & 0 & 0 & C_x & C_y & C_z \end{bmatrix}$$

$$[k_e] = \frac{EA_d}{L_d}\begin{bmatrix} C_x^2 & C_xC_y & C_xC_z & -C_x^2 & -C_xC_y & -C_xC_z \\ C_xC_y & C_y^2 & C_yC_z & -C_xC_y & -C_y^2 & -C_yC_z \\ C_xC_z & C_yC_z & C_z^2 & -C_xC_z & -C_yC_z & -C_z^2 \\ -C_x^2 & -C_xC_y & -C_xC_z & C_x^2 & C_xC_y & C_xC_z \\ -C_xC_y & -C_y^2 & -C_yC_z & C_xC_y & C_y^2 & C_yC_z \\ -C_xC_z & -C_yC_z & -C_z^2 & C_xC_z & C_yC_z & C_z^2 \end{bmatrix}$$

2.5 Eigenvalue analysis

Instinctively, any structure that has the mass and elasticity can vibrate to some degree. Therefore, the structural design usually entails to consider the oscillatory motions with increasing amplitudes. Although there have linear and nonlinear oscillatory motions, the nonlinear behavior is mostly found in the structures. There are generally three types of engineering vibrations, namely free vibration, forced vibration, and damped vibration. When a system vibrates harmonically with amplitudes at specific frequencies so-called natural frequencies via imposing properly with the initial displacement and then releasing it, the free vibration ensues. It produces the oscillatory motion and constructs distinctive mode shapes depending on the distribution of mass and stiffness of the system without exerting any external force. For instance, when a swing is pulled on the back and then released it, this swing moves back and forth itself owing to the initial pulling effect. This is free vibration.

The force vibration betides when a system enlivens under the excitation of time-dependent external forces that can possibly be periodic (harmonic or a non-harmonic) and steady-state, transient, or random. If the periodic harmonic external force is exerted to the system, the vibration of system is said to be forced, for example, the vibration of a building during an earthquake. To avoid the resonance and large oscillations effect, the frequencies from the excitation of external forces have to evade a concurrence with one of the natural frequencies (Chopra, 2007). The damped vibration transpires when friction and other resistances gradually deplete the energy of a vibrating system by the ways of the gradual change in frequency or intensity or ceasing and resting the system in its equilibrium position. It can be seen when the shock absorber inhibits the suspension of the vehicles (Den Hartog, 1985; Inman, 2001).

There has the specific natural frequency for every structure producing the distinctive mode shapes. At higher frequencies, modes become more complex. For all engineering structures, the natural frequency is one of the key parameters that has to be considered. The natural frequencies and modes of vibration of an undamped free vibration system can be determined from the satisfaction of the subsequent matrix eigenvalue problem equation:

$$K\phi_n = \omega_n^2 M\phi_n, \text{ where } w_n = \sqrt{\frac{K}{M}} \tag{3}$$

where K and M are the stiffness and mass matrices of the system; ω_n is the n-th natural frequency of vibration, and ϕ_n is the n-th natural mode of vibration of the structure, respectively. The stiffness and mass matrices of each member of the structure can be calculated as follow and transformed into global coordinate system to assemble those matrices of the entire dome truss structure. It should be noted that the mass matrices are constructed as consistent mass model for the structural mass (Rao, 2011).

$$m_e = \frac{\rho_d A_d L_d}{6}\begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix}$$

2.6 Structural optimization techniques

Since the structural optimization problems are non-smooth and non-convex problems and they also have some criteria, such as falling into the local minima’s trap and requiring excessive number of iterations to assurance the optimal solution, the efficient and robust algorithms are compulsory to fulfill the actual design requirements. Depending on the factors such as the computational efficiency, robustness, accuracy of solutions, simplicity and possibility of the algorithm, the choice of the suitable algorithm for structural optimization problems is done. These algorithms are generally grouped as deterministic and non-deterministic (stochastic) methods (Tushaj et al., 2017).

2.6.1 Deterministic methods

2.6.1.1 Mathematical programming techniques

In order to get the optimum solution, these techniques compute the gradient computations of the objective function and constraints starting the search from a preselected initial position. However, in some cases, the gradient functions will not exist if the continuous constraint functions are not applied. And also, if the constraint functions are mathematically complex, the computation of their gradients will be difficult to calculate. In addition, the performance of the algorithms mainly depends on the quality of the initial position selection. If initial design point is chosen remote from the optimum solution and the design problem also has various local optima, it is expected that these algorithms will stop finding at one of the local optima as the optimum solution. Occasionally if the initial design point is not a good estimate, the problem will face difficulties in convergency, and thus no solution can be obtained. (Saka, Hasançebi, Geem, & Computation, 2016). For solving the minimization problems, they move in the negative direction of the gradient of the objective function so that the next position is ascertained. This process will stop when the values of design variables do not change significantly in two chronological iterations (Saka & Geem, 2013). Mathematical programming techniques can be divided as three main groups: sequential linear programs, penalty function methods, and gradient-based methods.

Sequential linear programs use linearization concept to solve the optimization problem. Even though sequential linear-programming-based design algorithms have a benefit in the availability of reliable linear programming packages for users, they also have some drawbacks such as requiring numerous optimization cycles to get the optimum solution, and the choice of move limits depending on problem. In addition, they have difficulties in the convergency for the optimization problem with large numbers of design variables making large linearization errors of these problems. And also, the linearized problem may not have a feasible solution if the starting point is infeasible. In order to solve them, the move limits should be eased as well as the application of the method should be carried out to resume (Haftka, 1985; Kirsch, 1993).

Penalty function methods uses sequential unconstrained minimization techniques to convert the constrained problem into unconstrained one to solve the problem as the unconstrained methods are more general and efficient than the constrained ones. They use a penalty function as a violation constraint when the solution disrupts the limited constraint functions. The penalty function methods are numerically reliable for structural optimization problems of moderate complexity. Even though they are generally suitable for various structural optimization problems, they also have the same drawback of requiring large number of structural analyses as sequential linear programs. Therefore, numerical difficulties may ascend when the ill-conditioned problems are minimized. There have exterior penalty function method and interior penalty function method. The exterior penalty function method directly accompanies each by each for the corresponding constraint violation and thus the treatment of the inequality terms differs from that of the equality terms. In exterior penalty function method, all intermediate solutions occupy in the feasible region to find the solution starting from an infeasible design point so that it will eradicate the need for finding the initial feasible design point. Nevertheless, the algorithm cannot find a feasible solution until it reaches the optimum one and as a result all intermediate solutions become infeasible and finally, they do not provide a usable design. Interior penalty function method is applied if and only if inequality constraint is present in the problem. Differ from the exterior penalty function method, this method requires starting a feasible initial design point to find the solution such that all intermediate solutions become feasible and thus they can provide usable designs. In addition, the constraints become crucial only when the solution process approaches near the end so that this makes a chance to select the near optimal solution of a less critical design in preference to optimal design (Saka & Geem, 2013).

The gradient-based methods directly handle the mathematical programming problem step by step as the constrained one without transforming it into another form. At each step, the solution approaches from current design variables to the next ones along a suitable direction that is determined by the gradient vectors of the objective and constraint functions (Saka & Geem, 2013).

2.6.1.2 Optimality criteria

Optimality criteria methods have more computational effort than mathematical programming techniques even though the mathematical programming ones are more general to implement. They transform the original problem in the form of a Lagrangian function to provide a near optimum design within some structural analyses without depending on the number of design variables with an iterative manner. This approach also requires the continuous design variable assumption and preselection of an appropriate initial design point to start its iterations so that the performance of these algorithms depend on the quality of the preselected initial design point (Saka et al., 2016). They have two different approaches called physical and mathematical tools. In the physical tools, they generally derive a criterion residing on stress constraints (fully stressed design), displacement constraints (fully displacement design), and an integrate version of both stresses and displacements constraints (fully utility design) whereas the mathematical tools use Kuhn-Tucker conditions to derive (Rozvany, 2012). Nevertheless, these methods may not converge to the optimum solution in certain cases (Saka & Geem, 2013).

It can be concluded that the structural optimization algorithms based on deterministic concept have many difficulties to meet the requirements of designing real-size structures under design code provisions.

2.6.2 Non-deterministic (Stochastic) methods

The stochastic search techniques make use of randomness in exploring the search for the optimal or near-optimal solutions and usually they reach the different final solutions when the algorithm is executed. Meta-heuristics are nontraditional stochastic search methods that are efficient in finding the solution of combinatorial optimization problems without requiring gradient information and an explicit relationship between the objective function and the constraints. These techniques inspire from the natural phenomena such as survival of the fittest, immune system, social culture, and swarm intelligence for the purpose of guiding and modifying the search process of optimal solution during the iterations (Holland, 1992; Lu, Chen, & Zheng, 2012; Yang, 2010). In addition, meta-heuristic techniques are approximate based so that there has no guarantee that the resulted optimum solution is the global one.

The performance of a meta-heuristic algorithm depends on the assessment between diversification and intensification strategies. Diversification discovers the search space more thoroughly and finds promising regions with good solutions whereas intensification takes advantages of local information in the promising areas hoping to discover better ones within a reasonable computational time (Talbi, 2009). When the intensification strategy controls the algorithm, it may lead to premature convergence often to a local optimum. However, when the diversification strategy pedals the algorithm, it will decelerate the convergency process even if the likelihood of finding the global or near-optimum solution of the optimization problem is amplified (Saka et al., 2016). In general, there have many taxonomies of meta-heuristic techniques according to their inspiration such as evolutionary-based algorithms, swarm-intelligence-based algorithms and physical related-based algorithms. Owing to their efficiency and robustness, meta-heuristic techniques have been developed for the practical engineering design optimization problems with both continuous and discrete design variables (Yang & Koziel, 2011).

2.7 Application of meta-heuristic algorithms in structural optimization

Most of these meta-heuristic algorithms that were successfully tested to the optimum design of truss structures under frequency constraints, are briefly reviewed. The enhanced version of forensic-based investigation (EFBI) algorithm is modified with by Kaveh et al. (Ali Kaveh, Hamedani, & Kamalinejad, 2021) for optimal design of frequency-constrained dome-like trusses. The chaotic water strider algorithm is developed by Kaveh et al. (A. Kaveh, Amirsoleimani, Dadras Eslamlou, & Rahmani, 2021) to optimize large-scale dome-shaped trusses with limited frequency constraints. The fruit fly optimization algorithm (FOA) using a memory-based search strategy by adding both the vision search radius for each fruit fly and an improved deb rule to handle the constraints is modified by Liu et al. (Liu, Zhu, Chen, & Cao, 2019) to optimize truss structures with frequency constraints and demonstrated that the new algorithm finds significantly lighter designs than other variants. The chaos-based firefly algorithms utilizing two chaotic maps of Logistic and Gaussian maps to tune the attractiveness and light absorption coefficients of FA (CGFA) is proposed by Kaveh and Javadi (A. Kaveh & Javadi, 2019) for optimization of cyclically large-size braced steel domes with multiple frequency constraints. The modified simulated annealing (MSAA) algorithm is utilized by Millan-Paramo and Abdalla Filho (Millan-Paramo & Abdalla Filho, 2020) for size and shape optimization of truss structures with natural frequency constraints The cyclical parthenogenesis algorithm (CPA) is proposed by Kaveh and Zolghadr (A Kaveh & Zolghadr, 2018) for optimum design of large-scale cyclically symmetric dome trusses with frequency constraints using the block diagonalization technique. A hybrid charged system search (CSS) algorithm with a migration-based local search (MBLS) mechanism is utilized by Jalili and Talatahari for the optimum design of trusses with frequency constraints.

The vibrating particles System (VPS) inspired by the damped oscillation of a single degree of freedom systems, is investigated by Kaveh and Ilchi Ghazaan (Ali Kaveh & Ilchi Ghazaan, 2017) to deal with large-scale dome trusses. The enhanced colliding-bodies optimization (ECBO) algorithm incorporating with multi-stage cascading techniques are used by Kaveh and Ilchi Ghazaan (A. Kaveh & Ilchi Ghazaan, 2016) for the optimization of large-scale dome trusses with frequency constraints. The hypotrochoid spiral optimization (HSPO) algorithm is presented by Kaveh and Mahjoubi (A. Kaveh & Mahjoubi, 2019).The improved differential evolution (DE) algorithm based on roulette wheel selection is presented by Ho-Huu et al. (Vinh Ho-Huu, Truong, Le, & Vo-Duy, 2018) to optimize the shape and size optimization of truss structures with frequency constraints. The firefly algorithm (FA) and an adaptive hybrid evolutionary firefly algorithm (AHEFA) are proposed by Lieu et al. (Lieu, Do, & Lee, 2018) for shape and size optimization of truss structures with frequency constraints.

The orthogonal multi-gravitational search (OMGSA) algorithm for truss optimization on shape and sizing with frequency constraints is introduced by

Khatibinia and Naseralavi (Khatibinia & Sadegh Naseralavi, 2014). The harmony search and ray optimizer for enhancing the PSO algorithm (HRPSO) is utilized by Kaveh and Javadi (A. Kaveh & Javadi, 2014) for shape and size optimization of trusses with multiple frequency constraints. The democratic particle swarm optimization (DPSO) is introduced by Kaveh and Zolghadr (A. Kaveh & Zolghadr, 2014) to optimize truss layout and size with frequency constraints. The colliding bodies optimization for truss optimization with multiple frequency constraints is investigated by Kaveh and Mahdavi (A. Kaveh & Mahdavi Dahoei, 2015). The sequential harmony search (SHS) is applied by Gholizadeh and Barzegar (Gholizadeh & Barzegar, 2012) for shape optimization of structures for frequency constraints by sequential harmony search algorithm. The particle swarm optimization (PSO) for truss optimization with dynamic constraints is proposed Gomes (Gomes, 2011). The teaching-learning based optimization modified with sub-population (MS-TLBO) for design of truss structures with natural frequency constraints is presented by Tejani et al (Tejani, Savsani, & Patel, 2016). The improved version of symbiotic organisms search (ISOS) is applied by Tejani et al. (Tejani, Savsani, Patel, & Mirjalili, 2018) for truss optimization with natural frequency constraints. The Niche Hybrid Genetic Algorithm (NHGA) combined with a simplex search method is presented by Lingyun et al. (Lingyun, Mei, Guangming, & Guang, 2005) to reduce premature of GA.

2.8 Particle swarm Optimization

Particle swarm optimization is one of the stochastic population-based meta-heuristic techniques that is inspired from swarm intelligence. Particle swarm optimization and its variants have been employed in structural optimization research community for the sake of their simplicity of implementation and their aptitude of swift convergence to a good solution in terms of number of function evaluations as well as robustness (Kennedy & Eberhart, 1995). In addition, they can efficiently handle the nonlinear and nonconvex design spaces with discontinuities. Being a population-based algorithm, any particle in the swarm randomly instigates in the design space and communicates with the best trajectories of itself and its neighbors by adjusting their own positions and velocities derived from the best positions of all particles dynamically. All particles tend to flow not only towards the best position of itself in the earlier iterations (**pbest**) but also towards the historical one that the neighbor particles so far (**gbest**) during the iteration until the swarm approaches to an optimum solution of the objective function (Camp, Meyer, & Palazolo, 2004). This leads to good collaborative search capability of the algorithm (Liang, Qin, Suganthan, & Baskar, 2004).

In standard PSO approach, it randomly initializes with a population of *npop* particles called swarm in the *d*-th dimensional search space and appraises the position of each particle in the search space. To control its movement through the search space, each particle $i \in \{1, \ldots, npop\}$ in the swarm uses the information of the current position $X_i \in \Re^{n_d} = [X_{i,1}, X_{i,2}, \ldots, X_{i,n_d}]$, the current velocity $\mathbf{V}_i \in \Re^{n_d} = [V_{i,1}, V_{i,2}, \ldots, V_{i,n_d}]$, the distance between the best-known position of the particles $\boldsymbol{pbest}_i \in \Re^{n_d} = [pbest_{i,1}, pbest_{i,2}, \ldots, pbest_{i,n_d}]$ and its current position, and the distance between the global best position of the swarm $\boldsymbol{gbest} \in \Re^{n_d} = [gbest_1, gbest_2, \ldots, gbest_{n_d}]$ and its current position. The velocity $\boldsymbol{V}_i$ is chosen in the range of [$-V_{max}$, V_{max}] to avoid the particles from moving beyond the search space. For each iteration, the current velocity is updated as:

$$\boldsymbol{V}_{i,d}^{t+1} = \omega^t \mathbf{V}_{i,d}^t + c_1 rand1_{i,d}(\boldsymbol{pbest}_{i,d}^t - \boldsymbol{X}_{i,d}^t) + c_2 rand2_{i,d}(\boldsymbol{gbest}_d^t - \boldsymbol{X}_{i,d}^t) \quad (1)$$

where c_1 and c_2 are acceleration control coefficients, and $rand1_i^d$ and $rand2_i^d$ are random coefficients of two uniform random sequences in the range of (0, 1). To balance among the local and global search, the inertia weight factor (ω) is used for adjusting the impact of the previous velocities on the computation of the new velocity (Shi & Eberhart, 1998). During the optimization process, a linearly decreasing inertial weight strategy was developed to promote a global search (exploration) in early iterations and slowly reduce the impact of previous velocities focusing a local search (exploitation) around their **pbest**$_i$ and **gbest** (Camp, 2007). The inertial weight is calculated as:

$$\omega^t = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \text{ x } \frac{t}{t_{\max}} \quad (2)$$

where w_{min} and w_{max} are the initial and final value which were set as 0.9 and 0.4, respectively (Shi & Eberhart, 1998), t is the current iteration, and t_{max} is the total number of iteration. With the updated velocity, the particles' position is updated as follows:

$$\boldsymbol{X}_{i,d}^{t+1} = \boldsymbol{X}_{i,d}^t + \boldsymbol{V}_{i,d}^{t+1} \quad (3)$$

In PSO, all the particles attempt to improve the performance of the algorithm by updating their velocities and positions according to their **pbest**$_i$ and **gbest** for the new iteration.

$$\mathbf{pbest}_i^{t+1} = \begin{cases} \boldsymbol{pbest}_{i,d}^t, & \text{if} \quad f(\boldsymbol{pbest}_{i,d}^t) \le f(\boldsymbol{X}_{i,d}^{t+1}) \\ \boldsymbol{X}_{i,d}^{t+1}, & \text{otherwise} \end{cases} \quad \text{for } d \in \{1, \ldots, n_d\} \quad (4)$$

$$\mathbf{gbest}^{t+1} = \begin{cases} \boldsymbol{pbest}_{i,d}^{t+1}, & \text{if} \quad f(\boldsymbol{pbest}_{i,d}^{t+1}) \le f(\boldsymbol{gbest}_d^t) \\ \boldsymbol{gbest}_d^t, & \text{otherwise} \end{cases} \quad \text{for } d \in \{1, \ldots, n_d\} \quad (5)$$

However, PSO inherits the premature convergence which is the main vulnerability of the optimization technique. The reason is that all particles in the swarm trace only the current global best position as the social learning method, regardless of the current global best position is quite away from the global optimum one. It brings the particles to be trapped in a local optimal point with a dexterous manner and finally this leads PSO to converge quickly.

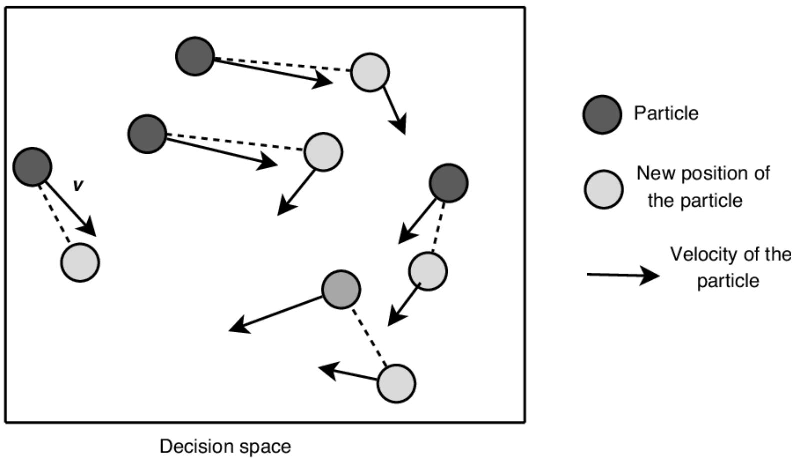


Figure 8. Particle swarm with their associated positions and velocities.

CHAPTER 3
Comprehensive Learning Phasor Particle Swarm Optimization Algorithm (CLPPSO)

3.1 Algorithm overview

The proposed CLPPSO method is developed by Ghasemi et al. (Ghasemi et al., 2019). It implements the direct combination of a phasor theory of mathematics and the comprehensive learning strategy to enhance the standard particle swarm optimization (PSO) techniques. The periodic functions of phase angle (θ) uphold the particle control parameters of PSO and simply make them to self-adaptive, balanced, and nonparametric ones (Ghasemi et al., 2019) whereas the comprehensive learning strategy discourages the premature convergence of the algorithm through preserving the diversity of the swarm (Liang et al., 2004).

As the proposed method is a new variant of standard PSO, it also resets with randomly generating an initial population. In the determination of the initial population, it is critical to deal with diversification that can lead to face the premature convergence for the algorithm. In addition, the initial population has the ability to progressive enhancement on searching the new positions and their velocities of the group of populations through the iterations. The newly generated best positions are chosen to act for the whole swarm or a part of it for next iteration without using gradient information during the search (Abdel-Basset, Abdel-Fatah, & Sangaiah, 2018; Loughlin & Ranjithan, 1997).

Moreover, the population size is preferred between 20 and 60 in most of the PSO implementations. And also, there has a contrary bond between number of populations and the number of required iterations of the algorithm. The more increase in the number of populations, the lesser the number of iterations is required to be converged near the solution. Even though the surge in the number of populations can improve the results, it also has the time-consuming effect since the algorithm requires to perform the number of structural analyses depending to the number of populations and the number of iterations. Therefore, it is important to compromise between the quality of results and the computational time as in any population-based meta-heuristic methods (Talbi, 2009). Thus, in this study, the number of populations are set to 30 which gives the optimal results with the proper convergence rate for all benchmarks.

3.2 Periodic Functions of phase angle

All control parameters of CLPPSO are incorporated with the periodic sine and cosine functions of one-dimensional phase angle (θ) lying within a range of 0–2π (6.2832 radians). The variation of these periodic functions (in an interval of [−1, 1]) and their absolute values (in [0, 1]) are presented in Fig. 9. In essence, the change in the function $\left|\cos\theta_i^t\right|^{2\sin\theta_i^t}$ generates adaptive search characteristics of CLPPSO with the phase angles of the particles and endorses a balance between global and local search of CLPPSO (Ghasemi et al., 2019).

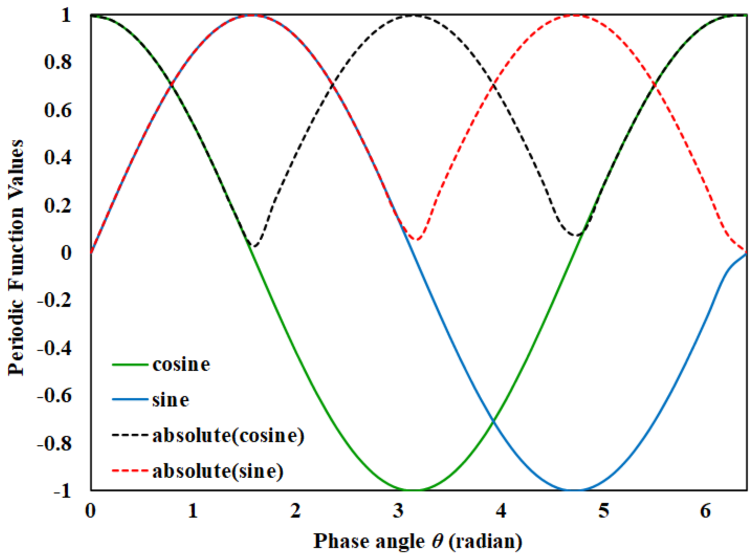


Figure 9. Changes in function based on their phase angles.

3.3 Comprehensive learning strategy

During the velocity updating process, the comprehensive learning strategy carries out a selection of $f_{i(d)} = \left[f_{i(1)}, f_{i(2)}, \ldots, f_{i(n_d)} \right]$ to identify the particles' pbests which the *i*-th particle should follow in the same *d*-th dimension. Contrary to the original PSO method, the current velocity of the *i*-th particle is updated as follows:

$$\boldsymbol{V}_{i,d}^{t+1} = \left(\frac{\left|\cos\theta_i^t\right|^{2\sin\theta_i^t}}{npop} \boldsymbol{V}_{i,d}^t \right) + \left|\cos\theta_i^t\right|^{2\sin\theta_i^t} (\boldsymbol{pbest}_{f_i(d),d}^t - \boldsymbol{X}_{i,d}^t) \tag{4}$$

where $\left|\cos\theta_i^t\right|^{2\sin\theta_i^t} / npop$ is the inertia weight factor, $\left|\cos\theta_i^t\right|^{2\sin\theta_i^t}$ is the acceleration control periodic function, $f_{i(d)}$ is the particles' pbests which the i-th particle should follow and $\boldsymbol{pbest}_{f_i(d),d}^t$ is the personal best position of the selected particle. The particles' position is also updated using the updated velocity as follow:

$$\boldsymbol{X}_{i,d}^{t+1} = \boldsymbol{X}_{i,d}^t + \boldsymbol{V}_{i,d}^{t+1} \tag{5}$$

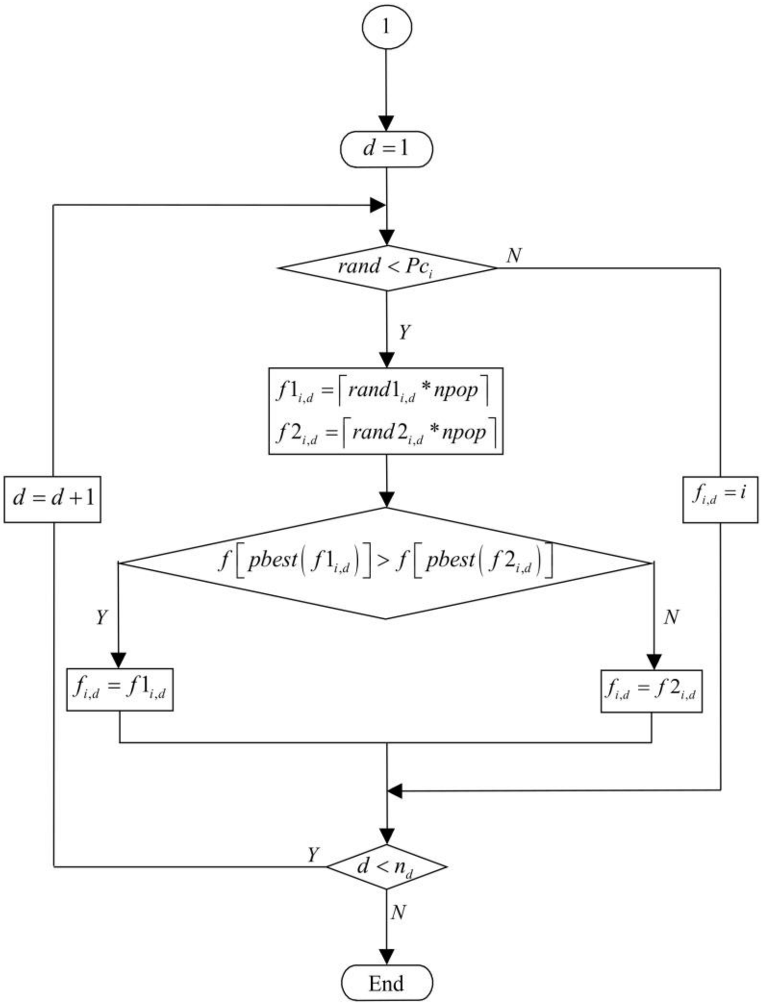


Figure 10. Identification of exemplar for i-th particle.

The selection of $f_{i(d)}$ is based on the randomly generated decision variable (within a range of 0 and 1) and the decision probability *Pc* to make sure that the particles release from blocking in a local optimum point (Liang, Qin, Suganthan, & Baskar, 2006). The *i*-th particle learns from its own pbest when the decision variable is larger than the decision probability; or else, it will learn from another particle's pbest. In case of learning from another particle's pbest, the comprehensive learning strategy employs two particles which are randomly chosen from the swarm and compares the corresponding objective functions of their *pbests*. For the *i*-th particle, these learning techniques are explored with the iterative manner for all *d-th* dimensions and then it adopts the one's *pbest* which has the larger objective function value as the new learning exemplar. Afterward, the new learning exemplar's *pbest* updates its searching route in the search domain. These steps will be recurrent for all particles in the swarm. In addition, on the occasion that all exemplars of the *i-th* particle are at its current *pbest* as the special case, the new learning exemplar's pbest (**pbest**$_{f_i(d)}$) can appoint one dimension from its search domain to do random learning from another particle's pbest at the corresponding dimension. The identification of exemplar is summarized in Fig. 10.

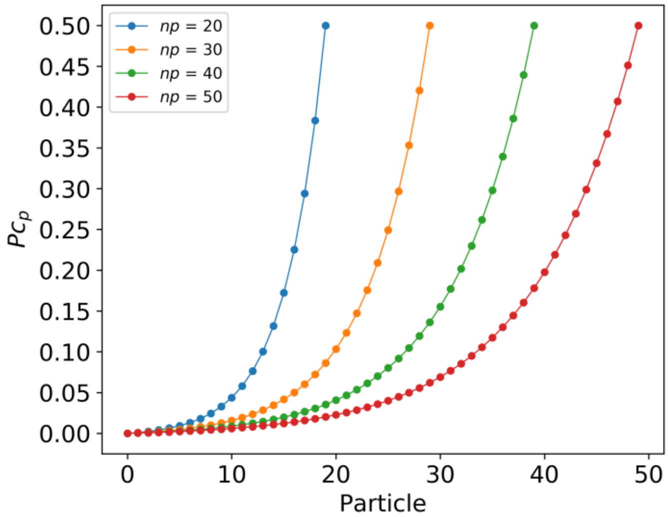


Figure 11. Decision probability for various no. of particles.

The *Pc* for the *i-th* particle is calculated as Eq. (6) and Fig. 11 presents *Pc* for various no. of particles.

$$Pc_i = 0.05 + 0.45\,\mathrm{x}\,\frac{\left(\exp\left(\dfrac{10(i-1)}{npop-1}\right)-1\right)}{(\exp(10)-1)} \tag{6}$$

With the condition that the *i*-th particle desists to improve on its objective function for more than a specified number of iterations, thus so called a refreshing gap *m*, it is permitted to learn from the exemplars and proceed the selection of $f_{i(d)}$ for the new learning exemplars to stay away from some excessive local optimal searches regarding with all dimensions in that *i*-th particle over the good exemplars to facilitate the computation of the accurate optimal design solutions.

The $pbest_i$ of each particle and the *gbest* of the swarm in CLPPSO will also be updated as Eq. (7) and (8) for the next iteration.

$$\mathbf{pbest}_i^{t+1} = \begin{cases} \boldsymbol{pbest}_{i,d}^{t}, & \text{if} \quad f(\boldsymbol{pbest}_{i,d}^{t}) \le f(\boldsymbol{X}_{i,d}^{t+1}) \\ \boldsymbol{X}_{i,d}^{t+1}, & \text{otherwise} \end{cases} \quad \text{for } d \in \{1,\ldots,n_d\} \tag{7}$$

$$\mathbf{gbest}^{t+1} = \begin{cases} \boldsymbol{pbest}_{i,d}^{t+1}, & \text{if} \quad f(\boldsymbol{pbest}_{i,d}^{t+1}) \le f(\boldsymbol{gbest}_{d}^{t}) \\ \boldsymbol{gbest}_{d}^{t}, & \text{otherwise} \end{cases} \quad \text{for } d \in \{1,\ldots,n_d\} \tag{8}$$

And the phase angle (θ) and the maximum velocity of each particle are recalculated with Eq. (9) and (10) for the next iterations. The algorithmic procedures are presented in Fig. 12.

$$\theta_i^{t+1} = \theta_i^t + \left|\cos\theta_i^t + \sin\theta_i^t\right| 2\pi \tag{9}$$

$$\boldsymbol{V}_{(\max)i,d}^{t+1} = \left|\cos\theta_i^{t+1}\right|^2 (\boldsymbol{X}_{(\max)d} - \boldsymbol{X}_{(\min)d}) \tag{10}$$

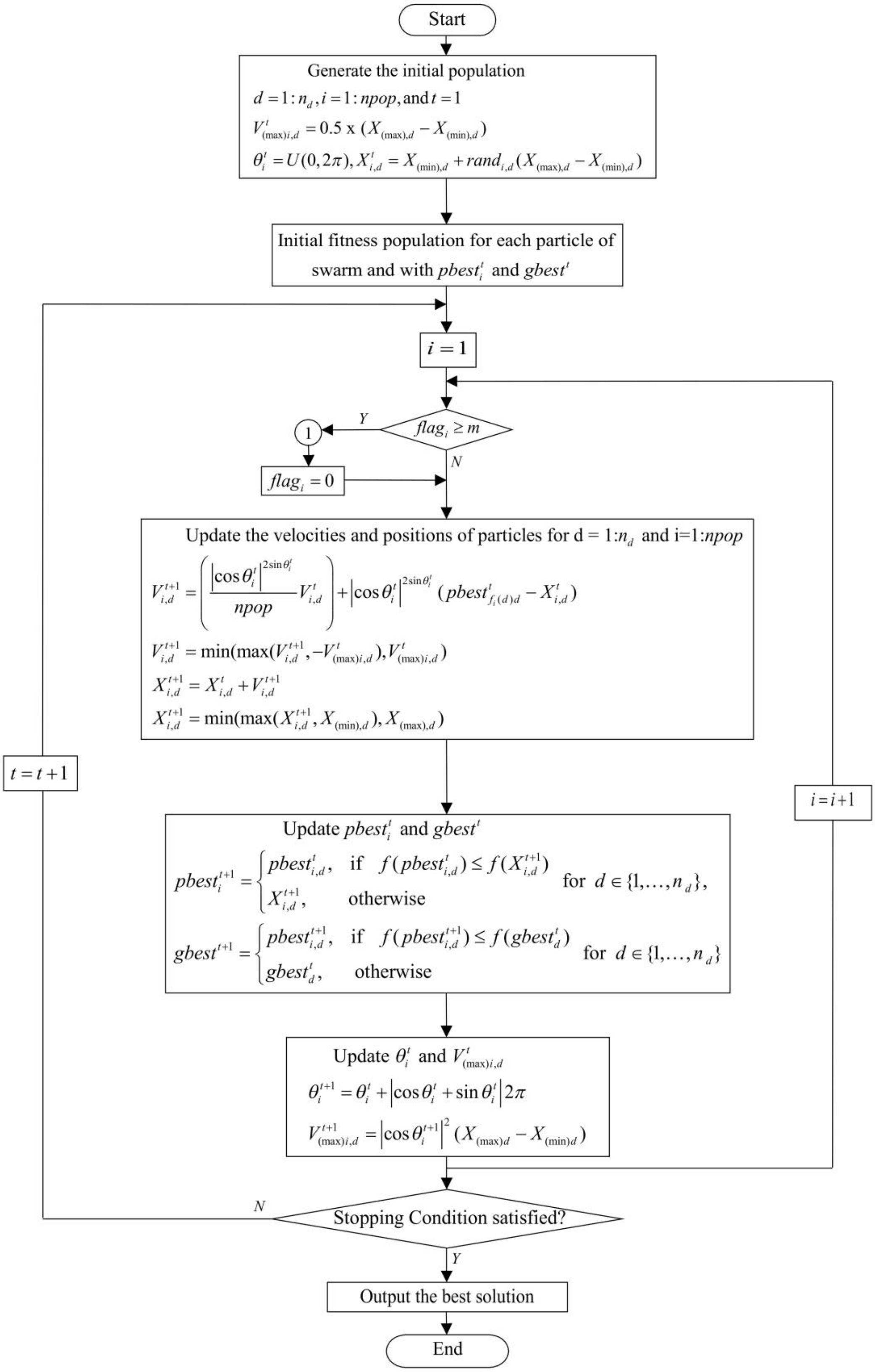


Figure 12. CLPPSO procedures.

CHAPTER 4
Design Examples

To investigate the effectiveness and robustness of the proposed CLPPSO method, the optimal sizing design of three prominent benchmarks of dome-like trusses will be performed. The first example is mid-scale dome-like truss of 120-bar while both the second and third ones are large-scale dome-like trusses of 600-bar and 1410-bar. The optimal sizing designs of all examples were successfully performed by the proposed CLPPSO method with the population of 30 particles and the total of 20 independent runs for 120-bar and 10 independent runs for 600-bar and 1410-bar dome-like trusses. The optimal results of the best and average runs of each example are mentioned to compare those of state-of-art meta-heuristic algorithms.

4.1 120-bar dome truss

The first benchmark example is the 120-member dome-like truss structure and its geometry in Fig. 13 was presented for the cost minimization of all member sizes under the limited natural frequency conditions. In this example, the additional mass of m_1 was assigned at node 1, m_2 at nodes 2 to 13, and m_3 at nodes 14 to 37. All design member areas were sorted into 7 design groups and the material properties and design parameters employed are listed in Table 1 whereas the node connectivity is presented in Table 2. The maximum number of iterations was set as 600 for this example so that the total number of structural analyses was 18000. The computational time for this problem is about 180 seconds (0.05 hours) for a run.

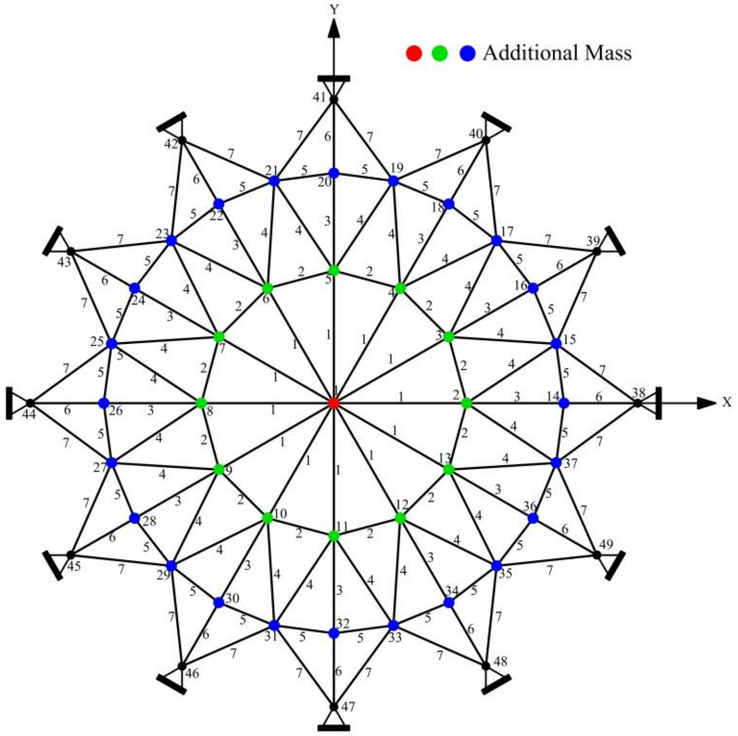
Table 1. Material properties and design parameters for 120-bar dome truss.

Parameters of 120-bar dome truss	Value
Modulus of elasticity E (N/m^2)	2.1×10^{11}
Material density ρ (kg/m^3)	7971.81
Additional mass (kg)	$m_1 = 3000$; $m_2 = 500$; $m_3 = 100$
Allowable range of cross-section (cm^2)	$1 \leq A \leq 129.3$
Constraints on the first two frequencies (Hz)	$\omega_1 \geq 9$; $\omega_2 \geq 11$

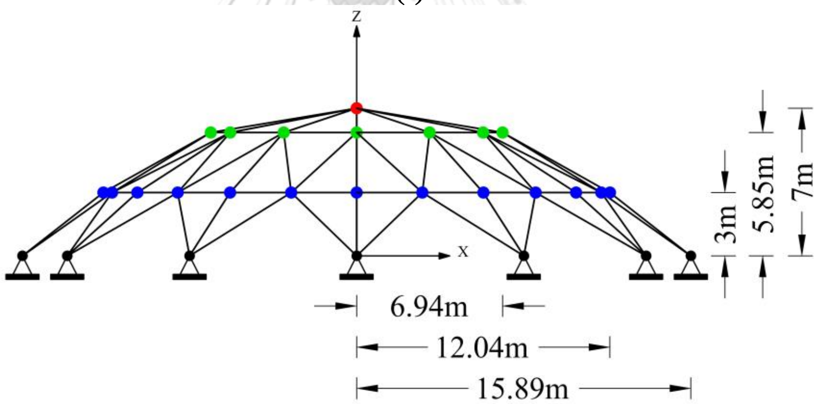
Table 3 describes the resulting member sizes of all 7 design groups and the total weight of $W = 8737.51$ kg in comparison with the optimal solutions from various meta-heuristic methods of vibrating particles system (VPS) (8888.74 kg) (Ali Kaveh & Ilchi Ghazaan, 2017), democratic particle swarm optimization (DPSO) (8890.48 kg) (A. Kaveh & Zolghadr, 2014), colliding-bodies optimization (CBO) (8889.13 kg) (A. Kaveh & Mahdavi Dahoei, 2015), harmony search-based mechanism into the particle swarm optimization with an Aging Leader and Challengers (HALC-PSO) (8889.96 kg) (A. Kaveh & Ilchi Ghazaan, 2015), improved symbiotic organisms search (ISOS) (8710.06 kg) (Tejani et al., 2018), modified sub-population teaching-learning-based optimization (MS-TLBO) (8708.73 kg) (Tejani et al., 2016), improved differential evolution (IDE) (8707.29 kg) (V. Ho-Huu, Vo-Duy, Luu-Van, Le-Anh, & Nguyen-Thoi, 2016), and adaptive hybrid evolutionary firefly (AHEFA) (8707.26 kg) (Lieu et al., 2018), respectively. More explicitly, the optimal weight computed by the proposed CLPPSO method has reduced than those of VPS by 1.701%, DPSO by 1.721%, CBO by 1.706%, and HALC-PSO by 1.715%. In addition, the performance of the proposed method agrees superbly with the referred meta-heuristic methods. with the comparable numerical efforts. For all the repeating CLPPSO solves, the convergency of the optimal solutions of both the best and mean values, see Fig. 14, occurred during the early number of iterations.

Table 2. Node connectivity of the 120-bar dome truss.

Elem. group	**Node**		**Elem. group**	**Node**		**Elem. group**	**Node**		**Elem. group**	**Node**		**Elem. group**	**Node**	
	1	2		1	2		1	2		1	2		1	2
	1	2		2	14		8	27		26	27		38	15
	1	3		3	16		27	9		27	28		15	39
	1	4		4	18		9	29		28	29		39	17
	1	5		5	20		29	10		29	30		17	40
	1	6		6	22		10	31		30	31		40	19
1	1	7	3	7	24	4	31	11	5	31	32		19	41
	1	8		8	26		11	33		32	33		41	21
	1	9		9	28		33	12		33	34		21	42
	1	10		10	30		12	35		34	35		42	23
	1	11		11	32		35	13		35	36		23	43
	1	12		12	34		13	37		36	37		43	25
	1	13		13	36		37	2		37	14		25	44
	2	3		2	15		14	15		14	38	7	44	27
	3	4		15	3		15	16		16	39		27	45
	4	5		3	17		16	17		18	40		45	29
	5	6		17	4		17	18		20	41		29	46
	6	7		4	19		18	19		22	42		46	31
	7	8		19	5		19	20		24	43		31	47
2	8	9	4	5	21	5	20	21	6	26	44		47	33
	9	10		21	6		21	22		28	45		33	48
	10	11		6	23		22	23		30	46		48	35
	11	12		23	7		23	24		32	47		35	49
	12	13		7	25		24	25		34	48		49	37
	13	2		25	8		25	26		36	49		37	38



(i)



(ii)

Figure 13. 120-bar dome truss geometry: (i) top view, (ii) side view

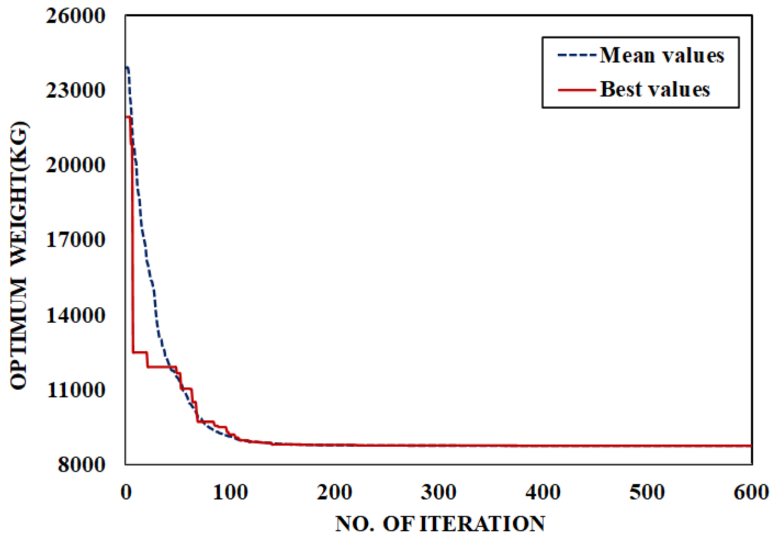


Figure 14. Solution convergence by CLPPSO method.

Table 3. Optimal design solutions of 120-bar dome truss by variations analysis methods.

Design Variables (Areas) cm^2	(Ali Kaveh & Ilchi Ghazaan, 2017)	(A. Kaveh & Zolghadr, 2014)	(A. Kaveh & Mahdavi Dahoei, 2015)	(A. Kaveh & Ilchi Ghazaan, 2015)	(Tejani et al., 2018)	(Tejani et al., 2016)	(V. Ho-Huu et al., 2016)	(Lieu et al., 2018)	Present
	VPS	DPSO	CBO	HALC-PSO	ISOS	MS-TLBO	IDE	AHEFA	CLPPSO
A_1	19.6836	19.607	19.6917	19.8905	19.6662	19.4886	19.4670	19.5094	19.2545
A_2	40.9581	41.290	41.1421	40.4045	39.8539	40.3949	40.5004	40.3867	40.5582
A_3	11.3325	11.136	11.1550	11.2057	10.6127	10.6921	10.6136	10.6033	10.8475
A_4	21.5387	21.025	21.3207	21.3768	21.2901	21.3139	21.1073	21.1168	20.9778
A_5	9.8867	10.060	9.8330	9.8669	9.7911	9.8943	9.8417	9.8221	9.9723
A_6	12.7116	12.758	12.8520	12.7200	11.7899	11.7810	11.7735	11.7735	12.0691
A_7	14.9330	15.414	15.1602	15.2236	14.7437	14.5979	14.8269	14.8405	14.7991
Best Weight (kg)	8888.74	8890.48	8889.1303	8889.96	8710.062	8708.729	8707.2898	8707.2559	8737.5055
Number of analyses	30000	6000	6000	17000	4000	4000	4060	3560	18000
Mean Weight(kg)	8896.04	8895.99	8891.254	8900.39	8728.5951	8734.7450	8707.8147	8707.5580	8738.2341
Standard Deviation	6.65	4.26	1.7926	6.38	14.2296	27.0503	0.5057	0.2535	0.4875
f_1 (Hz)	9.000	9.0001	9.000	9.000	9.001	9.0002	9.000	9.000	9.000
f_2 (Hz)	11.000	11.0007	11.0000	11.0000	10.998	11.0000	11.0000	11.0000	11.0000

4.2 600-bar dome truss

The second benchmark example is the 600-member dome-like truss structure and its geometry in Fig. 15 was presented for the cost minimization of all member sizes under the limited natural frequency conditions. In this example, the additional mass of 100 kg was assigned to all free nodes except the constraint ones. All design member areas were sorted into 25 design groups and the material properties and design parameters employed are listed in Table 4 whereas the nodes coordinates, and node connectivity are presented in Table 5 and 6, respectively. The 600-bar dome truss comprises with 24 substructures and each of these shares the angle of cyclic symmetry of 15˚. The maximum number of iterations was set as 1000 for this example so that the total number of structural analyses was 30000. The computational time for this problem is about 21,600 seconds (6 hours) for a run.

Table 7 describes the resulting member sizes of all 25 design groups and the total weight of $W = 6083.554$ kg in comparison with the optimal solutions from various meta-heuristic methods of vibrating particles system (VPS) (6120.01 kg) (Ali Kaveh & Ilchi Ghazaan, 2017), democratic particle swarm optimization (DPSO) (6344.55 kg) (A. Kaveh & Zolghadr, 2014), big bang-big crunch (BB-BC) (6394.64 kg) (A Kaveh & Zolghadr, 2018), harmony search algorithm (HS) (6357.59 kg) (A Kaveh & Zolghadr, 2018), cyclical parthenogenesis algorithm (CPA) (6336.85 kg) (A Kaveh & Zolghadr, 2018), chaotic water strider algorithm (Chaotic WSA) (6064.04 kg) (A. Kaveh et al., 2021), enhanced forensic-based investigation (EFBI) (6076.35 kg) (Ali Kaveh et al., 2021), and chaotic firefly algorithms with gaussian map (CGFA) (6058.49 kg) (A. Kaveh & Javadi, 2019). More explicitly, the optimal weight computed by the proposed CLPPSO method has reduced than those of VPS by 0.596%, DPSO by 4.114%, BB-BC by 4.865%, HS by 4.31%, and CPA by 3.997%. In addition, the performance of the proposed method agrees superbly with the referred meta-heuristic methods. with the comparable numerical efforts. For all the repeating CLPPSO solves, the convergency of the optimal solutions of both the best and mean values, see Fig. 16, occurred during the mid-number of iterations.

Table 4. Material properties and design parameters for 600-bar dome truss.

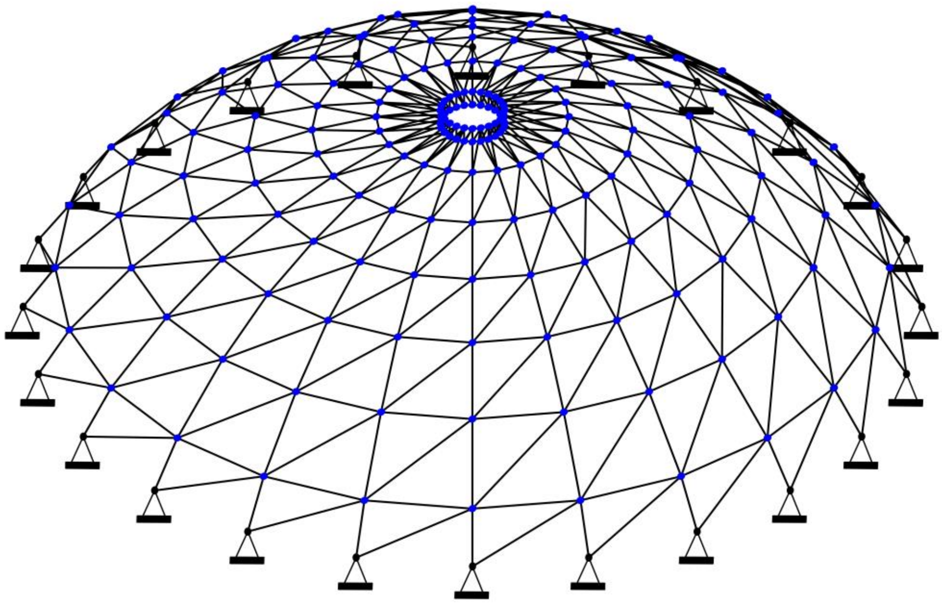
Parameters	Value
Modulus of elasticity E (N/m^2)	2×10^{11}
Material density ρ (kg/m^3)	7850
Additional mass (kg)	100
Allowable range of cross-section (cm^2)	$1 \leq A \leq 100$
Constraints on the first two frequencies (Hz)	$\omega_1 \geq 5$; $\omega_2 \geq 7$

Table 5. Coordinates of the nodes of the 600-bar dome truss.

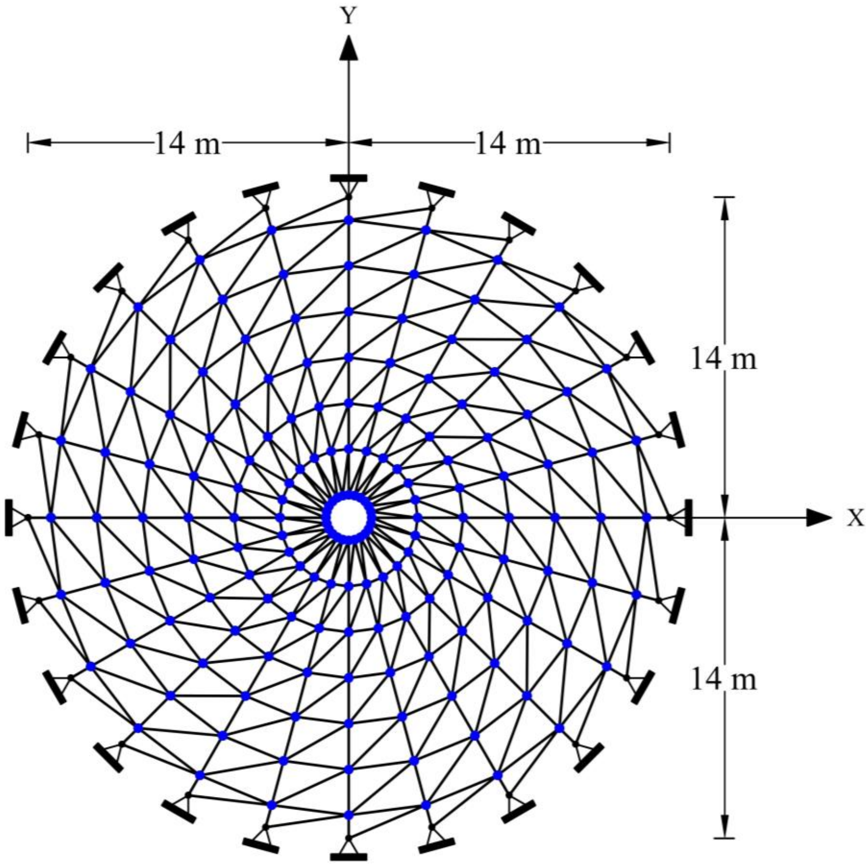
Node number	Coordinates	Node number	Coordinates	Node number	Coordinates
1	(1, 0, 7)	4	(5, 0, 6.75)	7	(11, 0, 3.5)
2	(1, 0, 7.5)	5	(7, 0, 6)	8	(13, 0, 1.5)
3	(3, 0, 7.25)	6	(9, 0, 5)	9	(14, 0, 0)

Table 6. Node connectivity of the typical substructure of 600-bar dome truss.

Elem.	**Node**		**Elem.**	**Node**		**Elem.**	**Node**		**Elem.**	**Node**	
group	1	2	**group**	1	2	**group**	1	2	**group**	1	2
1	1	2	8	3	11	15	5	14	22	8	9
2	1	3	9	3	12	16	6	7	23	8	16
3	1	10	10	4	5	17	6	14	24	8	17
4	1	11	11	4	12	18	6	15	25	9	17
5	2	3	12	4	13	19	7	8			
6	2	11	13	5	6	20	7	15			
7	3	4	14	5	13	21	7	16			



(i)



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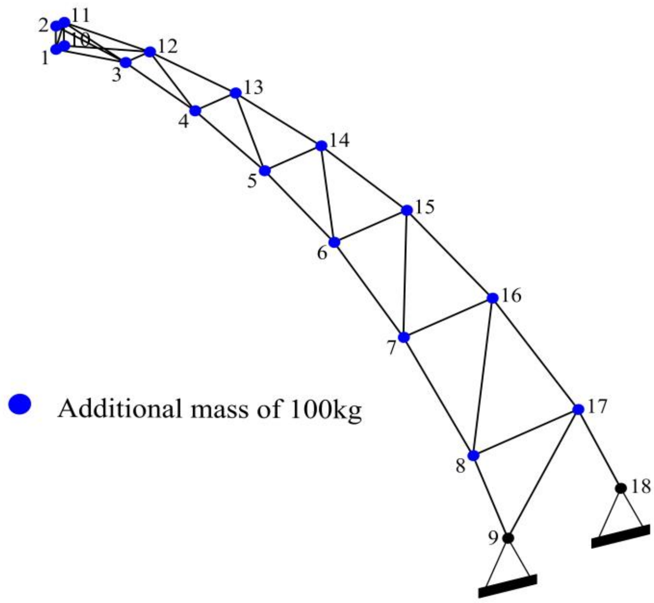


(iii)

Figure 15. 600-bar dome truss geometry(i) 3D view, (ii) Plan view, (iii) Substructure.

Table 7. Optimal design solutions of 600-bar dome truss by variations analysis methods.

Design Variables (Areas) cm^2	(Ali Kaveh & Ilchi Ghazaan, 2017)	(A. Kaveh & Zolghadr, 2014)	(A. Kaveh et al., 2021)	(Ali Kaveh et al., 2021)	(A. Kaveh & Javadi, 2019)	(A Kaveh & Zolghadr, 2018)			Present
	VPS	DPSO	Chaotic WSA	EFBI	CGFA	BB-BC	HS	CPA	CLPPSO
A_1	1.3155	1.365	1.4829	1.0999	1.3190	1.502	1.439	1.155	1.27
A_2	1.2299	1.391	1.2619	1.4922	1.3826	1.495	1.425	1.304	1.2054
A_3	5.5506	5.686	4.9784	6.0744	4.9379	7.077	4.942	4.178	4.3937
A_4	1.3867	1.511	1.4155	1.6234	1.3222	93.172	1.677	1.335	1.9413
A_5	17.4275	17.711	17.5189	17.4918	17.1285	18.93	18.331	18.375	15.9245
A_6	40.143	36.266	36.8573	37.2118	37.4657	32.699	36.074	39.914	40.6525
A_7	12.8848	13.263	13.0251	12.7873	12.7071	14.601	13.407	13.609	12.8582
A_8	15.5413	16.919	15.0761	14.8239	15.4252	15.492	17.066	16.47	15.6351
A_9	12.2428	13.333	11.6297	12.1764	11.3642	13.533	13.122	14.108	11.9356
A_{10}	9.3776	9.534	9.5607	9.0163	9.3343	10.424	10.061	10.038	9.3706
A_{11}	8.6684	9.884	8.2689	8.5044	8.3872	10.171	9.827	9.514	8.5435
A_{12}	9.1659	9.547	8.8515	8.9951	9.1101	11.374	9.388	9.329	9.7151
A_{13}	7.1664	7.866	7.0387	7.0357	7.1472	8.184	7.083	6.938	6.9767
A_{14}	5.217	5.529	5.2711	5.0993	5.1701	5.857	5.697	5.545	5.4602
A_{15}	6.5346	7.007	6.5632	6.1918	6.6239	7.669	7.139	6.763	6.5435
A_{16}	5.4741	5.462	5.1025	4.9514	5.2427	5.985	5.082	5.209	5.6899
A_{17}	3.6545	3.853	3.4304	3.9186	3.5213	3.807	3.295	3.842	3.4203
A_{18}	7.6034	7.432	7.7083	7.6312	7.6096	10.361	7.663	8.112	7.644
A_{19}	4.2251	4.261	4.3958	4.4271	4.2877	4.824	4.1	4.252	4.1368
A_{20}	1.9717	2.253	2.0435	2.3280	2.1684	2.506	1.882	2.227	1.9880
A_{21}	4.5107	4.337	4.4764	4.8534	4.6704	5.363	4.725	4.582	4.4873
A_{22}	3.5251	4.028	3.6590	3.9632	3.5380	5.353	3.86	3.336	3.6142
A_{23}	1.9255	1.954	1.9727	1.8527	1.8252	1.695	2.28	1.725	1.9907
A_{24}	4.7628	4.709	4.8843	4.7818	4.8110	5.762	4.912	4.675	4.8404
A_{25}	1.6854	1.410	1.6167	1.4354	1.6589	1.651	1.502	1.673	1.6092
Best Weight (kg)	6120.01	6344.55	6064.04	6076.35	6058.49	6394.64	6357.59	6336.85	6083.554
Number of analyses	30000	9000	30000	12000	10000	-	-	40000	30000
Mean Weight(kg)	6158.11	6674.71	6081.23	6098.52	6076.67	6704.11	6631.48	6376.01	6093.4281
Standard Deviation	28.49	473.21	8.29	11.95	22.42	551.65	304.09	90.39	12.0012
f_1 (Hz)	5.000	5.000	5.000	5.0001	5.000	5.001	5.000	5.000	5.0000

Design Variables (Areas) cm^2	(Ali Kaveh & Ilchi Ghazaan, 2017)	(A. Kaveh & Zolghadr, 2014)	(A. Kaveh et al., 2021)	(Ali Kaveh et al., 2021)	(A. Kaveh & Javadi, 2019)	(A Kaveh & Zolghadr, 2018)			Present
	VPS	DPSO	Chaotic WSA	EFBI	CGFA	BB-BC	HS	CPA	CLPPSO
f_2 (Hz)	7.000	7.000	7.000	7.0000	7.000	7.000	7.000	7.000	7.0000

4.3 1410-bar dome truss

The third benchmark example is the 1410-member dome-like truss structure and its geometry in Fig. 17 was presented for the cost minimization of all member sizes under the limited natural frequency conditions. In this example, the additional mass of 100 kg was assigned to all free nodes except the constraint ones. All design member areas were sorted into 47 design groups and the material properties and design parameters employed are listed in Table 8 whereas the nodes coordinates, and node connectivity are presented in Table 9 and 10, respectively. The 600-bar dome truss comprises with 30 substructures and each of these shares the angle of cyclic symmetry of 12˚. The maximum number of iterations was set as 1000 for this example so that the total number of structural analyses was 30000. The computational time for this problem is about 108,000 seconds (30 hours) for a run.

Table 11 describes the resulting member sizes of all 47 design groups and the total weight of $W = 10534.85$ kg in comparison with the optimal solutions from (10453.84 kg) (A. Kaveh & Zolghadr, 2014), enhanced colliding bodies optimization (ECBO) (10504.2 kg) (A. Kaveh & Ilchi Ghazaan, 2016), parameter free jaya algorithm (PFJA) (10326.296 kg) (Degertekin et al., 2021), big bang-big crunch (BB-BC) (10772.11 kg) (A Kaveh & Zolghadr, 2018), harmony search algorithm (HS) (10922.7 kg) (A Kaveh & Zolghadr, 2018), cyclical parthenogenesis algorithm (CPA) (10435.47 kg)(A Kaveh & Zolghadr, 2018), and chaotic water strider algorithm (Chaotic WSA) (10318.99 kg)(A. Kaveh et al., 2021). More explicitly, the optimal weight computed by the proposed CLPPSO method has reduced than those of BB-BC by 2.203%, and HS by 3.551%. In addition, the performance of the proposed method agrees superbly with the referred meta-heuristic methods. with the comparable numerical efforts. For all the repeating CLPPSO solves, the convergency of the optimal solutions of both the best and mean values, see Fig. 18, occurred during the mid-number of iterations.

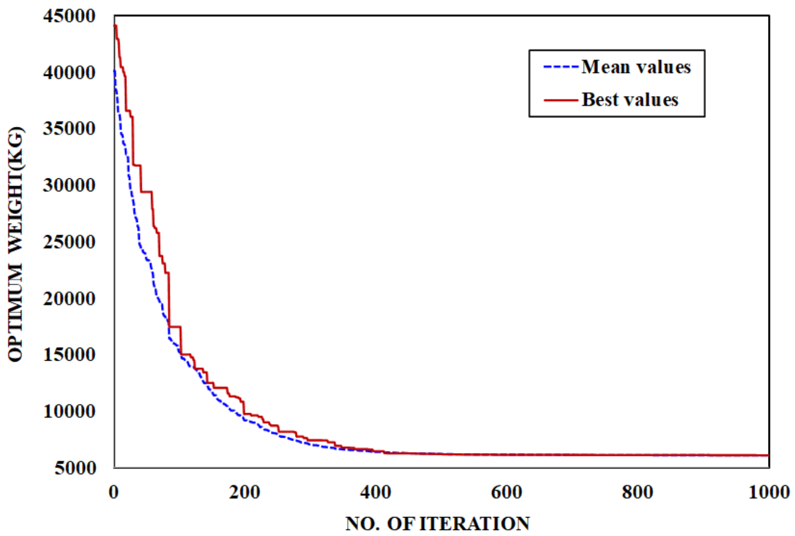


Figure 16. Solution convergence of 600-bar dome truss by CLPPSO method.

Table 8. Material properties and design parameters for 1410-bar dome truss.

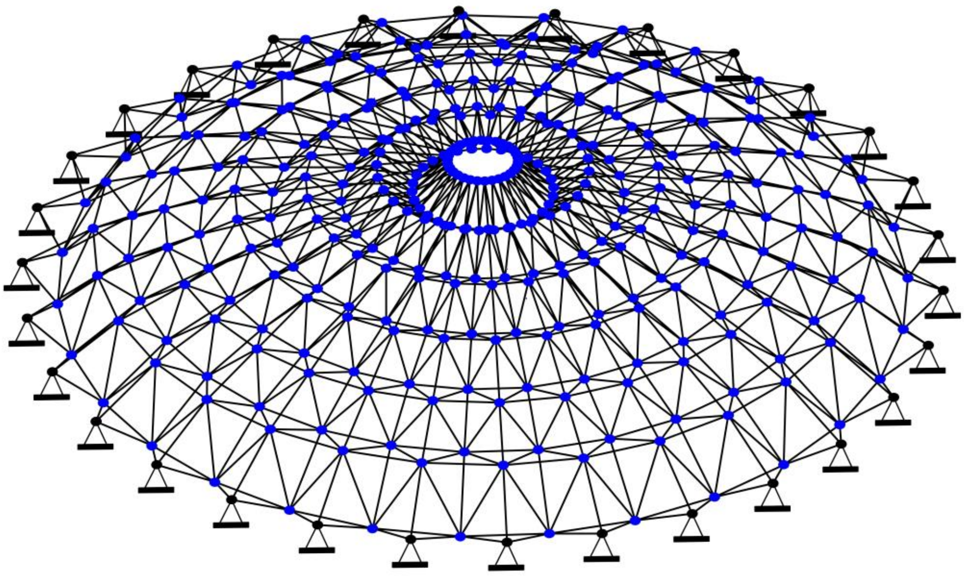
Parameters	Value
Modulus of elasticity *E* (N/m^2)	2×10^{11}
Material density ρ (kg/m^3)	7850
Additional mass (kg)	100
Allowable range of cross-section (cm^2)	$1 \leq A \leq 100$
Constraints on the first two frequencies (Hz)	$\omega_1 \geq 7$; $\omega_2 \geq 9$

Table 9. Coordinates of the nodes of the 1410-bar dome truss.

Node number	Coordinates	Node number	Coordinates
1	(1, 0, 4)	8	(1.989, 0.209, 3)
2	(3, 0, 3.75)	9	(3.978, 0.418, 2.75)
3	(5, 0, 3.25)	10	(5.967, 0.627, 2.25)
4	(7, 0, 2.75)	11	(7.956, 0.836, 1.75)
5	(9, 0, 2)	12	(9.945, 1.0453, 1)
6	(11, 0, 1.25)	13	(11.934, 1.2543, -0.5)
7	(13, 0, 0)		

Table 10. Node connectivity of the typical substructure of 600-bar dome truss.

Elem. group	**Node** 1	2	**Elem. group**	**Node** 1	2	**Elem. group**	**Node** 1	2	**Elem. group**	**Node** 1	2
1	1	2	13	4	10	25	8	9	37	11	12
2	1	8	14	4	11	26	8	14	38	11	17
3	1	14	15	4	17	27	8	15	39	11	18
4	2	3	16	5	6	28	8	21	40	11	24
5	2	8	17	5	11	29	9	10	41	12	13
6	2	9	18	5	12	30	9	15	42	12	18
7	2	15	19	5	18	31	9	16	43	12	19
8	3	4	20	6	7	32	9	22	44	12	25
9	3	9	21	6	12	33	10	11	45	13	19
10	3	10	22	6	13	34	10	16	46	13	20
11	3	16	23	6	19	35	10	17	47	13	26
12	4	5	24	7	13	36	10	23			



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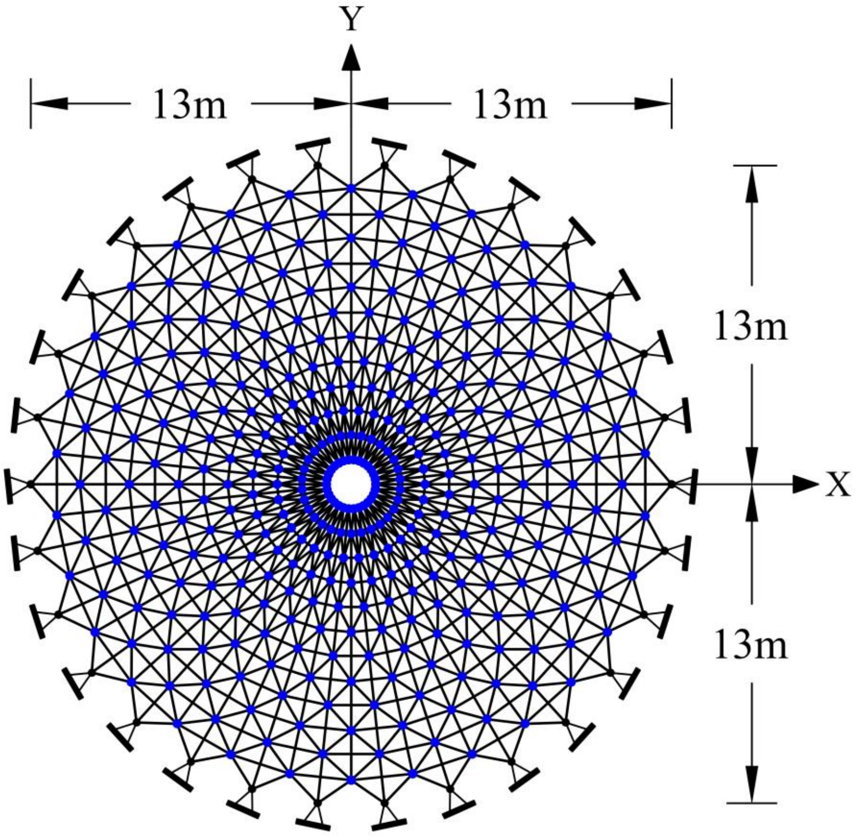

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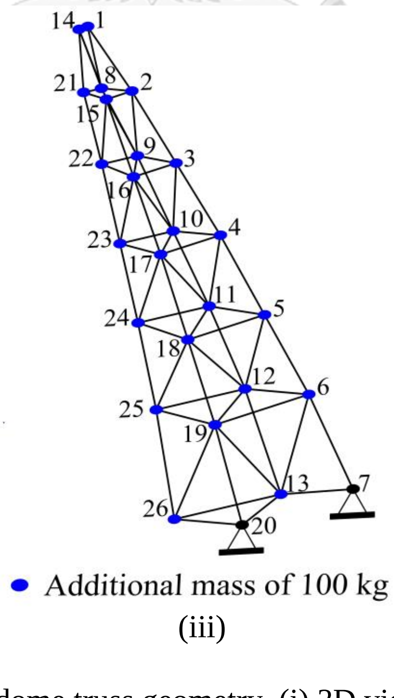

(iii)

Figure 17. 1410-bar dome truss geometry, (i) 3D view, (ii) Plan view, (iii) Substructure.

Table 11. Optimal design solutions of 1410-bar dome truss by variations analysis methods.

Design Variables (Areas) cm^2	(Ali Kaveh & Ilchi Ghazaan, 2017)	(A. Kaveh & Zolghadr, 2014)	(A. Kaveh & Ilchi Ghazaan, 2016)	(Degertekin et al., 2021)	(A. Kaveh et al., 2021)	(A Kaveh & Zolghadr, 2018)			Present
	VPS	DPSO	ECBO	PFJA	Chaotic WSA	BB-BC	HS	CPA	CLPPSO
A_1	5.633	7.209	7.9969	6.1902	6.3476	3.746	2.826	7.416	5.35
A_2	4.7628	5.006	6.1723	4.4036	5.188	3.857	3.109	4.768	3.4
A_3	37.7385	38.446	35.5011	31.2253	24.4074	40.493	26.874	38.993	26.5938
A_4	7.4927	9.438	10.251	8.4715	8.5241	8.869	8.358	8.966	8.0668
A_5	3.1824	4.313	5.3727	4.8590	5.5439	4.431	4.932	4.511	3.6831
A_6	1.0193	1.494	1.3488	1.5759	1.202	2.708	3.496	1.544	2.3176
A_7	8.9475	8.455	11.4427	12.9451	14.6949	35.164	46.153	8.371	21.3495
A_8	10.4272	9.488	9.7157	9.3263	9.3726	9.475	9.635	9.276	10.3539
A_9	4.1398	3.48	1.3005	3.2716	1.462	3.879	3.016	3.583	6.3859
A_{10}	3.1408	3.495	2.5046	3.2878	2.5768	5.345	2.409	3.476	4.5076
A_{11}	15.4194	16.037	10.7849	12.6719	10.722	17.692	8.074	15.531	13.8374
A_{12}	8.9931	9.796	10.1954	10.0979	8.7231	11.417	10.214	10.285	10.876
A_{13}	3.1988	2.413	2.23	2.5803	1.9054	3.097	1.794	2.497	3.0903
A_{14}	7.1565	5.681	5.1186	5.3769	3.8895	6.122	4.723	5.397	5.599
A_{15}	17.8564	15.806	14.0053	16.0581	12.8913	12.937	20.052	16.503	14.2288
A_{16}	9.2685	8.078	8.9713	8.6789	8.05	7.888	8.151	8.193	9.8232
A_{17}	3.3221	3.931	4.0756	3.3199	2.9941	4.82	4.052	3.829	2.9863
A_{18}	6.1486	6.099	5.9211	6.4966	7.2349	6.491	5.387	6.151	7.6236
A_{19}	8.4422	10.771	10.6915	10.8804	15.3852	9.399	8.506	10.465	8.6671
A_{20}	12.8578	13.775	10.622	14.0056	13.8992	12.493	12.763	13.925	12.1283
A_{21}	5.8031	4.231	4.5064	5.0843	5.6867	5.002	4.809	4.415	5.1126
A_{22}	7.5484	6.995	8.4086	6.9952	7.8515	7.055	8.002	6.863	6.4542
A_{23}	1.4805	1.837	5.8405	1.0270	1.0011	1.436	1.545	1.769	1.2039
A_{24}	4.5332	4.397	5.0342	4.3788	4.327	4.831	4.559	4.339	4.2053
A_{25}	2.0347	2.115	3.8932	2.1951	3.5281	1.807	2.387	2.115	2.2692
A_{26}	5.8589	4.923	6.1647	4.2562	4.5177	3.003	1.887	4.951	3.1514
A_{27}	2.4401	4.047	6.899	4.6605	6.8448	3.752	6.594	4.147	3.9958
A_{28}	6.925	5.906	11.6387	8.8694	12.9102	10.49	18.697	6.044	7.6868
A_{29}	3.3875	3.392	3.8343	3.2333	3.8706	3.658	4.142	3.222	3.3384

Design Variables (Areas) cm^2	(Ali Kaveh & Ilchi Ghazaan, 2017)	(A. Kaveh & Zolghadr, 2014)	(A. Kaveh & Ilchi Ghazaan, 2016)	(Degertekin et al., 2021)	(A. Kaveh et al., 2021)	(A Kaveh & Zolghadr, 2018)			Present
	VPS	DPSO	ECBO	PFJA	Chaotic WSA	BB-BC	HS	CPA	CLPPSO
A_{30}	1.5024	1.902	1.4772	1.7611	1.0192	2.534	3.67	1.97	3.4703
A_{31}	4.0498	4.381	1.3075	3.2831	1.2962	4.368	3.039	4.29	4.498
A_{32}	11.0886	8.442	4.4876	7.1936	2.5497	16.022	19.853	8.02	8.4914
A_{33}	5.4639	5.011	6.0196	4.9840	5.6478	5.056	5.415	4.857	6.07
A_{34}	2.8459	3.577	2.6729	3.6672	2.775	3.629	2.184	3.689	4.7849
A_{35}	2.3136	2.805	1.6342	2.4062	2.1062	3.812	2.55	2.831	4.1645
A_{36}	3.437	2.024	1.841	2.1576	2.5833	1.089	1.003	1.985	1.0546
A_{37}	8.0225	6.709	6.8841	7.1043	7.3146	6.624	8.139	6.373	7.2711
A_{38}	5.8009	5.054	4.1393	5.2070	3.7673	5.233	5.991	4.865	4.9319
A_{39}	4.4004	3.259	3.3264	3.6853	2.9003	3.306	3.964	3.412	4.3437
A_{40}	1.0005	1.063	1	1.0007	1	1.122	1.048	1.027	1.0317
A_{41}	7.7222	5.934	6.9376	6.6302	7.0355	6.62	6.642	6.218	6.1891
A_{42}	5.2574	7.057	4.4568	6.6773	6.9735	4.623	7.091	7.342	7.2491
A_{43}	4.5055	5.745	4.6758	5.2167	5.5549	4.99	5.56	5.458	4.8191
A_{44}	1.0005	1.185	1.0084	1.0016	1.0001	1.049	1.273	1.14	1
A_{45}	7.9383	7.274	7.5103	8.1289	8.5706	7.869	6.769	7.401	7.0979
A_{46}	4.7805	4.798	5.2449	4.5151	5.6116	3.942	4.557	4.578	4.3028
A_{47}	1.0054	1.515	1.055	1.0010	1.0127	1.008	1.247	1.561	1.0716
Best Weight (kg)	10491.83	10453.84	10504.20	10326.296	10318.99	10772.11	10922.7	10435.47	10534.85
Number of analyses	-	50000	7920	16900	30000	-	-	80000	30000
Mean Weight(kg)	10936.34	11100.57	10590.67	10399.828	10521.67	11534.63	11956.03	10658.48	10619.4875
Standard Deviation	158.39	334.2	52.51	75.441	122.146	219.31	501.16	129.9	109.3302
f_1 (Hz)	-	7.001	7.002	7.0009	7.0000	7.003	7.001	7.000	7.0000
f_2 (Hz)	-	9.003	9.001	9.0001	9.0021	9.000	9.002	9.000	9.0000

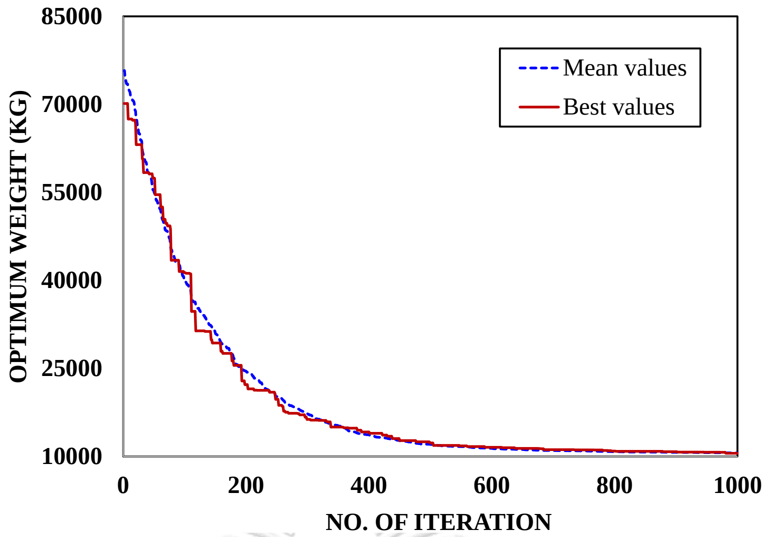

Figure 18. Solution convergence of 1410-bar dome truss by CLPPSO method.

CHAPTER 5
CONCLUSION

5.1 Conclusions

This research presents a new variant of particle swarm optimization called the phasor particle swarm optimization with comprehensive learning strategy (CLPPSO) for the optimal design of dome-like truss structures under the limited frequency-constraints. The approach mathematically adopts the direct combination of both the phasor theory in mathematics and comprehensive learning strategy to the particle swarm optimization that keeps the swarm's variability from eschewing premature convergence. The phase angle incorporating the periodic sine and cosine functions is essentially applied to model particle control parameters, through which the comprehensive strategy focuses only on the choice of the previous best positions of all particles for updating the exemplar particle’s velocity during the optimization process. The applications of the proposed method have been efficaciously shown through the optimal sizing design of the three various dome truss structures of 120-bars, 600-bars, and 1410-bars, respectively.

For all benchmarks, the number of populations of 30 gives the optimal results with the proper convergence rate making a balance between the optimal results and the computational time. The accuracy and robustness of the proposed techniques are remarkably evinced for all benchmark examples by competitive manners with those reported using different metaheuristic in the literature regarding their optimum solutions. In addition, the proposed method can also be maintained the likelihood of the premature convergence of the standard PSO exclusively in large-size dome-truss structures.

5.2 Future Research

Ongoing extension of this research are summarized as follows:

- The development of the CLPPSO method should be integrated on shape and topology optimization.
- In addition, it can be collaborated the CLPPSO method with some numerical techniques to upgrade the computational time for eigenvalue problems.

REFERENCES

Abdel-Basset, M., Abdel-Fatah, L., & Sangaiah, A. K. (2018). Chapter 10 - Metaheuristic Algorithms: A Comprehensive Review. In A. K. Sangaiah, M. Sheng, & Z. Zhang (Eds.), Computational Intelligence for Multimedia Big Data on the Cloud with Engineering Applications (pp. 185-231): Academic Press.

Bellagamba, L., & Yang, T. Y. (1981). Minimum-mass truss structures with constraints on fundamental natural frequency. 19(11), 1452-1458. doi:10.2514/3.7875

Blum, C., & Roli, A. J. A. c. s. (2003). Metaheuristics in combinatorial optimization: Overview and conceptual comparison. 35(3), 268-308.

Camp, C. V. (2007). Design of space trusses using big bang-big crunch optimization. 133(7), 999-1008. doi:doi:10.1061/(ASCE)0733-9445(2007)133:7(999)

Camp, C. V., Meyer, B. J., & Palazolo, P. J. (2004). Particle Swarm Optimization for The Design of Trusses. In Structures 2004 (pp. 1-10).

Chen, W.-F., & Lui, E. M. (2005). Handbook of structural engineering: CRC press.

Chopra, A. K. (2007). Dynamics of Structures: Pearson Education.

Degertekin, S. O., Yalcin Bayar, G., & Lamberti, L. (2021). Parameter free Jaya algorithm for truss sizing-layout optimization under natural frequency constraints. Computers & Structures, 245, 106461. doi:https://doi.org/10.1016/j.compstruc.2020.106461

Den Hartog, J. P. (1985). Mechanical Vibrations: Dover Publications.

Ghasemi, M., Akbari, E., Rahimnejad, A., Razavi, S. E., Ghavidel, S., & Li, L. (2019). Phasor particle swarm optimization: a simple and efficient variant of PSO. Soft Computing, 23(19), 9701-9718. doi:10.1007/s00500-018-3536-8

Gholizadeh, S., & Barzegar, A. (2012). Shape optimization of structures for frequency constraints by sequential harmony search algorithm. Engineering Optimization - ENG OPTIMIZ, 45, 1-20. doi:10.1080/0305215X.2012.704028

Gomes, H. M. (2011). Truss optimization with dynamic constraints using a particle swarm algorithm. 38(1 %J Expert Syst. Appl.), 957–968. doi:10.1016/j.eswa.2010.07.086

Grandhi, R. (1993). Structural optimization with frequency constraints - A review. 31(12), 2296-2303. doi:10.2514/3.11928

Haftka, R. T. J. A. j. (1985). Simultaneous analysis and design. 23(7), 1099-1103.

Ho-Huu, V., Truong, T., Le, L., & Vo-Duy, T. (2018). An improved differential evolution based on roulette wheel selection for shape and size optimization of truss structures with frequency constraints. Neural Computing and Applications, 29. doi:10.1007/s00521-016-2426-1

Ho-Huu, V., Vo-Duy, T., Luu-Van, T., Le-Anh, L., & Nguyen-Thoi, T. (2016). Optimal design of truss structures with frequency constraints using improved differential evolution algorithm based on an adaptive mutation scheme. Automation in Construction, 68, 81-94. doi:https://doi.org/10.1016/j.autcon.2016.05.004

Holland, J. H. (1992). Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence: MIT press.

Inman, D. J. (2001). Engineering Vibration: Prentice Hall.

Joines, J. A., & Houck, C. R. (1994, 27-29 June 1994). On the use of non-stationary penalty functions to solve nonlinear constrained optimization problems with GA's. Paper presented at the Proceedings of the First IEEE Conference on Evolutionary Computation. IEEE World Congress on Computational Intelligence.

Kaveh, A., Amirsoleimani, P., Dadras Eslamlou, A., & Rahmani, P. (2021). Frequency-constrained optimization of large-scale dome-shaped trusses using chaotic water strider algorithm. Structures, 32, 1604-1618. doi:https://doi.org/10.1016/j.istruc.2021.03.033

Kaveh, A., Hamedani, K. B., & Kamalinejad, M. (2021). An enhanced Forensic-Based Investigation algorithm and its application to optimal design of frequency-constrained dome structures. Computers & Structures, 256, 106643. doi:https://doi.org/10.1016/j.compstruc.2021.106643

Kaveh, A., & Ilchi Ghazaan, M. (2015). Hybridized optimization algorithms for design of trusses with multiple natural frequency constraints. Advances in Engineering Software, 79, 137-147. doi:https://doi.org/10.1016/j.advengsoft.2014.10.001

Kaveh, A., & Ilchi Ghazaan, M. (2016). Optimal design of dome truss structures with dynamic frequency constraints. Structural and Multidisciplinary Optimization, 53(3), 605-621. doi:10.1007/s00158-015-1357-2

Kaveh, A., & Ilchi Ghazaan, M. (2017). Vibrating particles system algorithm for truss optimization with multiple natural frequency constraints. Acta Mechanica, 228(1), 307-322. doi:10.1007/s00707-016-1725-z

Kaveh, A., & Javadi, S. M. (2014). Shape and size optimization of trusses with multiple frequency constraints using harmony search and ray optimizer for enhancing the particle swarm optimization algorithm. Acta Mechanica, 225(6), 1595-1605. doi:10.1007/s00707-013-1006-z

Kaveh, A., & Javadi, S. M. (2019). Chaos-based firefly algorithms for optimization of cyclically large-size braced steel domes with multiple frequency constraints. Computers & Structures, 214, 28-39. doi:https://doi.org/10.1016/j.compstruc.2019.01.006

Kaveh, A., & Mahdavi Dahoei, V. (2015). Colliding-Bodies Optimization for Truss Optimization with Multiple Frequency Constraints. Journal of Computing in Civil Engineering, 29. doi:10.1061/(ASCE)CP.1943-5487.0000402

Kaveh, A., & Mahjoubi, S. (2019). Hypotrochoid spiral optimization approach for sizing and layout optimization of truss structures with multiple frequency constraints. Engineering with Computers, 35(4), 1443-1462. doi:10.1007/s00366-018-0675-6

Kaveh, A., & Zolghadr, A. (2014). Democratic PSO for truss layout and size optimization with frequency constraints. Computers & Structures, 130, 10-21. doi:https://doi.org/10.1016/j.compstruc.2013.09.002

Kaveh, A., & Zolghadr, A. (2018). Optimal design of cyclically symmetric trusses with frequency constraints using cyclical parthenogenesis algorithm. 21(5), 739-755. doi:10.1177/1369433217732492

Kennedy, J., & Eberhart, R. (1995, 27 Nov.-1 Dec. 1995). Particle swarm optimization. Paper presented at the Proceedings of ICNN'95 - International Conference on Neural Networks.

Khatibinia, M., & Sadegh Naseralavi, S. (2014). Truss optimization on shape and sizing with frequency constraints based on orthogonal multi-gravitational search algorithm. Journal of Sound and Vibration, 333(24), 6349-6369. doi:https://doi.org/10.1016/j.jsv.2014.07.027
Kirsch, U. (1993). Structural optimization : fundamentals and applications. Berlin; New York: Springer-Verlag.
Liang, J. J., Qin, A. K., Suganthan, P. M., & Baskar, S. (2004, 10-13 Oct. 2004). Particle swarm optimization algorithms with novel learning strategies. Paper presented at the 2004 IEEE International Conference on Systems, Man and Cybernetics (IEEE Cat. No.04CH37583).
Liang, J. J., Qin, A. K., Suganthan, P. N., & Baskar, S. (2006). Comprehensive learning particle swarm optimizer for global optimization of multimodal functions. IEEE Transactions on Evolutionary Computation, 10(3), 281-295. doi:10.1109/TEVC.2005.857610
Lieu, Q. X., Do, D. T. T., & Lee, J. (2018). An adaptive hybrid evolutionary firefly algorithm for shape and size optimization of truss structures with frequency constraints. Computers & Structures, 195, 99-112. doi:https://doi.org/10.1016/j.compstruc.2017.06.016
Lingyun, W., Mei, Z., Guangming, W., & Guang, M. (2005). Truss optimization on shape and sizing with frequency constraints based on genetic algorithm. Computational Mechanics, 35(5), 361-368. doi:10.1007/s00466-004-0623-8
Liu, S., Zhu, H.-p., Chen, Z., & Cao, H. (2019). Frequency-constrained truss optimization using the fruit fly optimization algorithm with an adaptive vision search strategy. Engineering Optimization, 52, 1-21. doi:10.1080/0305215X.2019.1624738
Loughlin, D. H., & Ranjithan, S. R. (1997). The Neighborhood Constraint Method: A Genetic Algorithm-Based Multiobjective Optimization Technique. Paper presented at the ICGA.
Lu, P., Chen, S., & Zheng, Y. J. M. P. i. E. (2012). Artificial intelligence in civil engineering. 2012.
Millan-Paramo, C., & Abdalla Filho, J. E. (2020). Size and Shape Optimization of Truss Structures with Natural Frequency Constraints Using Modified Simulated Annealing Algorithm. Arabian Journal for Science and Engineering, 45(5), 3511-3525. doi:10.1007/s13369-019-04138-5
Rao, S. S. (2011). Chapter 12 - Dynamic Analysis. In S. S. Rao (Ed.), The Finite Element Method in Engineering (Fifth Edition) (pp. 427-469). Boston: Butterworth-Heinemann.
Rozvany, G. I. (2012). Structural design via optimality criteria: the Prager approach to structural optimization (Vol. 8): Springer Science & Business Media.
Saka, M. P., & Geem, Z. W. J. M. p. i. e. (2013). Mathematical and metaheuristic applications in design optimization of steel frame structures: an extensive review. 2013.
Saka, M. P., Hasançebi, O., Geem, Z. W. J. S., & Computation, E. (2016). Metaheuristics in structural optimization and discussions on harmony search algorithm. 28, 88-97.
Shi, Y., & Eberhart, R. (1998, 4-9 May 1998). A modified particle swarm optimizer. Paper presented at the 1998 IEEE International Conference on Evolutionary

Computation Proceedings. IEEE World Congress on Computational Intelligence (Cat. No.98TH8360).

Talbi, E.-G. (2009). Metaheuristics - From Design to Implementation.

Tejani, G. G., Savsani, V. J., & Patel, V. K. (2016). Modified sub-population teaching-learning-based optimization for design of truss structures with natural frequency constraints. Mechanics Based Design of Structures and Machines, 44(4), 495-513. doi:10.1080/15397734.2015.1124023

Tejani, G. G., Savsani, V. J., Patel, V. K., & Mirjalili, S. (2018). Truss optimization with natural frequency bounds using improved symbiotic organisms search. Knowledge-Based Systems, 143, 162-178. doi:https://doi.org/10.1016/j.knosys.2017.12.012

Tushaj, E., Lak, N. J. I. J. o. S., & Research, E. (2017). A review of structural size optimization techniques applied in the engineering design. 8(8), 706-714.

Wolpert, D. H., & Macready, W. G. J. I. t. o. e. c. (1997). No free lunch theorems for optimization. 1(1), 67-82.

Yang, X.-S. (2010). Nature-Inspired Metaheuristic Algorithms.

Yang, X.-S., & Koziel, S. (2011). Computational optimization and applications in engineering and industry (Vol. 359): Springer Science & Business Media.

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