การออกแบบระบบบนชิพสำหรับตัวควบคุมหุ่นยนต์หกแกนด้วยเอฟพีจีเอต้นทุนต่ำ


## A SYSTEM ON CHIP DESIGN OF A 6-AXIS ROBOTIC ARM CONTROLLER <br> IMPLEMENTED ON A LOW-COST FPGA



Accepted by the Faculty of Engineering, Chulalongkorn University in Partial Fulfillment of the Requirements for the Master's Degree
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(Associate Professor Boonsom Lerdhirunwong, Dr. Ing.)

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อากัส บีโจ : การออกแบบระบบบนชิพสำหรับตัวควบคุมหุ่นยนต์หกแกนด้วยเอฟพีจีเอต้นทุนต่ำ ( A System On Chip Design of a 6-Axis Robotic Arm Controller Implemented on a Low-Cost FPGA ) อ.ที่ปร็กษษาวิทยานิพนธ์หลัก : ผศ.ดร.วันเฉลิม โปรา, 101 หน้า.

วิทยานิพนธ์จบับนี้นำเสนอการพัฒนาตัวควบคุมแขนกลหุ่นยนต์ 6 แกน ซึ่งใช้พลังงานจากเครื่องศูบ ไฮโดรลิก โดยแฮนกลแต่ละแกนจะถูกขับเคลื่อนด้วยลิ้นเซอร์โว ตัวควบคุมนี้สร้างขึ้นโดยใช้ FPGA ราคาต่ำ หน่วยประมวลผลหลักของตัวควบคุมรองรับอุปกรณ์รอบข้างหลายประธภท เช่น ช่องทางอนุกรม (UART) วงจร ควบคุมสัญญาณนาฬิกา (Timer/Counter) ช่องทางเข้าออกอเนกประสงค์ (//O) สัญญาณขัดจังหวะ (Interrupt) ชุดสร้างสัญญาณกล้ำพัลส์ (PWM) และช่องทาง SPI ถูกรวบรวมและออกแบบไว้ใน FPGA

วิทยานิพนธ์นี้แบ่งออกเป็น 4 ส่วน โดยส่วนแรกจะกล่าวถึงการแก้สมการผกผันจลนศาสตร์ ส่วนที่สอง จะกล่าวถึงการออกแบบตัวควบคุม ส่วนที่สามจะกล่าวถึงการพัฒนาส่วนเครื่องและส่วนฮอฟต์แวร์ส่วนสุดท้าย จะกล่าวถึงการพัฒนาซอฟต์แวร์เชื่อมประสานแบบกราฟิก ด้วยการกำหนดเงื่อนไขบางประการ ทำให้แก้ สมการผกผันจลนศาสตร์โดยใช้วิธีแบบปิด (closed-form method) ได้ ตัวควบคุมชนิด 2-DOF PID ถูก จอกแบบขึ้นโดยใช้แบบจำลองที่ได้จากวิธีการระบุแบบจำลองของระบบ วิธีการวนซ้ำกำลังสองน้อยที่สุด และ วิธีการวางตำแหน่งขั้ว ถูกนำมาใช้ในการระบุแบบจำลองของระบบ และการออกแบบตัวควบคุม ส่วนเครื่องถูก สร้างขึ้นเพื่อทำหน้าที่เป็นตัวควบคุมหุ่นยนต์หกแกน ท้ายที่สุดซอฟต์แวร์เชื่อมประสานแบบกราฟักถูก พัพนาขึ้นลำหรับประสานการทำงานระหว่างผู้ใช้เครื่องของตัวควบคุม

ในวิทยานิพนธ์นี้ตัวควบคุมหุ่นยนต์ใหม่ถูกสร้างขึ้นเพื่อใช้แทนตัวควบคุมหุ่นยนต์เดิมซึ่งมีข้อจำกัด หลายประการเช่น หน่วยความจำน้อย การใช้งานยากลำบาก การติดตั้งไม่สะดวก และไม่สามารถติดต่อกับ ข่องทางสื่อสารสมัยใหม่ได้ โดยรวมแล้ว ตัวควบคุมใหม่นี้มีคุณลักษณะดีกว่าตัวควบคุมเดิม ซึ่งจะช่วยเพิ่ม ผลิตภาพให้ดียิ่งขึ้นได้
ศู่นย์วิทยทรัพยากร

ปีการศึกษา. 2551
\#\#50706550 21: MAJOR ELECTRICAL ENGINEERING

## KEYWORDS: ROBOTIC ARM / KINEMATICS / PID CONTROLLER DESIGN / MODELING SYSTEM / SYSTEM-ON-CHIP / FPGA

## AGUS BEJO: A SYSTEM ON CHIP DESIGN OF A 6-AXIS ROBOTIC ARM CONTROLLER IMPLEMENTED ON A LOW-COST FPGA. ADVISOR: ASST. PROF. WANCHALERM PORA, Ph.D., 101 pp.

In this work, we developed a 6-axis robotic arm controller. The robotic arm is powered by hydraulic pump. At each axis its arm is driven by a servo valve. This controller is implemented on a low-cost FPGA. The main control processor supported by some peripherals such as UART, Timer/Counter, Digital I/O, Interrupt, PWM Generator, SPI and SPI Reader is embedded on a FPGA chip.

This work is composed of four parts: solving the inverse kinematics problem, designing proper controller, developing required hardwares and developing GUI user application software. With some constraints, a set of formulae is developed to solve the inverse kinematics problem using closed-form method. A 2-DOF PID controller is designed using obtained model from system identification step. The Recursive Least Square method and Pole Placement method are respectively employed for the system identification and the controller design tasks. Required hardwares are then implemented to realize the controller system. Finally, a GUI user application software is developed to interface the controller system with human operator.

Through this work, a new robotic controller system is created to replace the old existing controller system which has many constraints such as limited storage space, uneasy usage, time consuming setup and incompatibility with modern communication channel. Overall, this new controller system provides better features which is possible to increase the productivity rate.
 Academic Year: 2008

## Acknowledgments

Firstly, I would like to express my profound gratitude to my advisor, Asst. Prof. Dr. Wanchalerm Pora, for his patient guidance, encouragement and excellent advice as well as giving me extraordinary experiences throughout the work. He has given me abundant attention and offered invaluable assistance during my study at Chulalongkorn University. I am very sure that this thesis would not have been successful without his total supervision from the very early stage until the end of the work.

My sincere thanks to Assoc. Prof. Dr. Ekachai Leelarasmee as a former head of Embedded System and IC Design (ESID) Laboratory who give me some encouragement to accomplish my master study as well as to pursue my further study.

Special thanks in particular to Sakonpong Buranawit, Torkiat Taithongchai, Kampanard Suwannawut and all of my friends in ESID Laboratory for their kindness and very good friendship. They always help me much on keeping the spirit up, sharing the literature during the work, discussing and joking in our spare time. The unforgotten thing is that they advise me much on scheduling my working time so I can accomplish my work on time.

Another thanks is addressed to Stainless Steel Home Equipment Manufacturing Co, Ltd for their generosity of letting me use their resources for conducting my experiments. I am indebted to Chaiwat Srivongchareon, Samnoek, Manote, and other factory staffs who have introduced me to this work, guided me and accompanied me during my work. I can not imagine how to accomplish this work without their assistance and facilities support. It is a very valuable thing that help me much on finishing the work properly.

I would also like to convey many thanks to the AUN/Seed-Net scholarship program under JICA who has given me a great opportunity to pursue my advanced study abroad and provided entire financial support during my study.

Finally, I wishes to express my love and gratitude to my beloved families; for their understanding and endless love, through the duration of my study. Their moral support has brought me to an earnest work and a desire to do the best for them.

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## CHAPTER I

## INTRODUCTION

### 1.1 Introduction

The term robot was first introduced by the Czech Karel Capek in 1920 [1]. It means an artificial humanoids - biped robots - which help human beings in physically difficult tasks [2]. Recently, the term robot is used to denote animated autonomous machines. Robot can be classified into two main classes, namely robot manipulators and mobile robots. This work will concern only on the robot manipulators which is defined as a computer-controlled industrial manipulator [1]. A robot manipulator is composed of links connected by joints into an open kinematic chain. Joints can be either rotary (revolute) or linear (prismatic). The number of joints determines the degrees-of-freedom (DOF). A manipulator with 6 axes posses 6 DOF , generally three for positioning and three for orientation. The joints may be actuated electrically, hydraulically or pneumatically.

Through this writing, the term of robotic arms will be used rather than robot manipulators. Robotic arms, especially the 6-axis ones, have been widespreadly used in several industrial processes [3] for many years. The robotic arm that will be emphasized in this work is a painting machine.


Figure 1.1: Robot system architecture

Nowadays there are many types of robotic arms that is functioned as painting machines [4-6]. Some of them have been employed for a long time. Perhaps, their service support cannot be extended. Usually, just the electronic parts of the machines which have been in service ten years or more are obsolete. On the other hands, most factory support the mechanic part of the machines themselves. It is a dilemma to choose whether to continue using the old machine with high risk and high maintenance cost due to lack of standard spare parts, or to invest in new machines which are costly. Moreover, most old machines have restrictions [7-9] such as limited storage space, incompatibility with modern technology media or communication channel.

This research was conducted to offer a solution towards some disadvantages above by designing a System-on-Chip (SoC) embedded controller based on a low-cost FPGA. The system developed in this research is shown in Figure 1.1. It is composed of a Microblaze module (32-bit RISC softcore processor), an internal memory module, a UART module, a 32-bit timer/counter module, a digital I/O module, a SPI module, 6 custom SPI readers for interfacing ADC and 6 custom PWM generators as DACs. By using FPGA, it will improve the computation power compared with that of 8051 microcontrollers. FPGA's processing power is closed to that of DSP, however its price is much cheaper than that of DSP. The other advantages of FPGA is that it has hardware-level reconfigurable, so any module with I/O terminal can be designed, modified or adjusted very flexibly. Overall, with integrating all mentioned module on chip this yields a high-performance system.

The controller controls angular movement of each axis, both direction and velocity, by sending digital data to the DAC modules in the form of PWM signals which connected to the servo valve via low-pass filters and amplifiers. The signals that are produced by the DAC modules represent the outputs of the controller. Their polarity and magnitude respectively determines the direction and velocity of movement. The position of each axis is measured from its potentiometer that is attached at each joint. This represents the input of the controller. A PC, supported by GUI application software, can be connected to the controller system via a USB/UART to make it more interactive and userfriendly.

### 1.2 Literature Review

For a few decades, some researches regarding to the development of robotic arms controller have been done, and are still being done. Inverse kinematics settlement and controller design are common issues to face.

Closed-form method, that is solving the inverse kinematics analytically, gives preferable solution [3]. Nonetheless, the closed-form method requires complex calculation and rely on the robot structure. An improvement of closed-form method was proposed to reduce the computational cost by avoiding a large amount of inverse matrix multiplication [10]. Another approach of closedform method was Product-of-Exponentials (POE) [11]. The other methods were numerical methods (nonclosed-form) such as geometrical solution [12] and iterative solution [13]. Nonclosed-form methods involve numerical iteration until a desired end-point position and orientation is reached to within a maximum allowable error. Nonclosed-form methods have some weaknesses as mentioned in
chapter 2.1.3.
Apart from the kinematics problem, robotic arm controllers also have been developed in various ways, both of algorithm and its hardware. PI controller [14], PID controller [15-17] or adaptive controller $[18,19]$ have been implemented by using PC [18], Microcontroller [17,20] and FPGA [15]. It is a bargain to choose which algorithm is suitable to solve inverse kinematics and controller regarding to the hardware requirement and the desired system performance. A new approach to improve the end-effector of robotic arm motion has been proposed by incorporating a controller algorithm such as PID [16] or MRAC [19] and an optimization algorithm such as Least Mean Square.

This work did entire mentioned issues both solving kinematics and designing controller from software up to hardware to replicate or re-design a 6-axis robotic arm controller system as a study case. In this case, a set of formulae to solve specific inverse kinematics problem was developed and 2-DOF PID controller was designed. A preliminary research to realize that system has been done by author using microcontroller [17].

### 1.3 Objectives

The main objective of this research is to develop a system-on-chip of a 6-axis robotic arm controller. To achieve said objective, one must fulfill the following aims:

1. Solving the inverse kinematics problem
2. Modeling the servo valves system of the robotic arm
3. Designing 2-DOF PID controller
4. Designing required hardwares
5. Implementing the controller on FPGA
6. Developing a GUI user application software

### 1.4 Scope of Thesis

The system designed by this research has specification as follows: ยากร

- It has an embedded PID controller
- It can communicate with PC via Serial UART/USB port
- All required modules/peripherals are integrated on chip
- It has GUI interface to control the robot through PC
- It has two mode, those are teaching mode and play back mode
- It can load or store the trajectory file from or to PC


### 1.5 Methodology

In this work, the inverse kinematics problem is solved analytically (closed-form method). After the desired trajectory pattern that contains the sequence of the robot movement in Cartesian coordinate had specified by an operator, corresponding file then is generated in Microsoft Excel format called trajectory file. This file is generated by solving the inverse kinematics problem with some (angular to digital value) conversions. This file is then loaded into memory of the controller system. At playback mode, this file will became the set-points of the controller to drive each position of joint so that the end-point of the robot moves to the position and orientation such the trajectory pattern had been specified.

The robot is powered by a hydraulic pump which is controlled through digital I/O connections. Its motion is driven by signals coming to the servo valves. To design proper controller, a good knowledge regarding to this servo valves must be known. The robotic servo valves are identified by using Recursive Least Square method. Some signals are given as input to the servo valves for driving the robot while the output signals are measured from the potentiometer attached in every joint. Both input and output signals are recorded and then processed by using mentioned method to obtain the model. This model represents a transfer function that states the relation between position toward input signal. Three types of input signals i.e. step, triangle and random signals are employed. The effect of the position and the load toward the model was studied. If the model does not change too much because of its load, a PID controller can be chosen as the controller. However, when the model change so much, it will be better to design an adaptive controller. By using adaptive controller, it can compensate the change of the plant model so that the controller performance kept well.

Having the servo valves model were obtained, proper 2-DOF PID controller then was designed based on the knowledge of that model. This controller with its apparatus peripheral will be implemented on FPGA. Before being implemented on FPGA, the controller will be applied by using microcontroller. This way is useful for observing the performance of the controller. If the controller performance is not good enough, it can be modified or can be improved easily without time consuming step. Eventually, after the controller performance has achieved the desired one then the controller can be implemented on FPGA. It can be said that an imitation of microcontroller system with fix features will be built on FPGA to replace the microcontroller as the main processing for the controller.

For the reason of interactivity, a GUI user application software is developed on a PC. This one can be used to re-adjust or to modify the controller system behavior such as updating the controller parameter, importing or exporting the trajectory file, updating the movement speed and so on by an operator easily.

### 1.6 Contributions

Through this research it is wished to yield a replicate of the old robotic arm controller which has some superiority such as:

1. Its hardware is easy to operate.
2. Its software is easy to use.
3. It can be modified to suit software needs quickly.
4. It provides better features and better capabilities.

### 1.7 Publications

1. Agus Bejo and Wanchalerm Pora, Combination of Model Reference Adaptive Control and Least Mean Square Algorithms for Robotic Arm Controllers, Asia International Symposium on Mechatronics 2008 (AISM 2008), Aug. 27-31, 2008, Hokkaido University, Japan.
2. Agus Bejo and Wanchalerm Pora, An Improvement of The End-point Error for Multiple-axis Robotic Arms Using The LMS Algorithm, 31 ${ }^{\text {st }}$ Electrical Engineering Conference (EECON31), Oct 29-30, 2008, Nakhon Nayok, Thailand.
3. Agus Bejo, Wanchalerm Pora and Hiroaki Kunieda, Development of a 6-Axis Robotic Arm Controller Implemented on a Low-Cost Microcontroller, Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology Conference 2009 (ECTI-Con 2009), May 6-9, 2009, Pattaya, Thailand.


## CHAPTER II

## BASIC THEORY AND PROPOSED METHODS

This chapter provides basic theory and some proposed methods which are used through this work.

### 2.1 Robot Kinematics

Robot arm kinematics deals with analytical study of the geometry of its motion with respect to a fixed reference coordinate system, particularly the relations between the joint-variable space and the position and orientation of the end-effector [1]. The robot arm used in this work has 6-DOF, which are all revolute-type. Hence, we use robotic arm term through this writing to refer a 6-axis robot arm.

### 2.1.1 Homogeneous Transformation Matrix

The homogeneous transformation matrix $\mathbb{T}$ is a $4 \times 4$ matrix which maps a position vector expressed in homogeneous coordinate system to another coordinate system. A large part of robot kinematics is concerned with the establishment of various coordinate systems to represent the positions and orientation of rigid object, and with transformations among these coordinate systems. Homogeneous transformations combine the operations of rotation and translation into a single matrix multiplication.

Suppose two coordinate systems are assigned to each link of a robot arm at link i-1 and link $i$ respectively. The link $i-1$ coordinate system is the reference coordinate system and the link $i$ coordinate system is the moving coordinate system when joint $i$ is activated. Using the homogeneous transformation matrix $\mathbb{T}$, a point $p_{i}$ can be expressed in the link $i$ coordinate system in terms of the link $i-1$ coordinate system as

$$
\begin{equation*}
p_{i-1}={ }^{i-1} \mathbb{T}_{i} p_{i} \tag{2.1}
\end{equation*}
$$

A homogeneous transformation matrix can be considered to consist of four submatrices:

$$
\mathbb{T}=\left[\begin{array}{c|c}
\mathbb{R} & \vec{T}  \tag{2.2}\\
\hline \vec{P} & s
\end{array}\right]=\left[\begin{array}{c|l}
\text { Rotation } & \text { Translation } \\
\hline \text { Perspective } & \text { ScaleFactor }
\end{array}\right]
$$

$\mathbb{R}$ is a $3 \times 3$ matrix that represents the relative rotation (orientation). Three basic rotation matrices $\mathbb{R}$ are given in (2.3) to (2.5). $\mathbb{R}_{x, \alpha}, \mathbb{R}_{y, \alpha}$ and $\mathbb{R}_{z, \alpha}$ respectively represent the rotation through an angel $\alpha$ about the x -axis, y -axis and z -axis.

$$
\begin{align*}
& \mathbb{R}_{x, \alpha}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha
\end{array}\right]  \tag{2.3}\\
& \mathbb{R}_{y, \alpha}=\left[\begin{array}{ccc}
\cos \alpha & 0 & \sin \alpha \\
0 & 1 & 0 \\
-\sin \alpha & 0 & \cos \alpha
\end{array}\right]  \tag{2.4}\\
& \mathbb{R}_{z, \alpha}=\left[\begin{array}{ccc}
\cos \alpha & -\sin \alpha & 0 \\
\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{array}\right] \tag{2.5}
\end{align*}
$$

$\vec{T}$ is a $3 \times 1$ column vector that represents the relative translation (position). Three basic translation vectors $\vec{T}$ are given in (2.6). $\vec{T}_{x, a}, \vec{T}_{y, b}$ and $\vec{T}_{z, c}$ respectively represent the translation in $a$ distance through the x -axis, $b$ distance through the y -axis and $c$ distance through the z -axis.

$$
\vec{T}_{x, a}=\left[\begin{array}{l}
a  \tag{2.6}\\
0 \\
0
\end{array}\right], \vec{T}_{y, b}=\left[\begin{array}{l}
0 \\
b \\
0
\end{array}\right], \vec{T}_{z, c}=\left[\begin{array}{l}
0 \\
0 \\
c
\end{array}\right]
$$

A $1 \times 3$ row vector $\vec{P}$ and a scalar $s$ are the perspective and scale factor parameter that generally given by $\vec{P}=\left[\begin{array}{lll}0 & 0 & 0\end{array}\right]$ and $s=1$, when the joint is a rotary type.

### 2.1.2 The Denavit-Hartenberg Representation

In 1955, Denavit and Hartenberg [21] proposed a systematic and generalized approach of utilizing matrix algebra to describe and represent the spatial geometry of the links of a robot arm with respect to a fixed reference frame. This method uses a homogeneous transformation matrix $\mathbb{T}$ to describe the spatial relationship between two adjacent rigid mechanical links. The Denavit-Hartenberg (DH) representation results a homogeneous transformation matrix representing each link's coordinate system at the joint with respect to the previous link's coordinate system.

Suppose $A_{i}$ is the homogeneous matrix that transforms the coordinate of a point from frame $i$ to frame $i$-1 i.e $A_{i}={ }^{i-1} \mathbb{T}_{i}$. In the D-H convention, each homogeneous transformation $A_{i}$ is represented as a product of four basic transformations

$$
\begin{gather*}
\text { a product of four basic transformations }  \tag{2.7}\\
A_{i}=\left[\begin{array}{cccc}
\cos \theta_{i} & -\sin \theta_{i} \cos \alpha_{i} & \sin \theta_{i} \sin \alpha_{i} & a_{i} \cos \theta_{i} \\
\sin \theta_{i} & \cos \theta_{i} \cos \alpha_{i} & \cos \theta_{i} \sin \alpha_{i} & a_{i} \sin \theta_{i} \\
0 & \sin \alpha_{i} & \cos \alpha_{i} & d_{i} \\
0 & 0 & 0 & 1
\end{array}\right] \tag{2.8}
\end{gather*}
$$

where the four quantities $\theta_{i}, d_{i}, a_{i}$ and $\alpha_{i}$ are parameters of link $i$ and joint $i$ as depicted in Figure 2.1. These parameters are obtained by step 7 in the D-H general procedure below. Since all joints of the robot used in this thesis are revolute joint so the three of above four quantities are constant while the fourth parameter $\theta_{i}$ is the joint variable.


Figure 2.1: Denavit-Hartenberg frame assignment

Through sequential transformations, the end-effector can be transformed and expressed in the base coordinates system. The homogeneous matrix ${ }^{0} \mathbb{T}_{i}$ which specifies the location of the $i^{t h}$ coordinate frame with respect to the base coordinate system is the chain product of successive coordinate transformation matrices of $A_{i}$ and is expressed as

$$
\begin{equation*}
{ }^{0} \mathbb{T}_{i}=A_{1} A_{2} A_{3} \ldots A_{i} \tag{2.9}
\end{equation*}
$$

This is the general procedure to derive the link parameters based on the D-H convention:
step 1: Locate and label the joint axes $z_{0}, \ldots, z_{n-1}$.
step 2: Establish the base frame. Set the origin anywhere on the $z_{0}$-axis.
The $x_{0}$ and $y_{0}$ axes are chosen conveniently to form a right-hand frame.
For $i=1, \ldots, n-1$ perform steps 3 to 5
step 3: Locate the origin $o_{i}$ where the common normal to $z_{i}$ and $z_{i-1}$ intersects $z_{i}$. If $z_{i}$ intersects $z_{i-1}$ locate $o_{i}$ at this intersection.
If $z_{i}$ and $z_{i-1}$ are parallel, locate $o_{i}$ at joint $i$.
step 4: Establish $x_{i}$ along the common normal between $z_{i-1}$ and $z_{i}$ through $0_{i}$ or in the direction normal to the $z_{i-1}-z_{i}$ plane if $z_{i-1}$ and $z_{i}$ intersect.
step 5: Establish $y_{i}$ to complete a right-hand frame.
step 6: Establish the end-effector $o_{n} x_{n} y_{n} z_{n}$
step 7: Create a table of link parameters $\theta_{i}, \alpha_{i}, d_{i}$ and $a_{i}$
$\theta_{i}=$ the angle between $x_{i-1}$ and $x_{i}$ measured about $z_{i-1}$.
$\alpha_{i}=$ the angle between $z_{i-1}$ and $z_{i}$ measured about $x_{i}$.
$d_{i}=$ the distance along $z_{i-1}$ from $o_{i-1}$ to the intersection of the $x_{i}$ and $z_{i-1}$ axes.
$a_{i}=$ the distance along $x_{i}$ from $o_{i}$ to the intersection of the $x_{i}$ and $z_{i-1}$ axes.
step 8: Form the homogeneous transformation matrix $A_{i}$ by substituting the above parameters into (2.8)
step 9: Form ${ }^{0} \mathbb{T}_{n}=A_{1} A_{2} A_{3} \ldots A_{n}$. This gives the position and orientation of the end-effector expressed in base coordinates.

### 2.1.3 Inverse Kinematics

There are two fundamental problems in robot arm kinematics namely forward kinematics and inverse kinematics. Fig 2.2 shows a simple block diagram of both forward and inverse kinematics problems. Forward kinematics problem is defined as how to compute the end-point position and orientation using known link parameters when joint variables are given. Conversely, inverse kinematics problem is defined as how to compute the joint variables using known link parameters when the end-point position and orientation is given.


Figure 2.2: Forward and Inverse Kinematics Problems

The general procedure described in section 2.1.2 yields the end-point position and orientation when the link parameters are known and a set of joint variables is given. This procedure is called forward kinematics solution. Generally, forward kinematics problem can be solved easier than inverse kinematics one. Nonetheless, inverse kinematics is more common faced in real application such as trajectory tracing.
S. Kucuk and Z. Bingul [3] mentioned that there are three types of method for solving the inverse kinematics problem as follows: $219 \cap \backsim 9 \mathrm{O}$

1. Complete analytical solution (closed-form method). The solutions of all joint variables solved analytically are derived from the basic equations. This method gives very accurate result but it requires complex calculation. $\because 19190 \cap \cap 9 \cap ?$
2. Numerical solution (nonclosed-form method). The solutions of joint variables solved numerically are obtained by iterative computational procedures. This method gives worse result than that of the analytical method but it can be applied generally. With bad initial values, correct solutions can not be guaranteed. It may yield only one solution or no answer at all.
3. Semi-analytical solutions. This solution combines analytical and numerical method. Some of the joint variables are determined analytically and some of them are computed numerically.

This thesis proposed a set of formulae to solve the inverse kinematics problem by using closedform method as shown in section 3.1.

### 2.2 System Identification Method

System identification is the method that estimate the mathematical model of a system from experimental data [22]. Estimate model obtained by identification is then used for designing the controller mathematically. The following are general procedures of system identification steps:

1. Select and define a model structure (containing unknown parameters).
2. Exciting the system by known signals (step, sinusoidal, pseudo-random signals).
3. Collect and record both inputs and outputs data over a certain interval time.
4. Estimate the parameters using some statistically based methods.
5. Validate the obtained model, repeat 1-4 if required.


Figure 2.3: System identification setup

The specific methods used at each step depend on the type of model desired (parametric or nonparametric, continuous-time or discrete-time) and on the experimental conditions. The validation is the verification step to decide whether the identified model is acceptable or not.

0 The equipment setup of system identification for discrete time models is illustrated in Figure 2.3. A sampled input sequence $u(k)$ (where $k$ is the discrete time) is applied to the physical system by means of a digital-to-analog converter (DAC). The measured sampled output sequence $y(k)$ is obtained by means of an analog-to-digital converter (ADC). An estimate model with adjustable parameters $\hat{\theta}$ is calculated on the computer. The error $e$, that is the difference between output $y(k)$ and the predicted output $\hat{y}(k)$, is used to update the estimate model parameters in order to minimize this error. The simple formula of adaptation algorithm is expressed:

$$
\begin{equation*}
\hat{\theta}_{k+1}^{T}=\hat{\theta}_{k}^{T}+\mu \phi^{T} e_{k} \tag{2.10}
\end{equation*}
$$

where $\hat{\theta}^{T}, \mu, \phi^{T}$ and $e$ are the estimated parameter, adaptation gain, measured data and error respectively. A non-recursive identification algorithms proceed a bulk of data input and output over a certain interval time to obtain the estimated parameter. The bigger data being processed the better estimated parameter is yielded. However, the increase of used data will be followed by the increase of computation cost. Recursive method is an alternative technique to counter that problem. Recursive method offers some advantages such as:

- Reducing the data processing since this algorithm only proceed a shape of data instead of the whole data.
- Estimating the real current model.
- Reducing the memory and computation requirement.
- Making possible to be applied as a real-time identification (on-line identification).


### 2.2.1 Least-Squares Method

The Least-Squares method was first introduced by Karl Gauss [23] for determining the orbits of planets. This method then becomes a major tool and the most popular estimation technique in parameter estimation because it is easy to comprehend and easy to be implemented. This algorithm is objected to minimize the sum of the squares of error between the actual data and that predicted by the model.

Let the process that will be estimated be described by an $n$th order difference equation as in (2.11) or be expressed as transfer function $G(q)$ as in (2.12). Assume that $N$ samples of measurements of $u_{k}, \ldots, u_{k-N}$ and $y_{k}, \ldots, y_{k-N}$ are obtained from experiment. Where $y_{k}$ and $u_{k}$ are the output and input sample at time $k$ respectively and $a_{1}, a_{2}, \ldots, a_{n}, b_{1}, b_{2}, \ldots, b_{n}$ is a set of constant parameters.

$$
\begin{align*}
& y_{k}+a_{1} y_{k-1}+\ldots+a_{n} y_{k-n}=b_{1} u_{k-1}+b_{2} u_{k-2}+\ldots+b_{n} u_{k-n}  \tag{2.11}\\
& \text { Q40 } \tag{2.12}
\end{align*}
$$

where
$\begin{aligned} & A(q)=1+a_{1} q^{-1}+\ldots+a_{n} q^{-n} \\ & B(q)=b_{1} q^{-1}+\ldots+b_{n} q^{-n}\end{aligned} \quad .6010 \%$ ?

To estimate the parameters $a_{i}$ and $b_{i}$ we introduce the residual error as

$$
\begin{equation*}
y_{k}+a_{1} y_{k-1}+\ldots+a_{n} y_{k-n}=b_{1} u_{k-1}+b_{2} u_{k-2}+\ldots+b_{n} u_{k-n}+\epsilon_{k} \tag{2.13}
\end{equation*}
$$

or

$$
\begin{equation*}
A(q) y_{k}=B(q) u_{k}+\epsilon_{k} \tag{2.14}
\end{equation*}
$$

This equation is called AutoRegresive with eXoganious (ARX) and will be used through this writing. Filling the data samples into (2.13) results a set of linear equation that can be expressed as a simple matrix form in (2.15)

$$
\begin{equation*}
Y=\Phi \theta+\epsilon \tag{2.15}
\end{equation*}
$$

where

$$
\begin{aligned}
& Y=\left[\begin{array}{l}
y_{k} \\
y_{k-1} \\
\ldots \\
y_{k-N}
\end{array}\right] \\
& \Phi=\left[\begin{array}{lllllll}
-y_{k-1} & \ldots & -y_{k-n} & x_{k} & x_{k-1} & \ldots & x_{k-n} \\
-y_{k-2} & \ldots & -y_{k-n-1} & x_{k-1} & x_{k-2} & \ldots & x_{k-n-1} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
-y_{k-N} & \ldots & -y_{k-n-N} & x_{k-N} & x_{k-1} & \ldots & x_{k-n-N}
\end{array}\right] \\
& \theta=\left[\begin{array}{l}
a_{1} \\
a_{2} \\
\ldots \\
b_{n}
\end{array}\right] \\
& \epsilon=\left[\begin{array}{l}
\epsilon(k) \\
\epsilon(k-1) \\
\ldots \\
\epsilon(k-N)
\end{array}\right]
\end{aligned}
$$

The estimate of $\theta$, denoted by $\hat{\theta}$, is obtained by solving (2.15). A necessary condition to solve this set of equations is $N \geq n$. When $N=n$, it has a unique solution given in (2.16).

$$
\begin{equation*}
\hat{\theta}=\Phi^{-1} Y \tag{2.16}
\end{equation*}
$$

where $\Phi^{-1}$ is the inverse of the square matrix $\Phi$. However, when $N>n$, generally it is not possible to find a $\hat{\theta}$ vector which can fit the data samples exactly. It may be caused by too low order of the model or a wrong model structure. One way to determine the parameters is to estimate them based on least-square error.

Let the error $\epsilon(k)$ and the sum of the squares error $V_{L S}$ are stated by (2.17) and (2.18) respectively.

$$
\begin{align*}
& \epsilon(k)=y(k)-\hat{y}(k)=y(t)-\Phi \hat{\theta} \\
& V_{L S}=\frac{1}{N} \sum_{j=k-N}^{k} \epsilon(j)^{2}=\frac{1}{N} \epsilon^{T} \epsilon \tag{2.17}
\end{align*}
$$

To carry out the minimization, we express

$$
\begin{equation*}
V_{L S}(\theta)=\frac{1}{N}(Y-\Phi \theta)^{T}(Y-\Phi \theta)=\frac{1}{N}\left[Y^{T} Y-\theta^{T} \Phi^{T} Y-Y^{T} \Phi \theta-\theta^{T} \Phi^{T} \Phi \theta\right] \tag{2.19}
\end{equation*}
$$

Taking the first derivative of $V_{L S}(\theta)$ with respect to $\theta$ and equating the result to zero, we have

$$
\begin{equation*}
\left.\frac{\partial V_{L S}(\theta)}{\partial \theta}\right|_{\theta=\hat{\theta}}=\frac{1}{N}\left[-2 \Phi^{T} Y+2 \Phi^{T} \Phi \hat{\theta}\right]=0 \tag{2.20}
\end{equation*}
$$

Hence the solution is given by the equation

$$
\begin{equation*}
\hat{\theta}=\left[\Phi^{T} \Phi\right]^{-1} \Phi^{T} Y \tag{2.21}
\end{equation*}
$$

### 2.2.2 Recursive Least Square Method

In some cases, such as adaptive control, it may be necessary to estimate a model on-line while the process is in operation. The model will be updated when the new observations are available. Hence for computation efficiency, it is desirable to arrange the algorithms in such a way that the results obtained previously can be used for on-line updating. This way of algorithm is called recursive. The recursive version of Least-Squares method is called Recursive Least-Square methods. Because of some advantages mentioned above, this thesis will concern on this method only.

In this methods, a square matrix $P_{k}$ is introduced as in (2.22). Suppose that we have found an estimate of parameters at time $k$ th, that is $\hat{\theta_{k}}$. Then we could update those estimated parameters using a new data obtained at time $k+1$ th $\left(y_{k+1}\right)$ by formulae (2.23), (2.24) and (2.25).

$$
\begin{equation*}
994 \Omega \hat{\theta}_{k+1}=\hat{\theta}_{k}+P_{k+1} \Phi \epsilon_{k+1} \| ? \tag{2.24}
\end{equation*}
$$

$F$ is forgetting factor which should be less than or equal to 1 . When the process is slowly time varying, the measurements obtained a long time ago contain less information than the recent one. In order to let the estimator follow the change of the process, it is desirable to truncate the old measurements in the estimation algorithm.

### 2.2.3 Order Selection

The purpose of order selection is to find the model order that is most accurate for its use in control. There are some order selection methods that do not need complete parameter estimaton of model like checking the rank of covariance matrices and plotting the singular value of the Hankel matrix [22].

However, these methods are not accurate for noisy data. In this thesis, models will be estimated with increasing orders and then the best order will be chosen using an error criterion. This method is so-called cross-validation.

A number of criteria exist for order selection. In identification literature [23], the prediction error criterion is often used. Denote prediction error criterion using a validation data set as in (2.26) and using the estimate data set as in (2.27).

$$
\begin{align*}
& V_{P E}^{V}=\frac{1}{N} \sum_{k=1}^{N} \hat{H}^{-1}(q)\left[y_{k}^{V}-\hat{G}(q) u_{k}^{V}\right]^{2}  \tag{2.26}\\
& V_{P E}^{E}=\frac{1}{N} \sum_{k=1}^{N} \hat{H}^{-1}(q)\left[y_{k}-\hat{G}(q) u_{k}\right]^{2} \tag{2.27}
\end{align*}
$$

where $u_{k}^{V}, y_{k}^{V}$ and $u_{k}, y_{k}$ are the input-output from the validation data set and the input-output from the estimate data set respectively. Assume that the white noise $e_{k}$ is Gaussian and the model order is correct, then Akaike $(1974,1981)$ has derived an asymptotically unbiased estimate of $V_{P E}^{V}$ using $V_{P E}^{E}$ :

$$
\begin{equation*}
F P E=V_{P E}^{V}=\frac{N+d}{N-d} V_{P E}^{E} \tag{2.28}
\end{equation*}
$$

where $N$ is the number of estimation data samples and $d$ is the number of model paramters. This is the famous Akaike's Final Prediction Error criterion (FPE). Here the factor $\frac{N+d}{N-d}$ is used to correct for the over-fit effect.

Remember that $H(q)=1$ for ARX model, therefore the term lost function $V_{O E}$ can be used instead of FPE.

$$
\begin{equation*}
V_{O E}=\frac{1}{N} \sum_{k=1}^{N} \hat{\epsilon}_{O E k} \tag{2.29}
\end{equation*}
$$

Note that output error $\hat{\epsilon}_{O E(k)}$ is not the same as residual error $\hat{\epsilon}_{(k)}$. The relation between them is

In general, the lost function $V_{O E}$ decreases as the order $n$ increases. The reduction of $V_{O E}$ will be insignificant when the order of the model is high enough for simulation purposes. Based on this principle, a procedure for order selection is so simple. The appropriate model order can be chosen as the one for which $V_{O E}$ stops decreasing significantly.

### 2.2.4 Validation

The goal of model validation is to check if the identifed model is good enough for use in control. This is done by carrying out a whiteness test on the sequence of residual errors. An acceptable identified model should verify the condition

$$
\begin{gather*}
|R N(i)| \leq \frac{2.17}{\sqrt{N}} ; \quad i=1,2, \ldots, \max \left(n_{A}, n_{B}+d\right)  \tag{2.33}\\
R N(i)=\frac{\frac{1}{N} \sum_{k=1}^{N} \epsilon_{k} \epsilon_{k-i}}{\frac{1}{N} \sum_{k=1}^{N} \epsilon_{k}^{2}} \tag{2.34}
\end{gather*}
$$

where $N$ is the number of samples.

### 2.3 Control Design Method

Controllers can be classified into feedback and feed forward controllers. The feedback controllers itself are divided into one degree of freedom and two degree of fredom controllers (abbreviated as 1DOF and 2-DOF respectively) [24]. The structure of both controllers (1-DOF and 2-DOF) in discrete time are shown in Figure 2.4.


1-DOF controller is simpler to design than 2-DOF controller because it has fewer parameters. However, 2-DOF controller has more capabilities to shape the responses both reference and disturbance signals simultaneously and independently.

Figure 2.5 shows the general principle of controller design. In order to design and tune a controller correctly, one needs:

1. Determination of the plant model.
2. Specification of the performance.
3. Computation of the controller parameters (the coefficients of the polynomials $R(z), S(z)$ and $T(z)$.
4. Verification of the achieved robustness margins and sensitivity functions.


Figure 2.5: Controller design principle

Step no.1, that is system identification, has been detailed previously in chapter 2.2 . This section will detail the other points.

### 2.3.1 2-DOF PID Controller

Proportional, integral and derivative (PID) controllers are widely used in industrial practice. Because of its merit and its simplicity to design, this thesis will be focused on 2-DOF Discrete PID Controller.

Consider the 2-DOF discrete control structure presented in Figure 2.4 (b). The control law is

$$
\begin{aligned}
& \text { implemented by: } \\
& \qquad \text { (0) } \\
& \text { (2) } 0 \text { ? } \frac{T(z)}{R(z)} r(k)-\frac{S(z)}{R(z)} y(k) \text { (2.35) }
\end{aligned}
$$

It is easy to arrive at the following relation between $r(k)$ and $y(k)$ :

$$
\begin{equation*}
y(k)=\frac{T(z)}{R(z)} \frac{B(z) / A(z)}{(1+B(z) S(z) / A(z) R(z))} r(k)=\frac{B(z) T(z)}{A(z) R(z)+B(z) S(z)} r(k) \tag{2.36}
\end{equation*}
$$

The error, that is the difference between the reference signal $r(k)$ and the actual value $y(k)$, is given by:

$$
\begin{equation*}
e(k)=r(k)-y(k)=\left(1-\frac{B(z) T(z)}{A(z) R(z)+B(z) S(z)}\right) r(k) \tag{2.37}
\end{equation*}
$$

Simplifying, we arrive at

$$
\begin{equation*}
e(k)=\frac{A(z) R(z)+B(z) S(z)-B(z) T(z)}{A(z) R(z)+B(z) S(z)} r(k) \tag{2.38}
\end{equation*}
$$

The discrete time form of the error expression in (2.38) is given by:

$$
\begin{equation*}
E(z)=\frac{A(z) R(z)+B(z) S(z)-B(z) T(z)}{A(z) R(z)+B(z) S(z)} r(z) \tag{2.39}
\end{equation*}
$$

where $E(z)$ and $r(z)$ are, respectively, the Z-transforms of $e(k)$ and $r(k)$. Using the final value theorem from discrete time systems, we obtain:

$$
\begin{equation*}
\lim _{k \rightarrow \infty} e_{k}=\lim _{z \rightarrow 1} \frac{z-1}{z} E(z) \tag{2.40}
\end{equation*}
$$

Substituting the expression for $E(z)$ in (2.39) with step function input, the above equation becomes

$$
\begin{equation*}
\lim _{k \rightarrow \infty} e_{k}=\lim _{z \rightarrow 1} \frac{z-1}{z} \frac{A(z) R(z)+B(z) S(z)-B(z) T(z)}{A(z) R(z)+B(z) S(z)} \frac{z}{z-1} \tag{2.41}
\end{equation*}
$$

where we have made use of the fact that $R(z)$ is the transfer function of unit step. When the controller has an integral action, $R(1)=0$. Using this, the above equation reduces to:

$$
\begin{equation*}
e(\infty)=\left.\frac{S(z)-T(z)}{S(z)}\right|_{z=1}=\frac{S(1)-T(1)}{S(1)} \tag{2.42}
\end{equation*}
$$

The above condition can be satisfied if one of the following condition is met:

$$
\begin{gather*}
S=T  \tag{2.43}\\
S(1)=T(1) \tag{2.44}
\end{gather*}
$$

If we use $S=T$ to solve (2.42) it mean that we use 1-DOF controller. We also need to assume that $S(1)$ is nonzero otherwise it is a 1-DOF controller too.

### 2.3.2 Pole Placement

The pole placement method allows the design of a 2-DOF discrete controller both for stable and unstable systems without restriction on the degrees of the polynomials $A(z)$ and $B(z)$, without restriction on the time delay and without restriction on the plant zeros (stable or unstable) because this method does not simplify the system zeros. The only restriction concerns the possible common factors of $A(z)$ and $B(z)$, which must be simplified before the computations are carried out.

Let the controlled plant is characterized by the transfer function:

$$
\begin{equation*}
H(z)=\frac{z^{-d} B(z)}{A(z)} \tag{2.46}
\end{equation*}
$$

where
$d:$ the integer number of sampling periods contained in the time delay

$$
\begin{aligned}
& A(z): 1+a_{1} z^{-1}+\ldots+a_{n A} z^{-n A} \\
& B(z): b_{1} z^{-1}+\ldots+b_{n B} z^{-n B}
\end{aligned}
$$

The closed loop transfer function is given by:

$$
\begin{equation*}
H_{C L}(z)=\frac{z^{-d} T(z) B(z)}{A(z) R(z)+z^{-d} B(z) S(z)}=\frac{z^{-d} T(z) B(z)}{P(z)} \tag{2.47}
\end{equation*}
$$

where

$$
\begin{aligned}
& P(z)=1+p_{1} z^{-1}+\ldots+p_{n_{P}} z^{-n_{P}} \\
& R(z)=1+r_{1} z^{-1}+\ldots+r_{n_{P}} z^{-n_{R}} \\
& S(z)=s_{0}+s_{1} z^{-1}+\ldots+s_{n_{P}} z^{-n_{S}}
\end{aligned}
$$

Polynomial $P(z)$ can be specified directly by defining the desired closed loop poles such as $P(z)=\left(1+p_{1}^{\prime} z^{-1}\right)\left(1+p_{2}^{\prime} z^{-1}\right) \ldots\left(1+p_{n P}^{\prime} z^{-1}\right)$. Generally, $P(z)$ is chosen in the form of a secondorder polynomial by discretization of a second-order continuous time system, specifying $\omega_{0}, \zeta$ and assuring that the condition in (2.48) is satisfied.

$$
\begin{equation*}
0.25 \leq \omega_{0} T_{s} \leq 1.5 ; \quad 0.7 \leq \zeta \leq 1 \mid \tag{2.48}
\end{equation*}
$$

Once $P(z)$ is specified, the following equation (known as Bezout Identity equation) must be solved to compute $R(z)$ and $S(z)$ in (2.47):


$$
\begin{equation*}
n_{A}=\operatorname{deg} A(z) ; \quad n_{B}=\operatorname{deg} B(z) \tag{2.50}
\end{equation*}
$$

This polynomial equation has a unique solution with minimal degree (when $A(z)$ and $B(z)$ ) do not have common factors) for

$$
\begin{gather*}
n_{P}=\operatorname{deg} P(z) \leq n_{A}+n_{B}+d-1  \tag{2.51}\\
n_{R}=\operatorname{deg} R(z)=n_{B}+d-1  \tag{2.52}\\
n_{S}=\operatorname{deg} S(z)=n_{A}-1 \tag{2.53}
\end{gather*}
$$

In order to solve equation (2.49) effectively, this is often represented in the matrix form

$$
\begin{equation*}
M x=p \tag{2.54}
\end{equation*}
$$

where

$$
\begin{align*}
& x=\left[1, r_{1}, \ldots, r_{n_{R}}, s_{0}, \ldots, s_{n_{S}}\right]^{T}  \tag{2.55}\\
& p=\left[1, p_{1}, \ldots, p_{i}, p_{n_{P}}, 0, \ldots, 0\right]^{T} \tag{2.56}
\end{align*}
$$

and the matrix $M$ has the following form

where:
$b_{i}^{\prime}=0$ for $i=0,1, \ldots, d ; \quad b_{i}^{\prime}=b_{i-d}$ for $i \geq d+1$
The vector $x$, contains the coefficients of the polynomial $S(z)$ and $R(z)$, is obtained using matrix inversion M by the formula
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where $M^{-1}$ is the matrix inverse of $M$. This inverse exists if the determinant of the matrix $M$ is not zero. One can prove that this is verified if and only if $A(z)$ and $B(z)$ are coprime polynomials (no simplifications between zeros and poles). Finally, parameters of polynomial $R(z)$ are obtained by equation (2.43), (2.44) or (2.45).

## CHAPTER III

## IMPLEMENTATION, HARDWARE AND SOFTWARE DESIGN

This chapter explains the implementation and the hardware design of controller system. At first, a set of formulae for solving the inverse kinematics problem and how to generate the trajectory files are described. The 2-DOF PID controllers then are conveyed based on servo valves models obtained from system identification. Finally, the required hardwares are constructed to realize the controller system.


## 

In order to develop formulae for solving the inverse kinematics problem described in section 2.1.3, a set of joint parameter represents the mechanical structure of the robot must be known. The mechanical structure of the robot used in this case study is shown in Figure 3.1. The coordinate $O i, i=0, . .7$ was specified. Then the parameters $l_{i}, \alpha_{i}, d_{i}$, and $\theta_{i}$ can be extracted following the steps in section 2.1.2 as shown in Table 3.1.

Table 3.1: Denavit-Hartenberg Parameters

| Axis | $l_{i}$ | $\alpha_{i}$ | $d_{i}$ | $\theta_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | $\frac{\pi}{2}$ | 63.2 | $\theta_{1}$ |
| 2 | 100 | 0 | 0 | $\theta_{2}$ |
| 3 | 164 | 0 | 0 | $\theta_{3}$ |
| 4 | 10.16 | $\frac{\pi}{2}$ | 0 | $\theta_{4}$ |
| 5 | 0 | $\frac{\pi}{2}$ | 7.6 | $\theta_{5}$ |
| 6 | 0 | $\frac{\pi}{2}$ | 8.5 | $\theta_{6}$ |
| 7 | 11 | $\frac{\pi}{2}$ | 0 | $0.7 \pi$ |

### 3.1.1 General inverse kinematics formulae when $O_{7}$ and its orientation are given

In this section, we develop a general inverse kinematics formulae when a desired end-point position $\left(O_{7}\right)$ and its orientation are given. Figure 3.2 and Figure 3.3 show the coordinate frame example and the zoomed area of end-point respectively.


Figure 3.2: Coordinate frame example for section 3.1.1
The end-point position is specified in original coordinate direction $O_{0}\left(O_{7}=\left[\begin{array}{lll}x_{7} & y_{7} & z_{7}\end{array}\right]^{T}\right)$ whereas the orientation is specified by $\alpha$ and $\beta$ as shown in Figure 3.3.


Figure 3.3: End-point position and orientation

Assume that $O_{7}, \alpha$ and $\beta$ are given, where $O_{i}=\left[x_{i} y_{i} z_{i}\right]^{T}$ is the position coordinate of joint- $i$ in $X_{0}$-axis, $Y_{0}$-axis and $Z_{0}$-axis respectively. The corresponding homogeneous matrix transformation of desired position and orientation $\left({ }^{0} \mathbb{T}_{7}\right)$ is stated by a rotation of $\alpha$ degrees about $Z_{0}$ axis followed by a rotation $\alpha$ degrees about $Y_{7}$ axis, a rotation 180 degrees about $X_{7}$ axis and translated by translation $O_{7}=\left[x_{7} y_{7} z_{7}\right]^{T}$.

$$
\begin{gather*}
{ }^{0} \mathbb{T}_{7}=\operatorname{Rot}_{z, \alpha} \operatorname{Rot}_{y, \beta} \text { Rot }_{x, 180} \text { Trans }_{x, x_{7}} \operatorname{Trans}_{y, y_{7}} \text { Trans }_{z, z_{7}}  \tag{3.1}\\
{ }^{0} \mathbb{T}_{7}=\left[\begin{array}{cccc}
\cos \alpha \cos \beta & \sin \alpha & -\cos \alpha \sin \beta & x_{7} \\
\sin \alpha \cos \beta & -\cos \alpha & -\sin \alpha \sin \beta & y_{7} \\
-\sin \beta & 0 & -\cos \beta & z_{7} \\
0 & 0 & 0 & 1
\end{array}\right] \tag{3.2}
\end{gather*}
$$

From Figure 3.3 we can easily obtain $O_{6}$ :

$$
O_{6}=O_{7}+\left[\begin{array}{c}
-l_{7} \cos \beta \cos \alpha  \tag{3.3}\\
-l_{7} \cos \beta \sin \alpha \\
l_{7} \sin \beta
\end{array}\right]
$$

$\theta_{7 a}$ is a constant which is determined by the mechanical structure of the robotic arm. The relation between $\theta_{7 a}$ and $\theta_{7}$ is given in (3.4).


Figure 3.4: $X_{r}-Z_{0}$ plane view

There are many possible points of $O_{5}$ so that the angle between link-6 and link-7 satisfies $\theta_{7 a}$. In order to get a unique position of $O_{5}$, we constrains this so that it is in the $X_{r}-Z_{0}$ plane and $O_{5}$ always in the left side of $O_{6}$. Figure 3.4 shows the $X_{r}-Z_{0}$ plane view. An auxiliary variable $\gamma$ is denoted by:

$$
\gamma=\pi-\beta-\theta_{T a}
$$

Then the position of $O_{5}$ in the original base coordinate is:

$$
O_{5}=O_{6}+\left[\begin{array}{c}
-d_{6} \cos \gamma \cos \alpha  \tag{3.6}\\
-d_{6} \cos \gamma \sin \alpha \\
-d_{6} \sin \gamma
\end{array}\right]
$$

Because link-5 and link-6 always be perpendicular, it satisfies the following rule:

$$
\begin{equation*}
\left(x_{4}-x_{5}\right)\left(x_{6}-x_{5}\right)+\left(y_{4}-y_{5}\right)\left(y_{6}-y_{5}\right)+\left(z_{4}-z_{5}\right)\left(z_{6}-z_{5}\right)=0 \tag{3.7}
\end{equation*}
$$

Using known position of $O_{6}$ and $O_{5}$ in (3.3) and (3.6), we introduce constants $c_{1}=x_{6}-x_{5}$, $c_{2}=y_{6}-y_{5}$ and $c_{3}=z_{6}-z_{5}$. Then equation (3.7) becomes:

$$
\begin{equation*}
\left(x_{4}-x_{5}\right) c_{1}+\left(y_{4}-y_{5}\right) c_{2}+\left(z_{4}-z_{5}\right) c_{3}=0 \tag{3.8}
\end{equation*}
$$



Figure 3.5: Top view of $X_{0}-Y_{0}$ plane
Figure 3.5 shows the top view of $X_{0}-Y_{0}$ plane. From this figure we can compute $\theta_{1}$ using formula in (3.10). We also have correlation between $x_{4}, y_{4}$ and $x_{41}$ as shown in (3.11) and (3.12), where $x_{i 1}(i=2,3,4,5)$ is the projection of corresponding position $O_{i}$ in $x_{1}$ direction.

$$
\begin{equation*}
\theta_{1}=\tan ^{-1}\left(\frac{y_{5}}{x_{5}}\right) \tag{3.10}
\end{equation*}
$$

$$
\begin{align*}
& x_{4}=x_{41} \cos \theta_{1}  \tag{3.11}\\
& y_{4}=x_{41} \sin \theta_{1} \tag{3.12}
\end{align*}
$$

Substituting (3.11) and (3.12) into (3.9) yields


Figure 3.6 shows the side view of $X_{1}-Z_{0}$ plane. Because link- 4 and link- 5 is perpendicular in this plane, it satisfies:

(9) 09 ( 0 (3.17)

$$
\begin{equation*}
\left(x_{41}-x_{31}\right)^{2}+\left(z_{41}-z_{31}\right)^{2}=l_{4}^{2} \tag{3.18}
\end{equation*}
$$

Substituting (3.15) into (3.17) yields

$$
\begin{equation*}
\left(\frac{c_{4}-c_{3} z_{4}}{c_{1} \cos \theta_{1}+c_{2} \sin \theta_{1}}-x_{51}\right)^{2}+\left(z_{41}-z_{51}\right)^{2}=d_{5}^{2} \tag{3.19}
\end{equation*}
$$

Because $z_{4}=z_{41}+d_{1}$ then

$$
\begin{equation*}
\left(\frac{c_{4}-c_{3}\left(z_{41}+d_{1}\right)}{c_{1} \cos \theta_{1}+c_{2} \sin \theta_{1}}-x_{51}\right)^{2}+\left(z_{41}-z_{51}\right)^{2}=d_{5}^{2} \tag{3.20}
\end{equation*}
$$

Simplifying this equation yields

$$
\begin{equation*}
p z_{41}^{2}+q z_{41}+r=0 \tag{3.21}
\end{equation*}
$$

where
$p=\left[\frac{c_{3}^{2}}{\left(c_{1} \cos \theta_{1}+c_{2} \sin \theta_{1}\right)^{2}}\right]+1$
$q=-2\left[\frac{c_{3}}{c_{1} \cos \theta_{1}+c_{2} \sin \theta_{1}}\left(\frac{c_{4}-c_{3} d_{1}}{c_{1} \cos \theta_{1}+c_{2} \sin \theta_{1}}-x_{51}\right)+z_{51}\right]$
$r=\left(\frac{c_{4}-c_{3} d_{1}}{c_{1} \cos \theta_{1}+c_{2} \sin \theta_{1}}-x_{51}\right)^{2}+z_{51}^{2}-d_{5}^{2}$

Using quadratic formula, we have 2 solutions for the problem in (3.21). However, we only choose the bigger one because we constrain that the position of $O_{4}$ always higher than the position of $O_{5}$. Therefore, the solution of $z_{41}$ is given by:

$$
\begin{equation*}
z_{41}=\frac{-q+\sqrt{q^{2}-4 p r}}{2 p} \tag{3.22}
\end{equation*}
$$

After $z_{41}$ is known then we can obtain $x_{41}$ using formula in (3.15). Now we can simplify formula in (3.16)

$$
\begin{equation*}
\left(x_{31}-x_{41}\right) c_{6}+\left(z_{31}-z_{41}\right) c_{7}=0 \tag{3.23}
\end{equation*}
$$

where $c_{6}=x_{51}-x_{41}$ and $c_{7}=z_{51}-z_{41}$. Substituting (3.25) into (3.18) yields
where

$s=\left(c_{6}^{2} / c_{7}^{2}\right)+1$
$s=\left(c_{6}^{2} c_{7}^{2}+1\right.$
$t=2\left[\frac{c_{6}}{7}\left(z_{41}-\frac{c_{6} x_{41}+c_{7} z_{41}}{c_{7}}\right)-x_{41}\right]$
$u=\left[\left(z_{41}-\frac{c_{6} x_{41}+c_{7} z_{41}}{c_{7}}\right)^{2}+x_{41}^{2}\right]-l_{5}^{2}$
Problems in (3.26) also has 2 solutions. Because the position of $O_{3}$ always in the left of $O_{4}$ then we chose the smaller one. The solution of $x_{31}$ is given by:

$$
\begin{equation*}
x_{31}=\frac{-t-\sqrt{t^{2}-4 s u}}{2 s} \tag{3.27}
\end{equation*}
$$

Then $z_{31}$ can be solved using formula in (3.25). From Figure 3.6 we can derive a formula to obtain $\theta_{2}, \theta_{3}$ and $\theta_{4}$ as in (3.29), (3.30) and (3.31) respectively, where $a=z_{31}, b=x_{31}$ and $d=\left\|O_{2}-O_{4}\right\|_{2}$ :

$$
\begin{gather*}
c=\sqrt{a^{2}+b^{2}}  \tag{3.28}\\
\theta_{2}=\tan ^{-1}\left(\frac{a}{b}\right)+\cos ^{-1}\left(-\frac{l_{3}^{2}-l_{2}^{2}-c^{2}}{2 l_{2} c}\right)  \tag{3.29}\\
\theta_{3}=-\pi+\cos ^{-1}\left(-\frac{c^{2}-l_{2}^{2}-l_{3}^{2}}{2 l_{2} l_{3}}\right)  \tag{3.30}\\
\theta_{4}=\pi-\cos ^{-1}\left(-\frac{d^{2}-l_{3}^{2}-l_{4}^{2}}{2 l_{3} l_{4}}\right) \tag{3.31}
\end{gather*}
$$

Figure 3.7: Zoomed area of $O_{3}, O_{4}, O_{5}$ and $O_{6}^{\prime}$ in $X_{1}-Z_{0}$ plane
Figure 3.7 shows the zoomed area of $O_{3}, O_{4}, O_{5}$ and $O_{6}^{\prime}$ in $X_{1}-Z_{0}$ plane. $O_{6}^{\prime}$ is the projection of $O_{6}$ in $X_{1}$ coordinate direction. From this figure, we derive the following formulae, where $\delta=\theta_{4}+$ $\theta_{3}+\theta_{2}$ :

$$
\begin{align*}
& z_{61^{\prime}}=z_{51}+d_{6} \sin \delta  \tag{3.32}\\
& x_{6^{\prime}}=x_{61^{\prime}} \cos \theta_{1}  \tag{3.33}\\
& y_{6^{\prime}}=x_{61}+d_{6} \cos \delta, ~(3)  \tag{3.35}\\
& z_{6^{\prime}}=z_{61^{\prime}}+d_{1}
\end{align*}
$$

Denotes vectors $\overrightarrow{v_{6}}$ and $\overrightarrow{v_{6^{\prime}}}$ are the relative position of $O_{6}$ and $O_{6}^{\prime}$ toward $O_{5}$ respectively.

$$
\begin{gather*}
\overrightarrow{v_{6}}=\left[\begin{array}{lll}
\left(x_{6}-x_{5}\right) & \left(y_{6}-y 5\right) & \left(z_{6}-z_{5}\right)
\end{array}\right]  \tag{3.37}\\
\overrightarrow{v_{6^{\prime}}}=\left[\begin{array}{lll}
\left(x_{6^{\prime}}-x_{5}\right) & \left(y_{6^{\prime}}-y_{5}\right) & \left(z_{6^{\prime}}-z_{5}\right)
\end{array}\right] \tag{3.38}
\end{gather*}
$$

By using dot product of two vector formula we obtain:

$$
\begin{equation*}
\theta_{5 a}=\cos ^{-1}\left(\frac{\overrightarrow{v_{6}} \overrightarrow{v_{6^{\prime}}}}{\left\|\overrightarrow{v_{6}}\right\|\left\|\overrightarrow{v^{\prime}}\right\|}\right)=\cos ^{-1}\left(\frac{\overrightarrow{v_{6}} \overrightarrow{v_{6^{\prime}}}}{d_{6}^{2}}\right) \tag{3.39}
\end{equation*}
$$

Then we get $\theta_{5}$ :

$$
\begin{equation*}
\theta_{5}=\frac{\pi}{2} \pm \theta_{5 a} \tag{3.40}
\end{equation*}
$$

We introduce $\Delta \vec{v}=\overrightarrow{v_{6}}-\overrightarrow{v^{\prime}}$. The sign in (3.40) is plus $(+)$ when $\Delta \vec{v}_{(2)}$ is negative, otherwise is minus $(-)$. Now, $\theta_{1} \ldots \theta_{5}$ have been known and $\theta_{7}$ is a constant. Therefore we can obtain $\theta_{6}$ using the following formula:

$$
\begin{equation*}
{ }^{0} \mathbb{T}_{7}={ }^{0} \mathbb{T}_{5}{ }^{5} \mathbb{T}_{6}{ }^{6} \mathbb{T}_{7} \tag{3.41}
\end{equation*}
$$

where ${ }^{5} \mathbb{T}_{6}=A_{6}$ and ${ }^{6} \mathbb{T}_{7}=A_{7}$ as defined in (2.8). Because the unknown parameter $\theta_{6}$ is stated in $A_{6}$, so the solution of problem (3.41) is:

$$
\begin{equation*}
A_{6}=0 \mathbb{T}_{5}^{-1}{ }^{0} \mathbb{T}_{7} A_{7}^{-1} \tag{3.42}
\end{equation*}
$$

Finally, we get $\theta_{6}$ from known matrix $A_{6}$

$$
\begin{equation*}
\theta_{6}=\cos ^{-1} A_{6(1,1)} \tag{3.43}
\end{equation*}
$$

### 3.1.2 Specific inverse kinematic formulae when $O_{7}$ is given and link-6 is perpendicular to the object plane ( $Y_{0}-Z_{0}$ plane)

This is a specific case for section 3.1.1. We develop another formulae of inverse kinematics when the end-point position $\left(O_{7}\right)$ is given and the orientation is fixed determined so that the link-6 always be perpendicular to the object plane ( $Y_{0}-Z_{0}$ plane).

In fact, this case can be solved using general inverse kinematics formulae as in previous section. However, the general inverse kinematics formulae requires much complex computation. Therefore, we develop another simplified formulae to satisfy this case. Figure 3.8 shows the coordinate frame example of section 3.1.2.

In this case, to make the end-point orientation constant, $\theta_{6}$ should be determined previously. If the end-point position is given then the appropriate angle $\theta_{1}$ to $\theta_{6}$ can be obtained by the formulae in (3.44) to (3.57). Assume that an end-point $O_{7}=\left[x_{7} y_{7} z_{7}\right]^{T}$ is given and $\theta_{6}$ is defined previously, where $x_{i}, y_{i}$ and $z_{i}$ are the position coordinate of joint- $i$ in $X_{0}$-axis, $Y_{0}$-axis and $Z_{0}$-axis respectively.


Figure 3.8: Coordinate frame example for section 3.1.2

Note that $\theta_{7}$ is also a constant that defined by its mechanical structure. From Figure 3.8, $O_{6}, O_{5}$ and $O_{4}$ can be computed by equation in (3.44) to (3.46)

$$
\begin{gather*}
O_{6}=O_{7}+\left[\begin{array}{c}
-l_{7} \sin \theta_{7} \\
\cos \theta_{6} \cos \theta_{7} \\
-l_{7} \sin \theta_{6} \cos \theta_{7}
\end{array}\right]  \tag{3.44}\\
O_{5}=O_{6}+\left[\begin{array}{c}
-d_{6} \\
0 \\
0 \\
0
\end{array}\right]  \tag{3.45}\\
0 \quad\left[\begin{array}{c}
0 \\
0
\end{array}\right]
\end{gather*}
$$

Using known position of $O_{4}$, then $\theta_{1}$ can be computed:


$$
\begin{equation*}
\theta_{1}=\tan ^{-1}\left(\frac{y_{4}}{x_{4}}\right) \tag{3.47}
\end{equation*}
$$

If the area around $O_{1}, O_{3}$ and $O_{4}$ is zoomed in with top view as shown in Figure 3.9, the next formula can be derived to compute $\theta_{5}$ and position $O_{3}$.

$$
\begin{equation*}
\theta_{5}=\frac{\pi}{2}+\theta_{1} \tag{3.48}
\end{equation*}
$$



Figure 3.9: Zooming in the area $O_{1}, O_{3}$ and $O_{4}$ with top view

$$
O_{3}=O_{4}+\left[\begin{array}{c}
-l_{4} \cos \theta_{1}  \tag{3.49}\\
-l_{4} \sin \theta_{1} \\
0
\end{array}\right]
$$

An auxiliary position, called $T_{1}$ and variables $a, b, c$ and $d$ are introduced. Figure 3.10 is the zoomed area of $O_{1}, O_{2}$ and $O_{3}$ in $X_{1}-Z_{0}$ plane. From this figure we can derive formulae to compute $\theta_{2}$ and $\theta_{3}$ as in (3.54) and (3.55), where $\|\cdot\|$ denotes norm order 2.


Figure 3.10: Zooming in the area $O_{1}, O_{2}$ and $O_{3}$ with side view

$$
9090 T_{1}=O_{3}+\left[\begin{array}{c}
0 \\
0 \\
-d_{1} \tag{3.52}
\end{array}\right]
$$

$$
\begin{gather*}
c=\left\|O_{3}-O_{1}\right\|=\sqrt{a^{2}+b^{2}}  \tag{3.53}\\
\theta_{2}=\tan ^{-1}\left(\frac{a}{b}\right)+\cos ^{-1}\left(-\frac{l_{3}^{2}-l_{2}^{2}-c^{2}}{2 l_{2} c}\right)  \tag{3.54}\\
\theta_{3}=-\pi+\cos ^{-1}\left(-\frac{c^{2}-l_{2}^{2}-l_{3}^{2}}{2 l_{2} l_{3}}\right) \tag{3.55}
\end{gather*}
$$

Figure 3.11 shows the zoomed area of $O_{2}, O_{3}$ and $O_{4} . \theta_{4}$ can be obtained by the following formula:


Figure 3.11: Zooming in the area $O_{2}, O_{3}$ and $O_{4}$ with side view

$$
\begin{gather*}
d=\left\|O_{2}-O_{4}\right\|_{2}=\sqrt{\left(x_{2}-x_{4}\right)^{2}+\left(y_{2}-y_{4}\right)^{2}+\left(z_{2}-z_{4}\right)^{2}}  \tag{3.56}\\
\theta_{4}=\pi-\cos ^{-1}\left(-\frac{d^{2}-l_{3}^{2}-l_{4}^{2}}{2 l_{3} l_{4}}\right) \tag{3.57}
\end{gather*}
$$

### 3.1.3 Moving end-point in $O_{0}$ coordinate direction with keeping its orientation

In this section, we design a strategy for moving the end-point of the robotic arm. The end-point will be moved left-right, up-down and in-out in $Y_{0}, Z_{0}$ and $X_{0}$ direction respectively while the orientation is kept constant. This strategy requires formulae which is almost the same with section 3.1.1. The only difference is that to keep its orientation, we move $O_{5}, O_{6}$ and $O_{7}$ parallelly with the same relative movement from the previous position. Figure 3.12 shows the coordinate frame example for this strategy.

Assume that present positions $O_{i P}=\left[\begin{array}{lll}x_{i P} & y_{i P} & z_{i P}\end{array}\right]^{T}$ are known by using forward kinematics formula $O_{i P}=$ ForwardKinematics $\left(\vec{\theta}_{P}\right)$, where $i=5,6,7$ and $\vec{\theta}_{P}$ are obtained from the measurement of potentiometers. If desired relative movement, $\Delta \vec{P}=\left[\Delta P_{x} \Delta P_{y} \Delta P_{z}\right]^{T}$, in original base $\left(O_{0}\right)$ coordinate direction is given then we can easily find the next position of $O_{7 N}, O_{6 N}$ and $O_{5 N}$ :

$$
\begin{align*}
& O_{7 N}=O_{7 P}+\Delta \vec{P}  \tag{3.58}\\
& O_{6 N}=O_{6 P}+\Delta \vec{P} \tag{3.59}
\end{align*}
$$

$$
\begin{equation*}
O_{5 N}=O_{5 P}+\Delta \vec{P} \tag{3.60}
\end{equation*}
$$



Figure 3.12: Coordinate frame example for section 3.1.3 strategy

Using known position of $O_{5}, O_{6}$ and $O_{7}$ then we can compute $\theta_{1} \ldots \theta_{5}$ with formulae in (3.10) to (3.40). In this case, there is a difference formula on how to compute $\theta_{6}$ compare to that one in section 3.1.1 because here we do not specify the orientation by $\alpha$ and $\beta$. Therefore we just need previous orientation as the next orientation.

Suppose that corresponding homogeneous matrix transformations of the present position ${ }^{0} \mathbb{T}_{7}$ is denoted by ${ }^{0} \mathbb{T}_{7 P}$ whereas the next position ${ }^{0} \mathbb{T}_{7},{ }^{0} \mathbb{T}_{5},{ }^{5} \mathbb{T}_{6}$ and ${ }^{6} \mathbb{T}_{7}$ are denoted by ${ }^{0} \mathbb{T}_{7 N},{ }^{0} \mathbb{T}_{5 N}$, $A_{6 N}$ and $A_{7 N}$ respectively. The next homogeneous matrix transformation ${ }^{0} \mathbb{T}_{7 N}$ is obtained from the present homogeneous matrix transformation ${ }^{0} \mathbb{T}_{7 P}$ using the following relation:

$$
0 \overbrace{0} \mathbb{T}_{7 N}=0 \mathbb{T}_{7 P}+\left[\begin{array}{cccc}
0 & 0 & 0 & \Delta \vec{P}_{x}  \tag{3.61}\\
0 & 0 & 0 & \Delta \vec{P}_{y} \\
0 & 0 & 0 & \Delta \vec{P}_{z} \\
0 & 0 & 0 & 0
\end{array}\right]
$$

To find $\theta_{6}$, we use the following formula:

$$
\begin{equation*}
{ }^{0} \mathbb{T}_{7 N}={ }^{0} \mathbb{T}_{5 N} A_{6 N} A_{7 N} \tag{3.62}
\end{equation*}
$$

$$
\begin{equation*}
A_{6 N}={ }^{0} \mathbb{T}_{5 N}^{-1}{ }^{0} \mathbb{T}_{7 N} A_{7 N}^{-1} \tag{3.63}
\end{equation*}
$$

Then, we get $\theta_{6}$ from this known matrix $A_{6 N}$

$$
\begin{equation*}
\theta_{6}=\cos ^{-1} A_{6 N(1,1)} \tag{3.64}
\end{equation*}
$$

This is the general procedure for moving the end-point of robotic arm in coordinate $O_{0}$ direction with keeping its orientation:

1. Obtain the present Polar position $\vec{\theta}_{P}$ by measuring the potentiometer attached at every axis.
2. Using known $\vec{\theta}_{P}$ and general homogeneous transformation matrix in (2.8), compute:
${ }^{0} \mathbb{T}_{7 P}=A_{1 P} A_{2 P} A_{3 P} \ldots A_{7 P}$.
3. Obtain the next position and its orientation ${ }^{0} \mathbb{T}_{7 N}$ using formula in (3.61).
4. Compute $\theta_{1 N} \ldots \theta_{5 N}$ using formulae in (3.10) to (3.40).
5. Solve $\theta_{6 N}$ using formula in (3.64).
6. Drive the robotic arm following the new Polar position $\vec{\theta}_{N}$.

### 3.1.4 Moving end-point in $O_{7}$ coordinate Direction with keeping its orientation

In this section, we design a similar strategy for moving the end-point of the robotic arm as in section 3.1.3. The same formulae and the same procedure are also employed. However, the relative movement will be specified in $O_{7}$ coordinate direction (not in $O_{0}$ coordinate direction). As addition, we constrain the movement only in $X_{7}$ or $Y_{7}$ or $Z_{7}$ exclusively.

The following procedure is a general step for moving the end-point of robotic arm in coordinate $O_{7}$ direction with keeping its orientation:

1. Obtain the present Polar position $\vec{\theta}_{P}$ by measuring the potentiometer attached at every axis.
2. Using known $\vec{\theta}_{P}$ and general homogeneous transformation matrix in (2.8), compute:
${ }^{0} \mathbb{T}_{7 P}=A_{1 P} A_{2 P} A_{3 P} \ldots A_{7 P}$.
3. Assume that we have link- 8 so that the position and orientation of $O_{8}$ is the same with $O_{7}$ exactly. Now we can derive a movement in $O_{7}$ coordinate direction using this pseudo link8. Moving $\Delta P_{x}$ distance in $X_{7}$ direction means specifies $\alpha_{8}=0, d_{8}=0, l_{8}=\Delta P_{x}$, and $\theta_{8}=0$. Corresponding homogeneous transformation matrix of axis- 8 can be stated by $A_{8}=$ $A_{i}\left(\alpha=0, d=0, l=\Delta P_{x}, \theta=0\right)$. Likewise, $\Delta P_{y}$ and $\Delta P_{z}$ distance movement in $Y_{7}$ and $Z_{7}$ direction respectively can be stated by $A_{8}=A_{i}\left(\alpha=0, d=0, l=\Delta P_{y}, \theta=\pi / 2\right)$ and $A_{8}=A_{i}\left(\alpha=0, d=\Delta P_{z}, l=0, \theta=0\right)$.
4. Compute the pseudo-position $O_{8}={ }^{0} \mathbb{T}_{7 P} A_{8}$.
5. Obtain the next position and its orientation ${ }^{0} \mathbb{T}_{7 N}$ using formula in (3.61), where $\Delta P_{x}=x_{8}-x_{7}$, $\Delta P_{y}=y_{8}-y_{7}$ and $\Delta P_{z}=z_{8}-X_{z}$.
6. Compute $\theta_{1 N} \ldots \theta_{5 N}$ using formulae in (3.10) to (3.40).
7. Solve $\theta_{6 N}$ using formula in (3.64).
8. Drive the robotic arm following the new Polar position $\vec{\theta}_{N}$.

### 3.2 Trajectory File Generator

A trajectory pattern that contains a sequence of the robot movement in Cartesian coordinate is obtained by solving the inverse kinematics problem with some angular-to-digital conversions. This trajectory pattern is specified by an human operator in Cartesian coordinate. Corresponding file represents the sequence of the polar coordinate in digital value then is generated. This file is stored in Microsoft Excel format and so-called trajectory file.

Using inverse kinematics formulae in (3.44) to (3.57), a Cartesian coordinate will be converted into polar coordinate. Since the controller is a digital system, the polar coordinate value needs to be converted into digital value before proceeded by the controller. The relation between the angle in polar coordinate and the digital value is regressed by Linear Least Square method using data from experiment. This is called angular to digital value conversion.

The following is the least squares regression line at joint- $i$ :

$$
\begin{equation*}
\hat{y_{i}}=m_{i} \theta_{i}+c_{i} \tag{3.65}
\end{equation*}
$$

where $\hat{y_{i}}, \theta_{i}, m_{i}, c_{i}, \bar{\theta}_{i}$ and $\bar{y}_{i}$ denote the estimate of the angle in digital value, the angle, regression coefficient, a constant, the mean of $\theta_{i}$ and the mean of $y$ at axis- $i . \sim$

In this robotic arm, all of joints using rotary potentiometers to measure the angle in every joint except joint-3 which uses linear potentiometer, Generally, the linear regression line in (3.65) is used to state the relation between the angle in polar coordinate and the digital value. Particularly in joint-3, this linear regression line is used to state the relation between the length of $L_{p}$ and the digital value as shown in Figure 3.13. The relation between the angle and the length of $L_{p}$ is given in (3.68).

$$
\begin{equation*}
L_{p}\left(\theta_{2}, \theta_{3}\right)=\sqrt{\left(l_{2} \cos \theta_{2}+l_{31} \cos \left(-\left(\theta_{3}+\theta_{2}\right)\right)-l_{21}\right)^{2}+\left(\left(l_{2} \sin \theta_{2}-l_{31} \sin \left(-\left(\theta_{3}+\theta_{2}\right)\right)\right)^{2}\right.} \tag{3.68}
\end{equation*}
$$

where $l_{21}=18 \mathrm{~cm}$ and $l_{31}=20 \mathrm{~cm}$.


Figure 3.13: The structure of linear potentiometer at joint-3
The experiment was set up to find $m_{i}$ and $c_{i}$ as explain in (3.66) and (3.67). After measuring $\theta_{i, j}$ and $y_{i, j}$, parameters $m_{i}$ and $c_{i}$ are obtained and shown in Table 3.2.


### 3.3 System Identification

The following section describes the system identification steps. The first step is usually the selection of a model structure. There are various possibilities of structure state-space and polynomial forms such as ARX, ARMAX, OE, BJ etc. The more complex the structure, the better estimate model is yielded. In this work, a high-accuracy model is not required, therefore ARX structure is chosen because of its simplicity. For a given choice of structure, the order of the model needs to be specified before the corresponding parameters are estimated.

In order to estimate the servo valves of the robot, a set of data input and output from experiment are required. As an illustration, the 6 -th axis servo valve estimation is given to explain its step using System Identification Toolbox provided by Matlab.

### 3.3.1 Delay Estimation

The amount of delay $(d)$ as shown in (2.46) also infulence the selection of the model order. Therefore we need to estimate the best delay before selecting the model order. There are various options available for determining the time delay from input to output. However, this work will focus only on one of them, that is using delayest utility. This function evaluates an ARX structure as in (3.69), where $u, y, n a, n b$ and $d$ respectively are the input signal, the output signal, the order of input, the order of output and the time delay.

$$
\begin{equation*}
y_{k}+a_{1} y_{k-1}+\ldots+a_{n a} y_{k-n a}=b_{1} u_{k-d}+\ldots+b_{n b} u_{k-n b-d+1} \tag{3.69}
\end{equation*}
$$



Assume that we have two set of data input and output ( $x 1, y 1$ and $x 2, y 2$ ) which are obtained from measurement at axis- 6 as shown in Figure 3.14. Here is the Matlab syntax to estimate the delay
using delayest toolbox.

```
>> cdata = iddata(y1,x1); % data test
>> vdata = iddata(y2,x2); % data validation
>> delay = delayest(cdata);
    delay = 1
```

This result shows that the best time delay $(d)$ that should be selected is 1 .

### 3.3.2 Order Selection

Once we have decided the model structure to use, the next task is to determine the order(s). In general, the selected order should give as close as possible to the real model. However, it should be not higher than necessary. This can be determined by analyzing the improvement in fit as a function of model order. When doing this, it is advisable to use a separate, independent dataset for validation. Choosing an independent validation dataset (vdata in this example) would improve the detection of over-fitting.


Figure 3.15: The ARX model structure selection


In addition to a progressive analysis of multiple model orders, explicit determination of optimum orders can be performed for some model structures. Functions arxstruc and selstruc are used for choosing the best order for ARX models. In this work, MATLAB 7.0 is employed to facilitate system identification. We check the fit for all 100 combinations of up to $10 B(z)$-parameters and up to $10 A(z)$-parameters as in (2.46), all with a delay value of 1 :

```
>> V = arxstruc(cdata,vdata,struc(1:10,1:10,1));
```

The best fit for the validation data set is obtained by:

```
>> nn = selstruc(V,0);
    nn=10}1010
```

For choosing the model order interactively, the following syntax can be used:

```
>> nns = selstruc(V);
```

Figure 3.15 shows the ARX model structure selection panel. From this figure we know that the best fit model is performed when $n a=10, n b=10$ and the time delay $d=1$ as shown by the red color bar. If we use Akaike Criterion, we get the best order model when $n a=1, n b=6$ and $d=1$ as shown by the blue color bar. A simple $2^{\text {nd }}$ order ARX yields only $3.5 \times 10^{-3}$ unexpected output variance. This structure has good enough approximate of the model and nowever can be well exploited to estimate the controller as proven by the experiment result in section 5.7.

### 3.4 Controller Design

Due to result in previous section, 2-DOF PID controller will be designed based on the approximate model of the robot. For the reason of simplicity, $2^{\text {nd }}$ order of plant model and $2^{\text {nd }}$ order of 2 -DOF PID controller with pole placement method are chosen. Assume that the approximate model, $H(z)$, obtained by identification is given in (3.70) and the controller structure is as shown in Figure 2.4 (b).

$$
\begin{equation*}
H(z)=\frac{B(z)}{A(z)}=\frac{b_{1} z^{-1}+b_{2} z^{-2}}{1+a_{1} z^{-1}+a_{2} z^{-2}} \tag{3.70}
\end{equation*}
$$

A 2-nd order controller parameter is stated by:


A polynomial $P(z)$ represents the desired pole locations is specified by:

$$
\begin{equation*}
P(z)=1+p_{1} z^{-1}+p_{2} z^{-2}+p_{3} z^{-3}+p_{4} z^{-4} \tag{3.73}
\end{equation*}
$$

In this design, polynomial $R(z)$ is not in general form. By reconfiguring (2.54) we have

$$
\begin{equation*}
x=\left[s_{0}, s_{1}, s_{2}, r\right]^{T} \tag{3.74}
\end{equation*}
$$

$$
\begin{gather*}
M=\left[\begin{array}{llll}
b_{1} & 0 & 0 & 1 \\
b_{2} & b_{1} & 0 & a_{1}-1 \\
0 & b_{2} & b_{1} & a_{2}-a_{1} \\
0 & 0 & b_{2} & -a_{2}
\end{array}\right]  \tag{3.75}\\
K=\left[\begin{array}{l}
p_{1}+1-a_{1} \\
p_{2}+a_{1}-a_{2} \\
p_{3}+a_{2} \\
p_{4}
\end{array}\right]  \tag{3.76}\\
x=M^{-1} K \tag{3.77}
\end{gather*}
$$

The following is an example of 2-DOF controller design for joint-1. The estimate model of plant is given in (3.78) and the desired pole locations are multiple pole at 0.8 as shown in (3.79). In order to guarantee that the controller system is stable, the poles must be chosen so that they are located inside unity circle. The closer of the poles to the zero, the faster of settling time to be stable. However the closer of the poles to the zero will affect to the vulnerability of its stability. Therefore, locating pole at 0.8 is one of good choices regarding to those reasons.

$$
\begin{gather*}
H(z)=\frac{B(z)}{A(z)}=\frac{-0.0026 z^{-1}-0.0116 z^{-2}}{1-1.4825 z^{-1}+0.4821 z^{-2}}  \tag{3.78}\\
P(z)=\left(1-0.8 z^{-1}\right)^{4}=1-3.2 z^{-1}+3.84 z^{-2}-2.048 z^{-3}+0.4096 z^{-4} \tag{3.79}
\end{gather*}
$$

Then the matrix $M$ and vector $K$ are stated by:

$$
M=\left[\begin{array}{llll}
-0.0026 & 0 & 0 & 1  \tag{3.80}\\
-0.0116 & -0.0026 & 0 & -2.4825 \\
0 & -0.0116 & -0.0026 & 1.9646 \\
0 & 0 & -0.0116 & -0.4821
\end{array}\right]
$$

The solution is:

$$
K=\left[\begin{array}{l}
-0.7175  \tag{3.81}\\
1.8754 \\
-1.5659 \\
0.4096
\end{array}\right]
$$



### 3.5 Hardware Design

This section explains the hardware requirement. In general, the hardware is classified into four parts i.e. main processor board, analog board, digital I/O board and power supply board.

### 3.5.1 Main Processor Board

The main processor board is the central processing unit of the controller system. This part is responsible on receiving, processing and transmitting data from and to external devices. This part composed of several internal peripherals such as 32-bit soft-core processor, UART, Digital I/O, Timer/Counter, SPI, SPI reader and PWM generator. All of mentioned peripherals are embedded in a Xilinx FPGA XC3S400.


Figure 3.16: MicroBlaze core block diagram


### 3.5.1.1 MicroBlaze 32-bit Soft-core Processor

Microblaze is a 32-bit soft-core processor developed by Xilinx. This is a reduced instruction set computer (RISC) optimized for implementation in Xilinx Field Programmable Gate Arrays (FPGAs). Figure 3.16 shows a functional block diagram of the MicroBlaze core.

The basic MicroBlaze architecture is consists of 32 general-purpose registers, an Arithmetic Logic Unit (ALU), a shift unit, and two levels of interrupt. This basic design can then be configured with more advanced features such as: memory management unit (MMU), barrel shifter, memory management/memory protection unit, floating-point unit (FPU), caches, exception handling, debug


Figure 3.17: MicroBlaze core with advanced features
logic, and others. This flexibility allows the user to balance the required performance of the target application against the logic area cost of the soft processor. Figure 3.17 shows a MicroBlaze system configured with advanced features.

Three types of memory interfaces are supported: Local Memory Bus, Processor Local Bus (PLB) / On-Chip Peripheral Bus (OPB) and Xilinx CacheLink (XCL). In this design, Microblaze system is configured with 16 KB local/internal memory, active low reset, 50 MHz bus clock frequency and several advanced feautures which are connected via PLB Bus as described in subsection 3.5.1.2 to 3.5.1.7.
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### 3.5.1.2 UART

The UART peripheral is connected to the PLB Bus to provide the controller interface for asynchronous serial data transfer as shown in Figure 3.18

This peripheral performs parallel to serial conversion on character reveived through PLB and serial to parallel conversion on characters received from external serial devices. The UART peripheral is capable of transmitting and receiving $8,7,6$ or 5 bit characters with 1 bit stop and 1 bit parity.

There are three sections inside the UART peripheral. First, the PLB interface module provides the interface to the PLB and implements PLB protocol logic. Second, the register module consist


Figure 3.18: Block diagram of UART peripheral
of an 8-bit status register, an 8 -bit control register and a pair of 8 -bit Transmit/Receive registers. All registers are accessed directly from the PLB using the PLB module. Third, the control module consists of a RX module, a TX module, a parameterized baud rate generator (BRG) and a control unit.


### 3.5.1.3 Digital I/O (a) Q

The digital I/O peripheral is a general purpose Input/Output (GPIO) to interface the PLB Bus with external devices. It is a 32-bit bi-directional data transfer which can be configured either as input or as output at a time.

An input port may be configured to take its external input either from bidirectional 3-state pin or from the dedcated input only pins. As a ouput port, the data is driven out through a 3-state buffer as
well as to an input only pin. The port can be configured dynamically for input or output by enabling or disabling the 3 -state buffer.

The bock diagram of GPIO is shown in Figure 3.19. It is composed of 3 modules i.e PLB interface, Interrupt Control and GPIO core. It can be configured as a single or a dual channel device. The channel width and the direction are configurable and can be enabled individually.

### 3.5.1.4 Timer/Counter



Figure 3.20: Block diagram of Timer/Counter peripheral

This Timer/Counter peripheral is a 32-bit timer module that attaches to the PLB bus. It is organized as two identical timer modules as shown in Figure 3.20. Each timer module has an associated load register that is used to hold either the initial value for the counter for event generation or a capture value depending on the mode of the timer.

There are three modes that can be used with the two Timer/Counter modules:

- Generate Mode. The value in the load register is loaded into the counter. When enabled, the counter beings to count up or down depending on the selection of the Timer Control Status Register. This mode is useful for generating repetitive interrupts or external signals with a specified interval.
- Capture Mode. The value of the counter is stored in the load register when the external capture signal is asserted. This mode is useful for time tagging external events while simultaneously generating an interrupt.
- Pulse Width Modulation (PWM) Mode. Two timer/counters are used as a pair to produce an output signal (PWMo) with a spefified frequency and duty factor. Timer0 and timer1 are used to set the period and the high interval time of the PWM respectively.


### 3.5.1.5 SPI

The SPI peripheral connects to the PLB bus and provides a serial interface to SPI devices. The SPI protocol provides a simple method for a master and as selected slave to exchange data. SPI is a full-duplex synchronous channel that supports four-wire interface (transmit, receive, clock and slaveselect) between a master and a selected slave. The block diagram of SPI peripheral is shown in Figure 3.21 .


Figure 3.21: Block diagram of SPI peripheral

When configured as a master, the SPI can communicate with both off-chip and on-chip slaves. However, when configured as a slaye, it can communicate only with on-chip masters. The number of slaves is limited to 32 by the size of the Slave Select Register. However, the number of slaves and masters will impact the achievable performance.

### 3.5.1.6 SPI reader

The SPI reader peripheral is a custom logic 16-bit shift register which is compatible with SPI protocol. However it is only able to perform read access and act as a slave module. As shown in Figure 1.1, there are 6 channels of ADC accessed by SPI protocol. The ADCs are desired to be accessed concurrently. It is redundant to use 6 SPI peripheral because it consumes more resources. Therefore a custom SPI reader is developed to optimize and reduce the resources usage. A single command can be sent to all


Figure 3.22: Block diagram of SPI reader
of ADCs while 6 SPI readers are used to read the data from every channel concurrently. Figure 3.22 shows the block diagram of SPI reader module with their connections to ADCs.


### 3.5.1.7 PWM Generator

The PWM generator peripheral is a custom logic hardware which is dedicated specifically to generate PWM signal. Figure 3.23 shows the block diagram of PWM generator.

The PWM generator composed of a 32-bit counter register and two 32-bit comparator registers. The counter data and the comparators data are defined by sending data through PLB bus. The PLB interface module then store that data into counter register and comparator registers. When rising clock is happened, the counter register value will be incremented by 1 and at the same time this
counter value is compared with both comparator registers value. When the value of counter register is greater than comparator register A value then PWM ouput signal is forced to be ' 0 '. When the value of counter register is greater than comparator register B value then PWM output signal is forced to ' 1 ' and register counter value is reset to 0 (zero). The PWM signal generation can be illustrated as shown in Figure 3.24.


Figure 3.24: PWM output signal

### 3.5.2 Analog Board

Analog board composed of three parts: digital isolator, filter-amplifier and analog to digital converter (ADC). The block diagram of Analog board is shown in Figure 3.25.


### 3.5.2.1 Digital Isolator

The digital isolator is used to isolate the power supply between the main processor board and the analog board. Those power supply must be separated in order to reduce and avoid the noise propagation from or to external devices. ADuM14xx digital isolator is chosen because it has very high switching speed (up to 90 Mbps ), low pulse width distortion (less than 2 ns ) and provides four independent
isolation channels in a variety of channel configurations and data rates. Figure 3.26 shows the block diagram of ADuM14xx.


Figure 3.26: Block diagram of ADuM14xx

### 3.5.2 2 Filter and Amplifier

The filter and amplifier circuit are respectively used to convert the PWM signal into analog signal and to amplify the analog signal. The filter is designed specifically to satisfy the PWM characteristics. In a typical PWM signal, the base frequency is fixed but the pulse width is a variable. The produced analog signal is directly proportional to the duty cycle $(D)$ of the PWM signal. A typical PWM signal is shown in Figure 3.27.


A Fourier analysis shows that there is a strong peak at frequency $f_{P W M}$ and other strongs harmonics at frequency $n f_{P W M}$ as shown in Figure 3.28, where $n$ is an integer and $f_{P W M}$ is the base frequency of PWM signal. These peaks are unwanted noise and should be eliminated. To eliminate these noise a low-pass filter with frequency cut off $\left(f_{c}\right)$ lower than $f_{P W M}$ must be employed.

In this design, the PWM base frequency $\left(f_{P W M}\right)$ is fix determined by loading 4096 at register comparator B in Figure 3.24. With 100 MHz source clock, it yields 40.96 us periode of PWM ( $T_{P W M}$ ) or equal to 25 KHz frequency of PWM $\left(f_{P W M}\right)$. Therefore the low-pass filter must have frequency cut off less than 25 KHz . The lower $f_{c}$ the better analog signal is yielded. However, one must be considered that a sinusoidal waveform will be generated at frequency 200 Hz . Hence the $f_{c}$ of the low-pass filter must be selected so that it is less than 25 KHz but greater than 200 Hz . In this case, $f_{c}=232.27 \mathrm{~Hz}$ is chosen. This low-pass filter is realized by using 2-nd order passive filter with $R_{1}=10 \mathrm{KOhm}, R_{2}=100 \mathrm{KOhm}, C_{1}=0.1 \mathrm{uF}$ and $C_{2}=4700 \mathrm{pF}$ as shown in Figure 3.29 and equation in (3.84).


Figure 3.28: The spectrum frequency of PWM

$$
\begin{equation*}
f_{c}=\frac{1}{2 \pi \sqrt{R_{1} R_{2} C_{1} C_{2}}}=\frac{1}{2 \pi \sqrt{10 * 100 * 0.1 * 4700 * 10^{-12}}}=232.27 \mathrm{~Hz} \tag{3.84}
\end{equation*}
$$



Figure 3.29: The circuit of filter and amplifier
The analog signal produced by low-pass filter will be used to drive the servo valve of the robotic arm. Since the PWM signal level are -2.5 V and +2.5 V , the output of analog signal needs to be amplified so that it is powerful enough to drive the servo valve. In this design, analog signal is amplified 4.6 times by choosing $R_{3}=36 \mathrm{KOhm}$ and $R_{4}=10 \mathrm{KOhm}$ as computed in equation (3.85), where $V_{i}$ and $V_{o}$ are the input and the output signal respectively. Therefore the analog signal output span becomes -11.5 V to +11.5 V .

$$
\begin{equation*}
V_{o}=\left(1+\frac{R_{3}}{R_{4}}\right) V_{i}=\left(1+\frac{36}{10}\right) V_{i}=4.6 V_{i} \tag{3.85}
\end{equation*}
$$

### 3.5.2.3 Analog to Digital Converter

The analog to digital converter is used to convert the analog signal obtained by measuring the potentiometer voltage into digital value. The potentiometer is attached in every joint of robotic arm to
represent the angle. In other word, the ADC is used to convert the measured angle of joint into digital value.


Figure 3.30: The circuit of analog to digital converter

A single channel ADC (MCP3201) is chosen in this design because its features satisfy the system requirement. Moreover, it has very low price compare to other multi-channels ADCs. MCP3201 has 12-bit resolution, 100 KSPS maximum sampling rate, SPI protocol compatible and single supply operation up to +5 V . The ADC circuit is shown in Figure 3.30.


A 2-nd order active low-pass filter with unity gain is placed between measured potentiometer voltage and the ADC. This filter is used to filter high frequency noise from external and functioned as a buffer to isolate the input load. Because the measured signal is changed very slow, frequency cut off $f_{c}=1592.4 \mathrm{~Hz}$ is a good value to be selected. This filter is realized by choosing $R_{1}=R_{2}=1 \mathrm{KOhm}$
and $C_{1}=C_{2}=0.1 \mathrm{uF}$ as computed in (3.86).

$$
\begin{equation*}
f_{c}=\frac{1}{2 \pi \sqrt{R_{1} R_{2} C_{1} C_{2}}}=\frac{1}{2 \pi \sqrt{1 * 1 * 0.1 * 0.1 * 10^{-12}}}=1592.4 \mathrm{~Hz} \tag{3.86}
\end{equation*}
$$

The frequency response and the phase response of the designed filter are shown in Figure 3.31. This data is obtained by giving some sinusoidal waveforms wit at point A and then measure the output at point B

### 3.5.3 Digital I/O Board

The digital I/O part is used to interface the digital input-output signal between main processor and external devices. Solid state relays LH150 and opto-couplers CNY173 are applied as output and input interfaces respectively as shown in Figure 3.32. This interface solves the DC level mismatch problem (main processor voltage is $0-3.3 \mathrm{~V}$; whereas those of $\mathrm{I} / \mathrm{Os}$ are either $0-12 \mathrm{~V}$ or $0-24 \mathrm{~V}$ ) and noisy environment problem.


Figure 3.32: Block diagram of Digital I/O

### 3.5.4 Power Supply Board

Several DC voltages are required to supply power for main processor board, analog board and digital I/O board. In order to eliminate the noise interference among those board, the DC voltages should be isolated from each others. In this design, a multi secondary side transformer was constructed to produce eight constant voltage supplies with four isolately ground as shown in Fig. 3.33. Furthermore, the current requirement for all supplies are less than 0.5 A , that linear regulation therefore seems to be a good selection for this purpose. By isolating the power supply, it will reduce the noise and interference from both external noisy factory environment and among internal parts.


Figure 3.33: Power supply unit

### 3.6 Software

This section concern with the software of the controller system. Physically, the hardware provides 3 buttons as input interface used to command the controller system software: 2 general buttons and 1 emergency button as shown in Figure 5.14.


### 3.6.1 Main Menu

Figure 3.34 shows the state machine diagram of the software. The main software consist of two main states: standalone mode and PC mode. At the main menu state, the state when the controller system just ran, some peripheral initializations are done. Initializations are required to set how each peripheral works. UART initializes the serial communication between controller system and PC such
as port, baudrate, stop bit and so on. SPI initializes the communication between controller system and ADC. $P W M$ initializes the duty cycle and the base frequency of PWMs. $I / O$ initializes general input and output pins. Controller initializes the PID controllers parameter in their initial value. $L C D$ initializes the LCD appearance. Interrupt initializes the interrupt service requests.

Having finished the initialization, the software come to a continuous-looping. The looping will never end unless one of command buttons is pressed. Pressing $S D$ button brings the state to standalone mode. Likewise, pressing PC button brings the state to PC mode. Conversely, pressing back button will back its state to the main menu.


Figure 3.35: Flowchart diagram of software main menu

### 3.6.2 Standalone Mode State

Figure 3.36 shows the flowchart diagram of standalone mode state. A second after standalone mode is chosen then the system will check the existence of the trajectory file. If there is no trajectory file available in the Flash ROM then it shows an empty message on LCD display then goes to main menu state. Otherwise, it goes to a continuous looping until either start button or back button is pressed. Pressing start button brings its state to start whereas pressing back button brings its state to the main тепи.


Figure 3.36: Flowchart diagram of standalone mode

### 3.6.3 Run State

In run state, the trajectory file will be loaded from the Flash ROM into main memory of the system. Then the robot will be driven so that it moves following the pattern stored in the trajectory file. Pressing stop button brings its state to stop whereas pressing pause button brings its state to pause.


### 3.6.4 Pause State

When pause button is pressed, controller system holds the present position and waits until the next command is given. Pressing continue button lets the robot continuing its motion and brings its state
back to run state whereas pressing back button brings it to standalone mode state.


### 3.6.5 Stop State

Stop state is similar with pause state. If stop button is pressed, controller system holds the present position. While the present position is hold, however, its trajectory sequence is resetted to the initial position of the trajectory file. Then it waits until the next command is given. If restart button is pressed, it goes to run state again. If back button is pressed, it backs to standalone mode state.


Figure 3.39: Flowchart diagram of stop state

### 3.6.6 PC Mode State



When PC Mode is chosen, controller system will activate the UART communication channel by enabling UART interrupt. The controller system can communicate with PC, either receiving command from PC or transmitting data to PC, using UART protocol. During PC mode, all of robotic operations are controlled through PC. When a certain command is received, its state goes to corresponding state as shown in Figure 3.34. Every command is known by a unique identity number called ID command. If there is no command received and back button is pressed then it brings the state back to the main menu.

### 3.6.7 Switch Pump State

This state is done when a command with $\mathrm{ID}=22$ is received. After the a byte command is received then it waits to receive the next byte for determining the next instruction. If the next received byte is ' 1 ' then controller system switch the pump ON. Conversely, the controller switch the pump OFF
when it receives '0' as the next received byte.


Figure 3.41: Flowchart diagram of switch pump state

### 3.6.8 Switch Valve State

This is a similar state with switch pump. The state comes to switch valve state when a command with $\mathrm{ID}=33$ is received. The next received byte determines the next instruction, either switch on the oil valve or switch off the oil valve. The next received byte $=' 1$ ' means switch the oil valve ON, otherwise switch the oil valve OFF.


### 3.6.9 Switch Control State

The same way as switch pump and switch valve is also used in switch control state. The state comes to switch control when a command with $\mathrm{ID}=44$ is received. The next received byte $={ }^{\prime} 1$ ' means the controller status is ON , otherwise is OFF .


Figure 3.43: Flowchart diagram of switch control state

### 3.6.10 Playback State

If a command with $\mathrm{ID}=47$ is received then it goes to playback state. Playback means the controller system drives the robot moving with certain pattern automatically. The second received byte determines the playback status. If the second received byte is ' 1 ' then playback status is ON , otherwise is OFF. When the playback status is ON, the controller system drives the robot following a sequence of data represents its position from the trajectory file.


### 3.6.11 Update Controller State

After a corresponding ID command of update controller is received, then series of data are received following the ID command. These series of data represent the coefficient of controller parameters $R(z), S(z)$ and $T(z)$ as shown in (3.71), (3.72) and (3.83). These data is begun by 2 bytes represents
the coefficient of $R_{0}$, followed by 2 bytes represents the coefficient of $R_{1}$ and so on up to $T_{2}$ as shown in flowchart diagram 3.45.


Figure 3.45: Flowchart diagram of update controller state

### 3.6.12 Get Single Reference State



Figure 3.46: Flowchart diagram of get single reference state

- The get single reference state is begun by receiving a byte of corresponding ID command then followed by 12 bytes data and ended by a byte of closing ID. These 12 bytes data divides into 6 parts which every part ( 2 bytes data) represents the reference value of angle/position in every axis of robotic arm. A closing ID is required to guarantee that these sequence of data are received properly. When the closing ID does not received correctly then the received data will never used to update the reference value.


### 3.6.13 Save to Flash State



Figure 3.47: Flowchart diagram of save to flash

Save to flash means a present trajectory file used by the controller system will be stored in the Flash ROM. Storing the trajectory file into Flash ROM enables the standalone mode menu because the controller system can load this trajectory file anytime without connected to a PC. As shown in Figure 3.47, when a command ID of save to flash is received then it does a looping to copy the trajectory data occupies the volatile memory at certain address to the Flash ROM of the controller system.

### 3.6.14 Identification State



Figure 3.48: Flowchart diagram of identification state

In identification state, two bytes of data are received firstly. The first byte is a corresponding ID command of identification and the second command determines the identification status. If the second byte is '0' then identification status is OFF, otherwise is ON. When the identification status is ON, 5 bytes of data are received following the previous command. These data consist of 3 parts: 1 byte to define the desired signal waveform, 2 bytes represents the desired period of signal and last 2 bytes represents the desired amplitude of signal that is used as input signal in system identification process.

### 3.6.15 Get Trajectory File State



Figure 3.49: Flowchart diagram of get trajectory file state

Get trajectory file state consists of multiple routines of get single reference and some additional routines as shown in flowchart diagram 3.49. Firstly, a byte ID command is received then followed by 2 bytes data represents the number of data rows that will be received later. Every row of received data composed of 12 bytes data represents the reference value of angle/position and 1 byte of closing ID as used in get single reference state.

### 3.6.16 Interrupt Routine

There are three available interrupts provided by the software: emergency interrupt, timer interrupt and UART interrupt.

- Emergency interrupt handles the emergency condition by pressing the emergency button. When the emergency button is pressed, the controller system forces to switch the servo valve and the oil valve OFF, re-initializes their peripherals and back to the main menu state. Figure 3.50 shows the flowchart diagram of emergency interrupt.


Figure 3.50: Flowchart diagram of interrupt timer routine

- Timer interrupt generates a certain interval time (in this case 10 ms ) that becomes the controller sampling rate. Every single sampling rate, a timer interrupt routine will be performed. This routine covers updating ADC value, getting current reference value, computing PID algorithm and generating signal for system identification. When the controller status is ON, the system generates control signal using PID algorithm and send it to DAC module for driving the robotic arm. The same way is treated to the identification status. When the identification status is ON, the system generates certain waveform based on its command and send the measured data to PC. In order to avoid an interleaving it must be guaranteed that this routine will be accomplished before the coming timer interrupt is occurred. Figure 3.51 shows the flowchart diagram of timer interrupt routine.


Figure 3.51: Flowchart diagram of timer interrupt routine

- UART interrupt handles a command that is sent from PC. This interrupt will be enabled only when PC mode is selected. There are 9 commands known by the controller system: switch control, switch pump, switch valve, get single reference, get trajectory file, playback, update
controller, save to flash and identification. Each command has unique command code to differentiate each other. For example command for getting single reference has command code $=$ 11 , switch pump has command code $=22$ and so on. Each command has a specific routine that will be executed immediately after the command is received.



## CHAPTER IV

## GUI USER APPLICATION SOFTWARE

This chapter concern with GUI user application software as an interface between the operator (human) and the controller system. This software is developed by using Delphi Integrated Development Equipment program. Basically, it has five main menus: Controller System, Identification System, Trajectory Generator, Teaching Mode and Simulation. The state machine diagram of the main menu and its screenshot display are shown in Figure 4.1 and Figure 4.2 respectively.


Figure 4.1: The state machine diagram of main menu


Figure 4.2: Main menu

Clicking one of five available menus above brings the state to the corresponding menu state and shows a new window panel. Then, closing the menu window panel will back its state to the main menu state. Particularly for the windows panel Controller System, Teaching Mode and Trajectory Generator respectively, they have a command to bring its state to Teaching Mode and Simulation state directly. Subsection 4.1 to 4.5 describe the detail of each those menu. Complete flowchart diagrams for each state in this state machine diagram are described in appendix.

### 4.1 Controller System

Controller system controls entire behavior of the controller system: how to operate the robot both manually and automatically, how to setup the controller parameter and so on. This menu has 3 tab panels: Controller, Setup Controller and Data Record. The state machine diagram of this menu is shown in Figure 4.3 whereas their screenshot displays are shown in Figure 4.4, 4.5 and 4.6.


Figure 4.3: The state machine diagram of controller system menu

## 

- Communication Setting sets the serial UART communication parameters such as port, baudrate and open-close communication status.
- Power Control controls the hydraulic pump and oil valve. The oil valve can be turned on only when the hydraulic pump has been turned on already. When the hydraulic pump is turned off, the oil valve will be turned off as well automatically.
- PID Control controls the PID controller status. When PID controller status is ON, the robotic arm moves automatically following the given position either single position or sequence of positions as in trajectory file that has been sent to the controller system previously.


Figure 4.4: Controller tab

- Select Mode provides two operation modes: Teaching Mode and Playback Mode. Teaching mode means the robot is operated under an human operator control. The trajectory yielded during teaching mode can be stored as a trajectory file and later can be loaded and played back. Playback mode means the robot operates automatically doing the trajectory file that has been stored in the controller system previously.
- Reference are the references value of the PID controller system. When the playback status is ON, these reference values will change following the trajectory file. Conversely, these one can be set manually and can be sent to the controller system by pressing Send Reference button.
- Potentiometer Measurement is the measured values that represent the angle in every joint of robotic arm.
- Movement Control control the relative movement of the robotic arm in Cartesian coordinate.
- Matrix Transformation represents the position and orientation of the end-point of robotic arm.


### 4.1.2 Setup Controller

- 2-DOF PID Controller Parameters adjusts the controller parameters. $R, S$ and $T$ are the polinomials of controller with structure as shown in Figure 2.4 (b).
- 1-DOF PID Controller Parameters adjusts the controller parameters with structure as shown in

Figure 2.4 (a). Send PID Parameters button is used to send the controller parameter at joint as in selected axis.

- Fuzzy Controller Membership adjusts the membership function of parameter input error, delta error and output if fuzzy algorithm is chosen. Generate Membership button and Send Member-


Figure 4.5: Setup controller tab
ship button are used to generate the membership function and send that membership function to the controller system respectively.

### 4.1.3 Data Record



Figure 4.6: Data record menu

Data record shows the recorded data during the robot operation. These data are stored in a Microsoft Excel file format. Pressing Refresh button will clear the recorded data and save button will save the recorded data into PC storage.

### 4.2 Identification System

Identification system is provided to support the data collection during experiment. The collected data is composed of data input and data output of the servo valve in every joint of robotic arm. In advance, these data are processed to yield the estimate model of the servo valve of robot. The state machine diagram of the identification system menu is shown in Figure 4.7.


Figure 4.7: The state machine diagram of identification system menu

### 4.2.1 Control

This menu controls the data collection process: when the data is began to be collected, when the data is stopped to be collected, what kind of input data is applied and so on. Figure 4.8 shows the screenshot of control submenu.

- Command is a panel to control the identification status. Start button and Stop button are used to begin and end the data collection process.
- Power Control controls the hydraulic pump and oil valve. The oil valve can be turned on only when the hydraulic pump has been turned on. However the oil valve will be turned off automatically when the hydraulic pump is turned off.
- Communication sets the serial UART communication parameters such as port, baudrate and open-close communication status.
- Potentiometer shows the measured values represent the angle in every joint of robotic arm.
- Input Signal gives choices; what kind of input signal will be employed in the identification system. There are three types of input signal: Square, Triangle and Pseudo Random Binary Se-


Figure 4.8: Control in Identification System
quence (PRBS). For Square and Triangle, its amplitude and period of the signal can be adjusted manually.

### 4.2.2 Data Record

Data record shows the collected data during the system identification process. These data will be stored in PC storage as a Microsoft Excel file format by pressing save button.


Figure 4.9: The state machine diagram of teaching mode menu

### 4.3 Teaching Mode

This menu provides a panel to control the robot in teaching mode. In this mode, the robot is fully controlled by an human operator. The robot position can be specified either in Cartesian or Polar coordinate. During teaching process, the positions of the robot can be stored as a trajectory file. This trajectory file then can be loaded and executed in playback mode. The state machine diagram and the sreenshot display of teaching mode menu are shown in Figure 4.9 and Figure 4.10 respectively.


Figure 4.10: Teaching mode menu

- Movement Control control the relative movement of the robotic arm in Cartesian coordinate. The relative movement distance and the direction are respectively determined by the movement resolution and the direction button.
- Communication Setup sets the serial UART communication parameters such as port, baudrate and open-close communication status.
- Command is a panel to control the robot moving to the specified position. Button Go C and Go $P$ respectively forces the robot moves to a specified point in Cartesian and in Polar coordinate. Pressing confirm button will save current position to the trajectory file.
- Power Control controls the hydraulic pump and oil valve. The oil valve can be turned on only when the hydraulic pump has been turned on already. When the hydraulic pump is turned off, the oil valve will be turned off as well automatically.
- PID Control controls the PID controller status. When PID controller status is ON, the robotic arm moves automatically following the given position either single position or sequence of positions as in trajectory file that has been sent to the controller system previously.
- Trajectory Control modifies current trajectory file during teaching mode such as loading trajectory file, inserting and deleting a data.


### 4.4 Trajectory Generator

This panel provides an interface to do everything related to the trajectory file. Creating a new trajectory file, loading trajectory file, editing trajectory file, sending trajectory file and saving trajectory file are some available commands in this menu. Figure 4.11 shows the state machine diagram of trajectory generator menu whereas Figure 4.12 shows its screenshot display.


Figure 4.11: The state machine of trajectory generator menu

- Set Initial Position specifies the initial position of the robotic arm end-point in Cartesian coordinate.
- Add New Position adds a new position in trajectory file.
- Insert Row inserts a new data in selected row of trajectory file.

- Delete Row deletes a data in selected row of trajectory file.
- Load Trajectory opens the trajectory file that has been stored in the PC storage previously.
- Generate Trajectory converts the sequences of position in Cartesian coordinate into polar coordinate (angle in every joint) and converts the polar coordinate into digital value.


Figure 4.12: Trajectory generator

- Refresh clears the current trajectory data.
- Simulate calls the simulation panel.
- Send Trajectory sends the trajectory file to the controller system. This trajectory file will be stored at the RAM of controller system.
- Save to Flash ROM saves the trajectory file that is stored in RAM into Flash ROM of the controller system. When a trajectory file has been sent to the RAM and its trajectory has been run well then it can be stored into flash ROM to keep it reside in the controller system permanently.
- Save stores current trajectory file to the PC storage as a Microsoft Excel file format.


### 4.5 Simulation

This menu acts as a tool to simulate the trajectory file before loaded in the real controller system. It is highly recommended to do a simulation by this tool to guarantee that the trajectory file will drive the robotic arm following the desired pattern properly. Figure 4.13 shows the state machine diagram of simulation menu whereas Figure 4.14 shows its screenshot display.


Figure 4.13: The state machine diagram of simulation menu

- Command is a panel to control the simulation status. Run button and Stop button are used to begin and stop the simulation respectively. Run button only available when the simulated trajectory file has been loaded.
- Simulation Speed is used to tune the speed of robotic arm motion in simulation.
- Camera View varies the way of view in many styles. Three basic views: top view, front view and side view are available. It also can be tuned manually by adjusting the value in each parameters such as azimuth, elevation and distance.


Figure 4.14: Inverse kinematics simulation with IDE Delphi

## CHAPTER V

## TEST AND EXPERIMENT RESULTS

### 5.1 End-point Position Calibration

This experiment is purposed to calibrate the end-point position of the robotic arm by comparing a targeted position and the real one obtained from measurement. Because a specific tool to measure the real position is not available then the real position is measured by the method described below.


Figure 5.1: End-point measurement method
Suppose a base point $O(0,0,0)$ and three auxiliary points with known coordinate are given: point $A(\mathrm{a}, 0,0)$, point $B(0, \mathrm{~b}, 0)$ and point $C(0,0, \mathrm{c})$ as shown in Figure 5.1. Any point $P$ with coordinate ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) can be known by measuring the distance between point $P$ and each point of $O, A$, $B$ and $C$ namely $d o, d a, d b$ and $d c$. There are four equations that represent each of those distance as in (5.1) to (5.4).

$$
\begin{gather*}
x^{2}+y^{2}+z^{2}=d o^{2}  \tag{5.1}\\
(x-a)^{2}+y^{2}+z^{2}=d a^{2} \tag{5.2}
\end{gather*}
$$

$$
\begin{equation*}
x^{2}+(y-b)^{2}+z^{2}=d b^{2} \tag{5.3}
\end{equation*}
$$

$$
\begin{equation*}
x^{2}+y^{2}+(z-c)^{2}=d c^{2} \tag{5.4}
\end{equation*}
$$

By substituting equation (5.1) into equation (5.2), (5.3) and (5.4) it yields:

$$
\begin{align*}
& \Delta x=\frac{d a^{2}-d o^{2}-a^{2}}{-2 a}  \tag{5.5}\\
& \Delta y=\frac{d b^{2}-d o^{2}-b^{2}}{-2 b}  \tag{5.6}\\
& \Delta z=\frac{d c^{2}-d o^{2}-c^{2}}{-2 c} \tag{5.7}
\end{align*}
$$

Practically, we use $O=\left[\begin{array}{lll}46 & 37 & 0\end{array}\right]^{T}, A=\left[\begin{array}{lll}268 & 37 & 0\end{array}\right]^{T}, B=\left[\begin{array}{llll}46 & -37 & 0\end{array}\right]^{T}$, and $C=\left[\begin{array}{lll}46 & 37 & 49\end{array}\right]^{T}$ in this measurement. Therefore we have variables of $a, b$ and $c$ are $222,-74$ and 49 respectively. Using equation (5.5), (5.6) and (5.7) we obtain the measured end-point position $P$ by formula in (5.8). Table 5.1 shows the complete measurement results for this experiment.

$$
P=O+\left[\begin{array}{c}
\Delta x  \tag{5.8}\\
\Delta y \\
\Delta z
\end{array}\right]
$$

Table 5.1: End-point measurement results in cm unit

| Target Point |  |  |  | Measurement |  |  |  | Measured Point |  |  | Error |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{d}$ | $y_{d}$ | $z_{d}$ | $d o$ | $d a$ | $d b$ | $d c$ | $x_{m}$ | $y_{m}$ | $z_{m}$ | $x$ | $y$ | $z$ |  |
| 180 | 0 | 170 | 220 | 220 | 185 | 193 | 182.11 | 0 | 169.14 | -2.11 | 0 | 0.85 |  |
| 180 | -60 | 170 | 238 | 218 | 206 | 213 | 182.39 | -61.62 | 169.47 | -2.39 | 1.62 | 0.52 |  |
| 180 | 60 | 170 | 218 | 238 | 183 | 190 | 182.72 | 61.62 | 167.71 | -2.72 | -1.62 | 2.28 |  |
| 190 | 0 | 170 | 226 | 226 | 192 | 188 | 192.43 | 0 | 169.52 | -2.43 | 0 | 0.47 |  |
| 190 | 0 | 150 | 212 | 212 | 180 | 171 | 192.36 | 0 | 152.5 | -2.36 | 0 | -2.5 |  |
| 190 | 0 | 120 | 194 | 194 | 168 | 149 | 191.76 | 0 | 120.54 | -1.76 | 0 | -0.54 |  |
| 190 | 0 | 80 | 170 | 170 | 154 | 115 | 192.30 | 0 | 77.39 | -2.30 | 0 | 2.60 |  |
| 160 | 0 | 170 | 208 | 207 | 171 | 202 | 162.54 | -2.80 | 167.59 | -2.54 | 2.80 | 2.40 |  |
| 140 | 0 | 170 | 197 | 197 | 157 | 214 | 141.26 | 0 | 165.77 | -1.26 | 0 | 1.01 |  |
| 170 | 0 | 170 | 213 | 213 | 176 | 197 | 171.77 | 0 | 171.36 | -1.77 | 0 | -1.36 |  |

### 5.2 Analog Board Tests

These experiments test whether the analog board has worked properly or not. The experiment setup is shown in Figure 5.2.


Figure 5.2: The circuit for matching the ADC and DAC level
The first test is done by generating sinusoidal and sawtooth waveform independently. Microblaze is programmed to produce digital values represent the sinusoidal and sawtooth waveform. Those signals then converted into analog signal by DAC part which is composed of PWM generator, filter and amplifier circuit. Analog signals at point A are observed by oscilloscope. The results are shown in Figure 5.3 and 5.4.


Figure 5.3: Generating sinusoidal waveform


Figure 5.4: Generating sawtooth waveform

The second test is done by looping back the signal that has been generated by DAC part into ADC part. Both of DAC and ADC values are recorded by a PC and displayed in the screen. Since the DAC level $(-12 \mathrm{~V}$ to $+12 \mathrm{~V})$ is unequal to the ADC level $(0 \mathrm{~V}$ to $+5 \mathrm{~V})$, it needs an adjustment to make it works at proper operation point for each. Figure 5.2 is a simple voltage divider circuit that can be used for this purpose. By choosing $R_{1}=6.8 \mathrm{KOhm}$ and $R_{2}=5 \mathrm{KOhm}$ it can adjust the DAC level $(-12$ V to $+12 \mathrm{~V})$ to the lower level $(-5 \mathrm{~V}$ to $+5 \mathrm{~V})$. The diode $D 1$ is functioned to protect the negative level so that it only has 0 V to +5 V level which is match with the ADC level. Figure 5.5 and 5.6 show the test results. The red line is the DAC data whereas the blue line is the ADC data.


Figure 5.5: ADC and DAC looping back test with sawtooth waveform


Figure 5.6: ADC and DAC looping back test with triangle waveform

### 5.3 Inverse Kinematics Simulation

Figure 5.7 and Figure 4.14 show the simulation of inverse kinematics using Matlab and Delphi. Those simulation demonstrate how the robotic arm move following a given trajectory pattern in Cartesian coordinate.


Figure 5.7: Inverse kinematics simulation with Matlab

### 5.4 Playback Tests

In order to observe the controller performance, we also test the robotic operation at playback mode. Firstly, a sequence of targeted positions in Cartesian coordinate is specified. In this experiment we specify a sequence of points forming a square pattern. By using trajectory generator menu, the corresponding trajectory file is then generated. After being simulated by simulation tool, finally the trajectory file is transferred to the controller system and the controller is operated based on the given trajectory file. Figure 5.8 shows the experiment result. The red-line square represents the ideal trajectory pattern whereas the blue-line square represents the real trajectory pattern.


### 5.5 Servo Valves Model

The servo valve in every axis is identified using Recursive Least Square (RLS) Algorithm. Square signals is given as the inputs and the outputs are recorded as shown in Figure 3.14. The estimate model has structure as shown in (3.70). Complete estimated models of the six servo valves are shown in Table 5.2.

Table 5.2: Estimate Model of Servo Valves

| Axis | Estimate Model |  |
| :---: | :---: | :---: |
|  | $A\left(z^{-1}\right)$ | $B\left(z^{-1}\right)$ |
| 1 | $1-1.4825 z^{-1}+0.4821 z^{-2}$ | $-0.0026 z^{-1}-0.0116 z^{-2}$ |
| 2 | $1-1.6972 z^{-1}+0.6973 z^{-2}$ | $-0.0008 z^{-1}-0.0036 z^{-2}$ |
| 3 | $1-1.715 z^{-1}+0.715 z^{-2}$ | $-0.0011 z^{-1}-0.026 z^{-2}$ |
| 4 | $1-1.7023 z^{-1}+0.7023 z^{-2}$ | $-0.0003 z^{-1}-0.0039 z^{-2}$ |
| 5 | $1-1.7339 z^{-1}+0.7339 z^{-2}$ | $-0.0006 z^{-1}-0.0043 z^{-2}$ |
| 6 | $1-1.9267 z^{-1}+0.9268 z^{-2}$ | $-0.0004 z^{-1}-0.0011 z^{-2}$ |

### 5.6 Controller Coefficient

The PID controller structure is described in Figure 2.4 (b). By solving (3.77) and (3.83), the controller coeficients of $R\left(z^{-1}\right), S\left(z^{-1}\right)$ and $T\left(z^{-1}\right)$ are obtained as shown in Table 5.3.


### 5.7 Controller Performance

The performance of PID controller at every axis is shown in Figure 5.9. Step input is employed to observe the PID controller performance individually. Solid line curve is the response of PID controller from simulation based on the estimate model obtained by identification whereas dotted line curve is the measured response of PID controller from the experiment. Figure 5.10 shows the error curves, that is the difference between measured values and reference values.



Figure 5.11 shows the end-point error curves when it moves in every axis. Experiments were done by moving the end-point position for every axis individually. The positions are moved respectively from $(191,6,149)$ to $(189,23,149),(154,0,206)$ to $(124,0,233),(180,0,135)$ to $(187,0,121)$, $(173,0,134)$ to $(161,0,132),(191,1,148)$ to $(187,-11,148)$ and $(180,0,135)$ to $(187,0,121)$.



# Figure 5.11: The end-point position error curves <br> ศูนย์วิทยทรัพยากร <br> จุหาลงกรณ์มหาวิทยาลัย 

### 5.8 Hardware Synthesis

Table 5.4 shows the synthesis report of the device utilization.

Table 5.4: Device utilization summary

| Item | Number | Percentage |
| :--- | :--- | :---: |
| Number of Slices | 3149 out of 3584 | 87 |
| Number of Slice Flip Flops | 3670 out of 7168 | 51 |
| Number of 4 input LUTs | 4283 out of 7168 | 59 |
| Number of bonded IOBs | 35 out of 97 | 36 |
| Number of BRAMs | 8 out of 16 | 50 |
| Number of MULT18X18s | 3 out of 16 | 18 |
| Number of GCLKs | 8 out of 8 | 100 |
| Number of DCMs | 2 out of 4 | 50 |

### 5.9 Demonstration Video

In order to show the experiment results on controlling the robotic arm clearly, demonstration videos were taken. These videos are also shared to public at http://www.youtube.com/user/arumdapta98.

### 5.10 Hardware Assembly

Figure 5.12 to 5.14 show the developed hardwares to realize the controller system.


Figure 5.12: The developed hardware of controller system based on FPGA


Figure 5.13: The developed hardware of controller system based on microcontroller


Figure 5.14: The assembled hardware

## CHAPTER VI

## DISCUSSION AND CONCLUSIONS

### 6.1 Discussion

This section discuss some experiment results both hardware and software as depicted in chapter 5.

### 6.1.1 Hardware

In this work, a 6-axis robotic arm controller is developed. Each axis has its own 2-DOF PID controller which control each servo valve at every axis independently. A 32-bit RISC soft-core processor (microblaze) with some peripherals and custom peripherals are constructed to realize those controller on a FPGA chip. The device utilization summary of hardware synthesis for controller implementation on Xilinx FPGA XC3S400 is shown in Table 5.4. From this table it looks that the developed hardware spends $87 \%$ of total resources provided by Xilinx FPGA XC3S400. The two biggest procentage of resource usage is allocated for custom peripheral and microblaze circuit which spend $21 \%$ and $19 \%$ respectively. The $47 \%$ procentage left is allocated for other peripherals such as timer/counter, SPI, UART, I/O etc. Custom peripheral is a dedicated 6 PWM generators and 6 SPI readers. This peripheral is designed to handle a specific job that is generating PWM signal and reading SPI data input. Using this costum peripheral we can save much more resources compare to when we use general 6 timer/counter and 6 SPI peripheral to handle the same job. This can reduce resources usage from $96 \%$ to only $21 \%$.

The main clock of the controller system is driven by an external 25 MHz source clock. The microblaze processor clock, bus clock and most of peripheral clock use this 25 MHz source clock. Only PWM generator peripheral which uses 100 MHz source clock. This clock is obtained by multiplying the 25 MHz source clock 4 times through Digital Clock Manager. PWM generator needs faster clock because the faster clock is used the better analog output is produced.

Analog Board is an external device which needs more attention on hardware readiness test compare to other external devices such as I/O board and power supply board. Basically, analog board test consist of two tests: DAC test and ADC test. The DAC test can be done by generating any signals such as square, sawtooth, triangle or sinusoidal waveform then observe the analog output through oscilloscope. Whereas the ADC test is done by providing certain analog signal then read and display it on PC. Figure 5.3, 5.4, 5.5 and 5.6 results show that the analog board has work properly. It can produce analog signal through DAC and read analog signal through ADC satisfically.

A specific test is also done to check the performance of low pass filter circuit. Figure 3.31 shows that the frequency response and phase response of the designed filter has satisfied response as desired one in section 3.5.2.3.

Finally an overall hardware test is conducted to ensure that all hardwares are ready to use. The overall test and some partially tests above show that the designed hardwares are work well and ready in use. Unfortunately, those hardware never have tried to control the robotic arm in real because of the limited time on working schedule. However the 2-DOF PID controller design ever tried and implemented on microcontroller at preliminary work. The result as shown in Figure 5.9 has proved that the designed controller work properly even using microcontroller. Therefore we are sure that those designed controller will work properly as well if it is implemented on the FPGA based.

### 6.1.2 Controller

An inverse kinematics simulation must be performed before loading and running the trajectory file at the controller system. In this experiments, 3 kind of trajectory files are generated and simulated: square pattern, triangle pattern, and zig-zag pattern. To generate this trajectory files, some points must be defined previously in Cartesian coordinate by an operator. For example, points $P_{1}(160,-$ $50,180), P_{2}(160,-50,130), P_{3}(160,50,130)$ and $P_{4}(160,50,180)$ are determined as a via points to generate square trajectory pattern in cm unit length. Every 5 cm movement in any direction, the Cartesian coordinate will be converted into Polar coordinate using closed-form inverse kinematics formula as given in 3.44 to 3.57. An polar to digital conversion then is employed to convert the corresponding Polar coordinate into digital value. Finally, the sequence of the digital value are stored in a Microsoft Excel format namely trajectory file.

This simulation is required to guarantee that the trajectory file can drive the robotic arm move following the desired pattern properly. By assuming that the controller performance is ideal, this simulation provides a good enough testing tool represents the real motion of robotic arm. Chosen trajectory file then can be loaded, ran and stored in the controller system safely. Figure 5.7 and 4.14 show an example of this simulation. It looks that desired trajectory file could drive the robotic arm satisfically.

Having generated and simulated, the trajectory file is loaded into controller system. Now it is the controller responsibility to drive the robot so that the angle in every axis following the sequence values in the trajectory file. The performance of 2-DOF PID controller in each axis is shown in Figure 5.9. Step input is employed to observe these performance individually and independently. Solid line curve represents the response of the controller system from simulation which is simulated by the model obtained from identification whereas dotted line curve represents the real response of the controller system which is obtained from. From this figure it looks that the controller responses both from simulation and from experiment resemble one another. It means that the estimate model of the servo valves are closed enough to the real model.

Controller error, that is the difference between measured angle and the reference input in each axis, is less than 6 (digital value) or less than $0.5^{\circ}$ at steady state condition as shown in Figure 5.10. End-point error is the difference between the targeted position and the measured position in Cartesian coordinate. The measured end-point error is less than 3 cm as shown in Table 5.1. This error is also proven by experiment result as shown in Figure 5.8. The error yielded by that experiment (the
difference between the real square pattern at playback mode and the ideal one) is less than 1.5 cm . This error can be said small enough when this error can be tolerated and acceptable such that the painting target do not need rework.

As shown in Figure 5.9, the axis-2 requires much more time to reach steady state. Most axes require 10 up to 30 sampling time to be stable. Only axis- 2 that requires more than 100 sampling time or equal with more than 1 second to achieve the steady state. From the experiments, axis- 2 can not be forced to reach the reference too fast. If this is done, it yields oscillation and it never be stable. This problem is happened because the center of mass of the whole machine, i.e weight of the robot is on this axis, hence the velocity of this axis is not directly proportional to the DC component of the servo valve. PID controller will think that the weight is the disturbance of the system i.e that have not formulated in the model. Moreover when the angle of the axis changes, the center of mass moves and the force that counter weight changes too. It mean that the model may change dependent on the angle at a particular time. Therefore we have to choose slower rise-time for this axis.

### 6.2 Conclusions

This work is conducted to develop an embedded system of 6-axis robotic arm controller. This means that both hardware and software are designed here. The controller system itself is integrated and embedded in a single chip. This is usually called a system on chip. Xilinx FPGA XC3S400 is chosen to implement that controller system because its features satisfy the requirement and its price is so cheap.

The angle of robotic arm in every axis is measured from the potentiometer attached in every joint. It then be converted from analog value to digital value by ADC part. This measured angle in digital value will be compared with reference values. The difference between measured value and the reference value, namely error, is proceeded by PID algorithm to produce control signal. This control signal is then used to drive the servo valve of robotic arm such that the angle in every joint following its reference value.

To design proper PID controller, the servo valves were identified. The estimate model of servo valves and the PID controller are simulated using matlab tool. After it satisfied the desired specification, the controller is then implemented on microcontroller. Finally, the microcontroller will be replaced by FPGA. Experiment results show that the designed controller has worked very well.

A set of formulas were developed to solve the inverse kinematics problem. With some constraints such as by keeping the end-point orientation constant, the inverse kinematics problem can be solved with closed-form method. This method has been worked correctly as shown in the simulation and the experiment results.

In this work, a GUI user application software is developed as well. This GUI application software can be used to setup and control the robotic arm controller system interactively. Through this GUI application software, an operator can command the robot easily. Some interactive menus are designed as easy as possible to use. Moreover, with its user-friendliness it can increase the productivity because the operator can modify and adjust anything regarding the robotic arm operation easily and
quickly.

### 6.3 Suggestion for future work

In this work, we have not considered about the disturbance effect against the servo valves model. As mentioned in the discussion session, particularly in axis-2, its models may change depend on the endpoint position. When its model change too much, it makes the PID controller unstable and oscillate. Therefore other identification methods which include the disturbance variable could be considered in the future future work to get better estimate model and to get better controller performance. Adaptive controllers such as MRAC or repetitive-path optimization are other points that can be further considered.


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## APPENDIX

## Flowchart Diagram of GUI User Application Software

1. Communication Setting


## 2. Movement Control



## 3. Command in System Identification



## 4. Command in Teaching Mode


5. Data Record


## 6. Input Signal


7. Mode Selection


## 8. PID Control


9. Power Control

10. Setup Controller


## 11. Trajectory Control


12. Trajectory Generator

13. Simulation Speed

14. Command in Simulation

15. Camera View


## Biography

Agus Bejo was born in Sleman, Indonesia, in 1980. He entered Gadjah Mada University, Indonesia in 1998 and received his Bachelor's degree majoring in Electrical Engineering in 2003. He joined LG Electronics Company as a research and development staff in 2003. In the end of year 2005, he decided to revert to Gadjah Mada University as a faculty staff. Since 2007, he has been granted a scholarship from the AUN/SEED-Net to pursue his Master's degree in Electrical Engineering Department at Chulalongkorn University, Thailand. Now He is a member of Embedded System and Digital IC Design Laboratory, Chulalongkorn University. His current research is related to the design a system on chip of robotic arm controller.


