CHAPTER 1

Introduction

1.1 Preliminary

In formulating a path integral theory of an electron gas in a random potential, Thaveesomboon (1), Bezak (2) was faced with the evaluation of the following one-dimensional path integral:

$$K(x_b, T ; x_a, 0) = \int_{x(0)=x_a}^{x(T)=x_b} \mathcal{D}[x(t)] \exp\left\{ \frac{i}{\hbar} S[x] \right\} \tag{1.1}$$

where $S[x]$ is the action of an oscillator with a memory term defined by

$$S[x] = \frac{m}{2}\int_0^T \dot{x}^2(t)\,dt - \frac{m\omega^2}{4T}\int_0^T dt \int_0^T d\sigma\,[x(t) - x(\sigma)]^2 \tag{1.2}$$

To evaluate eq. (1.1), Papadopoulos (3) factored out the memory term and transformed tha path integral in eq. (1.1) to be that of the path integral of the forced harmonic oscillator with constant force. Using a different approach, Sebastian (4) assumed that the memory term of this path integral depended only on the value of $\int_0^T x(t)dt$ (i.e., not on the detailed behavior of $x(t)$). He considered the integration over all paths that have $\int_0^T x(t)dt$ equal to a definite value, say u, (i.e. all paths which obey $\int_0^T x(t)dt = u$). He then changed the form of the path integral so that it became that of forced harmonic oscillator with constant force. After performing the integration over all possible value of u he found the same result as those of Papadopoulos.

Chetverikov (5) expressed all possible paths as being the sum of the free particle path and all other possible paths which could be represented by two suitable functions. Following Bezak's idea, Drchal and Masek (6) generated the action function in eq. (1.2) from a differential operator. They did use the eigen functions of this operator to represent all possible paths. Gifeisman (7) expressed all possible paths as the sum of the free particle path and the deviations from this path which could be represented by a Fourier sine series.

Dhara, Khandekar, and Lawande (8) performed directed integration to the path integral in eq. (1.1). They obtained a set of recursion formulas which could be transformed into another set of non-linear different equations. After performing some transformation, they obtained a set of differential equations which could be solved to yield values for their path integral. Finally, by generating eq. (1.1) from a shifted origin of the harmonic-oscillator path integral, Sa-yakanit (9) could obtain the same result after applying the free-particle technique.

1.2 Outline of the Thesis

The purpose of this thesis is to formulate a systematic technique for calculating the path integral in eq. (1.1). The basic ideas of our techniques come from Feynman (10) and Devies (11) which will be discussed in the following chapters.

In chapter 2 we discuss briefly the meaning of the Feynman path integrals and show how the path integrals can be constructed. In chapter 3 we present Feynman's technique for evaluating the pre-factor of the harmonic oscillator and discuss some of the mathematical difficulties of this approach. Feynman was able to avoid these difficulties by using the free-particle limit technique. In chapter 4 we present Devies' technique for obtaining the harmonic oscillator propagator and discuss his work on the classical action. In chapter 5 we develop our techniques by combining Feynman's and Devies' ideas. We obtain the harmonic oscillator propagator as an example of our techniques. In chapter 6 we apply our technique to obtain of the non-local harmonic oscillator propagator. The conclusions and discussions are presented in the last chapter.