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APPENDIX

APPENDIX A

GENERAL EXPRESSION FOR ELECTROELASTIC FIELDS

A.1 Mechanical Displacement

$$u_r = \frac{\partial}{\partial \rho}\left(\psi_1 + \psi_2 + \psi_3\right) \tag{A.1}$$

$$= \frac{A_{01}}{\rho} + \sum_{i=1}^{3}\left[\begin{array}{l} 2\rho B_{i0} - \sum_{n=1}^{\infty}\frac{\cos(n\pi\eta)}{\zeta_n^2}\left[p_i\zeta_n A_{in} I_1(p_i\zeta_n\rho) - p_i\zeta_n B_{in} K_1(p_i\zeta_n\rho)\right] \\ -\sum_{s=1}^{\infty}\frac{\cosh(q_i\gamma_s\eta)}{\lambda_s^2}\left[-\lambda_s C_{is} J_0(\lambda_s\rho) - \lambda_s D_{is} Y_0(\lambda_s\rho)\right] \end{array}\right]$$

$$= \frac{A_{01}}{\rho} + \sum_{i=1}^{3}\left[\begin{array}{l} 2\rho B_{i0} + \sum_{n=1}^{\infty}\frac{p_i\cos(n\pi\eta)}{\zeta_n}\left[-A_{in} I_1(p_i\zeta_n\rho) + B_{in} K_1(p_i\zeta_n\rho)\right] \\ +\sum_{s=1}^{\infty}\frac{\cosh(q_i\gamma_s\eta)}{\lambda_s}\left[C_{is} J_0(\lambda_s\rho) + \lambda_s D_{is} Y_0(\lambda_s\rho)\right] \end{array}\right] \tag{A.2}$$

$$u_z = k_{11}\frac{\partial \psi_1}{\partial \xi} + k_{12}\frac{\partial \psi_2}{\partial \xi} + k_{13}\frac{\partial \psi_3}{\partial \xi} \tag{A.3}$$

$$= \sum_{i=1}^{3} k_{1i}\left[\begin{array}{l} -4q_i^2\kappa\eta B_{i0} + \sum_{n=1}^{\infty}\frac{\zeta_n\sin(n\pi\eta)}{\zeta_n^2}\left[A_{in} I_0(p_i\zeta_n\rho) + B_{in} K_0(p_i\zeta_n\rho)\right] \\ -\sum_{s=1}^{\infty}\frac{q_i\lambda_s\sinh(q_i\gamma_s\eta)}{\lambda_s^2}\left[C_{is} J_0(\lambda_s\rho) + D_{is} Y_0(\lambda_s\rho)\right] \end{array}\right]$$

$$= \sum_{i=1}^{3} k_{1i}\left[\begin{array}{l} -4q_i^2\kappa\eta B_{i0} + \sum_{n=1}^{\infty}\frac{\sin(n\pi\eta)}{\zeta_n}\left[A_{in} I_0(p_i\zeta_n\rho) + B_{in} K_0(p_i\zeta_n\rho)\right] \\ +\sum_{s=1}^{\infty}\frac{q_i\sinh(q_i\gamma_s\eta)}{\lambda_s}\left[-C_{is} J_0(\lambda_s\rho) - D_{is} Y_0(\lambda_s\rho)\right] \end{array}\right] \tag{A.4}$$

A.2 Electric potential

$$\tilde{\phi} = k_{21}\frac{\partial \psi_1}{\partial \xi} + k_{22}\frac{\partial \psi_2}{\partial \xi} + k_{23}\frac{\partial \psi_3}{\partial \xi} \tag{A.5}$$

$$= \sum_{i=1}^{3} k_{2i}\begin{bmatrix} -4q_i^2\kappa\eta B_{i0} + \sum_{n=1}^{\infty}\frac{\zeta_n \sin(n\pi\eta)}{\zeta_n^2}\left[A_{in}I_0(p_i\zeta_n\rho) + B_{in}K_0(p_i\zeta_n\rho)\right] \\ -\sum_{s=1}^{\infty}\frac{q_i\lambda_s \sinh(q_i\gamma_s\eta)}{\lambda_s^2}\left[C_{is}J_0(\lambda_s\rho) + D_{is}Y_0(\lambda_s\rho)\right] \end{bmatrix}$$

$$= \sum_{i=1}^{3} k_{2i}\begin{bmatrix} -4q_i^2\kappa\eta B_{i0} + \sum_{n=1}^{\infty}\frac{\sin(n\pi\eta)}{\zeta_n}\left[A_{in}I_0(p_i\zeta_n\rho) + B_{in}K_0(p_i\zeta_n\rho)\right] \\ +\sum_{s=1}^{\infty}\frac{q_i \sinh(q_i\gamma_s\eta)}{\lambda_s}\left[-C_{is}J_0(\lambda_s\rho) - D_{is}Y_0(\lambda_s\rho)\right] \end{bmatrix} \tag{A.6}$$

A.2 Electric fields

$$E_r = -\frac{\partial \tilde{\phi}}{\partial \rho} \tag{A.7}$$

$$= \sum_{i=1}^{3}(-k_{2i})\begin{bmatrix} \sum_{n=1}^{\infty}\frac{\sin(n\pi\eta)}{\zeta_n}\left[A_{in}p_i\zeta_n I_1(p_i\zeta_n\rho) - B_{in}p_i\zeta_n K_0(p_i\zeta_n\rho)\right] \\ +\sum_{s=1}^{\infty}\frac{q_i \sinh(q_i\gamma_s\eta)}{\lambda_s}\left[C_{is}\lambda_s J_0(\lambda_s\rho) + D_{is}\lambda_s Y_0(\lambda_s\rho)\right] \end{bmatrix}$$

$$= \sum_{i=1}^{3} k_{2i}\begin{bmatrix} \sum_{n=1}^{\infty}\sin(n\pi\eta)\left[-A_{in}p_i I_1(p_i\zeta_n\rho) + B_{in}p_i K_0(p_i\zeta_n\rho)\right] \\ +\sum_{s=1}^{\infty}q_i \sinh(q_i\gamma_s\eta)\left[-C_{is}J_0(\lambda_s\rho) - D_{is}Y_0(\lambda_s\rho)\right] \end{bmatrix} \tag{A.8}$$

$$E_z = -\frac{\partial \tilde{\phi}}{\partial \xi} \tag{A.9}$$

$$= \sum_{i=1}^{3}(-k_{2i})\begin{bmatrix} -4q_i^2 B_{i0} + \sum_{n=1}^{\infty}\frac{\zeta_n \cos(n\pi\eta)}{\zeta_n}\left[A_{in}I_0(p_i\zeta_n\rho) + B_{in}K_0(p_i\zeta_n\rho)\right] \\ +\sum_{s=1}^{\infty}\frac{q_i^2\lambda_s \sinh(q_i\gamma_s\eta)}{\lambda_s}\left[-C_{is}J_0(\lambda_s\rho) - D_{is}Y_0(\lambda_s\rho)\right] \end{bmatrix}$$

$$= \sum_{i=1}^{3} k_{2i} \begin{bmatrix} 4q_i^2 B_{i0} + \sum_{n=1}^{\infty}\cos(n\pi\eta)\left[-A_{in}I_0(p_i\zeta_n\rho) - B_{in}K_0(p_i\zeta_n\rho)\right] \\ +\sum_{s=1}^{\infty} q_i^2 \sinh(q_i\gamma_s\eta)\left[C_{is}J_0(\lambda_s\rho) + D_{is}Y_0(\lambda_s\rho)\right] \end{bmatrix} \quad \text{(A.10)}$$

A.3 Stresses

$$\sigma_{rr} = \left(c_{11}\frac{\partial^2}{\partial\rho^2} + c_{12}\frac{1}{\rho}\frac{\partial}{\partial\rho}\right)\psi_i + \left(c_{13}k_{1i} + e_{31}k_{2i}\right)\frac{\partial^2\psi_i}{\partial\xi^2} \quad \text{(A.11)}$$

$$\begin{aligned}
= {} & (c_{12} - c_{11})\frac{1}{\rho^2}A_{01} + \sum_{i=1}^{3}\left[2c_{11} + 2c_{12} - 4\left(c_{13}k_{1i} + e_{31}k_{2i}\right)q_i^2\right]B_{0i} \\
& -\sum_{i=1}^{3}\sum_{n=1}^{\infty}\frac{\cos(n\pi\eta)}{\zeta_n^2}\begin{bmatrix} A_{in}\left(-\frac{p_i\zeta_n}{\rho}I_1(p_i\zeta_n\rho) + p_i^2\zeta_n^2 I_0(p_i\zeta_n\rho)\right) + \\ B_{in}\left(\frac{p_i\zeta_n}{\rho}K_1(p_i\zeta_n\rho) + p_i^2\zeta_n^2 K_0(p_i\zeta_n\rho)\right) \end{bmatrix} \\
& -\sum_{i=1}^{3}\sum_{s=1}^{\infty}\frac{\cosh(q_i\gamma_s\eta)}{\lambda_s^2}\begin{bmatrix} C_{is}\left(\frac{\lambda_s}{\rho}J_1(\lambda_s\rho) - \lambda_s^2 J_0(\lambda_s\rho)\right) + \\ D_{is}\left(\frac{\lambda_s}{\rho}Y_1(\lambda_s\rho) - \lambda_s^2 Y_0(\lambda_s\rho)\right) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
= {} & (c_{12} - c_{11})\frac{1}{\rho^2}A_{01} + \sum_{i=1}^{3}B_{0i}\left[2\left(c_{11} + c_{12}\right) - 4\left(c_{13}k_{1i} + e_{31}k_{2i}\right)q_i^2\right] \\
& -c_{11}\sum_{i=1}^{3}\sum_{n=1}^{\infty}\frac{\cos(n\pi\eta)}{\zeta_n^2}\begin{bmatrix} A_{in}\left(-\frac{p_i\zeta_n}{\rho}I_1(p_i\zeta_n\rho) + p_i^2\zeta_n^2 I_0(p_i\zeta_n\rho)\right) + \\ B_{in}\left(\frac{p_i\zeta_n}{\rho}K_1(p_i\zeta_n\rho) + p_i^2\zeta_n^2 K_0(p_i\zeta_n\rho)\right) \end{bmatrix} \\
& -c_{11}\sum_{i=1}^{3}\sum_{s=1}^{\infty}\frac{\cosh(q_i\gamma_s\eta)}{\lambda_s^2}\begin{bmatrix} C_{is}\left(\frac{\lambda_s}{\rho}J_1(\lambda_s\rho) - \lambda_s^2 J_0(\lambda_s\rho)\right) + \\ D_{is}\left(\frac{\lambda_s}{\rho}Y_1(\lambda_s\rho) - \lambda_s^2 Y_0(\lambda_s\rho)\right) \end{bmatrix} \\
& -c_{12}\sum_{i=1}^{3}\sum_{n=1}^{\infty}\frac{\cos(n\pi\eta)}{\zeta_n^2}\left[A_{in}\frac{p_i\zeta_n}{\rho}I_1(p_i\zeta_n\rho) - B_{in}\frac{p_i\zeta_n}{\rho}K_1(p_i\zeta_n\rho)\right] \\
& -c_{12}\sum_{i=1}^{3}\sum_{s=1}^{\infty}\frac{\cosh(q_i\gamma_s\eta)}{\lambda_s^2}\left[-C_{is}\frac{\lambda_s}{\rho}J_1(\lambda_s\rho) - D_{is}\frac{\lambda_s}{\rho}Y_1(\lambda_s\rho)\right] \\
& -\sum_{i=1}^{3}\left(c_{13}k_{1i} + e_{31}k_{2i}\right)\sum_{n=1}^{\infty}\frac{\zeta_n^2\cos(n\pi\eta)}{\zeta_n^2}\left[-A_{in}I_0(p_i\zeta_n\rho) - B_{in}K_0(p_i\zeta_n\rho)\right] \\
& -\sum_{i=1}^{3}\left(c_{13}k_{1i} + e_{31}k_{2i}\right)\sum_{s=1}^{\infty}\frac{\lambda_s^2\cosh(q_i\gamma_s\eta)}{\lambda_s^2}\left[C_{is}J_0(\lambda_s\rho) + D_{is}Y_0(\lambda_s\rho)\right]
\end{aligned}$$

$$
\begin{aligned}
&= (c_{12}-c_{11})\frac{1}{\rho^2}A_{01}+\sum_{i=1}^{3}B_{0i}\left[2(c_{11}+c_{12})-4(c_{13}k_{1i}+e_{31}k_{2i})q_i^2\right]\\
&\quad -\sum_{i=1}^{3}\sum_{n=1}^{\infty}A_{in}\frac{\cos(n\pi\eta)}{\zeta_n^2}\left\{\begin{array}{l}(-c_{11}+c_{12})\dfrac{p_i\zeta_n}{\rho}I_1(p_i\zeta_n\rho)+\\ \left[c_{11}p_i^2-(c_{13}k_{1i}+e_{31}k_{2i})\right]\zeta_n^2 I_0(p_i\zeta_n\rho)\end{array}\right\}\\
&\quad -\sum_{i=1}^{3}\sum_{n=1}^{\infty}B_{in}\frac{\cos(n\pi\eta)}{\zeta_n^2}\left\{\begin{array}{l}(c_{11}-c_{12})\dfrac{p_i\zeta_n}{\rho}K_1(p_i\zeta_n\rho)+\\ \left[c_{11}p_i^2-(c_{13}k_{1i}+e_{31}k_{2i})\right]\zeta_n^2 K_0(p_i\zeta_n\rho)\end{array}\right\}\\
&\quad -\sum_{i=1}^{3}\sum_{s=1}^{\infty}C_{is}\frac{\cosh(q_i\gamma_s\eta)}{\lambda_s^2}\left\{\begin{array}{l}(c_{11}-c_{12})\dfrac{\lambda_s}{\rho}J_1(\lambda_s\rho)+\\ \left[-c_{11}+(c_{13}k_{1i}+e_{31}k_{2i})q_i^2\right]\lambda_s^2 J_0(\lambda_s\rho)\end{array}\right\}\\
&\quad -\sum_{i=1}^{3}\sum_{s=1}^{\infty}D_{is}\frac{\cosh(q_i\gamma_s\eta)}{\lambda_s^2}\left\{\begin{array}{l}(c_{11}-c_{12})\dfrac{\lambda_s}{\rho}Y_1(\lambda_s\rho)+\\ \left[-c_{11}+(c_{13}k_{1i}+e_{31}k_{2i})q_i^2\right]\lambda_s^2 Y_0(\lambda_s\rho)\end{array}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= (c_{12}-c_{11})\frac{1}{\rho^2}A_{01}+\sum_{i=1}^{3}B_{0i}\left[2(c_{11}+c_{12})-4(c_{13}k_{1i}+e_{31}k_{2i})q_i^2\right]\\
&\quad +\sum_{i=1}^{3}\sum_{n=1}^{\infty}A_{in}\left[(-c_{11}p_i^2+c_{13}k_{1i}+e_{31}k_{2i})I_0(p_i\zeta_n\rho)+(c_{11}-c_{12})p_i\frac{I_1(p_i\zeta_n\rho)}{\zeta_n\rho}\right]\cos(n\pi\eta)\\
&\quad +\sum_{i=1}^{3}\sum_{n=1}^{\infty}B_{in}\left[(-c_{11}p_i^2+c_{13}k_{1i}+e_{31}k_{2i})K_0(p_i\zeta_n\rho)+(c_{12}-c_{11})p_i\frac{K_1(p_i\zeta_n\rho)}{\zeta_n\rho}\right]\cos(n\pi\eta)\\
&\quad +\sum_{i=1}^{3}\sum_{s=1}^{\infty}C_{is}\left[(c_{11}-c_{13}k_{1i}q_i^2-e_{31}k_{2i}q_i^2)J_0(\lambda_s\rho)+(c_{12}-c_{11})\frac{J_1(\lambda_s\rho)}{\lambda_s\rho}\right]\cosh(q_i\gamma_s\eta)\\
&\quad +\sum_{i=1}^{3}\sum_{s=1}^{\infty}D_{is}\left[(c_{11}-c_{13}k_{1i}q_i^2-e_{31}k_{2i}q_i^2)Y_0(\lambda_s\rho)+(c_{12}-c_{11})\frac{Y_1(\lambda_s\rho)}{\lambda_s\rho}\right]\cosh(q_i\gamma_s\eta)
\end{aligned}
\tag{A.12}
$$

$$
\sigma_{\theta\theta} = \left(c_{12}\frac{\partial^2}{\partial\rho^2}+c_{11}\frac{1}{\rho}\frac{\partial}{\partial\rho}\right)\psi_i+(c_{13}k_{1i}+e_{31}k_{2i})\frac{\partial^2\psi_i}{\partial\xi^2} \tag{A.13}
$$

$$
\begin{aligned}
&= (c_{11}-c_{12})\frac{1}{\rho^2}A_{01}+\sum_{i=1}^{3}B_{0i}\left[2(c_{11}+c_{12})-4(c_{13}k_{1i}+e_{31}k_{2i})q_i^2\right]\\
&\quad +\sum_{i=1}^{3}\sum_{n=1}^{\infty}A_{in}\left[(-c_{12}p_i^2+c_{13}k_{1i}+e_{31}k_{2i})I_0(p_i\zeta_n\rho)+(c_{12}-c_{11})p_i\frac{I_1(p_i\zeta_n\rho)}{\zeta_n\rho}\right]\cos(n\pi\eta)\\
&\quad +\sum_{i=1}^{3}\sum_{n=1}^{\infty}B_{in}\left[(-c_{12}p_i^2+c_{13}k_{1i}+e_{31}k_{2i})K_0(p_i\zeta_n\rho)+(c_{11}-c_{12})p_i\frac{K_1(p_i\zeta_n\rho)}{\zeta_n\rho}\right]\cos(n\pi\eta)\\
&\quad +\sum_{i=1}^{3}\sum_{s=1}^{\infty}C_{is}\left[(c_{12}-c_{13}k_{1i}q_i^2-e_{31}k_{2i}q_i^2)J_0(\lambda_s\rho)+(c_{11}-c_{12})\frac{J_1(\lambda_s\rho)}{\lambda_s\rho}\right]\cosh(q_i\gamma_s\eta)
\end{aligned}
$$

$$+\sum_{i=1}^{3}\sum_{s=1}^{\infty} D_{is}\left[\left(c_{12}-c_{13}k_{1i}q_i^2-e_{31}k_{2i}q_i^2\right)Y_0(\lambda_s\rho)+\left(c_{11}-c_{12}\right)\frac{Y_1(\lambda_s\rho)}{\lambda_s\rho}\right]\cosh(q_i\gamma_s\eta)$$

(A.14)

$$\sigma_{zz} = c_{13}\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\right)\psi_i+\left(c_{33}k_{1i}+e_{33}k_{2i}\right)\frac{\partial^2\psi_i}{\partial\xi^2} \quad \text{(A.15)}$$

$$\begin{aligned}
= & \sum_{i=1}^{3} B_{0i}\left[2\left(c_{13}+c_{13}\right)-4\left(c_{33}k_{1i}+e_{33}k_{2i}\right)q_i^2\right] \\
& +\sum_{i=1}^{3}\sum_{n=1}^{\infty} A_{in}\left[\left(-c_{13}p_i^2+c_{33}k_{1i}+e_{33}k_{2i}\right)I_0(p_i\zeta_n\rho)\right]\cos(n\pi\eta) \\
& +\sum_{i=1}^{3}\sum_{n=1}^{\infty} B_{in}\left[\left(-c_{13}p_i^2+c_{33}k_{1i}+e_{33}k_{2i}\right)K_0(p_i\zeta_n\rho)\right]\cos(n\pi\eta) \\
& +\sum_{i=1}^{3}\sum_{s=1}^{\infty} C_{is}\left[\left(c_{13}-c_{33}k_{1i}q_i^2-e_{33}k_{2i}q_i^2\right)J_0(\lambda_s\rho)\right]\cosh(q_i\gamma_s\eta) \\
& +\sum_{i=1}^{3}\sum_{s=1}^{\infty} D_{is}\left[\left(c_{13}-c_{33}k_{1i}q_i^2-e_{33}k_{2i}q_i^2\right)Y_0(\lambda_s\rho)\right]\cosh(q_i\gamma_s\eta)
\end{aligned} \quad \text{(A.16)}$$

$$\sigma_{rz} = \left(c_{44}+c_{44}k_{1i}+e_{15}k_{2i}\right)\frac{\partial^2\psi_i}{\partial\rho\partial\xi} \quad \text{(A.17)}$$

$$\begin{aligned}
= & -\sum_{i=1}^{3}\sum_{n=1}^{\infty}\left(c_{44}+c_{44}k_{1i}+e_{15}k_{2i}\right)\frac{-\zeta_n\sin(n\pi\eta)}{\zeta_n^2}\left[A_{in}p_i\zeta_n I_1(p_i\zeta_n\rho)-B_{in}p_i\zeta_n K_1(p_i\zeta_n\rho)\right] \\
& -\sum_{i=1}^{3}\sum_{s=1}^{\infty}\left(c_{44}+c_{44}k_{1i}+e_{15}k_{2i}\right)\frac{q_i\lambda_s\sinh(q_i\gamma_s\eta)}{\lambda_s^2}\left[-C_{is}\lambda_s J_1(\lambda_s\rho)-D_{is}\lambda_s Y_0(\lambda_s\rho)\right]
\end{aligned}$$

$$\begin{aligned}
= & \sum_{i=1}^{3}\sum_{n=1}^{\infty}\left(c_{44}+c_{44}k_{1i}+e_{15}k_{2i}\right)\left[A_{in}p_iI_1(p_i\zeta_n\rho)-B_{in}p_iK_1(p_i\zeta_n\rho)\right]\sin(n\pi\eta) \\
& +\sum_{i=1}^{3}\sum_{s=1}^{\infty}\left(c_{44}+c_{44}k_{1i}+e_{15}k_{2i}\right)\left[C_{is}q_iJ_1(\lambda_s\rho)+D_{is}q_iY_0(\lambda_s\rho)\right]\sinh(q_i\gamma_s\eta)
\end{aligned} \quad \text{(A.18)}$$

A.4 Dielectric Displacements

$$D_r = \left(e_{15}+e_{15}k_{1i}-\tilde{\kappa}_{11}k_{2i}\right)\frac{\partial^2\psi_i}{\partial\rho\partial\xi} \quad \text{(A.19)}$$

$$\begin{aligned}
= & \sum_{i=1}^{3}\sum_{n=1}^{\infty}\left(e_{15}+e_{15}k_{1i}-\tilde{\kappa}_{11}k_{2i}\right)\left[A_{in}p_iI_1(p_i\zeta_n\rho)-B_{in}p_iK_1(p_i\zeta_n\rho)\right]\sin(n\pi\eta) \\
& +\sum_{i=1}^{3}\sum_{s=1}^{\infty}\left(e_{15}+e_{15}k_{1i}-\tilde{\kappa}_{11}k_{2i}\right)\left[C_{is}q_iJ_1(\lambda_s\rho)+D_{is}q_iY_0(\lambda_s\rho)\right]\sinh(q_i\gamma_s\eta)
\end{aligned} \quad \text{(A.20)}$$

$$D_z = e_{13}\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\right)\psi_i+\left(e_{33}k_{1i}-\tilde{\kappa}_{33}k_{2i}\right)\frac{\partial^2\psi_i}{\partial\xi^2} \quad \text{(A.21)}$$

$$
\begin{aligned}
= \quad & \sum_{i=1}^{3} B_{0i}\left[2\left(e_{31}+e_{31}\right)-4\left(e_{33}k_{1i}-\tilde{\kappa}_{33}k_{2i}\right)q_i^2\right] \\
& +\sum_{i=1}^{3}\sum_{n=1}^{\infty} A_{in}\left[\left(-e_{31}p_i^2+e_{33}k_{1i}-\tilde{\kappa}_{33}k_{2i}\right)I_0(p_i\zeta_n\rho)\right]\cos(n\pi\eta) \\
& +\sum_{i=1}^{3}\sum_{n=1}^{\infty} B_{in}\left[\left(-e_{31}p_i^2+e_{33}k_{1i}-\tilde{\kappa}_{33}k_{2i}\right)K_0(p_i\zeta_n\rho)\right]\cos(n\pi\eta) \qquad \text{(A.22)} \\
& +\sum_{i=1}^{3}\sum_{s=1}^{\infty} C_{is}\left[\left(e_{31}-e_{33}k_{1i}q_i^2+\tilde{\kappa}_{33}k_{2i}q_i^2\right)J_0(\lambda_s\rho)\right]\cosh(q_i\gamma_s\eta) \\
& +\sum_{i=1}^{3}\sum_{s=1}^{\infty} D_{is}\left[\left(e_{31}-e_{33}k_{1i}q_i^2+\tilde{\kappa}_{33}k_{2i}q_i^2\right)Y_0(\lambda_s\rho)\right]\cosh(q_i\gamma_s\eta)
\end{aligned}
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BIOGRAPHY

Mr. Wichairat Kaewjuea was born on November 22, 1978 in Krabi, Thailand. He attended Ammartpanichnukul School in Krabi and graduated in 1996. He received his Bachelor's Degree in Civil Engineering from Faculty of Engineering, Prince of Songkla University in 2000. He pursued his Master Degree studies in Civil Engineering at Chulalongkorn University, Bangkok, Thailand in May 2000.