CHAPTER III

MODEL PREDICTIVE CONTROL

Controlling a nonlinear, unstable, limited manipulated variable process such the process in this research, sulphur control of Hydro-desulphurisation process, the result of PID control algorithm would give high over shoot process response because the manipulated variables adjustment would not be fast enough for the change of the nonlinear process. This leads to this research by making use of model predictive in control.

In this chapter describes model predictive control theory, algorithm and application used in this research.

3.1 Theory

From the need to control Control-Variables (CV) at their set points, Model Predictive Control, MPC, is a technique that based on process model in order to predict the behavior of the process output in the future. Proper calculating the Manipulated-Variables (MV) make use of optimizing the objective function in which minimizes the difference of the predicted values and required setpoints in the manipulated variable and state variable form. This technique is an online optimal control, which it is both controller and optimizer. The model predictive control structure shows in the figure 3.1

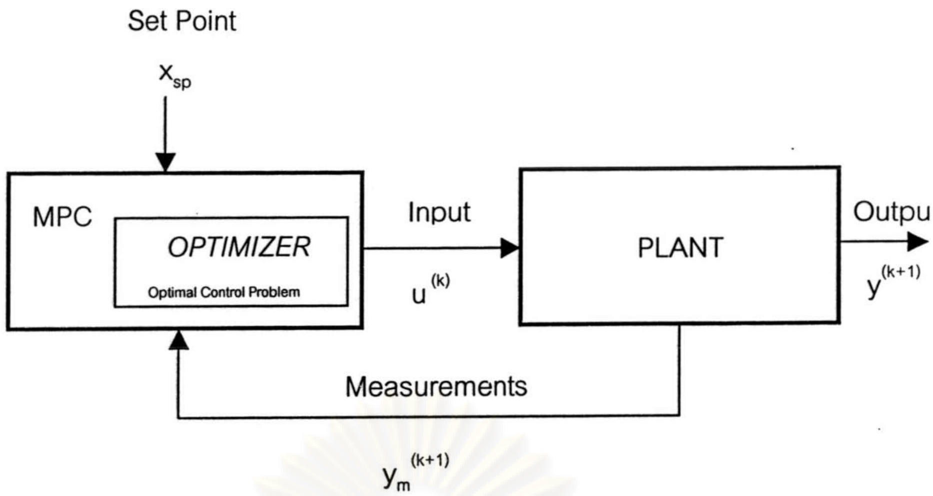

Figure 3.1 Model predictive control system

From the diagram in figure 3.1, the model predictive controller controls plant variables at their set points by monitoring the deviation of the process variables from the set points first, which is in the form of measurement signal and the output from prediction of process model and their set points. In order to calculate proper and in-line with the process behavior manipulated variables. Calculation uses dynamic optimization principle.

Model Predictive Control can be applied both linear and nonlinear process model. Moreover, the model predictive control performance can be fixed by proper objective function selection. This is however depending on the process response. The application of the model predictive is limited to the process or system which its models and parameters must be reliable and accurate at a certain level. By measuring the control variables and outputs, we can know the current condition of the system.

In addition, using model predictive control together with state estimator and parameter estimation are able to manage the error of the model and the disturbance in the process, whenever the model still can represent the real process. As a result, the model predictive control performance will be able to give a better control than primitive control algorithm like PID.

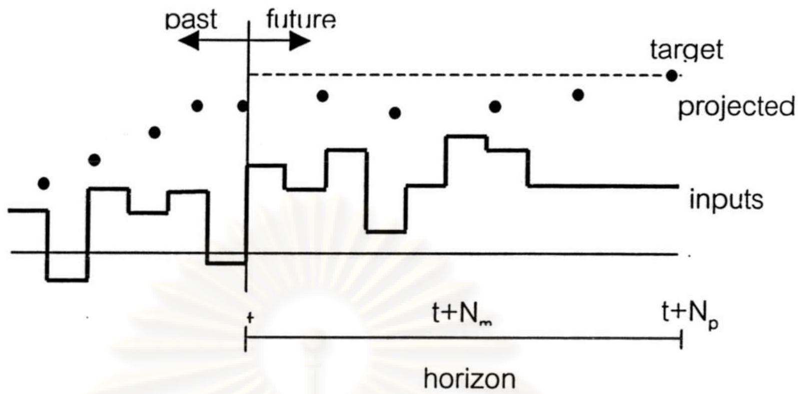

Figure 3.2 Receding Horizon Strategy

Model predictive structure, using optimization objective function for finding proper manipulated variables to control controlled variables to the desired set points by using Moving Horizon or Receding Horizon Strategy is shown in Figure 3.2.

In order to calculate a set of manipulated variables at time t, model predictive calculates a set of manipulated variables for N_m steps. This composes of current MV and future MV over the input horizon. From the model, the algorithm can predict the behavior of the output at time t (y(t+k/t)) for N_p steps. Optimizing the objective function leads to minimizing the error deviation between controlled variables and their set points within constraints of inputs and outputs of the process. This method is called Open Loop Optimal Control. Only the first calculated manipulated variable from the optimization will be used for controlling the process at time t. After that, the model predictive control system measures and estimates the state variables again for finding the next manipulated variable. This will be looped until the end time t_f.

3.2 Model predictive control structure

Because the model predictive structure requires objective function optimization for finding the solution, important components composes of

1. Objective function (A mathematic form represents the problem that we wants to find the solution.)
2. Equal constrain problem
3. Unequal constrain problem

Generally, to be able to calculate a future manipulated variable data set from state equations within constraints, this is performed by minimizing the objective function. Next is an example of the equation form for finding the manipulated variable data set.

In a real process, manipulated variables are limited in a certain range of operation. This is also included in constrain problems. For controlled variable is equated to its set point at the horizon.

Objective function
$$\min \int_{0}^{t_f} \left\{ W_1 \left(X - X^{sp} \right)^2 + W_2 \left(\Delta U \right)^2 \right\} dt \qquad (3.1)$$

State equation
$$\dot{X} = f\left(X(t), U(t)\right) \qquad (3.2)$$

Manipulated variable constraint
$$U_{\min} < U(t) < U_{\max} \qquad (3.3)$$

Controlled variable constraint
$$X\left(t + t_f\right) = X_{sp} \qquad (3.4)$$

Where

W_1, W_2	weighting factors
U_{min}, U_{max}	minimum and maximum manipulated variable
t_f	time at horizon

3.2.1 Process model

General form of process for representing the real process in model predictive control can be written as below:

$$\dot{x} = f(x,u) \\ y = g(x,d) \tag{3.5}$$

where

f	dynamic of the controlled process
x	state vector variable
u	manipulated variable vector (process input signal)
y	output variable vector

State equation in this research is in state space form, which is a linear model. In case the process model is a nonlinear type, we have to do locally linearization for every variable first. After linearization, state equation of the model, both continuous and discrete form, can be written as below.

Continuous equation
$$\dot{x} = Ax + Bu \\ y = Cx \tag{3.6}$$

Where

A,B and C	constant matrix size nxn
n	amount of state variables

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_n} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_n} \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \cdots & \frac{\partial g_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial g_n}{\partial x_1} & \frac{\partial g_n}{\partial x_2} & \cdots & \frac{\partial g_n}{\partial x_n} \end{bmatrix}$$

For discrete equation

$$\begin{aligned} x_{k+1} &= Gx_k + Hu_k \\ y_k &= Cx_k \end{aligned} \qquad (3.7)$$

where

G, H and C constant matrix

Note that, in this thesis uses discrete form as equation (3.7)

3.2.2 Manipulated variable and state variable constraints

Model predictive control system can control the system within constraints of manipulated variable and state variable. The equation boundaries can be categorized to equality constraint and inequality constraint.

1. The equality constraint is an equation in which value can be clearly defined. The equation can be written in the form:

$$h(x,u) = 0 \qquad (3.8)$$

2. The inequality constraint can be written in the form:

$$g(x,u) > 0 \qquad (3.9)$$

The inequality constraint can be divided to Hard and Soft constraints

2.1 Hard inequality constraint

$$u_{\min} \leq u \leq u_{\max} \tag{3.10}$$

2.2 Soft inequality constraint

$$\begin{aligned} & u_{\min} \pm \varepsilon \leq u_{\max} \pm \varepsilon \\ & 0 \leq \varepsilon \leq \varepsilon_{\max} \end{aligned} \tag{3.11}$$

Note that, in the thesis uses hard constraint as equation (3.10)

3.2.3 Objective Function

For model predictive control system, mostly, objective function is written in quadratic form i.e. square of state and manipulated variable difference. Objective function is a function that depicts the performance of optimization in which the function solution is only positive value in case of finding minimum value (and minus value in case of finding maximum value). Objective function can be written in many forms e.g. Dynamic Matrix Control (DMC), Prett and Gillette (1979) wrote objective function in form of power of one quadratic dynamic function for controlled and manipulated variables, Ricker (1985) and Eaton and Rawling (1990) wrote objective function in form of power of two of controlled and manipulated variables. Objective function in form of power of two of controlled and manipulated variables can be written as below:

$$J = \frac{1}{2}\left[\left(x_{sp} - x\right)^T Q\left(x_{sp} - x\right) + \left(u_k - u_{k-1}\right)^T R\left(u_k - u_{k-1}\right)\right] \tag{3.12}$$

or

$$J = \frac{1}{2}\left[x^T Q x + u^T R u\right] \tag{3.13}$$

Q and R are state variables and manipulated variables weighted matrix respectively. These two matrix are tuning parameters depict the priority of controlled and manipulated variable movement.

For model predictive controller, to be able to control the control variables to be at their set points within N_m steps, the process response calculation for N_p step can be written in performance index form as below.

Continuous form: $$J = \int_{t}^{t+N_p} \frac{1}{2}\left[x^T Qx + u^T Ru\right] dt \qquad (3.14)$$

Because model predictive control is able to control the controlled variables to be at their set points within time k+Nm, the delta move of manipulated variables and state variables change are zero after time k+Nm. From equation 3.14, the performance index can be rewritten in the form below.

Discrete form: $$J = \sum_{k}^{k+N_m} \frac{1}{2}\left[x^T Qx + u^T Ru\right] \qquad (3.15)$$

3.2.4 Process optimization with constraints

In reality, process manipulated variables are limited in a certain operating range e.g. the reactor outlet temperature for this thesis is limited to 370 degree Celsius. Model predictive control system can be cooperate with equality and inequality constraints in its objective function.

Consider a set of manipulated variables at time j, which is u_{k+i}^{j}, where $i = 0,1,\ldots,N_m$ and in line with objective function, see figure 3.2. The objective function is in quadratic form as equation 3.15.

Where N_m number of steps with pass through the control axis.

j interested time for calculating manipulated variable.

k discrete time

From process model equations in state space form, equation 3.7

$$\begin{aligned} x_{k+1} &= Gx_k + Hu_k \\ y_k &= Cx_k \end{aligned} \qquad (3.7)$$

Equality constraint: $$Gx_k + Hu_k - x_{k+1} = 0 \qquad (3.16)$$

Inequality constraint: $$u_{k,\min} \le u_k \le u_{k,\max} \qquad (3.17)$$

From minimum's principle, the most typical method for optimal control problem with boundary equations is Lagrange Multiplier (White, 1977).

From 3.17, we see that the manipulated variable still does not include constraints in objective function. When we apply Lagrange Multiplier's principle, equation 3.7, 3.16 and 3.17 can be combined with objective function 3.15. The new performance index called augmented cost function. It can be written as follow:

Continuous form: $$L(x,u) = \sum_{t}^{t+N_m} \frac{1}{2}\left[\left(x^T Qx + u^T Ru\right) + \lambda\left(G_x + H_u - \dot{x}\right)\right] \qquad (3.18)$$

Discrete form: $$L(x,u) = \sum_{k}^{k+N_m} \frac{1}{2}\left[\left(x_k^T Qx + u_k^T Ru_k\right) + \lambda_{k+1}\left(Gx_k + Hu_k - x_{k+1}\right)\right] \qquad (3.19)$$

Where $\lambda(t) \in R^n$ is an n-equations Lagrange Multiplier.

When solving the equations, we got Ricati Eqaultion in helping solving equation in order to get weighting matrix P for calculating controller gain and find proper trajectory of manipulated variable for process control. In case of weighting matrix P Q and R are constants, this controller will be called Steady State Optimal Control. The Ricati equation in order to find P_k which run to a constant.

$$P_k = Q + GP_{k+1}G - GP_{k+1}H\left(R + H^T P_{k+1} H\right)^{-1} H^T P_{k+1} G^T \qquad (3.20)$$

And the gain finding equation for state feedback control in equation 3.21 will be calculated only once at the control time.

$$K_k = R^{-1}H^T\left(G^T\right)^{-1}(P_k - Q) \tag{3.21}$$

The equation for finding manipulated variable value (only once at the control time $k\Delta t$) is as below:

$$u_k = -K_k x_k \tag{3.22}$$

Where $\lambda(t) \in R^n$ is an n-equations Lagrange Multiplier.

3.2.5 Model Predictive Control Algorithm

In this thesis, the model predictive control algorithm can be written in the form below:

1. Measure the measured variables
2. Take measured and manipulated variables to the observer for finding the state variable of the process.
3. Use state variable from the observer feeding to model predictive controller at the control time
4. Calculate constant process matrix G, H and C at control time
5. Use matrix G, H, C, Q and R for finding P_k starting from P_{k+M} to P_k by using Ricati equation until $P_k = P_{k+1}$

   $$P_k = Q + GP_{k+1}G - GP_{k+1}H\left(R + H^T P_{k+1} H\right)^{-1} H^T P_{k+1} G^T$$
6. Use the found P_k to calculate controller gain by using equation:

   $$K_k = R^{-1}H^T\left(G^T\right)^{-1}(P_k - Q)$$

7. Multiply gain with state variable at control time
8. Define u_k if $u_k > const.$ let $u_k = const.$ and if $u_k < const.$ let $u_k = const.$
9. Feeding the manipulated variable from the model predictive controller to the lower level controller e.g. PID to control the process
10. Measure the measured variable again and follow step 1.

3.3 Model Predictive Controller for Hydrodesulphurisation Process

The hydro-desulphurisation process is exothermal process unit which is used for treating sulphur out of mixed ranged gas oil. With tight sulphur specification according to the law and the requirement to minimize the cost of production, sulphur content in product need to be controlled. In the industry, model based controllers has been used widely.

In order to control sulphur content in product, we need to know the reactor operating temperature. The dynamic relation between the temperature and sulphur content is the main variables to understand the process. Because, on the assumption that there was no online sulphur analyzer in the process line, in this thesis has proposed an online sulphur content in product estimator with Kalman Filter lab result update technique, which will be explained in Chapter 5. The process model for formulating the controller will be explained in Chapter 4.

The model used in this thesis is written in the state space form, which based on the mass and heat balance study in Chapter 4. As a result of the model open loop response in figure 6.3C, Chapter VI, the amount of released heat change was little, approximately 0.25% when the reactor inlet temperature changed from 347 to 352 °C. The assumption is made for the rate of heat change that it is constant.

$$\frac{dQ}{dt} = 0 \tag{3.28}$$

Therefore, the differential equation form from mass and heat balance study in Chapter 4 can be written in the state space form as:

$$x_{k+1} = Gx_k + Hu_k \tag{3.29}$$

$$y_k = Cx_k \tag{3.30}$$

or:

$$\begin{bmatrix} Sp_{k+1} \\ Tro_{k+1} \\ Tri_{k+1} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & g_{33} \end{bmatrix} \cdot \begin{bmatrix} Sp_k \\ Tro_k \\ Tri_k \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ h_{31} \end{bmatrix} \cdot [Trisp_k] \tag{3.31}$$

$$[y_k] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Sp_k \\ Tro_k \\ Tri_k \end{bmatrix} \tag{3.32}$$

Where

$$g_{11} = -\frac{W}{\rho V_r} - \alpha k_0 e^{-E/R(WABT+273.15)} \cdot 2Sp$$

$$g_{12} = -\alpha k_0 e^{-E/R(WABT+273.15)} \cdot \frac{E}{R}(WABT+273.15)^{-2} \cdot \frac{2}{3} \cdot Sp^2$$

$$g_{12} = -\alpha k_0 e^{-E/R(WABT+273.15)} \cdot \frac{E}{R}(WABT+273.15)^{-2} \cdot \frac{1}{3} \cdot Sp^2$$

$$g_{21} = \frac{-\Delta H}{\rho V_r C_p} \alpha k_0 e^{-E/R(WABT+273.15)} \cdot 2Sp$$

$$g_{22} = -\frac{W}{\rho V_r} + \frac{-\Delta H}{\rho V_r C_p} \alpha k_0 e^{-E/R(WABT+273.15)} \cdot \frac{E}{R}(WABT+273.15)^{-2} \cdot \frac{2}{3} \cdot Sp^2$$

$$g_{23} = -\frac{W}{\rho V_r} + \frac{-\Delta H}{\rho V_r C_p} \alpha k_0 e^{-E/R(WABT+273.15)} \cdot \frac{E}{R}(WABT+273.15)^{-2} \cdot \frac{1}{3} \cdot Sp^2$$

$$g_{33} = -\frac{W}{\rho V_f}$$

$$h_{31} = \frac{W}{\rho V_f}$$

$$WABT = \frac{1}{3} Tri + \frac{2}{3} Tro$$