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ภาคผนวก

ภาคผนวก ก

รายละเอียดของไฟไนต์เอลิเมนต์เมตริกซ์ที่ประดิษฐ์ขึ้น

ในบทนี้จะเป็นการแสดงรายละเอียดของไฟไนต์เอลิเมนต์เมตริกซ์ที่ประดิษฐ์ขึ้นในบทที่ 3 โดยจะเริ่มจากค่าฟังก์ชันการประมาณภายในสำหรับความเร็วและความดันซึ่งอยู่ในรูปดังต่อไปนี้

สำหรับความเร็วที่จุดต่อ

$$N_1 = L_1 \tag{3.12ก}$$

$$N_2 = L_2 \tag{3.12ข}$$

$$N_3 = L_3 \tag{3.12ค}$$

สำหรับความเร็วไร้จุดต่อ

$$N_4 = 4L_2L_3 \tag{3.12ง}$$

$$N_5 = 4L_1L_3 \tag{3.12จ}$$

$$N_6 = 4L_1L_2 \tag{3.12ฉ}$$

สำหรับความดัน

$$H_1 = L_1 \tag{3.13ก}$$

$$H_2 = L_2 \tag{3.13ข}$$

$$H_3 = L_3 \tag{3.13ค}$$

เมื่อ L_i คือค่าพิกัดพื้นที่สำหรับจุดต่อ i ซึ่งเป็นฟังก์ชันของพิกัด x และ y ดังนี้

$$L_i(x,y) = \frac{1}{2A}(a_i + b_i x + c_i y) \qquad i = 1,2,3 \tag{3.10}$$

เมื่อ A = พื้นที่ของเอลิเมนต์ที่พิจารณา

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \tag{3.11}$$

$$a_1 = x_2y_3 - x_3y_2 \qquad b_1 = y_2 - y_3 \qquad c_1 = x_3 - x_2$$

$$a_2 = x_3y_1 - x_1y_3 \qquad b_2 = y_3 - y_1 \qquad c_2 = x_1 - x_3$$

$$a_3 = x_1y_2 - x_2y_1 \qquad b_3 = y_1 - y_2 \qquad c_3 = x_2 - x_1$$

สามารถเขียนค่าอนุพันธ์อันดับหนึ่งของฟังก์ชันการประมาณภายในของความเร็วให้อยู่ในรูปดังนี้

$$\frac{\partial}{\partial x}\left\{N_i\right\}=\frac{\partial}{\partial x}\begin{Bmatrix}L_1\\L_2\\L_3\\4L_2L_3\\4L_1L_3\\4L_1L_2\end{Bmatrix}=\frac{1}{2A}\begin{Bmatrix}b_1\\b_2\\b_3\\4\left(b_2L_3+b_3L_2\right)\\4\left(b_3L_1+b_1L_3\right)\\4\left(b_1L_2+b_2L_1\right)\end{Bmatrix} \qquad (ก.1)$$

$$\frac{\partial}{\partial y}\left\{N_i\right\}=\frac{\partial}{\partial y}\begin{Bmatrix}L_1\\L_2\\L_3\\4L_2L_3\\4L_1L_3\\4L_1L_2\end{Bmatrix}=\frac{1}{2A}\begin{Bmatrix}c_1\\c_2\\c_3\\4\left(c_2L_3+c_3L_2\right)\\4\left(c_3L_1+c_1L_3\right)\\4\left(c_1L_2+c_2L_1\right)\end{Bmatrix} \qquad (ก.2)$$

และการหาค่าอินทิกรัลของพิกัดพื้นที่สามารถคำนวณได้จากสูตร

$$\int_A L_1^{\alpha}L_2^{\beta}L_3^{\gamma}\,dA=\frac{\alpha!\,\beta!\,\gamma!}{(\alpha+\beta+\gamma+2)!}\cdot 2A \qquad (ก.3)$$

สามารถคำนวณหาค่าไฟไนต์เอลิเมนต์เมตริกซ์ที่ประดิษฐ์ขึ้นในบทที่ 3 ได้ดังต่อไปนี้

จาก $$K_{\alpha\beta\gamma'}=\int_{\Omega}N_{\alpha}N_{\beta}N_{\gamma,x}\,d\Omega \qquad (3.23ก)$$

จะได้ว่า $$K_{\alpha\beta 1'}=\int_A N_{\alpha}N_{\beta}N_{1,x}\,dA$$

$$=\int_A N_{\alpha}N_{\beta}\frac{b_1}{2A}\,dA$$

$$=\frac{b_1}{360}\begin{bmatrix}30&15&15&12&24&24\\15&30&15&24&12&24\\15&15&30&24&24&12\\12&24&24&32&16&16\\24&12&24&16&32&16\\24&24&12&16&16&32\end{bmatrix}$$

$$K_{\alpha\beta 2^*} = \int_A N_\alpha N_\beta N_{2,x}\, dA$$

$$= \int_A N_\alpha N_\beta \frac{b_2}{2A} dA$$

$$= \frac{b_2}{360}\begin{bmatrix} 30 & 15 & 15 & 12 & 24 & 24 \\ 15 & 30 & 15 & 24 & 12 & 24 \\ 15 & 15 & 30 & 24 & 24 & 12 \\ 12 & 24 & 24 & 32 & 16 & 16 \\ 24 & 12 & 24 & 16 & 32 & 16 \\ 24 & 24 & 12 & 16 & 16 & 32 \end{bmatrix}$$

$$K_{\alpha\beta 3^*} = \int_A N_\alpha N_\beta N_{3,x}\, dA$$

$$= \int_A N_\alpha N_\beta \frac{b_3}{2A} dA$$

$$= \frac{b_3}{360}\begin{bmatrix} 30 & 15 & 15 & 12 & 24 & 24 \\ 15 & 30 & 15 & 24 & 12 & 24 \\ 15 & 15 & 30 & 24 & 24 & 12 \\ 12 & 24 & 24 & 32 & 16 & 16 \\ 24 & 12 & 24 & 16 & 32 & 16 \\ 24 & 24 & 12 & 16 & 16 & 32 \end{bmatrix}$$

$$K_{\alpha\beta 4^*} = \int_A N_\alpha N_\beta N_{4,x}\, dA$$

$$= \frac{2b_2}{A}\int_A N_\alpha N_\beta L_3\, dA + \frac{2b_3}{A}\int_A N_\alpha N_\beta L_2\, dA$$

$$= \frac{b_2}{630}\begin{bmatrix} 42 & 21 & 42 & 28 & 56 & 28 \\ 21 & 42 & 42 & 56 & 28 & 28 \\ 42 & 42 & 126 & 84 & 84 & 28 \\ 28 & 56 & 84 & 96 & 48 & 32 \\ 56 & 28 & 84 & 48 & 96 & 32 \\ 28 & 28 & 28 & 32 & 32 & 32 \end{bmatrix} + \frac{b_3}{630}\begin{bmatrix} 42 & 42 & 21 & 28 & 28 & 56 \\ 42 & 126 & 42 & 84 & 28 & 84 \\ 21 & 42 & 42 & 56 & 28 & 28 \\ 28 & 84 & 56 & 96 & 32 & 48 \\ 28 & 28 & 28 & 32 & 32 & 32 \\ 56 & 84 & 28 & 48 & 32 & 96 \end{bmatrix}$$

$$K_{\alpha\beta 5^*} = \int_A N_\alpha N_\beta N_{5,x}\, dA$$

$$= \frac{2b_1}{A}\int_A N_\alpha N_\beta L_3\, dA + \frac{2b_3}{A}\int_A N_\alpha N_\beta L_1\, dA$$

$$= \frac{b_1}{630}\begin{bmatrix} 42 & 21 & 42 & 28 & 56 & 28 \\ 21 & 42 & 42 & 56 & 28 & 28 \\ 42 & 42 & 126 & 84 & 84 & 28 \\ 28 & 56 & 84 & 96 & 48 & 32 \\ 56 & 28 & 84 & 48 & 96 & 32 \\ 28 & 28 & 28 & 32 & 32 & 32 \end{bmatrix} + \frac{b_3}{630}\begin{bmatrix} 126 & 42 & 42 & 28 & 84 & 84 \\ 42 & 42 & 21 & 28 & 28 & 56 \\ 42 & 21 & 42 & 28 & 56 & 28 \\ 28 & 28 & 28 & 32 & 32 & 32 \\ 84 & 28 & 56 & 32 & 96 & 48 \\ 84 & 56 & 28 & 32 & 48 & 96 \end{bmatrix}$$

$$K_{\alpha\beta 6^x} = \int_A N_\alpha N_\beta N_{6,x}\, dA$$

$$= \frac{2b_1}{A}\int_A N_\alpha N_\beta L_2\, dA + \frac{2b_2}{A}\int_A N_\alpha N_\beta L_1\, dA$$

$$= \frac{b_1}{630}\begin{bmatrix} 42 & 42 & 21 & 28 & 28 & 56 \\ 42 & 126 & 42 & 84 & 28 & 84 \\ 21 & 42 & 42 & 56 & 28 & 28 \\ 28 & 84 & 56 & 96 & 32 & 48 \\ 28 & 28 & 28 & 32 & 32 & 32 \\ 56 & 84 & 28 & 48 & 32 & 96 \end{bmatrix} + \frac{b_2}{630}\begin{bmatrix} 126 & 42 & 42 & 28 & 84 & 84 \\ 42 & 42 & 21 & 28 & 28 & 56 \\ 42 & 21 & 42 & 28 & 56 & 28 \\ 28 & 28 & 28 & 32 & 32 & 32 \\ 84 & 28 & 56 & 32 & 96 & 48 \\ 84 & 56 & 28 & 32 & 48 & 96 \end{bmatrix}$$

จาก $$K_{\alpha\beta\gamma^y} = \int_\Omega N_\alpha N_\beta N_{\gamma,y}\, d\Omega \qquad (3.23ข)$$

จะได้ว่า $$K_{\alpha\beta 1^x} = \int_A N_\alpha N_\beta N_{1,y}\, dA$$

$$= \int_A N_\alpha N_\beta \frac{c_1}{2A} dA$$

$$= \frac{c_1}{360}\begin{bmatrix} 30 & 15 & 15 & 12 & 24 & 24 \\ 15 & 30 & 15 & 24 & 12 & 24 \\ 15 & 15 & 30 & 24 & 24 & 12 \\ 12 & 24 & 24 & 32 & 16 & 16 \\ 24 & 12 & 24 & 16 & 32 & 16 \\ 24 & 24 & 12 & 16 & 16 & 32 \end{bmatrix}$$

$$K_{\alpha\beta 2^y} = \int_A N_\alpha N_\beta N_{2,y}\, dA$$

$$= \int_A N_\alpha N_\beta \frac{c_2}{2A} dA$$

$$= \frac{c_2}{360}\begin{bmatrix} 30 & 15 & 15 & 12 & 24 & 24 \\ 15 & 30 & 15 & 24 & 12 & 24 \\ 15 & 15 & 30 & 24 & 24 & 12 \\ 12 & 24 & 24 & 32 & 16 & 16 \\ 24 & 12 & 24 & 16 & 32 & 16 \\ 24 & 24 & 12 & 16 & 16 & 32 \end{bmatrix}$$

$$K_{\alpha\beta 3^y} = \int_A N_\alpha N_\beta N_{3,y}\, dA$$

$$= \int_A N_\alpha N_\beta \frac{c_3}{2A} dA$$

$$= \frac{c_3}{360}\begin{bmatrix} 30 & 15 & 15 & 12 & 24 & 24 \\ 15 & 30 & 15 & 24 & 12 & 24 \\ 15 & 15 & 30 & 24 & 24 & 12 \\ 12 & 24 & 24 & 32 & 16 & 16 \\ 24 & 12 & 24 & 16 & 32 & 16 \\ 24 & 24 & 12 & 16 & 16 & 32 \end{bmatrix}$$

$$K_{\alpha\beta 4^y} = \int_A N_\alpha N_\beta N_{4,y}\, dA$$

$$= \frac{2c_2}{A}\int_A N_\alpha N_\beta L_3\, dA + \frac{2c_3}{A}\int_A N_\alpha N_\beta L_2\, dA$$

$$= \frac{c_2}{630}\begin{bmatrix} 42 & 21 & 42 & 28 & 56 & 28 \\ 21 & 42 & 42 & 56 & 28 & 28 \\ 42 & 42 & 126 & 84 & 84 & 28 \\ 28 & 56 & 84 & 96 & 48 & 32 \\ 56 & 28 & 84 & 48 & 96 & 32 \\ 28 & 28 & 28 & 32 & 32 & 32 \end{bmatrix} + \frac{c_3}{630}\begin{bmatrix} 42 & 42 & 21 & 28 & 28 & 56 \\ 42 & 126 & 42 & 84 & 28 & 84 \\ 21 & 42 & 42 & 56 & 28 & 28 \\ 28 & 84 & 56 & 96 & 32 & 48 \\ 28 & 28 & 28 & 32 & 32 & 32 \\ 56 & 84 & 28 & 48 & 32 & 96 \end{bmatrix}$$

$$K_{\alpha\beta 5^y} = \int_A N_\alpha N_\beta N_{5,y}\, dA$$

$$= \frac{2c_1}{A}\int_A N_\alpha N_\beta L_3\, dA + \frac{2c_3}{A}\int_A N_\alpha N_\beta L_1\, dA$$

$$= \frac{c_1}{630}\begin{bmatrix} 42 & 21 & 42 & 28 & 56 & 28 \\ 21 & 42 & 42 & 56 & 28 & 28 \\ 42 & 42 & 126 & 84 & 84 & 28 \\ 28 & 56 & 84 & 96 & 48 & 32 \\ 56 & 28 & 84 & 48 & 96 & 32 \\ 28 & 28 & 28 & 32 & 32 & 32 \end{bmatrix} + \frac{c_3}{630}\begin{bmatrix} 126 & 42 & 42 & 28 & 84 & 84 \\ 42 & 42 & 21 & 28 & 28 & 56 \\ 42 & 21 & 42 & 28 & 56 & 28 \\ 28 & 28 & 28 & 32 & 32 & 32 \\ 84 & 28 & 56 & 32 & 96 & 48 \\ 84 & 56 & 28 & 32 & 48 & 96 \end{bmatrix}$$

$$K_{\alpha\beta 6^y} = \int_A N_\alpha N_\beta N_{6,y}\, dA$$

$$= \frac{2c_1}{A}\int_A N_\alpha N_\beta L_2\, dA + \frac{2c_2}{A}\int_A N_\alpha N_\beta L_1\, dA$$

$$= \frac{c_1}{630}\begin{bmatrix} 42 & 42 & 21 & 28 & 28 & 56 \\ 42 & 126 & 42 & 84 & 28 & 84 \\ 21 & 42 & 42 & 56 & 28 & 28 \\ 28 & 84 & 56 & 96 & 32 & 48 \\ 28 & 28 & 28 & 32 & 32 & 32 \\ 56 & 84 & 28 & 48 & 32 & 96 \end{bmatrix} + \frac{c_2}{630}\begin{bmatrix} 126 & 42 & 42 & 28 & 84 & 84 \\ 42 & 42 & 21 & 28 & 28 & 56 \\ 42 & 21 & 42 & 28 & 56 & 28 \\ 28 & 28 & 28 & 32 & 32 & 32 \\ 84 & 28 & 56 & 32 & 96 & 48 \\ 84 & 56 & 28 & 32 & 48 & 96 \end{bmatrix}$$

จาก $$H_{\alpha\mu'} = \frac{1}{\rho}\int_{\Omega} N_{\alpha,x} H_{\mu}\, d\Omega \qquad (3.23ค)$$

$$= \frac{1}{6\rho}\begin{bmatrix} b_1 & b_1 & b_1 \\ b_2 & b_2 & b_2 \\ b_3 & b_3 & b_3 \\ b_2+b_3 & b_2+2b_3 & 2b_2+b_3 \\ b_1+2b_3 & b_1+b_3 & 2b_1+b_3 \\ b_1+2b_2 & 2b_1+b_2 & b_1+b_2 \end{bmatrix}$$

จาก $$H_{\alpha\mu'} = \frac{1}{\rho}\int_{\Omega} N_{\alpha,y} H_{\mu}\, d\Omega \qquad (3.23ง)$$

$$= \frac{1}{6\rho}\begin{bmatrix} c_1 & c_1 & c_1 \\ c_2 & c_2 & c_2 \\ c_3 & c_3 & c_3 \\ c_2+c_3 & c_2+2c_3 & 2c_2+c_3 \\ c_1+2c_3 & c_1+c_3 & 2c_1+c_3 \\ c_1+2c_2 & 2c_1+c_2 & c_1+c_2 \end{bmatrix}$$

จาก $$S_{\alpha\beta^{xx}} = 2\nu\int_A N_{\alpha,x} N_{\beta,x}\, dA + \nu\int_A N_{\alpha,y} N_{\beta,y}\, dA \qquad (3.23จ)$$

จะได้ว่า $$= 2\nu M_{\alpha\beta^{xx}} + \nu M_{\alpha\beta^{yy}}$$

เช่นเดียวกัน $$S_{\alpha\beta^{yy}} = \nu\int_A N_{\alpha,y} N_{\beta,x}\, dA \qquad (3.23ฉ)$$

$$= \nu M_{\alpha\beta^{yy}}$$

$$S_{\alpha\beta^{xy}} = \nu\int_A N_{\alpha,x} N_{\beta,y}\, dA \qquad (3.23ช)$$

$$= \nu M_{\alpha\beta^{xy}}$$

$$S_{\alpha\beta xx} = \nu \int_A N_{\alpha,x} N_{\beta,x}\, dA + 2\nu \int_A N_{\alpha,y} N_{\beta,y}\, dA \qquad (3.23ช)$$
$$= \nu M_{\alpha\beta xx} + 2\nu M_{\alpha\beta yy}$$

ซึ่งเมื่อทำการอินทิเกรต จะได้ผลลัพธ์ดังนี้

สำหรับ $M_{\alpha\beta xx}$ จะได้ว่า

$$M_{11xx} = \frac{b_1 b_1}{4A} \qquad M_{12xx} = \frac{b_1 b_2}{4A} \qquad M_{13xx} = \frac{b_1 b_3}{4A}$$
$$M_{21xx} = \frac{b_2 b_1}{4A} \qquad M_{22xx} = \frac{b_2 b_2}{4A} \qquad M_{23xx} = \frac{b_2 b_3}{4A}$$
$$M_{31xx} = \frac{b_3 b_1}{4A} \qquad M_{32xx} = \frac{b_3 b_2}{4A} \qquad M_{33xx} = \frac{b_3 b_3}{4A}$$
$$M_{14xx} = \frac{b_1}{3A}\left(b_2 + b_3\right) \qquad M_{15xx} = \frac{b_1}{3A}\left(b_3 + b_1\right) \qquad M_{16xx} = \frac{b_1}{3A}\left(b_1 + b_2\right)$$
$$M_{24xx} = \frac{b_2}{3A}\left(b_2 + b_3\right) \qquad M_{25xx} = \frac{b_2}{3A}\left(b_3 + b_1\right) \qquad M_{26xx} = \frac{b_2}{3A}\left(b_1 + b_2\right)$$
$$M_{34xx} = \frac{b_3}{3A}\left(b_2 + b_3\right) \qquad M_{35xx} = \frac{b_3}{3A}\left(b_3 + b_1\right) \qquad M_{36xx} = \frac{b_3}{3A}\left(b_1 + b_2\right)$$
$$M_{41xx} = \frac{b_1}{3A}\left(b_2 + b_3\right) \qquad M_{42xx} = \frac{b_2}{3A}\left(b_2 + b_3\right) \qquad M_{43xx} = \frac{b_3}{3A}\left(b_2 + b_3\right)$$
$$M_{51xx} = \frac{b_1}{3A}\left(b_3 + b_1\right) \qquad M_{52xx} = \frac{b_2}{3A}\left(b_3 + b_1\right) \qquad M_{53xx} = \frac{b_3}{3A}\left(b_3 + b_1\right)$$
$$M_{61xx} = \frac{b_1}{3A}\left(b_1 + b_2\right) \qquad M_{62xx} = \frac{b_2}{3A}\left(b_1 + b_2\right) \qquad M_{63xx} = \frac{b_3}{3A}\left(b_1 + b_2\right)$$
$$M_{44xx} = \frac{2}{3A}\left(b_2^2 + b_2 b_3 + b_3^2\right) \qquad M_{54xx} = \frac{1}{3A}\left(2b_1 b_2 + b_1 b_3 + b_2 b_3 + b_3^2\right)$$
$$M_{45xx} = \frac{1}{3A}\left(2b_1 b_2 + b_1 b_3 + b_2 b_3 + b_3^2\right) \qquad M_{55xx} = \frac{2}{3A}\left(b_1^2 + b_1 b_3 + b_3^2\right)$$
$$M_{46xx} = \frac{1}{3A}\left(2b_1 b_3 + b_1 b_2 + b_3 b_2 + b_2^2\right) \qquad M_{56xx} = \frac{1}{3A}\left(2b_2 b_3 + b_1 b_2 + b_1 b_3 + b_1^2\right)$$
$$M_{64xx} = \frac{1}{3A}\left(2b_1 b_3 + b_1 b_2 + b_3 b_2 + b_2^2\right)$$
$$M_{65xx} = \frac{1}{3A}\left(2b_2 b_3 + b_1 b_2 + b_1 b_3 + b_1^2\right)$$
$$M_{66xx} = \frac{2}{3A}\left(b_1^2 + b_1 b_2 + b_2^2\right)$$

สำหรับ $M_{\alpha\beta^{n}}$ จะได้ว่า

$$M_{11^{n}} = \frac{b_1 c_1}{4A} \qquad M_{12^{n}} = \frac{b_1 c_2}{4A} \qquad M_{13^{n}} = \frac{b_1 c_3}{4A}$$

$$M_{21^{n}} = \frac{b_2 c_1}{4A} \qquad M_{22^{n}} = \frac{b_2 c_2}{4A} \qquad M_{23^{n}} = \frac{b_2 c_3}{4A}$$

$$M_{31^{n}} = \frac{b_3 c_1}{4A} \qquad M_{32^{n}} = \frac{b_3 c_2}{4A} \qquad M_{33^{n}} = \frac{b_3 c_3}{4A}$$

$$M_{14^{n}} = \frac{b_1}{3A}\left(c_2 + c_3\right) \qquad M_{15^{n}} = \frac{b_1}{3A}\left(c_3 + c_1\right) \qquad M_{16^{n}} = \frac{b_1}{3A}\left(c_1 + c_2\right)$$

$$M_{24^{n}} = \frac{b_2}{3A}\left(c_2 + c_3\right) \qquad M_{25^{n}} = \frac{b_2}{3A}\left(c_3 + c_1\right) \qquad M_{26^{n}} = \frac{b_2}{3A}\left(c_1 + c_2\right)$$

$$M_{34^{n}} = \frac{b_3}{3A}\left(c_2 + c_3\right) \qquad M_{35^{n}} = \frac{b_3}{3A}\left(c_3 + c_1\right) \qquad M_{36^{n}} = \frac{b_3}{3A}\left(c_1 + c_2\right)$$

$$M_{41^{n}} = \frac{c_1}{3A}\left(b_2 + b_3\right) \qquad M_{42^{n}} = \frac{c_2}{3A}\left(b_2 + b_3\right) \qquad M_{43^{n}} = \frac{c_3}{3A}\left(b_2 + b_3\right)$$

$$M_{51^{n}} = \frac{c_1}{3A}\left(b_3 + b_1\right) \qquad M_{52^{n}} = \frac{c_2}{3A}\left(b_3 + b_1\right) \qquad M_{53^{n}} = \frac{c_3}{3A}\left(b_3 + b_1\right)$$

$$M_{61^{n}} = \frac{c_1}{3A}\left(b_1 + b_2\right) \qquad M_{62^{n}} = \frac{c_2}{3A}\left(b_1 + b_2\right) \qquad M_{63^{n}} = \frac{c_3}{3A}\left(b_1 + b_2\right)$$

$$M_{44^{n}} = \frac{1}{3A}\left(2b_2c_2 + b_2c_3 + b_3c_2 + 2b_3c_3\right)$$

$$M_{45^{n}} = \frac{1}{3A}\left(2b_2c_1 + b_2c_3 + b_3c_1 + b_3c_3\right)$$

$$M_{46^{n}} = \frac{1}{3A}\left(b_2c_1 + b_2c_2 + 2b_3c_1 + b_3c_2\right)$$

$$M_{54^{n}} = \frac{1}{3A}\left(2b_1c_2 + b_1c_3 + b_3c_2 + b_3c_3\right)$$

$$M_{55^{n}} = \frac{1}{3A}\left(2b_1c_1 + b_1c_3 + b_3c_1 + 2b_3c_3\right)$$

$$M_{56^{n}} = \frac{1}{3A}\left(b_1c_1 + b_1c_2 + b_3c_1 + 2b_3c_2\right)$$

$$M_{64^{n}} = \frac{1}{3A}\left(b_1c_2 + 2b_1c_3 + b_2c_2 + b_2c_3\right)$$

$$M_{65^{n}} = \frac{1}{3A}\left(b_1c_1 + b_1c_3 + b_2c_1 + 2b_2c_3\right)$$

$$M_{66^{n}} = \frac{1}{3A}\left(2b_1c_1 + b_1c_2 + b_2c_1 + 2b_2c_2\right)$$

สำหรับ $M_{\alpha\beta^{v}}$ จะได้ว่า

$$M_{\alpha\beta^{v}} = M_{\beta\alpha^{n}}$$

สำหรับ $M_{\alpha\beta''}$ จะได้ว่า

$$M_{11''} = \frac{c_1 c_1}{4A} \qquad M_{12''} = \frac{c_1 c_2}{4A} \qquad M_{13''} = \frac{c_1 c_3}{4A}$$

$$M_{21''} = \frac{c_2 c_1}{4A} \qquad M_{22''} = \frac{c_2 c_2}{4A} \qquad M_{23''} = \frac{c_2 c_3}{4A}$$

$$M_{31''} = \frac{c_3 c_1}{4A} \qquad M_{32''} = \frac{c_3 c_2}{4A} \qquad M_{33''} = \frac{c_3 c_3}{4A}$$

$$M_{14''} = \frac{c_1}{3A}\left(c_2 + c_3\right) \qquad M_{15''} = \frac{c_1}{3A}\left(c_3 + c_1\right) \qquad M_{16''} = \frac{c_1}{3A}\left(c_1 + c_2\right)$$

$$M_{24''} = \frac{c_2}{3A}\left(c_2 + c_3\right) \qquad M_{25''} = \frac{c_2}{3A}\left(c_3 + c_1\right) \qquad M_{26''} = \frac{c_2}{3A}\left(c_1 + c_2\right)$$

$$M_{34''} = \frac{c_3}{3A}\left(c_2 + c_3\right) \qquad M_{35''} = \frac{c_3}{3A}\left(c_3 + c_1\right) \qquad M_{36''} = \frac{c_3}{3A}\left(c_1 + c_2\right)$$

$$M_{41''} = \frac{c_1}{3A}\left(c_2 + c_3\right) \qquad M_{42''} = \frac{c_2}{3A}\left(c_2 + c_3\right) \qquad M_{43''} = \frac{c_3}{3A}\left(c_2 + c_3\right)$$

$$M_{51''} = \frac{c_1}{3A}\left(c_3 + c_1\right) \qquad M_{52''} = \frac{c_2}{3A}\left(c_3 + c_1\right) \qquad M_{53''} = \frac{c_3}{3A}\left(c_3 + c_1\right)$$

$$M_{61''} = \frac{c_1}{3A}\left(c_1 + c_2\right) \qquad M_{62''} = \frac{c_2}{3A}\left(c_1 + c_2\right) \qquad M_{63''} = \frac{c_3}{3A}\left(c_1 + c_2\right)$$

$$M_{44''} = \frac{2}{3A}\left(c_2^2 + c_2 c_3 + c_3^2\right) \qquad M_{54''} = \frac{1}{3A}\left(2c_1 c_2 + c_1 c_3 + c_2 c_3 + c_3^2\right)$$

$$M_{45''} = \frac{1}{3A}\left(2c_1 c_2 + c_1 c_3 + c_2 c_3 + c_3^2\right) \qquad M_{55''} = \frac{2}{3A}\left(c_1^2 + c_1 c_3 + c_3^2\right)$$

$$M_{46''} = \frac{1}{3A}\left(2c_1 c_3 + c_1 c_2 + c_3 c_2 + c_2^2\right) \qquad M_{56''} = \frac{1}{3A}\left(2c_2 c_3 + c_1 c_2 + c_1 c_3 + c_1^2\right)$$

$$M_{64''} = \frac{1}{3A}\left(2c_1 c_3 + c_1 c_2 + c_3 c_2 + c_2^2\right)$$

$$M_{65''} = \frac{1}{3A}\left(2c_2 c_3 + c_1 c_2 + c_1 c_3 + c_1^2\right)$$

$$M_{66''} = \frac{2}{3A}\left(c_1^2 + c_1 c_2 + c_2^2\right)$$

ภาคผนวก ข

รายละเอียดของโปรแกรม NVNL

โปรแกรมคอมพิวเตอร์ NVNL ที่ประดิษฐ์ขึ้นดังที่ได้กล่าวไว้ในบทที่ 4 มีรายละเอียดดังนี้

```
C
C     PROGRAM NVNL
C     A FINITE ELEMENT COMPUTER PROGRAM FOR SOLVING NAVIER-STOKES
C     EQUATIONS OF VISCOUS INCOMPRESSIBLE FLOW
C
      PARAMETER  (MXPOIV=2000, MXPOIP=1000, MXELE=2000)
      PARAMETER  (MXNEQ=2*MXPOIV+MXPOIP)
C
      IMPLICIT   REAL*8 (A-H,O-Z)
      DIMENSION  COORD(MXPOIV,2), TEXT(20)
      DIMENSION  UVEL(MXPOIV), VVEL(MXPOIV), PRES(MXPOIV)
      DIMENSION  SYSK(MXNEQ,MXNEQ), SYSR(MXNEQ)
      DIMENSION  SOL(MXNEQ), DSOL(MXNEQ)
      DIMENSION  ACHECK(MXPOIP,MXPOIP,2)
      CHARACTER*20  INPUT, OUTPUT
C
      INTEGER  INTMAT(MXELE,6)
      INTEGER  IBCU(MXPOIV), IBCV(MXPOIV), IBCP(MXPOIV)
      INTEGER  NUMBER(MXPOIP,MXPOIP)
C
   10 WRITE(6,20)
   20 FORMAT(/,' PLEASE ENTER THE INPUT FILE NAME:', /)
      READ(5,'(A)',ERR=10) INPUT
      OPEN(UNIT=7, FILE=INPUT, STATUS='OLD', ERR=10)
   30 WRITE(6,40)
   40 FORMAT(/, ' PLEASE ENTER FILE NAME FOR VELOCITY & PRESSURE',
     *          ' SOLUTIONS:', /                    )
      READ(5, '(A)', ERR=30) OUTPUT
      OPEN(UNIT=8, FILE=OUTPUT, STATUS='NEW', ERR=30)
C
C     READ TITLE OF COMPUTATION:
C
      READ(7,*)  NLINES
      DO 50  ILINE=1,NLINES
      READ(7,1) TEXT
    1 FORMAT(20A4)
   50 CONTINUE
C
C     READ INPUT DATA
C
      READ(7,1)  TEXT
```

```
      READ(7,*)  NPOIP, NELEM, NITER, TOL
      IF(NPOIP.GT.MXPOIP)  WRITE(6,60) NPOIP
   60 FORMAT(/,' PLEASE INCREASE THE PARAMETER MXPOIP TO',I5)
      IF(NPOIP.GT.MXPOIP)  STOP
      IF(NELEM.GT.MXELE)  WRITE(6,70) NELEM
   70 FORMAT(/,' PLEASE INCREASE THE PARAMETER MXELE TO',I5)
      IF(NELEM.GT.MXELE)  STOP
C
C     READ FLUID PROPERTIES
C
      READ(7,1)  TEXT
      READ(7,*)  DEN, VIS
C
C     READ NODAL COORDINATES, BOUNDARY CONDITIONS, THEIR VALUES
C
      READ(7,1)  TEXT
      DO 90  IP=1,NPOIP
      READ(7,*) I, IBCU(I), IBCV(I), IBCP(I),
     *          (COORD(I,K), K=1,2), UVEL(I), VVEL(I), PRES(I)
      IF(I.NE.IP)  WRITE(6,80) IP
   80 FORMAT(/, ' NODE NO.', I5, ' IN DATA FILE IS MISSING')
      IF(I.NE.IP) STOP
   90 CONTINUE
C
C     READ ELEMENT NODAL CONNECTIONS
C
      READ(7,1)  TEXT
      DO 110  IE=1,NELEM
      READ(7,*) I, (INTMAT(I,J), J=1,3)
      IF(I.NE.IE)  WRITE(6,120) IE
  120 FORMAT(/, ' ELEMENT NO.', I5, ' IN DATA FILE IS MISSING')
      IF(I.NE.IE) STOP
  110 CONTINUE
C
C     CREATE NODELESS VARIABLES ON EACH ELEMENT
C
      NPOIV = NPOIP
      DO 130 I=1,NPOIP
      DO 130 J=1,NPOIP
      DO 130 K=1,2
      ACHECK(I,J,K) = 0.
  130 CONTINUE
      DO 200 IE=NELEM,1,-1
      II = INTMAT(IE,1)
      JJ = INTMAT(IE,2)
      KK = INTMAT(IE,3)
      ACHECK(JJ,KK,2) = ACHECK(JJ,KK,2) + 1.
      ACHECK(KK,JJ,2) = ACHECK(KK,JJ,2) + 1.
      IF(ACHECK(JJ,KK,1).EQ.1.) GO TO 140
      ACHECK(JJ,KK,1) = 1.
```

```
      ACHECK(KK,JJ,1) = 1.
      NPOIV = NPOIV + 1
      INTMAT(IE,4) = NPOIV
      NUMBER(JJ,KK) = NPOIV
      NUMBER(KK,JJ) = NPOIV
      COORD(NPOIV,1) = (COORD(JJ,1)+COORD(KK,1))/2.
      COORD(NPOIV,2) = (COORD(JJ,2)+COORD(KK,2))/2.
      IF(IBCU(JJ).EQ.1.AND.IBCU(KK).EQ.1) IBCU(NPOIV) = 1.
      IF(IBCV(JJ).EQ.1.AND.IBCV(KK).EQ.1) IBCV(NPOIV) = 1.
      GO TO 150
  140 CONTINUE
      INTMAT(IE,4) = NUMBER(JJ,KK)
  150 CONTINUE
      ACHECK(II,KK,2) = ACHECK(II,KK,2) + 1.
      ACHECK(KK,II,2) = ACHECK(KK,II,2) + 1.
      IF(ACHECK(II,KK,1).EQ.1.) GO TO 160
      ACHECK(II,KK,1) = 1.
      ACHECK(KK,II,1) = 1.
      NPOIV = NPOIV + 1
      INTMAT(IE,5) = NPOIV
      NUMBER(II,KK) = NPOIV
      NUMBER(KK,II) = NPOIV
      COORD(NPOIV,1) = (COORD(II,1)+COORD(KK,1))/2.
      COORD(NPOIV,2) = (COORD(II,2)+COORD(KK,2))/2.
      IF(IBCU(II).EQ.1.AND.IBCU(KK).EQ.1) IBCU(NPOIV) = 1.
      IF(IBCV(II).EQ.1.AND.IBCV(KK).EQ.1) IBCV(NPOIV) = 1.
      GO TO 170
  160 CONTINUE
      INTMAT(IE,5) = NUMBER(II,KK)
  170 CONTINUE
      ACHECK(II,JJ,2) = ACHECK(II,JJ,2) + 1.
      ACHECK(JJ,II,2) = ACHECK(JJ,II,2) + 1.
      IF(ACHECK(II,JJ,1).EQ.1.) GO TO 180
      ACHECK(II,JJ,1) = 1.
      ACHECK(JJ,II,1) = 1.
      NPOIV = NPOIV + 1
      INTMAT(IE,6) = NPOIV
      NUMBER(II,JJ) = NPOIV
      NUMBER(JJ,II) = NPOIV
      COORD(NPOIV,1) = (COORD(II,1)+COORD(JJ,1))/2.
      COORD(NPOIV,2) = (COORD(II,2)+COORD(JJ,2))/2.
      IF(IBCU(II).EQ.1.AND.IBCU(JJ).EQ.1) IBCU(NPOIV) = 1.
      IF(IBCV(II).EQ.1.AND.IBCV(JJ).EQ.1) IBCV(NPOIV) = 1.
      GO TO 190
  180 CONTINUE
      INTMAT(IE,6) = NUMBER(II,JJ)
  190 CONTINUE
  200 CONTINUE
      DO 210 I=1,NPOIP
      DO 210 J=1,NPOIP
```

```
      IF(ACHECK(I,J,2).LT.2.) GO TO 210
      NUM = NUMBER(I,J)
      IBCU(NUM) = 0.
      IBCV(NUM) = 0.
  210 CONTINUE
C
      IF(NPOIV.GT.MXPOIV)  WRITE(6,220) NPOIV
  220  FORMAT(/,' PLEASE INCREASE THE PARAMETER MXPOIV TO',I5)
      IF(NPOIV.GT.MXPOIV)  STOP
C
      WRITE(6,230)  NPOIV, NPOIP, NELEM, NITER, TOL
  230  FORMAT(/,' *** THE FINITE ELEMENT MODEL CONSISTS OF:', /,
     *          '      NUMBER OF VELOCITY         =', I6, /,
     *          '      NUMBER OF PRESSURE         =', I6, /,
     *          '      NUMBER OF ELEMENTS         =', I6, /,
     *          '  WITH NUMBER OF ITERATIONS REQUIRED =', I6, /,
     *          '  OR SPECIFIED STOPPING TOLERANCE   =', F6.2 )
C
      DO 240 I=1,NPOIV
      SOL(I) = UVEL(I)
      SOL(I+NPOIV) = VVEL(I)
  240 CONTINUE
      DO 250 I=1,NPOIP
      SOL(I+NPOIV+NPOIV) = PRES(I)
  250 CONTINUE
C
      NEQ = 2*NPOIV + NPOIP
C
C     ENTER ITERATION LOOP
C
      DO 380 ITER=1,NITER
C
C     RESET THE SYSTEM EQUATIONS
C
      DO 260 I=1,NEQ
      SYSR(I) = 0.
  260 CONTINUE
      DO 270 I=1,NEQ
      DO 270 J=1,NEQ
      SYSK(I,J) = 0.
  270 CONTINUE
C
      WRITE(6,280) ITER
  280 FORMAT(/, 3X, ' * PERFORMING COMPUTATION AT ITERATION NUMBER',
     *          I16, ':')
C
C     ESTABLISH ELEMENT MATRICES AND ASSEMBLE ELEMENT EQUATIONS
C
      WRITE(6,290)
  290 FORMAT(/,' *** ESTABLISHING ELEMENT MATRICES AND',
```

```
     *              ' ASSEMBLING ELEMENT EQUATIONS ***'    )
           CALL TRI( NPOIV, NELEM,  DEN, VIS,  COORD,
     *               INTMAT,  SYSK, SYSR, SOL, MXPOIV,
     *               MXELE, MXNEQ               )
C
C     APPLY BOUNDARY CONDITIONS OF NODAL VELOCITIES AND PRESSURE
C
           WRITE(6,300)
  300  FORMAT(/,' *** APPLYING BOUNDARY CONDITIONS OF NODAL',
     *              ' INCREMENTS ***'          )
           CALL APPLYBC(NPOIV,  NEQ,  IBCU,  IBCV, IBCP,
     *                      SYSK, SYSR, MXPOIV, MXNEQ      )
C
C     SOLVE A SET OF SIMULTANEOUS SYSTEM EQUATIONS FOR SOLUTIONS
C
           WRITE(6,310)
  310  FORMAT(/,' *** SOLVING A SET OF SIMULTANEOUS EQS. FOR',
     *              ' NODAL INCREMENTS ***'      )
           WRITE(6,320)  NEQ
  320  FORMAT(5X,'( TOTAL OF', I5,' EQUATIONS TO BE SOLVED )')
           CALL GAUSS(NEQ, SYSK, SYSR, DSOL, MXNEQ)
C
C     CHECK FOR CONVERGENCE
C
           UP = 0.
           DOWN = 0.
           DO 330 I=1,NEQ
           ERROR = DSOL(I)
           UP = UP + ABS(ERROR)
           VALUE = SOL(I)
           DOWN = DOWN + ABS(VALUE)
  330  CONTINUE
           RATIO = UP*100./DOWN
           WRITE(6,340) RATIO
  340  FORMAT(6X, 'CURRENT SOLUTION HAS GLOBAL ERROR OF',
     *              F8.2, '%')
           IF(RATIO.GT.TOL) GO TO 360
C
C     SOLUTION CONVERGED WITHIN THE SPECIFIED TOLERANCE
C
           WRITE(6,350)
  350  FORMAT(/, 3X, ' *** SOLUTION CONVERGED WITHIN SPECIFIED',
     *              'TOLERANCE ***', //)
           GO TO 400
  360  CONTINUE
C
C     UPDATE NODAL SOLUTIONS
C
           DO 370 I=1,NEQ
           SOL(I) = SOL(I) + DSOL(I)
```

```
 370  CONTINUE
 380  CONTINUE
C
C     SOLUTION NOT CONVERGED WITHIN THE SPECIFIED TOLERANCE
C
      WRITE(6,390)
 390  FORMAT(/, 3X, ' ??? SOLUTION NOT CONVERGED WITHIN'
     *          ' SPECIFIED TOLERANCE ???', //)
C
 400  CONTINUE
C
C     PRINT OUT SOLUTIONS OF NODAL VELOCITIES AND PRESSURES
C
      WRITE(8,410)
 410  FORMAT(2X, 'NODE', 6X, 'U-VELOCITY', 6X, 'V-VELOCITY',
     *          8X, 'PRESSURE'                    )
C
C     ROUND-OFF SOLUTION VALUES FOR NEAT OUTPUT
C
      ROFF = 1.E-6
      DO 420  IEQ=1,NEQ
      VALUE = SOL(IEQ)
      IF(ABS(VALUE).LT.ROFF)  SOL(IEQ) = 0.
 420  CONTINUE
C
      DO 440  IP=1,NPOIP
      IEQU = IP
      IEQV = NPOIV + IP
      IEQP = 2*NPOIV + IP
      WRITE(8,430)  IP, SOL(IEQU), SOL(IEQV), SOL(IEQP)
 430  FORMAT(I6, 3E16.7)
 440  CONTINUE
C
      STOP
      END
C
C-----------------------------------------------------------------------
C
      SUBROUTINE APPLYBC(NPOIV,  NEQ,  IBCU,  IBCV, IBCP,
     *                   SYSK, SYSR, MXPOIV,  MXNEQ     )
C
C     APPLY BOUNDARY CONDITIONS FOR NODAL VELOCITIES AND PRESSURES
C      WITH CONDITION CODES OF
C           0 = FREE TO CHANGE (TO BE COMPUTED)
C           1 = FIXED AS SPECIFIED
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  SYSK(MXNEQ,MXNEQ), SYSR(MXNEQ)
C
      INTEGER  IBCU(MXPOIV), IBCV(MXPOIV), IBCP(MXPOIV)
```

```
C
C     APPLY BOUNDARY CONDITIONS FOR U-VELOCITIES
C
      IEQ1 = 1
      IEQ2 = NPOIV
      DO 100  IEQ=IEQ1,IEQ2
      IEQU = IEQ
      IF(IBCU(IEQU).EQ.0)  GO TO 100
C
      DO 110  IR=1,NEQ
      IF(IR.EQ.IEQ)  GO TO 110
      SYSK(IR,IEQ) = 0.
  110 CONTINUE
C
      DO 120  IC=1,NEQ
      SYSK(IEQ,IC) = 0.
  120 CONTINUE
      SYSK(IEQ,IEQ) = 1.
      SYSR(IEQ) = 0.
C
  100 CONTINUE
C
C     APPLY BOUNDARY CONDITIONS FOR V-VELOCITIES
C
      IEQ1 = NPOIV + 1
      IEQ2 = 2*NPOIV
      DO 200  IEQ=IEQ1,IEQ2
      IEQV = IEQ - NPOIV
      IF(IBCV(IEQV).EQ.0)  GO TO 200
C
      DO 210  IR=1,NEQ
      IF(IR.EQ.IEQ)  GO TO 210
      SYSK(IR,IEQ) = 0.
  210 CONTINUE
C
      DO 220  IC=1,NEQ
      SYSK(IEQ,IC) = 0.
  220 CONTINUE
      SYSK(IEQ,IEQ) = 1.
      SYSR(IEQ) = 0.
C
  200 CONTINUE
C
C     APPLY BOUNDARY CONDITIONS FOR PRESSURES
C
      IEQ1 = 2*NPOIV + 1
      IEQ2 = NEQ
      DO 300  IEQ=IEQ1,IEQ2
      IEQP = IEQ - 2*NPOIV
      IF(IBCP(IEQP).EQ.0)  GO TO 300
```

```
C
      DO 310  IR=1,NEQ
      IF(IR.EQ.IEQ)  GO TO 310
      SYSK(IR,IEQ) = 0.
 310  CONTINUE
C
      DO 320  IC=1,NEQ
      SYSK(IEQ,IC) = 0.
 320  CONTINUE
      SYSK(IEQ,IEQ) = 1.
      SYSR(IEQ) = 0.
C
 300  CONTINUE
C
      RETURN
      END
C
C-----------------------------------------------------------------------
C
      SUBROUTINE ASSMBLE( IE, INTMAT, AKELE, RELE, SYSK, SYSR,
     *                                NPOIV, MXNEQ, MXELE                )
C
C     ASSEMBLE ELEMENT EQUATIONS INTO SYSTEM EQUATIONS
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  AKELE(15,15), RELE(15)
      DIMENSION  SYSK(MXNEQ,MXNEQ), SYSR(MXNEQ)
C
      INTEGER  INTMAT(MXELE,6)
C
C     ASSEMBLING SYSTEM STIFFNESS MATRIX
C
C     CONTRIBUTION OF COEFFICIENTS ASSOCIATED WITH U & V VELOCITIES
C
      DO 100  I=1,6
      DO 100  J=1,6
      II = INTMAT(IE,I)
       JJ = INTMAT(IE,J)
      K  = I + 6
      L  = J + 6
      KK = NPOIV + II
      LL = NPOIV + JJ
       SYSK(II,JJ) = SYSK(II,JJ) + AKELE(I,J)
      SYSK(II,LL) = SYSK(II,LL) + AKELE(I,L)
      SYSK(KK,JJ) = SYSK(KK,JJ) + AKELE(K,J)
      SYSK(KK,LL) = SYSK(KK,LL) + AKELE(K,L)
 100  CONTINUE
C
C     CONTRIBUTION OF COEFFICIENTS ASSOCIATED WITH PRESSURE
C
```

```
      DO 200  I=1,6
      DO 200  J=1,3
      II = INTMAT(IE,I)
      JJ = INTMAT(IE,J)
      K  = I + 6
      L  = J + 12
      KK = NPOIV + II
      LL = 2*NPOIV + JJ
      SYSK(II,LL) = SYSK(II,LL) + AKELE(I,L)
      SYSK(KK,LL) = SYSK(KK,LL) + AKELE(K,L)
      SYSK(LL,II) = SYSK(LL,II) + AKELE(L,I)
      SYSK(LL,KK) = SYSK(LL,KK) + AKELE(L,K)
  200 CONTINUE
C
C     ASSEMBLING SYSTEM LOAD VECTOR
C
C     CONTRIBUTION OF VALUES ASSOCIATED WITH U & V VELOCITIES
C
      DO 300  I=1,6
      II = INTMAT(IE,I)
      K  = I + 6
      KK = NPOIV + II
      SYSR(II) = SYSR(II) + RELE(I)
      SYSR(KK) = SYSR(KK) + RELE(K)
  300 CONTINUE
C
C     CONTRIBUTION OF VALUES ASSOCIATED WITH PRESSURE
C
      DO 400  I=1,3
      II = INTMAT(IE,I)
      K  = I + 12
      KK = 2*NPOIV + II
      SYSR(KK) = SYSR(KK) + RELE(K)
  400 CONTINUE
C
      RETURN
      END
C
C-----------------------------------------------------------------
C
      SUBROUTINE GAUSS(N, A, B, X, MXNEQ)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  A(MXNEQ,MXNEQ), B(MXNEQ), X(MXNEQ)
C
C     PERFORM SCALING
C
      CALL SCALE(N, A, B, MXNEQ)
C
C     FORWARD ELIMINATION
C
```

```
C       PERFORM ACCORDING TO ORDER OF 'PRIME' FROM 1 TO N-1
C
        DO 100  IP=1,N-1
C
C       PERFORM PARTIAL PIVOTING
C
        CALL PIVOT(N, A, B, MXNEQ, IP)
C
C       LOOP OVER EACH EQUATION STARTING FROM THE ONE THAT CORRESPONDS
C       WITH THE ORDER OF 'PRIME' PLUS ONE
C
        DO 200  IE=IP+1,N
        RATIO = A(IE,IP)/A(IP,IP)
C
C       COMPUTE NEW COEFFICIENTS OF THE EQUATION CONSIDERED
C
        DO 300  IC=IP+1,N
        A(IE,IC) = A(IE,IC) - RATIO*A(IP,IC)
 300    CONTINUE
        B(IE) = B(IE) - RATIO*B(IP)
 200    CONTINUE
C
C       SET COEFFICIENTS ON LOWER LEFT PORTION TO ZERO
C
        DO 400  IE=IP+1,N
        A(IE,IP) = 0.
 400    CONTINUE
 100    CONTINUE
C
C       BACK SUBSTITUTION
C
C       COMPUTE SOLUTION OF THE LAST EQUATION
C
        X(N) = B(N)/A(N,N)
C
C       THEN COMPUTE SOLUTIONS FROM EQUATION N-1 TO 1
C
        DO 500  IE=N-1,1,-1
        SUM = 0.
        DO 600  IC=IE+1,N
        SUM = SUM + A(IE,IC)*X(IC)
 600    CONTINUE
        X(IE) = (B(IE) - SUM)/A(IE,IE)
 500    CONTINUE
        RETURN
        END
C
C-----------------------------------------------------------------------
C
        SUBROUTINE PIVOT(N, A, B, MXNEQ, IP)
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  A(MXNEQ,MXNEQ), B(MXNEQ)
C
C     PERFORM PARTIAL PIVOTING
C
      JP = IP
      BIG = ABS(A(IP,IP))
      DO 10  I=IP+1,N
      AMAX = ABS(A(I,IP))
      IF(AMAX.GT.BIG)  THEN
      BIG = AMAX
      JP  = I
      ENDIF
   10 CONTINUE
      IF(JP.NE.IP)  THEN
      DO 20  J=IP,N
      DUMY   = A(JP,J)
      A(JP,J) = A(IP,J)
      A(IP,J) = DUMY
   20 CONTINUE
      DUMY  = B(JP)
      B(JP) = B(IP)
      B(IP) = DUMY
      ENDIF
      RETURN
      END
C
C-----------------------------------------------------------------------
C
      SUBROUTINE SCALE(N, A, B, MXNEQ)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  A(MXNEQ,MXNEQ), B(MXNEQ)
C
C     PERFORM SCALING:
C
      DO 10  IE=1,N
      BIG = ABS(A(IE,1))
      DO 20  IC=2,N
      AMAX = ABS(A(IE,IC))
      IF(AMAX.GT.BIG)  BIG = AMAX
   20 CONTINUE
      DO 30  IC=1,N
      A(IE,IC) = A(IE,IC)/BIG
   30 CONTINUE
      B(IE) = B(IE)/BIG
   10 CONTINUE
      RETURN
      END
C
C-----------------------------------------------------------------------
```

```
C
      SUBROUTINE TRI( NPOIV,  NELEM, DEN, VIS, COORD,
     *                INTMAT, SYSK, SYSR, SOL, MXPOIV,
     *                MXELE, MXNEQ                    )
C
C     ESTABLISH ALL ELEMENT MATRICES AND ASSEMBLE THEM TO FORM
C     UP SYSTEM EQUATIONS
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  COORD(MXPOIV,2), SYSK(MXNEQ,MXNEQ)
      DIMENSION  SYSR(MXNEQ), SOL(MXNEQ)
      DIMENSION  B(3), C(3)
      DIMENSION  UELE(6), VELE(6), PELE(3)
      DIMENSION  HX(6,3), HXT(3,6), HY(6,3), HYT(3,6)
      DIMENSION  AMXX(6,6), AMYY(6,6), AMXY(6,6), AMYX(6,6)
      DIMENSION  SXX(6,6), SXY(6,6), SYX(6,6), SYY(6,6)
      DIMENSION  AKX(6,6,6), AKY(6,6,6)
      DIMENSION  GXX(6,6), GYY(6,6), ALX(6,6), ALY(6,6)
      DIMENSION  FX(6), FY(6), FI(3)
      DIMENSION  AKELE(15,15), RELE(15)
C
      INTEGER  INTMAT(MXELE,6)
C
C     COMPUTE KINEMATIC VISCOSITY
C
      ANEW = VIS/DEN
C
C     LOOP OVER THE NUMBER OF ELEMENTS
C
      DO 500  IE=1,NELEM
C
C     FIND ELEMENT LOCAL COORDINATES
C
      II = INTMAT(IE,1)
      JJ = INTMAT(IE,2)
      KK = INTMAT(IE,3)
      LL = INTMAT(IE,4)
      MM = INTMAT(IE,5)
      NN = INTMAT(IE,6)
C
      XG1 = COORD(II,1)
      XG2 = COORD(JJ,1)
      XG3 = COORD(KK,1)
      YG1 = COORD(II,2)
      YG2 = COORD(JJ,2)
      YG3 = COORD(KK,2)
      AREA= 0.5*(XG2*(YG3-YG1) + XG1*(YG2-YG3) + XG3*(YG1-YG2))
      IF(AREA.LE.0.)  WRITE(6,5)  IE
    5 FORMAT(/,'  !!! ERROR !!!  ELEMENT NO.', I5,
     *          ' HAS NEGATIVE OR ZERO AREA ', /,
```

```
     *          ' --- CHECK F.E. MODEL FOR NODAL COORDINATES',
     *          ' AND ELEMENT NODAL CONNECTIONS ---'      )
          IF(AREA.LE.0.)  STOP
C
          B(1) = YG2 - YG3
          B(2) = YG3 - YG1
          B(3) = YG1 - YG2
          C(1) = XG3 - XG2
          C(2) = XG1 - XG3
          C(3) = XG2 - XG1
C
C         SET UP [MXX] MATRIX
C
          DO 10 I=1,3
          DO 10 J=1,3
          AMXX(I,J) = B(I)*B(J)/4./AREA
   10     CONTINUE
          DO 20 I=1,3
          DO 20 J=4,6
          K = J-3
          AMXX(I,J) = B(I)*(B(1)+B(2)+B(3)-B(K))/3./AREA
          AMXX(J,I) = AMXX(I,J)
   20     CONTINUE
          AMXX(4,4) = 2.*(B(2)*B(2)+B(2)*B(3)+B(3)*B(3))/3./AREA
          AMXX(5,5) = 2.*(B(1)*B(1)+B(1)*B(3)+B(3)*B(3))/3./AREA
          AMXX(6,6) = 2.*(B(2)*B(2)+B(2)*B(1)+B(1)*B(1))/3./AREA
          AMXX(4,5) = (2.*B(1)*B(2)+B(1)*B(3)+B(2)*B(3)+B(3)*B(3))/3./AREA
          AMXX(5,4) = AMXX(4,5)
          AMXX(4,6) = (2.*B(1)*B(3)+B(1)*B(2)+B(2)*B(3)+B(2)*B(2))/3./AREA
          AMXX(6,4) = AMXX(4,6)
          AMXX(5,6) = (2.*B(2)*B(3)+B(1)*B(3)+B(2)*B(1)+B(1)*B(1))/3./AREA
          AMXX(6,5) = AMXX(5,6)
C
C         SETUP [MYY] MATRIX
C
          DO 30 I=1,3
          DO 30 J=1,3
          AMYY(I,J) = C(I)*C(J)/4./AREA
   30     CONTINUE
          DO 40 I=1,3
          DO 40 J=4,6
          K = J-3
          AMYY(I,J) = C(I)*(C(1)+C(2)+C(3)-C(K))/3./AREA
          AMYY(J,I) = AMYY(I,J)
   40     CONTINUE
          AMYY(4,4) = 2.*(C(2)*C(2)+C(2)*C(3)+C(3)*C(3))/3./AREA
          AMYY(5,5) = 2.*(C(1)*C(1)+C(1)*C(3)+C(3)*C(3))/3./AREA
          AMYY(6,6) = 2.*(C(2)*C(2)+C(2)*C(1)+C(1)*C(1))/3./AREA
          AMYY(4,5) = (2.*C(1)*C(2)+C(1)*C(3)+C(2)*C(3)+C(3)*C(3))/3./AREA
          AMYY(5,4) = AMYY(4,5)
```

```
      AMYY(4,6) = (2.*C(1)*C(3)+C(1)*C(2)+C(2)*C(3)+C(2)*C(2))/3./AREA
      AMYY(6,4) = AMYY(4,6)
      AMYY(5,6) = (2.*C(2)*C(3)+C(1)*C(3)+C(2)*C(1)+C(1)*C(1))/3./AREA
      AMYY(6,5) = AMYY(5,6)
C
C     SETUP [MYX] MATRIX
C
      DO 50 I=1,3
      DO 50 J=1,3
      AMYX(I,J) = B(I)*C(J)/4./AREA
   50 CONTINUE
      DO 60 I=1,3
      DO 60 J=4,6
      K = J-3
      AMYX(I,J) = B(I)*(C(1)+C(2)+C(3)-C(K))/3./AREA
      AMYX(J,I) = C(I)*(B(1)+B(2)+B(3)-B(K))/3./AREA
   60 CONTINUE
      AMYX(4,4) = (2.*B(2)*C(2)+B(2)*C(3)+B(3)*C(2)+2.*B(3)*C(3))/3.
     *            /AREA
      AMYX(4,5) = (2.*B(2)*C(1)+B(2)*C(3)+B(3)*C(1)+B(3)*C(3))/3./AREA
      AMYX(4,6) = (2.*B(3)*C(1)+B(3)*C(2)+B(2)*C(1)+B(2)*C(2))/3./AREA
      AMYX(5,4) = (2.*B(1)*C(2)+B(1)*C(3)+B(3)*C(2)+B(3)*C(3))/3./AREA
      AMYX(5,5) = (2.*B(1)*C(1)+B(1)*C(3)+B(3)*C(1)+2.*B(3)*C(3))/3.
     *            /AREA
      AMYX(5,6) = (2.*B(3)*C(2)+B(3)*C(1)+B(1)*C(2)+B(1)*C(1))/3./AREA
      AMYX(6,4) = (2.*B(1)*C(3)+B(1)*C(2)+B(2)*C(3)+B(2)*C(2))/3./AREA
      AMYX(6,5) = (2.*B(2)*C(3)+B(2)*C(1)+B(1)*C(3)+B(1)*C(1))/3./AREA
      AMYX(6,6) = (2.*B(1)*C(1)+B(1)*C(2)+B(2)*C(1)+2.*B(2)*C(2))/3.
     *            /AREA
C
C     SETUP [MXY] MATRIX
C
      DO 70 I=1,6
      DO 70 J=1,6
      AMXY(I,J) = AMYX(J,I)
   70 CONTINUE
C
C     SETUP [SXX], [SXY], [SYX], AND [SYY] MATRICES
C
      DO 80 I=1,6
      DO 80 J=1,6
      SXX(I,J) = 2.*ANEW*AMXX(I,J) + ANEW*AMYY(I,J)
   80 CONTINUE
C
      DO 90 I=1,6
      DO 90 J=1,6
      SXY(I,J) = ANEW*AMXY(I,J)
   90 CONTINUE
C
      DO 100 I=1,6
```

```
      DO 100 J=1,6
      SYX(I,J) = ANEW*AMYX(I,J)
  100 CONTINUE
C
      DO 110 I=1,6
      DO 110 J=1,6
      SYY(I,J) = ANEW*AMXX(I,J) + 2.*ANEW*AMYY(I,J)
  110 CONTINUE
C
C     SETUP [HX] AND [HXT] MATRICES
C
      DO 120 I=1,3
      DO 120 J=1,3
      HX(I,J) = B(I)/6./DEN
  120 CONTINUE
      HX(4,1) = B(2)/6./DEN + B(3)/6./DEN
      HX(4,2) = B(2)/6./DEN + B(3)/3./DEN
      HX(4,3) = B(2)/3./DEN + B(3)/6./DEN
      HX(5,1) = B(3)/3./DEN + B(1)/6./DEN
      HX(5,2) = B(3)/6./DEN + B(1)/6./DEN
      HX(5,3) = B(3)/6./DEN + B(1)/3./DEN
      HX(6,1) = B(1)/6./DEN + B(2)/3./DEN
      HX(6,2) = B(1)/3./DEN + B(2)/6./DEN
      HX(6,3) = B(1)/6./DEN + B(2)/6./DEN
      DO 130 I=1,3
      DO 130 J=1,6
      HXT(I,J) = HX(J,I)
  130 CONTINUE
C
C     SETUP [HY] AND [HYT] MATRICES
C
      DO 140 I=1,3
      DO 140 J=1,3
      HY(I,J) = C(I)/6./DEN
  140 CONTINUE
      HY(4,1) = C(2)/6./DEN + C(3)/6./DEN
      HY(4,2) = C(2)/6./DEN + C(3)/3./DEN
      HY(4,3) = C(2)/3./DEN + C(3)/6./DEN
      HY(5,1) = C(3)/3./DEN + C(1)/6./DEN
      HY(5,2) = C(3)/6./DEN + C(1)/6./DEN
      HY(5,3) = C(3)/6./DEN + C(1)/3./DEN
      HY(6,1) = C(1)/6./DEN + C(2)/3./DEN
      HY(6,2) = C(1)/3./DEN + C(2)/6./DEN
      HY(6,3) = C(1)/6./DEN + C(2)/6./DEN
      DO 150 I=1,3
      DO 150 J=1,6
      HYT(I,J) = HY(J,I)
  150 CONTINUE
C
C     SET UP [AKX-1] MATRIX
```

```
C
      FAC = B(1)/360.
      I = 1
      AKX(1,1,I) = 30.*FAC
      AKX(1,2,I) = 15.*FAC
      AKX(1,3,I) = 15.*FAC
      AKX(1,4,I) = 12.*FAC
      AKX(1,5,I) = 24.*FAC
      AKX(1,6,I) = 24.*FAC
      AKX(2,1,I) = 15.*FAC
      AKX(2,2,I) = 30.*FAC
      AKX(2,3,I) = 15.*FAC
      AKX(2,4,I) = 24.*FAC
      AKX(2,5,I) = 12.*FAC
      AKX(2,6,I) = 24.*FAC
      AKX(3,1,I) = 15.*FAC
      AKX(3,2,I) = 15.*FAC
      AKX(3,3,I) = 30.*FAC
      AKX(3,4,I) = 24.*FAC
      AKX(3,5,I) = 24.*FAC
      AKX(3,6,I) = 12.*FAC
      AKX(4,1,I) = 12.*FAC
      AKX(4,2,I) = 24.*FAC
      AKX(4,3,I) = 24.*FAC
      AKX(4,4,I) = 32.*FAC
      AKX(4,5,I) = 16.*FAC
      AKX(4,6,I) = 16.*FAC
      AKX(5,1,I) = 24.*FAC
      AKX(5,2,I) = 12.*FAC
      AKX(5,3,I) = 24.*FAC
      AKX(5,4,I) = 16.*FAC
      AKX(5,5,I) = 32.*FAC
      AKX(5,6,I) = 16.*FAC
      AKX(6,1,I) = 24.*FAC
      AKX(6,2,I) = 24.*FAC
      AKX(6,3,I) = 12.*FAC
      AKX(6,4,I) = 16.*FAC
      AKX(6,5,I) = 16.*FAC
      AKX(6,6,I) = 32.*FAC
C
C     SET UP [AKY-1] MATRIX
C
      FAC = C(1)/360.
      I = 1
      AKY(1,1,I) = 30.*FAC
      AKY(1,2,I) = 15.*FAC
      AKY(1,3,I) = 15.*FAC
      AKY(1,4,I) = 12.*FAC
      AKY(1,5,I) = 24.*FAC
      AKY(1,6,I) = 24.*FAC
```

```
      AKY(2,1,I) = 15.*FAC
      AKY(2,2,I) = 30.*FAC
      AKY(2,3,I) = 15.*FAC
      AKY(2,4,I) = 24.*FAC
      AKY(2,5,I) = 12.*FAC
      AKY(2,6,I) = 24.*FAC
      AKY(3,1,I) = 15.*FAC
      AKY(3,2,I) = 15.*FAC
      AKY(3,3,I) = 30.*FAC
      AKY(3,4,I) = 24.*FAC
      AKY(3,5,I) = 24.*FAC
      AKY(3,6,I) = 12.*FAC
      AKY(4,1,I) = 12.*FAC
      AKY(4,2,I) = 24.*FAC
      AKY(4,3,I) = 24.*FAC
      AKY(4,4,I) = 32.*FAC
      AKY(4,5,I) = 16.*FAC
      AKY(4,6,I) = 16.*FAC
      AKY(5,1,I) = 24.*FAC
      AKY(5,2,I) = 12.*FAC
      AKY(5,3,I) = 24.*FAC
      AKY(5,4,I) = 16.*FAC
      AKY(5,5,I) = 32.*FAC
      AKY(5,6,I) = 16.*FAC
      AKY(6,1,I) = 24.*FAC
      AKY(6,2,I) = 24.*FAC
      AKY(6,3,I) = 12.*FAC
      AKY(6,4,I) = 16.*FAC
      AKY(6,5,I) = 16.*FAC
      AKY(6,6,I) = 32.*FAC
C
C     SET UP [AKX-2] MATRIX
C
      FAC = B(2)/360.
      I = 2
      AKX(1,1,I) = 30.*FAC
      AKX(1,2,I) = 15.*FAC
      AKX(1,3,I) = 15.*FAC
      AKX(1,4,I) = 12.*FAC
      AKX(1,5,I) = 24.*FAC
      AKX(1,6,I) = 24.*FAC
      AKX(2,1,I) = 15.*FAC
      AKX(2,2,I) = 30.*FAC
      AKX(2,3,I) = 15.*FAC
      AKX(2,4,I) = 24.*FAC
      AKX(2,5,I) = 12.*FAC
      AKX(2,6,I) = 24.*FAC
      AKX(3,1,I) = 15.*FAC
      AKX(3,2,I) = 15.*FAC
      AKX(3,3,I) = 30.*FAC
```

```
      AKX(3,4,I) = 24.*FAC
      AKX(3,5,I) = 24.*FAC
      AKX(3,6,I) = 12.*FAC
      AKX(4,1,I) = 12.*FAC
      AKX(4,2,I) = 24.*FAC
      AKX(4,3,I) = 24.*FAC
      AKX(4,4,I) = 32.*FAC
      AKX(4,5,I) = 16.*FAC
      AKX(4,6,I) = 16.*FAC
      AKX(5,1,I) = 24.*FAC
      AKX(5,2,I) = 12.*FAC
      AKX(5,3,I) = 24.*FAC
      AKX(5,4,I) = 16.*FAC
      AKX(5,5,I) = 32.*FAC
      AKX(5,6,I) = 16.*FAC
      AKX(6,1,I) = 24.*FAC
      AKX(6,2,I) = 24.*FAC
      AKX(6,3,I) = 12.*FAC
      AKX(6,4,I) = 16.*FAC
      AKX(6,5,I) = 16.*FAC
      AKX(6,6,I) = 32.*FAC
C
C     SET UP [AKY-2] MATRIX
C
      FAC = C(2)/360.
      I = 2
      AKY(1,1,I) = 30.*FAC
      AKY(1,2,I) = 15.*FAC
      AKY(1,3,I) = 15.*FAC
      AKY(1,4,I) = 12.*FAC
      AKY(1,5,I) = 24.*FAC
      AKY(1,6,I) = 24.*FAC
      AKY(2,1,I) = 15.*FAC
      AKY(2,2,I) = 30.*FAC
      AKY(2,3,I) = 15.*FAC
      AKY(2,4,I) = 24.*FAC
      AKY(2,5,I) = 12.*FAC
      AKY(2,6,I) = 24.*FAC
      AKY(3,1,I) = 15.*FAC
      AKY(3,2,I) = 15.*FAC
      AKY(3,3,I) = 30.*FAC
      AKY(3,4,I) = 24.*FAC
      AKY(3,5,I) = 24.*FAC
      AKY(3,6,I) = 12.*FAC
      AKY(4,1,I) = 12.*FAC
      AKY(4,2,I) = 24.*FAC
      AKY(4,3,I) = 24.*FAC
      AKY(4,4,I) = 32.*FAC
      AKY(4,5,I) = 16.*FAC
      AKY(4,6,I) = 16.*FAC
```

```
      AKY(5,1,I) = 24.*FAC
      AKY(5,2,I) = 12.*FAC
      AKY(5,3,I) = 24.*FAC
      AKY(5,4,I) = 16.*FAC
      AKY(5,5,I) = 32.*FAC
      AKY(5,6,I) = 16.*FAC
      AKY(6,1,I) = 24.*FAC
      AKY(6,2,I) = 24.*FAC
      AKY(6,3,I) = 12.*FAC
      AKY(6,4,I) = 16.*FAC
      AKY(6,5,I) = 16.*FAC
      AKY(6,6,I) = 32.*FAC
C
C     SET UP [AKX-3] MATRIX
C
      FAC = B(3)/360.
      I = 3
      AKX(1,1,I) = 30.*FAC
      AKX(1,2,I) = 15.*FAC
      AKX(1,3,I) = 15.*FAC
      AKX(1,4,I) = 12.*FAC
      AKX(1,5,I) = 24.*FAC
      AKX(1,6,I) = 24.*FAC
      AKX(2,1,I) = 15.*FAC
      AKX(2,2,I) = 30.*FAC
      AKX(2,3,I) = 15.*FAC
      AKX(2,4,I) = 24.*FAC
      AKX(2,5,I) = 12.*FAC
      AKX(2,6,I) = 24.*FAC
      AKX(3,1,I) = 15.*FAC
      AKX(3,2,I) = 15.*FAC
      AKX(3,3,I) = 30.*FAC
      AKX(3,4,I) = 24.*FAC
      AKX(3,5,I) = 24.*FAC
      AKX(3,6,I) = 12.*FAC
      AKX(4,1,I) = 12.*FAC
      AKX(4,2,I) = 24.*FAC
      AKX(4,3,I) = 24.*FAC
      AKX(4,4,I) = 32.*FAC
      AKX(4,5,I) = 16.*FAC
      AKX(4,6,I) = 16.*FAC
      AKX(5,1,I) = 24.*FAC
      AKX(5,2,I) = 12.*FAC
      AKX(5,3,I) = 24.*FAC
      AKX(5,4,I) = 16.*FAC
      AKX(5,5,I) = 32.*FAC
      AKX(5,6,I) = 16.*FAC
      AKX(6,1,I) = 24.*FAC
      AKX(6,2,I) = 24.*FAC
      AKX(6,3,I) = 12.*FAC
```

```
      AKX(6,4,I) = 16.*FAC
      AKX(6,5,I) = 16.*FAC
      AKX(6,6,I) = 32.*FAC
C
C     SET UP [AKY-3] MATRIX
C
      FAC = C(3)/360.
      I = 3
      AKY(1,1,I) = 30.*FAC
      AKY(1,2,I) = 15.*FAC
      AKY(1,3,I) = 15.*FAC
      AKY(1,4,I) = 12.*FAC
      AKY(1,5,I) = 24.*FAC
      AKY(1,6,I) = 24.*FAC
      AKY(2,1,I) = 15.*FAC
      AKY(2,2,I) = 30.*FAC
      AKY(2,3,I) = 15.*FAC
      AKY(2,4,I) = 24.*FAC
      AKY(2,5,I) = 12.*FAC
      AKY(2,6,I) = 24.*FAC
      AKY(3,1,I) = 15.*FAC
      AKY(3,2,I) = 15.*FAC
      AKY(3,3,I) = 30.*FAC
      AKY(3,4,I) = 24.*FAC
      AKY(3,5,I) = 24.*FAC
      AKY(3,6,I) = 12.*FAC
      AKY(4,1,I) = 12.*FAC
      AKY(4,2,I) = 24.*FAC
      AKY(4,3,I) = 24.*FAC
      AKY(4,4,I) = 32.*FAC
      AKY(4,5,I) = 16.*FAC
      AKY(4,6,I) = 16.*FAC
      AKY(5,1,I) = 24.*FAC
      AKY(5,2,I) = 12.*FAC
      AKY(5,3,I) = 24.*FAC
      AKY(5,4,I) = 16.*FAC
      AKY(5,5,I) = 32.*FAC
      AKY(5,6,I) = 16.*FAC
      AKY(6,1,I) = 24.*FAC
      AKY(6,2,I) = 24.*FAC
      AKY(6,3,I) = 12.*FAC
      AKY(6,4,I) = 16.*FAC
      AKY(6,5,I) = 16.*FAC
      AKY(6,6,I) = 32.*FAC
C
C     SET UP [AKX-4] MATRIX
C
      FAC1 = B(2)/630.
      FAC2 = B(3)/630.
      I = 4
```

```
      AKX(1,1,I) = 42.*FAC1+42.*FAC2
      AKX(1,2,I) = 21.*FAC1+42.*FAC2
      AKX(1,3,I) = 42.*FAC1+21.*FAC2
      AKX(1,4,I) = 28.*FAC1+28.*FAC2
      AKX(1,5,I) = 56.*FAC1+28.*FAC2
      AKX(1,6,I) = 28.*FAC1+56.*FAC2
      AKX(2,1,I) = 21.*FAC1+42.*FAC2
      AKX(2,2,I) = 42.*FAC1+126.*FAC2
      AKX(2,3,I) = 42.*FAC1+42.*FAC2
      AKX(2,4,I) = 56.*FAC1+84.*FAC2
      AKX(2,5,I) = 28.*FAC1+28.*FAC2
      AKX(2,6,I) = 28.*FAC1+84.*FAC2
      AKX(3,1,I) = 42.*FAC1+21.*FAC2
      AKX(3,2,I) = 42.*FAC1+42.*FAC2
      AKX(3,3,I) = 126.*FAC1+42.*FAC2
      AKX(3,4,I) = 84.*FAC1+56.*FAC2
      AKX(3,5,I) = 84.*FAC1+28.*FAC2
      AKX(3,6,I) = 28.*FAC1+28.*FAC2
      AKX(4,1,I) = 28.*FAC1+28.*FAC2
      AKX(4,2,I) = 56.*FAC1+84.*FAC2
      AKX(4,3,I) = 84.*FAC1+56.*FAC2
      AKX(4,4,I) = 96.*FAC1+96.*FAC2
      AKX(4,5,I) = 48.*FAC1+32.*FAC2
      AKX(4,6,I) = 32.*FAC1+48.*FAC2
      AKX(5,1,I) = 56.*FAC1+28.*FAC2
      AKX(5,2,I) = 28.*FAC1+28.*FAC2
      AKX(5,3,I) = 84.*FAC1+28.*FAC2
      AKX(5,4,I) = 48.*FAC1+32.*FAC2
      AKX(5,5,I) = 96.*FAC1+32.*FAC2
      AKX(5,6,I) = 32.*FAC1+32.*FAC2
      AKX(6,1,I) = 28.*FAC1+56.*FAC2
      AKX(6,2,I) = 28.*FAC1+84.*FAC2
      AKX(6,3,I) = 28.*FAC1+28.*FAC2
      AKX(6,4,I) = 32.*FAC1+48.*FAC2
      AKX(6,5,I) = 32.*FAC1+32.*FAC2
      AKX(6,6,I) = 32.*FAC1+96.*FAC2
C
C     SET UP [AKY-4] MATRIX
C
      FAC1 = C(2)/630.
      FAC2 = C(3)/630.
      I = 4
      AKY(1,1,I) = 42.*FAC1+42.*FAC2
      AKY(1,2,I) = 21.*FAC1+42.*FAC2
      AKY(1,3,I) = 42.*FAC1+21.*FAC2
      AKY(1,4,I) = 28.*FAC1+28.*FAC2
      AKY(1,5,I) = 56.*FAC1+28.*FAC2
      AKY(1,6,I) = 28.*FAC1+56.*FAC2
      AKY(2,1,I) = 21.*FAC1+42.*FAC2
      AKY(2,2,I) = 42.*FAC1+126.*FAC2
```

```
      AKY(2,3,I) = 42.*FAC1+42.*FAC2
      AKY(2,4,I) = 56.*FAC1+84.*FAC2
      AKY(2,5,I) = 28.*FAC1+28.*FAC2
      AKY(2,6,I) = 28.*FAC1+84.*FAC2
      AKY(3,1,I) = 42.*FAC1+21.*FAC2
      AKY(3,2,I) = 42.*FAC1+42.*FAC2
      AKY(3,3,I) = 126.*FAC1+42.*FAC2
      AKY(3,4,I) = 84.*FAC1+56.*FAC2
      AKY(3,5,I) = 84.*FAC1+28.*FAC2
      AKY(3,6,I) = 28.*FAC1+28.*FAC2
      AKY(4,1,I) = 28.*FAC1+28.*FAC2
      AKY(4,2,I) = 56.*FAC1+84.*FAC2
      AKY(4,3,I) = 84.*FAC1+56.*FAC2
      AKY(4,4,I) = 96.*FAC1+96.*FAC2
      AKY(4,5,I) = 48.*FAC1+32.*FAC2
      AKY(4,6,I) = 32.*FAC1+48.*FAC2
      AKY(5,1,I) = 56.*FAC1+28.*FAC2
      AKY(5,2,I) = 28.*FAC1+28.*FAC2
      AKY(5,3,I) = 84.*FAC1+28.*FAC2
      AKY(5,4,I) = 48.*FAC1+32.*FAC2
      AKY(5,5,I) = 96.*FAC1+32.*FAC2
      AKY(5,6,I) = 32.*FAC1+32.*FAC2
      AKY(6,1,I) = 28.*FAC1+56.*FAC2
      AKY(6,2,I) = 28.*FAC1+84.*FAC2
      AKY(6,3,I) = 28.*FAC1+28.*FAC2
      AKY(6,4,I) = 32.*FAC1+48.*FAC2
      AKY(6,5,I) = 32.*FAC1+32.*FAC2
      AKY(6,6,I) = 32.*FAC1+96.*FAC2
C
C     SET UP [AKX-5] MATRIX
C
      FAC1 = B(1)/630.
      FAC2 = B(3)/630.
      I = 5
      AKX(1,1,I) = 42.*FAC1+126.*FAC2
      AKX(1,2,I) = 21.*FAC1+42.*FAC2
      AKX(1,3,I) = 42.*FAC1+42.*FAC2
      AKX(1,4,I) = 28.*FAC1+28.*FAC2
      AKX(1,5,I) = 56.*FAC1+84.*FAC2
      AKX(1,6,I) = 28.*FAC1+84.*FAC2
      AKX(2,1,I) = 21.*FAC1+42.*FAC2
      AKX(2,2,I) = 42.*FAC1+42.*FAC2
      AKX(2,3,I) = 42.*FAC1+21.*FAC2
      AKX(2,4,I) = 56.*FAC1+28.*FAC2
      AKX(2,5,I) = 28.*FAC1+28.*FAC2
      AKX(2,6,I) = 28.*FAC1+56.*FAC2
      AKX(3,1,I) = 42.*FAC1+42.*FAC2
      AKX(3,2,I) = 42.*FAC1+21.*FAC2
      AKX(3,3,I) = 126.*FAC1+42.*FAC2
      AKX(3,4,I) = 84.*FAC1+28.*FAC2
```

```
      AKX(3,5,I) = 84.*FAC1+56.*FAC2
      AKX(3,6,I) = 28.*FAC1+28.*FAC2
      AKX(4,1,I) = 28.*FAC1+28.*FAC2
      AKX(4,2,I) = 56.*FAC1+28.*FAC2
      AKX(4,3,I) = 84.*FAC1+28.*FAC2
      AKX(4,4,I) = 96.*FAC1+32.*FAC2
      AKX(4,5,I) = 48.*FAC1+32.*FAC2
      AKX(4,6,I) = 32.*FAC1+32.*FAC2
      AKX(5,1,I) = 56.*FAC1+84.*FAC2
      AKX(5,2,I) = 28.*FAC1+28.*FAC2
      AKX(5,3,I) = 84.*FAC1+56.*FAC2
      AKX(5,4,I) = 48.*FAC1+32.*FAC2
      AKX(5,5,I) = 96.*FAC1+96.*FAC2
      AKX(5,6,I) = 32.*FAC1+48.*FAC2
      AKX(6,1,I) = 28.*FAC1+84.*FAC2
      AKX(6,2,I) = 28.*FAC1+56.*FAC2
      AKX(6,3,I) = 28.*FAC1+28.*FAC2
      AKX(6,4,I) = 32.*FAC1+32.*FAC2
      AKX(6,5,I) = 32.*FAC1+48.*FAC2
      AKX(6,6,I) = 32.*FAC1+96.*FAC2
C
C     SET UP [AKY-5] MATRIX
C
      FAC1 = C(1)/630.
      FAC2 = C(3)/630.
      I = 5
      AKY(1,1,I) = 42.*FAC1+126.*FAC2
      AKY(1,2,I) = 21.*FAC1+42.*FAC2
      AKY(1,3,I) = 42.*FAC1+42.*FAC2
      AKY(1,4,I) = 28.*FAC1+28.*FAC2
      AKY(1,5,I) = 56.*FAC1+84.*FAC2
      AKY(1,6,I) = 28.*FAC1+84.*FAC2
      AKY(2,1,I) = 21.*FAC1+42.*FAC2
      AKY(2,2,I) = 42.*FAC1+42.*FAC2
      AKY(2,3,I) = 42.*FAC1+21.*FAC2
      AKY(2,4,I) = 56.*FAC1+28.*FAC2
      AKY(2,5,I) = 28.*FAC1+28.*FAC2
      AKY(2,6,I) = 28.*FAC1+56.*FAC2
      AKY(3,1,I) = 42.*FAC1+42.*FAC2
      AKY(3,2,I) = 42.*FAC1+21.*FAC2
      AKY(3,3,I) = 126.*FAC1+42.*FAC2
      AKY(3,4,I) = 84.*FAC1+28.*FAC2
      AKY(3,5,I) = 84.*FAC1+56.*FAC2
      AKY(3,6,I) = 28.*FAC1+28.*FAC2
      AKY(4,1,I) = 28.*FAC1+28.*FAC2
      AKY(4,2,I) = 56.*FAC1+28.*FAC2
      AKY(4,3,I) = 84.*FAC1+28.*FAC2
      AKY(4,4,I) = 96.*FAC1+32.*FAC2
      AKY(4,5,I) = 48.*FAC1+32.*FAC2
      AKY(4,6,I) = 32.*FAC1+32.*FAC2
```

```
      AKY(5,1,I) = 56.*FAC1+84.*FAC2
      AKY(5,2,I) = 28.*FAC1+28.*FAC2
      AKY(5,3,I) = 84.*FAC1+56.*FAC2
      AKY(5,4,I) = 48.*FAC1+32.*FAC2
      AKY(5,5,I) = 96.*FAC1+96.*FAC2
      AKY(5,6,I) = 32.*FAC1+48.*FAC2
      AKY(6,1,I) = 28.*FAC1+84.*FAC2
      AKY(6,2,I) = 28.*FAC1+56.*FAC2
      AKY(6,3,I) = 28.*FAC1+28.*FAC2
      AKY(6,4,I) = 32.*FAC1+32.*FAC2
      AKY(6,5,I) = 32.*FAC1+48.*FAC2
      AKY(6,6,I) = 32.*FAC1+96.*FAC2
C
C     SET UP [AKX-6] MATRIX
C
      FAC1 = B(1)/630.
      FAC2 = B(2)/630.
      I = 6
      AKX(1,1,I) = 42.*FAC1+126.*FAC2
      AKX(1,2,I) = 42.*FAC1+42.*FAC2
      AKX(1,3,I) = 21.*FAC1+42.*FAC2
      AKX(1,4,I) = 28.*FAC1+28.*FAC2
      AKX(1,5,I) = 28.*FAC1+84.*FAC2
      AKX(1,6,I) = 56.*FAC1+84.*FAC2
      AKX(2,1,I) = 42.*FAC1+42.*FAC2
      AKX(2,2,I) = 126.*FAC1+42.*FAC2
      AKX(2,3,I) = 42.*FAC1+21.*FAC2
      AKX(2,4,I) = 84.*FAC1+28.*FAC2
      AKX(2,5,I) = 28.*FAC1+28.*FAC2
      AKX(2,6,I) = 84.*FAC1+56.*FAC2
      AKX(3,1,I) = 21.*FAC1+42.*FAC2
      AKX(3,2,I) = 42.*FAC1+21.*FAC2
      AKX(3,3,I) = 42.*FAC1+42.*FAC2
      AKX(3,4,I) = 56.*FAC1+28.*FAC2
      AKX(3,5,I) = 28.*FAC1+56.*FAC2
      AKX(3,6,I) = 28.*FAC1+28.*FAC2
      AKX(4,1,I) = 28.*FAC1+28.*FAC2
      AKX(4,2,I) = 84.*FAC1+28.*FAC2
      AKX(4,3,I) = 56.*FAC1+28.*FAC2
      AKX(4,4,I) = 96.*FAC1+32.*FAC2
      AKX(4,5,I) = 32.*FAC1+32.*FAC2
      AKX(4,6,I) = 48.*FAC1+32.*FAC2
      AKX(5,1,I) = 28.*FAC1+84.*FAC2
      AKX(5,2,I) = 28.*FAC1+28.*FAC2
      AKX(5,3,I) = 28.*FAC1+56.*FAC2
      AKX(5,4,I) = 32.*FAC1+32.*FAC2
      AKX(5,5,I) = 32.*FAC1+96.*FAC2
      AKX(5,6,I) = 32.*FAC1+48.*FAC2
      AKX(6,1,I) = 56.*FAC1+84.*FAC2
      AKX(6,2,I) = 84.*FAC1+56.*FAC2
```

```
      AKX(6,3,I) = 28.*FAC1+28.*FAC2
      AKX(6,4,I) = 48.*FAC1+32.*FAC2
      AKX(6,5,I) = 32.*FAC1+48.*FAC2
      AKX(6,6,I) = 96.*FAC1+96.*FAC2
C
C     SET UP [AKY-6] MATRIX
C
      FAC1 = C(1)/630.
      FAC2 = C(2)/630.
      I = 6
      AKY(1,1,I) = 42.*FAC1+126.*FAC2
      AKY(1,2,I) = 42.*FAC1+42.*FAC2
      AKY(1,3,I) = 21.*FAC1+42.*FAC2
      AKY(1,4,I) = 28.*FAC1+28.*FAC2
      AKY(1,5,I) = 28.*FAC1+84.*FAC2
      AKY(1,6,I) = 56.*FAC1+84.*FAC2
      AKY(2,1,I) = 42.*FAC1+42.*FAC2
      AKY(2,2,I) = 126.*FAC1+42.*FAC2
      AKY(2,3,I) = 42.*FAC1+21.*FAC2
      AKY(2,4,I) = 84.*FAC1+28.*FAC2
      AKY(2,5,I) = 28.*FAC1+28.*FAC2
      AKY(2,6,I) = 84.*FAC1+56.*FAC2
      AKY(3,1,I) = 21.*FAC1+42.*FAC2
      AKY(3,2,I) = 42.*FAC1+21.*FAC2
      AKY(3,3,I) = 42.*FAC1+42.*FAC2
      AKY(3,4,I) = 56.*FAC1+28.*FAC2
      AKY(3,5,I) = 28.*FAC1+56.*FAC2
      AKY(3,6,I) = 28.*FAC1+28.*FAC2
      AKY(4,1,I) = 28.*FAC1+28.*FAC2
      AKY(4,2,I) = 84.*FAC1+28.*FAC2
      AKY(4,3,I) = 56.*FAC1+28.*FAC2
      AKY(4,4,I) = 96.*FAC1+32.*FAC2
      AKY(4,5,I) = 32.*FAC1+32.*FAC2
      AKY(4,6,I) = 48.*FAC1+32.*FAC2
      AKY(5,1,I) = 28.*FAC1+84.*FAC2
      AKY(5,2,I) = 28.*FAC1+28.*FAC2
      AKY(5,3,I) = 28.*FAC1+56.*FAC2
      AKY(5,4,I) = 32.*FAC1+32.*FAC2
      AKY(5,5,I) = 32.*FAC1+96.*FAC2
      AKY(5,6,I) = 32.*FAC1+48.*FAC2
      AKY(6,1,I) = 56.*FAC1+84.*FAC2
      AKY(6,2,I) = 84.*FAC1+56.*FAC2
      AKY(6,3,I) = 28.*FAC1+28.*FAC2
      AKY(6,4,I) = 48.*FAC1+32.*FAC2
      AKY(6,5,I) = 32.*FAC1+48.*FAC2
      AKY(6,6,I) = 96.*FAC1+96.*FAC2
C
C     EXTRACT ELEMENT NODAL U, V, P
C
      UELE(1) = SOL(II)
```

```
      UELE(2) = SOL(JJ)
      UELE(3) = SOL(KK)
      UELE(4) = SOL(LL)
      UELE(5) = SOL(MM)
      UELE(6) = SOL(NN)
      VELE(1) = SOL(II+NPOIV)
      VELE(2) = SOL(JJ+NPOIV)
      VELE(3) = SOL(KK+NPOIV)
      VELE(4) = SOL(LL+NPOIV)
      VELE(5) = SOL(MM+NPOIV)
      VELE(6) = SOL(NN+NPOIV)
      PELE(1) = SOL(II+NPOIV+NPOIV)
      PELE(2) = SOL(JJ+NPOIV+NPOIV)
      PELE(3) = SOL(KK+NPOIV+NPOIV)
C
C     SET UP [GX], [GY], [ALX], [ALY] MATRICES
C
      DO 160 I=1,6
      DO 160 J=1,6
      GXX(I,J) = 0.
      GYY(I,J) = 0.
      ALX(I,J) = 0.
      ALY(I,J) = 0.
      DO 170 K=1,6
      GXX(I,J) = GXX(I,J) + AKX(I,J,K)*UELE(K) + AKX(I,K,J)*UELE(K) +
     *           AKY(I,K,J)*VELE(K)
      GYY(I,J) = GYY(I,J) + AKY(I,J,K)*VELE(K) + AKY(I,K,J)*VELE(K) +
     *           AKX(I,K,J)*UELE(K)
      ALX(I,J) = ALX(I,J) + AKX(I,J,K)*VELE(K)
      ALY(I,J) = ALY(I,J) + AKY(I,J,K)*UELE(K)
  170 CONTINUE
      GXX(I,J) = GXX(I,J) + SXX(I,J)
      GYY(I,J) = GYY(I,J) + SYY(I,J)
      ALX(I,J) = ALX(I,J) + SXY(I,J)
      ALY(I,J) = ALY(I,J) + SYX(I,J)
  160 CONTINUE
C
C     THEN THE MATRIX 15*15 ON LHS OF THE ELEMENT EQS. IS
C
      DO 180 I=1,6
      DO 190 J=1,6
      AKELE(I  ,J  ) = GXX(I,J)
      AKELE(I+6,J+6) = GYY(I,J)
      AKELE(I  ,J+6) = ALY(I,J)
      AKELE(I+6,J  ) = ALX(I,J)
  190 CONTINUE
      DO 200 J=1,3
      AKELE(I  ,J+12)  = -1.*HX(I,J)
      AKELE(I+6,J+12) = -1.*HY(I,J)
  200 CONTINUE
```

```
 180  CONTINUE
      DO 210 I=1,3
      DO 220 J=1,6
      AKELE(I+12,J  ) = HXT(I,J)
      AKELE(I+12,J+6) = HYT(I,J)
 220  CONTINUE
      DO 230 J=1,3
      AKELE(I+12,J+12) = 0.
 230  CONTINUE
 210  CONTINUE
C
C     COMPUTE [FX] MATRIX
C
      DO 240 I=1,6
      SUMKX  = 0.
      SUMKY  = 0.
      SUMHX  = 0.
      SUMSXX = 0.
      SUMSXY = 0.
      DO 250 J=1,6
      DO 250 K=1,6
      SUMKX  = SUMKX + AKX(I,J,K)*UELE(J)*UELE(K)
      SUMKY  = SUMKY + AKY(I,J,K)*VELE(J)*UELE(K)
 250  CONTINUE
      DO 260 J=1,3
      SUMHX  = SUMHX + HX(I,J)*PELE(J)
 260  CONTINUE
      DO 270 J=1,6
      SUMSXX = SUMSXX + SXX(I,J)*UELE(J)
      SUMSXY = SUMSXY + SXY(I,J)*VELE(J)
 270  CONTINUE
      FX(I) = SUMKX + SUMKY - SUMHX + SUMSXX + SUMSXY
 240  CONTINUE
C
C     COMPUTE [FY] MATRIX
C
      DO 280 I=1,6
      SUMKX  = 0.
      SUMKY  = 0.
      SUMHY  = 0.
      SUMSYX = 0.
      SUMSYY = 0.
      DO 290 J=1,6
      DO 290 K=1,6
      SUMKX  = SUMKX + AKX(I,J,K)*UELE(J)*VELE(K)
      SUMKY  = SUMKY + AKY(I,J,K)*VELE(J)*VELE(K)
 290  CONTINUE
      DO 300 J=1,3
      SUMHY  = SUMHY + HY(I,J)*PELE(J)
 300  CONTINUE
```

```
      DO 310 J=1,6
      SUMSYX = SUMSYX + SYX(I,J)*UELE(J)
      SUMSYY = SUMSYY + SYY(I,J)*VELE(J)
  310 CONTINUE
      FY(I) = SUMKX + SUMKY - SUMHY + SUMSYX + SUMSYY
  280 CONTINUE
C
C     COMPUTE [FI] MATRIX
C
      DO 320 I=1,3
      FI(I) = 0.
      DO 330 J=1,6
      FI(I) = FI(I) + HXT(I,J)*UELE(J) + HYT(I,J)*VELE(J)
  330 CONTINUE
  320 CONTINUE
C
C     THUS THE RESIDUAL VECTOR ON RHS OF ELEMENT EQS. IS
C
      DO 340 I=1,6
      RELE(I  ) = -1.*FX(I)
      RELE(I+6) = -1.*FY(I)
  340 CONTINUE
      DO 350 I=1,3
      RELE(I+12) = -1.*FI(I)
  350 CONTINUE
C
C     ASSEMBLE THESE ELEMENT MATRICES TO FORM SYSTEM EQUATIONS
C
      CALL ASSMBLE( IE, INTMAT, AKELE,  RELE,  SYSK, SYSR,
     *                  NPOIV,  MXNEQ, MXELE                  )
C
  500 CONTINUE
C
      RETURN
      END
```

ประวัติผู้เขียนวิทยานิพนธ์

นายอาชว์ ปวีณวัฒน์ เกิดเมื่อวันที่ 24 เดือนกันยายน พุทธศักราช 2521 จังหวัดกรุงเทพมหานคร สำเร็จการศึกษาปริญญาวิศวกรรมศาสตรบัณฑิตจากภาควิชาวิศวกรรมเครื่องกล คณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย เมื่อปีการศึกษา 2542 เข้าศึกษาต่อในหลักสูตรวิศวกรรมศาสตรมหาบัณฑิต ภาควิชาวิศวกรรมเครื่องกล คณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย เมื่อปีการศึกษา 2543