CHAPTER II

PROPERTIES OF A TWO-DIMENSIONAL ELECTRON SYSTEM

Introduction

The two-dimensional electron systems we consider in this thesis constitute only a part of the large class of dynamically two-dimensional systems (2DS) that have been widely studied in the last two decades. By dynamically two-dimensional we mean that the components of the system are free to move in two spatial dimensions but have their motion constrained in the third dimension. Thus the wave vector is a good quantum number for two dimensions, but not for the third. These systems are not two-dimensional in a strict sense, both because wave functions have a finite spatial extent in the third dimension and because electromagnetic fields are not confined to a plane but spill out into the third dimension. Theoretical predictions for idealized two-dimensional systems must therefore be modified before they can be compared with experiment.

Two-Dimensional Electron Gas

The discovery of the quantum Hall effect was the result of systematic measurement on silicon field effect transistors-the most important device not only for applications but also for basic research. The pioneering work by Fowler, Fang, Howard and stiles(1966) has shown that new quantum phenomena become visible if the electrons of a conductor are confined within a typical length of order nm. Their discoveries opened the field of two-dimensional electron systems (2DES).

Two-dimensional electron systems can be realized in several classes of system. One example is the electrons trapped on the surface of liquid helium below 4.2 K. Since we cannot make the concentration of electrons too high in this system, chiefly because the liquid surface cannot sustain too many electrons pressed to the surface by an electric field, the electrons form a classical gas with a Boltzman distribution. There are two classes of system in which we can make a degenerate two dimensional electron gas with electrons occupied up to a Fermi energy E_F: at the surface of a semiconductor like silicon or gallium arsenide where the surface is usually in contact with a material which acts as an insulator like SiO_2 for a metal-oxide-semiconductor as shown in Fig. 2 and $Al_xGa_{1-x}As$ for heterostructures as shown in Fig. 3.

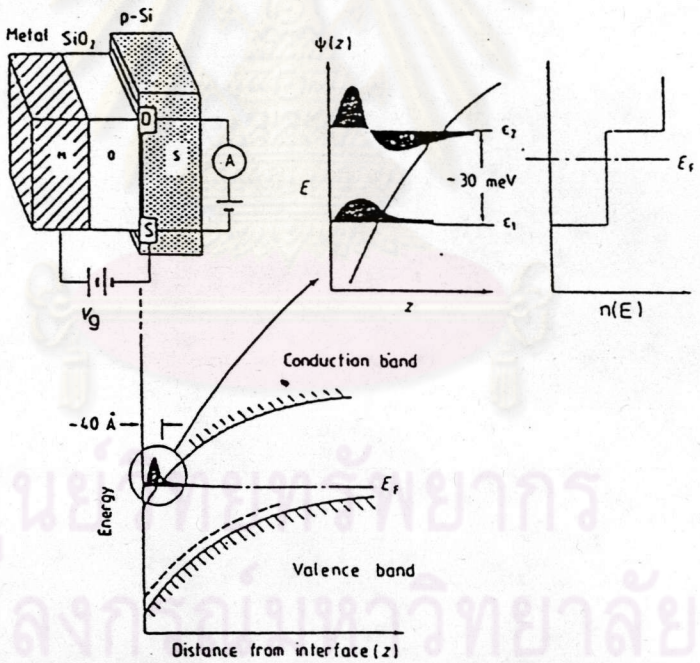

Fig. 2 Two - dimensional electron system in the metal-oxide-semiconductor inversion layer. Here S and D represent source and drain electrodes (usually n-type doped regions) respectively, V_G gate voltage and E_F Fermi energy. Bending in valence and conduction bands is depicted together with wave function ψ (z) and density of states n(E).

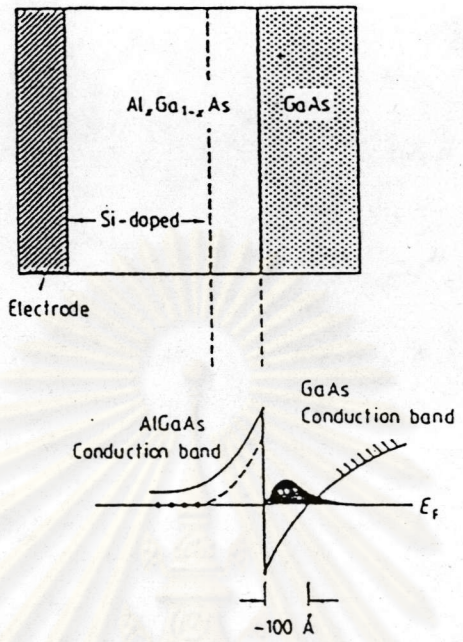

Fig. 3 Two-dimensional electron system in the semiconductor heterostructure. The figure depicts the case of selective doping in AlGaAs.

A MOS inversion layer consists of a metallic layer as an electrode, an oxide layer as an insulator and a semiconductor layer which is made from p-type Silicon. When we apply a voltage V_G, which is called the gate voltage, across the metal and the semiconductor, the valence and conduction bands of the semiconductor are bent as shown in Fig. 2.

When the bottom of the conduction band is pushed down below E_F near the interface of p-type Si and SiO_2, electrons are accumulated at the bottom of the conduction band there. The electron system may be regarded as a two-dimensional system, since the electrons are confined within the interface region and move relatively freely along the interface. This type of MOS system is called an inversion layer,

because the carrier in this example is the electron while the bulk semiconductor is p-type. The motion of an electron in this situation is described by Schrödinger's equation

$$H\psi(\vec{r}) = E\psi(\vec{r}) \qquad (2.1)$$

where

$$H = \frac{1}{2m^*}p^2 + V(z) \qquad (2.2)$$

and m* is the effective mass of the carrier, z is the direction perpendicular to the interface and V(z) represents the bottom of the conduction band and a Schottky barrier of the oxide layer. If we ignore scatterers or other imperfections in the system, the motion within xy plane is free with the wave function given by

$$\psi(\vec{r}) \propto \exp\{ i(k_x x + k_y y) \} f_n(z) \qquad (2.3)$$

where $f_n(z)$ is the wave function in a potential well formed by V(z) with a quantum number n. For the bound states quantized within V(z), $f_n(z)$ has discrete energy levels $\varepsilon_1, \varepsilon_2, ..., \varepsilon_n$. Since the density of states for free electrons in the two-dimensional space is a constant, the total density of states comprises a space of step functions as shown in Fig. 2. When $E_F < \varepsilon_1$ (or, more precisely, when kT is smaller than $\varepsilon_1 - E_F$ or other relevant energies), the electrons behave as a purely two-dimensional system. The current within the two-dimensional system is driven by a voltage applied across the two electrodes denoted in Fig. 2 by S(source) and D(drain). The MOS system is quite convenient in that the concentration (n) of 2D electrons can be varied in the same sample in a range as wide as $n \sim 0 - 10^{13}$ cm^{-2} by varying the gate voltage, which changes the degree of bending of the conduction band.

Two-Dimensional Electrons in Magnetic Fields

The quantum Hall effect was discovered on about the hundredth anniversary of Hall's original work, and the finding was announced in 1980 by von Klitzing, Dorda and Pepper [6]. It is found under certain conditions in an effectively two-dimensional system of electrons subjected to a strong magnetic field $\vec{B}$. We first take up the simplest quantum problem, that of independent two dimensional spinless electrons in a perpendicular magnetic field. This is, of course, old and standard stuff, but it will serve to establish the notation. To write down Schrödinger's equation explicitly it is necessary to choose a gauge. Two gauges which are particularly convenient are the Landau gauge, and the rotational invariant symmetric gauge. The latter gauge is most useful in the study of interacting electrons. In the Landau gauge, the vector potential $\vec{A}$ is

$$A_x = -yB, \quad A_y = 0, \quad A_z = 0 . \tag{2.4}$$

The Hamiltonian of a free electron system in a magnetic field is given by

$$H_0 = \frac{1}{2m^*}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2 . \tag{2.5}$$

Then Schrödinger's equation for the wave function is, in the Landau gauge,

$$\frac{\hbar^2}{2m^*}\left\{\left[\frac{\ell}{i}\frac{\partial}{\partial x} - \frac{eB}{\hbar c}y\right]^2 - \frac{\partial^2}{\partial y^2}\right\}\psi = H_0\psi = E\psi. \tag{2.6}$$

We take $\psi \propto e^{ikx}\varphi(y)$. Then

$$\frac{\hbar\omega}{2}\left\{-\ell^2\frac{\partial^2}{\partial y^2}+\left(\frac{y}{\ell}-\ell k\right)^2\right\}\varphi = E\varphi. \qquad (2.7)$$

Thus φ satisfies a shifted harmonic oscillator equation. We have introduced the magnetic length

$$\ell \equiv (\hbar c/eB)^{1/2}$$

which is a fundamental scale of the problem. It is in the range 50-100 Å and is comparable which several other lengths of the system. It is worth noting that ℓ is independent of material parameters. The different oscillator levels are labelled by $n = 0, 1, 2, \ldots$ and define the Landau levels. These solutions are

$$\varphi_n(y) \propto H_n(y/\ell - \ell k)\exp\left[-\left(y-\ell^2 k\right)/2\ell^2\right] \qquad (2.8)$$

where H_n is a Hermite polynomial. The energy levels are

$$E_n = (n+1/2)\hbar\omega, \quad (n = 0, 1, 2, \ldots) \qquad (2.9)$$

independent of k and are called *Landau levels* with Landau index n (Landau and Lifshitz 1977). The wave function in the y-direction is centred at $y = \ell^2 k$. If the y dimensions of the system are confined to $0 < y < W$ we see that $0 < k < W/\ell^2$. Let us impose periodic boundary conditions $\psi(x, y) = \psi(x + L, y)$. Then $k = 2\pi p/L$ with p an integer. The key aspect of this quantization is obviously the completely discrete energy spectrum (Fig. 4), which is hard to conceive in normal bulk systems.

If we apply a magnetic field to a three-dimensional system, for instance, an electron can move freely along the direction of the magnetic field with the corresponding classical orbit being a helix. The density of states of the 3D quantum system, is shown in Fig. 4, comprises a series of continuous bands arising from the motion along the magnetic field, unlike the two-dimensional case. In this sense the Landau quantization is perfect in two dimensions.

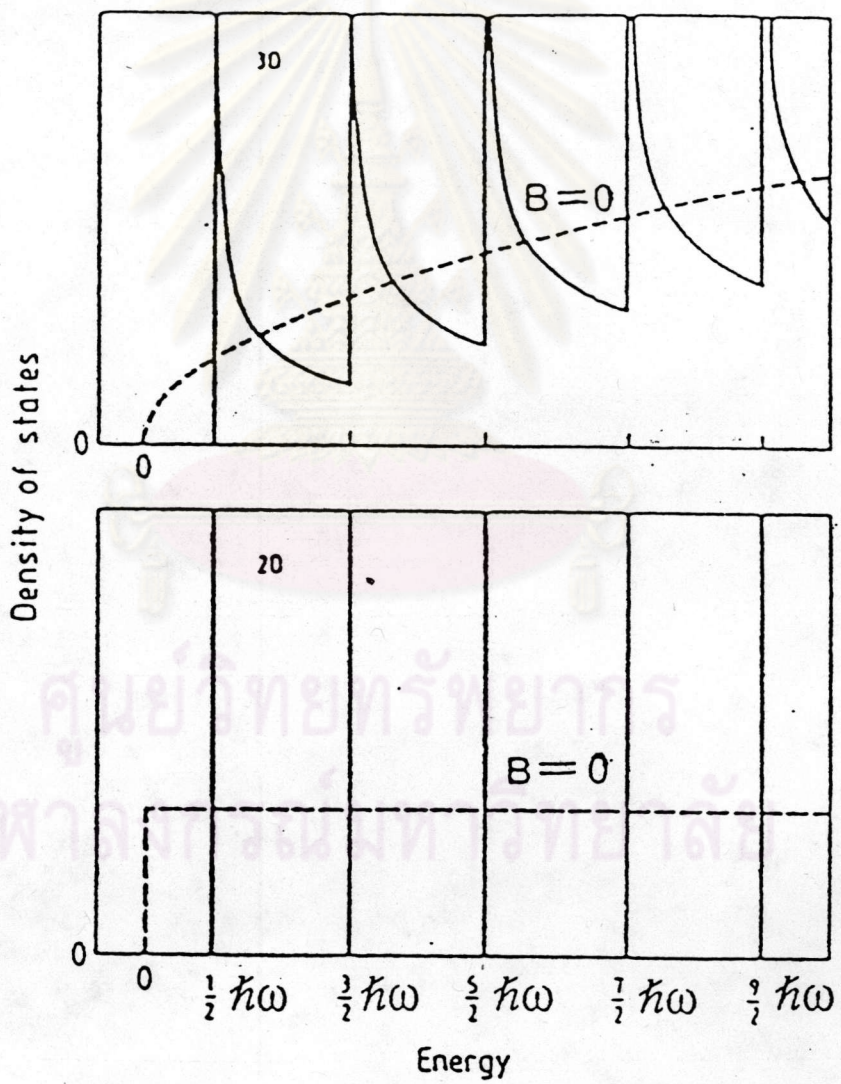

Fig. 4 Density of states for two and three-dimensional electron systems in magnetic fields.

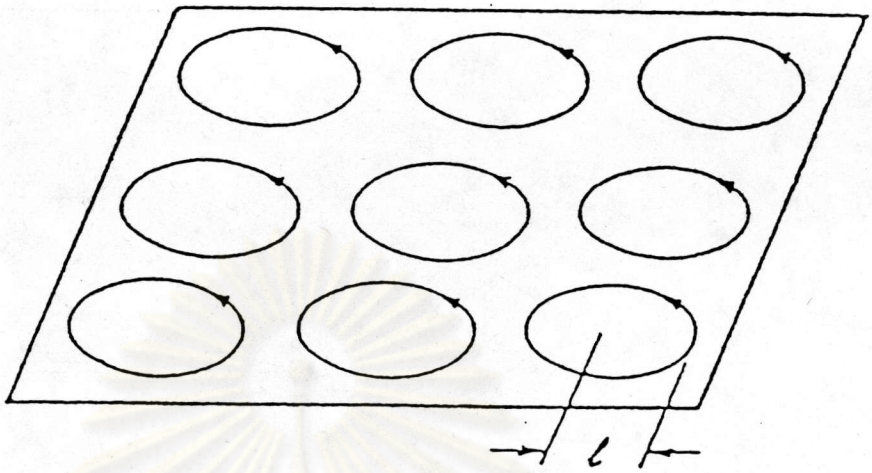

Fig. 5 The degeneracy (total number of states) in a Landau level in a two-dimensional system is roughly equal to the number of circles of radius, ℓ (cyclotron radius) covering the system.

Together with the condition on the range of k this implies that for every Landau level in a two-dimensional system, the degeneracy (number of states belonging to the level) is given by (Fig. 5)

$$N = LW / 2\pi \ell^2. \qquad (2.10)$$

The number of states per unit area of a full Landau level is

$$N_B = 1 / 2\pi \ell^2 = eB / \hbar c \ . \qquad (2.11)$$

In this idealized model, then, the electronic energies lie in equally spaced but highly degenerate levels. The spacing and degeneracy are inversely proportional so that if the degenerate states were broadened into a uniform distribution, the density of states

of the distribution would be $N_B / \hbar\omega = m^*/ 2\pi\hbar^2$, which is just the density of states of free electron of mass m* in two dimensions.

We take this opportunity to remark that the states ψ are extended in the x direction, but confined in the y direction. However, because of the massive degeneracy, linear combinations of states ψ exist which are confined in both directions: they are localized. Applied electric field and/or impurity potentials lift the Landau level degeneracy and with it the freedom to choose between extended and localized states.

If a Landau level is full, the Fermi level must lie in the gap between occupied levels. It is plausible that there is no scattering. We denote the *filling factor* ν, by

$$\nu = N/N_B = 2\pi\ell^2 n \qquad (2.12)$$

This dimensionless quantity indicates the filling of Landau levels; e.g. $\nu = 3.0$ means that only the lowest three Landau levels are full.

Impurity Effect (Random systems)

So far, we have concentrated on the free system. In real, say, semiconductor systems, there always exists randomness arising from impurities and the roughness of the semiconductor interfaces. In the presence of randomness, the hamiltonian is given by

$$H = H_0 + V(\vec{r})$$

where $V(\vec{r})$ is the random potential. The system is dominated by the dynamics of the centre coordinate (X, Y) of cyclotron motion. The equation of motion for (X, Y) is given by

$$\dot{X} = (i/\hbar)[V, X] = \frac{\ell^2}{\hbar}\frac{\partial V}{\partial y}, \quad \dot{Y} = (i/\hbar)[V, Y] = -\frac{\ell^2}{\hbar}\frac{\partial V}{\partial x} \tag{2.13}$$

which follows directly from the commutation relation

$$[x, \eta] = -[y, \xi] = i\ell^2.$$

where (ξ, η) is the relation coordinate around the centre.

A succinct way to represent this state is to express the coordinate of an electron as

$$x = X + \xi \qquad y = Y + \eta.$$

The degeneracy of the states with different (X, Y) in a Landau level is now lifted. This implies that the density of states is no longer a series of sharp lines: each Landau level becomes a band of finite width. We call this band a Landau sub-band (Fig. 6)

As is evident from the equation of motion above, the nature of these sub-bands depends quite strongly on the mode of spatial variation of the random potential. Let us assume that $V(\vec{r})$ varies rapidly within the length scale of the cyclotron radius ℓ. Such a potential may be constructed as an assembly of scatterers as

$$V(\vec{r}) = V_0 \sum \delta(\vec{r} - \vec{r}_i), \tag{2.15}$$

where $\vec{r}_i$ is the position of the ith scatterer. In this case, the motion of an electron may be regarded as quantum as hopping of the centre of the cyclotron motion with the hopping distance being $\sim \ell$ for each jump (Ando and Uemura 1974). The potential, eq. (2.15), is characterized by the dimensionless concentration of scatterers

$$c_i = 2\pi \ell^2 n_i \qquad (2.16)$$

where n_i is the original concentration of the scatterers and c_i represents the average number of scatterers within a circle of radius ℓ. The electronic structure of the system depends strongly on c_i. When $c_i >> 1$ (dense scatterers), the description of the system by the self-consistent Born Approximation (Ando and Uemura 1974) becomes applicable (Fig. 7). In this case, the density of states of each Landau sub-band is semi-elliptic with a width given by

$$\Gamma = 2c_i^{1/2} (V_0 / 2\pi \ell^2).$$

The ratio of Γ to the spacing of the Landau levels ($\hbar\omega$) is expressed as

$$\frac{\Gamma}{\hbar\omega} \sim \frac{1}{(\omega\tau_0)^{1/2}}$$

where τ_0 is the scattering relaxation time (mean free time) in the absence of magnetic field and $\hbar / \tau_0 \propto n_i V_0^2$. For strong magnetic fields of H ~ 10 T in Si MOS systems, which is a typical experimental condition, we have $\Gamma / \hbar\omega < 1$ i.e. the Landau sub-bands are separated.

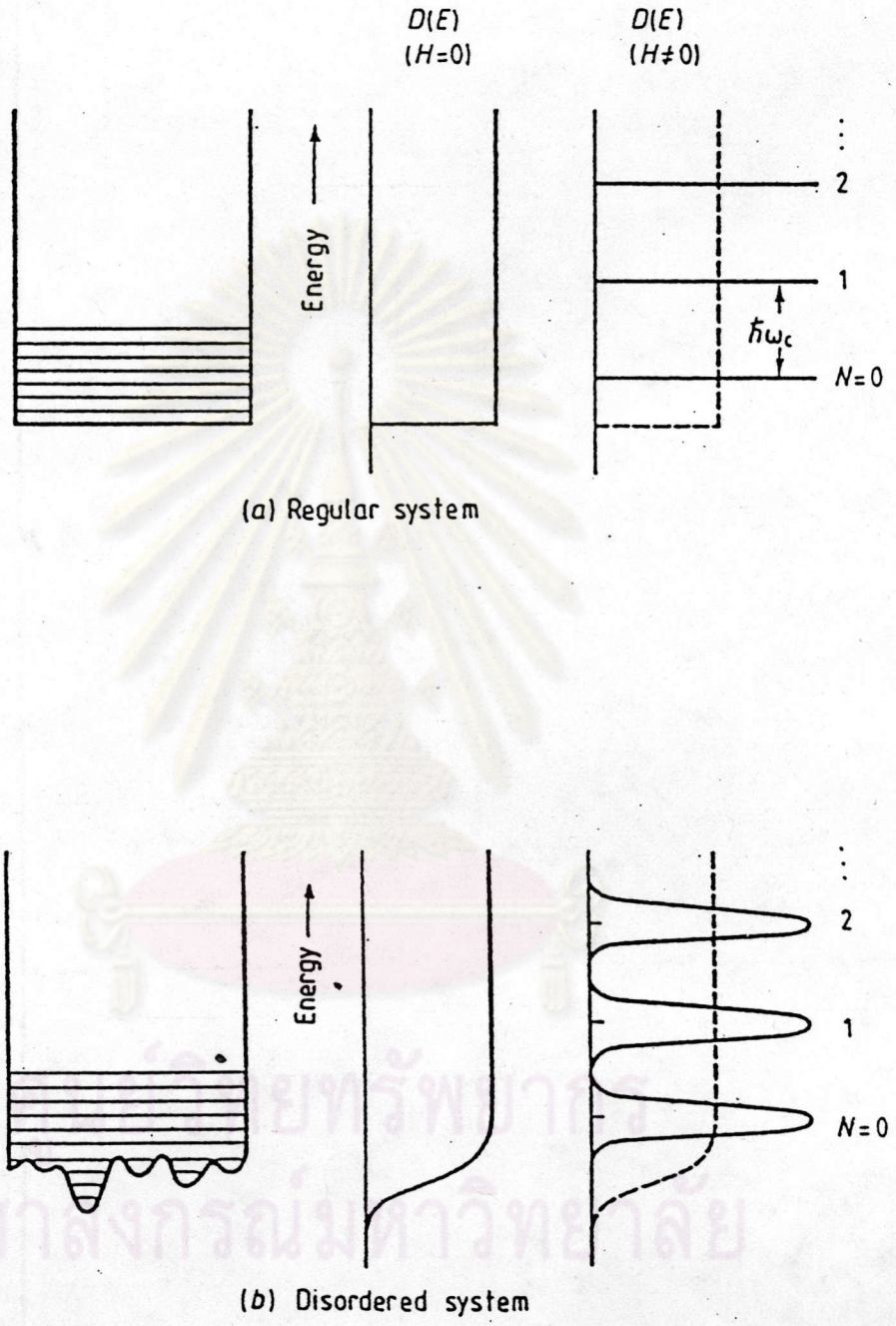

Fig. 6 Energy spectrum of a two-dimensional electron system with and without disorder.

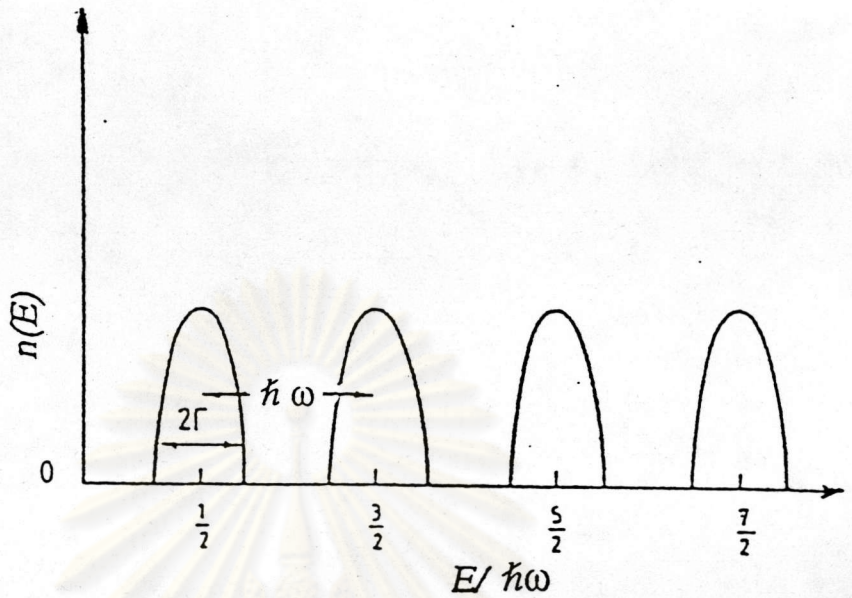

Fig. 7 Density of states (semi-elliptic) for the disordered two-dimensional electron system in a strong magnetic field in the self-consistent Born approximation (SCBA).

It is well known that the broadening of the LL's is due to disorder. There were several theoretical predictions concerning the broadening of the LL's. But we will propose here the method of Feynman path integration with a Gaussian model of disorder to find the density of states and predict the broadening of the LL's. In the next chapter we would like to introduce the Feynman path integral.