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A Comparison of Normal and Elliptical Estimation Methods in Structural Equation Models

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ABSTRACT

Monte Carlo simulation compared chi-square statistics, parameter estimates, and root mean square error of approximation values using normal and elliptical estimation methods. Three research conditions were imposed on the simulated data: sample size, population contamination percent, and kurtosis. A Bentler-Weeks structural model established the relationship between the sample variance-covariance matrix and the specified population model. The elliptical generalized least squares estimation method provided the better chi-square results in the presence of kurtosis. The parameter estimates were similar across research conditions for both the normal and elliptical estimation methods. The root mean square error of approximation values were robust in the presence of kurtosis for the elliptical estimation methods. The root mean square error of approximation is therefore the preferred inferential approach to assessing model fit in the presence of kurtosis because of known distributional properties and determination of confidence intervals for hypothesis testing.

Some type of estimation method is used in all parametric statistics, e.g., regression analysis, factor analysis, discriminant analysis, and canonical correlation analysis (Ferguson & Takane, 1989). The various estimation methods are used to derive sample estimates of population parameters (Marcoulides & Hershberger, 1997). The estimation methods however produce different results depending upon assumptions made by the researcher. In structural equation modeling, various normal and elliptical estimation methods can be used to estimate population parameters from sample data. Least squares (LS), generalized least squares (GLS), and maximum likelihood (ML) estimation procedures assume a normal distribution (Bollen, 1989). Elliptical LS (ELS), Elliptical GLS (EGLS), and Elliptical re-weighted least squares (ERLS) procedures assume an elliptical distribution (Bentler, 1992).

Related Research Literature

In practice, one typically does not know the population variance-covariance and the population parameter(s). Hence, an estimation method is used to obtain sample estimates of the unknown population parameter(s) based on the sample variance-covariance matrix. Once sample parameter estimates are derived, one can compute the model implied sample variance-covariance matrix, Σ. Sample parameter estimates are derived such that is as close to S as possible. The difference between S and Σ is typically indicated by a chi-square statistic, although the root mean square of approximation is also recommended (Schumacker & Lomax, 1996). Obviously, if S - = 0 , then the sample parameter estimates derived from the estimation method perfectly reflect the population parameters based on the fit function, $F(S, \Sigma(\theta))$, and chi-square equals zero.

Normal Distribution Theory

The normal distribution with certain statistical assumptions has played a fundamental role in multivariate statistical analysis (Muirhead, 1982). A sufficient condition for the underlying normal distribution assumption to hold is that the observed variables do not have excessive kurtosis. Basically, the kurtosis of each observed variable should equal zero, which is the kurtosis of a normal distribution (Bollen, 1989; Browne, 1974). In structural equation modeling, several normal estimation methods are available depending upon the fit function.

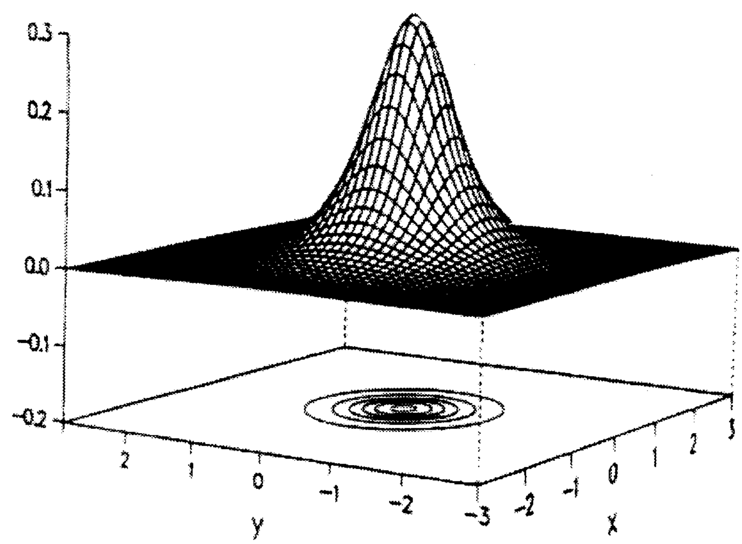
The least squares estimation method (LS) which assumes multivariate normal distributed variables minimizes the following fit function: $F_{LS} = .5 \operatorname{tr} [(S - \Sigma)^2]$ where the degrees of freedom are: $\underline{df} = .5 (p + q)(p + q + 1) - \underline{t}$, and $\underline{t}$ = the number of independent parameters to be estimated, n = the number of observations or sample size, (p + q) = the number of observed variables analyzed, and tr = the trace or diagonal sum of the matrix elements (Schumacker & Lomax, 1996). The fit function is equal to (n - 1) F_{LS}, which yields a chi-square statistic. The generalized least squares estimation method (GLS) yields the following fit function: $F_{GLS} = .5\ tr[(S - \Sigma) S^{-1}]^2$, where S^{-1} is a positive definite weight matrix of residuals derived from differences in the matrix elements (i.e., S - Σ). The default estimation method in most computer programs is the maximum likelihood estimation method, which can be derived by assuming that the observed variables are multivariate normal distributed. The ML parameter estimates are obtained by minimizing the following discrepancy function: $F_{ML} = tr (S \Sigma^{-1}) - (p + q) + \ln | \Sigma | - \ln | S |$. If the covariance matrix, S, is close to the predicted population matrix, Σ, then the sample data fits the model, and F_{ML} approaches zero (i.e., if S, then ln | | - ln | S | 0). Likewise, if S, then the trace or sum of the diagonals will be approximately equal to (p + q), the number of observed variables analyzed, and the value of tr $(S \Sigma^{-1}) - (p + q)$ will approach zero. In large samples and under specific conditions (Browne, 1974, 1984; Jöreskog, 1967), $(n - 1)F_{ML} \sim X^2$, where (p* - q) and p* = p(p+1)/2 are the degrees of freedom and q is the number of parameters to be estimated. Therefore, the ML fit function yields a chi-square statistic.

The multivariate normal distribution of z variables has a mean vector, μ, and a covariance matrix, Σ, described by the density function:

$$\underline{Y} = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

where Y = height of the normal curve for z variables, π = a constant 3.1416, and ∂ = base of Napierian logarithm = 2.7183 (Ferguson & Takane, 1989). Standard score variables have a mean = 0 and a standard deviation = 1, so μ = 0 and = 1. The area under the normal distribution is unity (see Figure 1).

Figure 1 Normal Distribution



A general formula to derive sample parameter estimates in a structural equation model given the normal distribution assumption is (Bentler, 1992):

$$Q_N = 2^{-1} tr\left[(S - \Sigma)W_2\right]^2 .$$

The weight matrix, denoted as W_2 in this general formula, is replaced by any of the three normal theory estimators of Σ^{-1}:

(a) $W_2 =$ I (identity matrix) gives normal least squares(LS)

(b) $W_2 = S^{-1}$ gives normal generalized least squares(GLS)

(c) $W_2 = \Sigma^{-1}$ gives normal re-weighted least squares(ML).

Elliptical Distribution Theory

Elliptical distributions are based on a broad class of distributions that include both heavy and light tailed symmetric distributions relative to the normal distribution. The characteristic function of an elliptical distribution for some function Ψ (Muirhead,1982) is of the form:

$$\phi(t) = e^{i\mu' t}\psi(t'V_t).$$

For m ≥2, Berkane and Bentler (1986) defined

$$\mu_i^{2m} = \frac{(2m)!}{2^m m!}(\kappa(m)+1)(\mu_i^{(2)})^m$$

where $\Psi^{(m)}(0)$ and $\Psi'(0)$, respectively, are the m^{th} and the first derivative of Ψ, evaluated at zero. Assume μ=0 without loss of generality, and $\mu_{i1i2...i2m} = E(X_{i1} X_{i2...} X_{i2m})$. Berkane and Bentler (1986) showed that, if $i_1 = i_2 = ... = i_{2m} = I$, then:

$$\kappa(m)+1 = \frac{\psi^{(m)}(0)}{(\psi'(0))^m}$$

This relationship characterizes the elliptical distribution, i.e., if a random variable y has density $f_y(y)$, if all odd moments are zero, and if the $(2m)^{th}$ moment exists and is defined by $\mu_{2m} = C(m)(\mu_2)^m$, for some constrained C depending on m, then y is elliptically distributed.

The multivariate elliptical distribution of y variables has a mean vector, μ, and a covariance matrix, Σ, described by the following density function (Bentler, 1992):

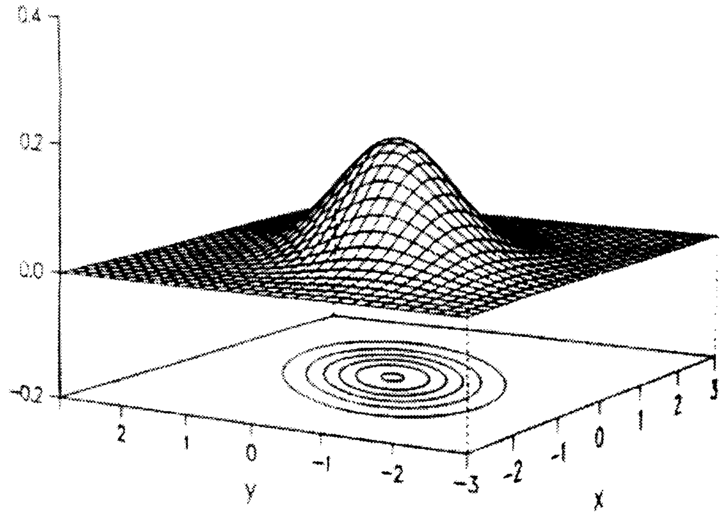
$$k_1 \det(\Sigma)^{-1/2} g(k_2 (z-\mu)'\Sigma^{-1}(z-\mu))$$

where k_1 and k_2 are constants and g is a non-negative function. This density function yields an elliptical distribution. The y variables have a common kurtosis parameter of:

$$\kappa = \frac{\sigma_{iiii}}{3\sigma_{ii}^{\ 2} - 1}$$

which describes the tails of the distribution relative to the multivariate normal distribution. The multivariate normal distribution is therefore a special case of the multivariate elliptical distribution when $\kappa = 0$. Values for the parameter κ, other than 0 (zero), characterize elliptical distributions (Berkane & Bentler, 1987a; 1987b).

Figure 2 Elliptical Distribution



A general formula to derive sample parameter estimates given an elliptical distribution assumption (Bentler, 1992) is:

$$Q_E = 2^{-1}(k+1)^{-1} tr\left[(S-S)W_2\right]^2 - d\left[tr(S-S)W_2\right]^2$$

The weight matrix, denoted as $W_{2,}$ in this general formula is replaced by any of three elliptical estimators of Σ^{-1}:

(a) $W_2 =$ I (identity matrix) gives elliptical least squares(ELS) estimates;

(b) $W_2 =$ S^{-1} (fixed) gives elliptical generalized least squares(EGLS) estimates;

(c) $W_2 =$ Σ^{-1} (iteratively updated) gives elliptical re-weighted least squares (ERLS) estimates.

The Mardia-based κ coefficient (Mardia, 1970; 1974) can be used in computing elliptical computations (Bentler, 1992). The default computation of κ (Shapiro & Browne, 1987) is given by:

$$k_1 = \frac{g_{2,p}}{p(p+2)},$$

where, $$g_{2,p} = N^{-1}\Sigma_1^N \left[(z_t - \bar{z})' S^{-1} (z_t - \bar{z}) \right]^2 - p(p+2)$$

is the deviation from the expected multivariate Mardia-based K kurtosis value. The z notation references raw score and mean vectors, respectively. The normalized (standard score) estimate is given by:

$$\frac{g_{2,p}}{(8p(p+2)/\underline{N})^{1/2}} ,$$

which, in large samples, operates the same as the unit normal variate in the normal distribution. The normalized estimate can be used to test the null hypothesis of multivariate normality.

The relative merits of alternative estimators of K has not yet been established (Bentler, 1992). In non-elliptical populations, these estimators do not necessarily converge. The Mardia-based coefficient, however, does have asymptotic expectation and variance, such that:

$$E(k_1) = k .$$

The use of normal or elliptical distributions in structural equation modeling is based on theoretical considerations. It is possible that failures of normal or elliptical estimation methods can be associated with the estimation of K (Tyler, 1982 & 1983). In most estimation methods, however, an assumption underlying the fit function is that the variables have some particular multivariate distribution, either normal or elliptical. Consequently, the chi-square (X^2) test is used as a goodness-of-fit test (fit function) between S and Σ, given optimal sample weight estimates.

Chi-Square, Parameter Estimates and Kurtosis

Chi-Square

A number of studies have investigated the chi-square statistic in normal and non-normal data samples. In non-normal samples containing kurtosis, the chi-square statistic based on the ML estimation method was too large, causing the rejection of a true structural equation model too often (Bentler, 1992; Harlow and Newcomb, 1984; La Du and Tanaka, 1989; Muthen and Kaplan, 1985; Tanaka, 1984). In studies using

ML estimation with normal samples, the chi-square statistic had little bias with samples ranging from n > 30 (Geweke & Singleton, 1980), to n = 200 (Boomsma, 1983), to n = 500 (Browne, 1982, 1984), to n = 1000 (Muthen & Kaplan, 1985). Wang, Fan, and Willson (1996) explained that the adjusted chi-square test (Satorra-Bentler re-scaled chi-square) reported in the presence of elliptical distributed data can provide acceptable conclusions given an appropriate sample size that balances the statistical power of the test with sampling variation. Hoogland and Boomsma (1998) suggested that the ML chi-square statistic often rejected the true model when the sample size was smaller than five times the number of degrees of freedom of the model. When the observed variables had an average positive kurtosis as large as 5.0, the sample size may have to be increased by up to10 times the size of the model. Given that the model is appropriate, the GLS chi-square statistic may have an acceptable performance for a sample size that is two times smaller than the sample size needed for an acceptable performance of the ML chi-square statistic.

Weng and Cheng (1997) recommended that although chi-square values given by ML, LS, and GLS estimators differ, the effects of this discrepancy on relative fit indices may diminish as sample size increases. For example, if a model fits the data and the sample size is very large, ML and GLS estimation methods yield a very similar chi-square statistic (Browne,1974).

Parameter Estimates

The effects of various estimation methods on the parameter estimates in structural equation models has also been studied. Harlow(1985) concluded that ML and ERLS parameter estimates were comparable in a Monte Carlo factor analysis simulation study. Muthen and Kaplan(1985) found no difference between parameter estimates using the ML and GLS estimation methods. Henly(1993) pointed out a striking similarity between ML and GLS estimates. Wang, Fan, & Willson(1996) also found the results from the ML and GLS methods to be practically identical, except for some insignificant differences.

Boomsma(1983) in a Monte Carlo study using ML estimation with normal continuous data, found that Generally for N ≥ 200 there is little bias in estimating parameters... (p.116). Boomsma also examined categorical, skewed, and kurtotic data, and he concluded that parameter estimates were unbiased for N = 400 using the ML estimation method. Boomsma's findings were supported in a Monte Carlo study by Muthen and Kaplan(1985) which studied estimates based on ordered categorical data using ML, GLS and ADF estimation methods. Muthen and Kaplan found that ML, GLS, and ADF methods were unbiased when using a sample size of 1000. Browne (1982, 1984) conducted a Monte Carlo study of ML and ADF estimation in both normal and non-normal continuous data with N = 400 and N = 500. Browne further suggested that parameter estimates were unbiased when using ML estimation in normal samples.

Hoogland and Boomsma (1998) found that the bias of ML parameter estimates increased when the level of univariate skewness and kurtosis deviated increasingly from normal theory values. Hoogland & Boomsma also suggested that a larger sample size($\underline{n}$ > 500) was a remedy for obtaining unbiased parameter estimates. Wang, Fan, and Willson(1996) concluded that population parameter mean estimates across 100 replications approached the population values as the sample size increased from 200 to 1000. The differences between the minimum and maximum parameter estimates decreased remarkably with an increased sample size. The quality of parameter estimates was not of much concern even with non-normal data, provided that appropriately large samples were used. Wang, Fan, and Willson also found that the parameter estimates appeared to stabilize when the sample size reached 500. Weng and Cheng(1997) compared the three normal theory estimators and found that ML and LS estimation methods yielded identical parameter estimates, which were slightly different from GLS estimates.

Kurtosis

A number of studies have examined the impact of kurtosis in non-normal data. Browne (1982, 1984) developed an asymptotic distribution free index which permitted the use of a generalized least squares estimator even when the variables exhibited excessive kurtosis (peakness) or insignificant kurtosis(flatness) in the multivariate normal distribution. Social scientists frequently are concerned about the skewness in their data; however, Browne indicated that it is kurtosis, *not* skewness, that was critical because kurtosis is a term in the mathematical expression for the covariances. That is, when data are not normally distributed, the researcher must know about the variables kurtosis, as well as the variable means and covariances, in order to make inferences about individual patterns of scores.

Harlow (1985) studied elliptical distributed data in factor analysis and found the ERLS (Elliptical reweighted least squares) estimation method performed the best under various levels of kurtosis ($\kappa > 0$). Hoogland and Boomsma (1998) further concluded that the bias in parameter estimates increased when the absolute value of kurtosis increased. They discovered a remarkable effect on the sign of the kurtosis, namely, the bias of ML estimates is positive for platykurtic distributions and negative for leptokurtic distributions. Bias becomes most extreme when the underlying distribution is highly leptokurtic.

The elliptical distribution differs from the normal distribution based on kurtosis in the sample data. One would therefore expect the chi-square statistics, parameter estimates, and root mean square error of approximation values to differ when comparing results from these two distributions. It is anticipated that, normal estimation methods in structural equation modeling would yield biased results when using non-normal sample data. Moreover, elliptical estimation methods should out perform normal estimation methods given elliptical data distributions.

Methods and Procedures

The EQS 5.7 software program (see appendix) permitted the specification of different population data contamination percentages (.05 and .10), sample sizes (1000, 5000, 10000), and kurtoses (1, 2, 3), which followed suggestions by Mattson (1997) and Mooney (1997). This yielded a 2 X 3 X 3 design with 18 unique research conditions. The EQS 5.7 program generated a sampling distribution based on 100 replications of these conditions. The fit function (X^2), structural coefficient (γ), and root mean square error of approximation values were saved in separate files and compared in tables across these research conditions.

Simulated Data Sets

The EQS 5.7 software program (Bentler & Wu, 1995) was used to generate pseudo-random samples of data to compute the sample variance-covariance matrix. Previous research by Bang and Schumacker (1998) has indicated that pseudo-random number generators don't produce normal distributions of data with sample sizes less than 10,000. Three sample sizes of 1000, 5000, and 10000 were chosen for the study to reflect this lack of normality in pseudo-random number generators when comparing the estimation methods.

Non-normal distributions were created by generating a normal distribution with μ and , and adding a smaller percent non-normal distribution with the same μ, but with a variance-covariance equal to $\kappa * \Sigma$. The scale factor, , which creates the non-normal population, ranges between 1 and 10. The present study used values of κ = 1, 2, or 3, because the use of values greater than 3 generated elliptical data which failed to converge using either normal or elliptical estimation methods in the study.

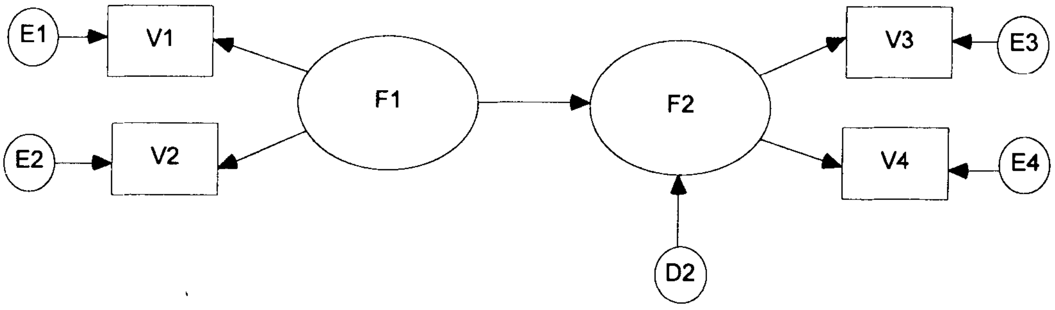
The smaller percent non-normal distribution which is added to a normal distribution can range between 0% and 100%, but is usually 10% or less; 5% and 10% were used in the study. Structural equation modeling estimates are typically asymptotic, meaning that they approach the true population value as sample size increases. These sample sizes are therefore suitable, especially since several researchers (e.g., Bentler,1992; Browne, 1982,1984) have suggested that larger sample sizes may be needed when estimation methods are based on fourth-order moments (kurtosis).

Structural Model

Gerbing and Anderson (1992, 1993) suggested that using substantively meaningful models in Monte Carlo simulation may increase our understanding of the results and that most simulation studies in structural equation modeling have used from two to six latent variables, with two to six indicators for each latent variable. In this study, a specific population model was simulated based on the Bentler-Weeks (Bentler & Weeks, 1980) structural equation model (see Figure 3).

The number of distinct values in the sample variance-covariance matrix is ten (10). This can be calculated as: .5 (p + q) (p + q + 1), where p = the number of dependent variables and q = the number of independent variables. The degrees of freedom for the chi-square statistic is calculated as the number of distinct values in the sample variance-covariance matrix minus the number of parameters to be estimated. Since there are ten distinct values in the sample variance-covariance matrix and six parameters to be estimated in the model (four E's, D2, and), the degrees of freedom is equal to four.

Figure 3 Bentler-Weeks Model



The Bentler-Weeks structural model is specified in the EQS 5.7 software program using the /EQUATION command to generate the population variance-covariance matrix. The EQS program /EQUATION command specifies fixed factor loadings of .80 (validity coefficients) for the observed variables that identify both the exogenous factor, F1, and the endogenous factor, F2. The /EQUATION command further indicates that V1 and V2 are two observed variables that are indicator (manifest) variables of F1 (exogenous factor) and that V3 and V4 are two observed variables that are indicator (manifest) variables of F2 (endogenous factor). A structural coefficient indicates that F1 predicts F2. The following set of /EQUATION command lines indicate the Bentler-Weeks structural equation model in the program:

```
/EQUATIONS
 V1 = .8*F1 + E1;
 V2 = .8*F1 + E2;
 V3 = .8*F2 + E3;
 V4 = .8*F2 + E4;
 F2 =   *F1 + D2;
```

where V1-V4 are observed variables, E1-E4 are measurement errors of the observed variables, F1and F2 are factors (latent variables), and D2 is the error of prediction for F2.

Data Analysis

The EQS 5.7 software program (Bentler & Wu, 1995) was used to simulate normal and elliptical distributions of data (see Figures 1 and 2) and estimate chi-square, structural coefficient, and root mean square error of approximation values for 18 unique research conditions based on sample size, population contamination percent, and kurtosis. The EQS 5.7 software program is annotated to indicate which command lines were changed for each of the research conditions. For example, CASES was used to specify the different sample sizes, METHODS was used to specify pairs of normal and elliptical estimation methods, and CONTAMINATION was used to indicate the smaller percent non-normal distribution and kurtosis factor. A sampling distribution based on 100 replications using a pseudo-random number generator with different seed values produced a point estimate for chi-square, parameter, and root mean square error of approximation values. The EQS 5.7 software program provided the necessary summary statistics.

The model chi-square values can be compared against a critical chi-square value of 9.488 at the .05 level of statistical significance for four degrees of freedom and a root mean square error of approximation value equal to or less than .05, implying a close fit. Kurtosis values should be greater than the value of $k > -2/(p+2)$, where p is the number of measured variables (Bentler and Berkane,1986; Tyler,1982). Given 4 measured variables, $k > -.25$. The user should be aware that the application of elliptical distributions to structural equation modeling is based on theoretical considerations. There is little experience that can be used to provide guidance on how to avoid breakdowns in the method, i.e., misleading results. It is possible that potential failures of elliptical estimation methods can be associated with poor estimation of k, hence poor estimation of the sample variance-covariance matrix.

Monte Carlo simulations were conducted based on generating data from a known population model, then estimating this true population model under different research conditions. Consequently, power determination was not required in the study. In practice, testing a null hypothesis of model fit requires power and sample size considerations. Schumacker and Lomax (1996) and MacCallum, Browne, and Sugawara (1996) provide programs and recommendations for power calculations and sample size. For example, the Hoelter critical N, which is $CN = (\chi^2 / F) + 1$, gives the sample size at which F would lead to a rejection of the null hypothesis. Their programs also use modification index values and root-mean-square error of approximation (RMSEA) values. The RMSEA values, together with the degrees of freedom (df) for the model, the sample size (n), and Type I error rate (alpha) are used to calculate power. RMSEA <= .05 are considered a close fit; values between .05-.08 are considered fair fit, between .08-.10, mediocre fit, and RMSEA > .10, poor fit.

Results

The chi-square values at k = 1 for both normal and elliptical estimation methods yielded similar results across the research conditions. These findings were expected because only sample size effects were present, with percent contamination having no impact. The results more clearly reflect the outcome of data generated using a pseudo-random number generator (An average chi-square value of 3.84 was obtained from the sampling distribution based on 100 replications using a normal distribution with sample sizes greater than 10,000). The structural coefficients were similar for both normal and elliptical estimation methods across the research conditions. The root mean square error of approximation (RMSEA) was robust across the research conditions for all estimation methods, except under extreme levels of contamination (10%) and kurtosis (k=3).

Least Squares Estimation

The normal least squares (LS) and elliptical least squares (ELS) estimation methods are compared in Tables 1 to 6. As the percent non-normal data and kurtosis increased, the chi-square values increased, but the elliptical least squares estimation method computed lower chi- square values. The structure coefficients and root mean square error of approximation values (RMSEA) remained similar, but were more distorted under conditions of extreme percent contamination (10%) and kurtosis (k=3). The least squares estimation method failed to yield a solution (lacked convergence) under these conditions, returning fewer than the required 100 replications.

TABLE 1 LS versus ELS method: Contamination = 5%, n = 1000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	1000	1	4.2305	4.2393	-.0087	.0087	.0315	.0316
			(2.716)	(2.731)	(1.011)	(1.011)	(.024)	(.024)
		2	9.7972	7.2139	-.0184	.0184	.0795	.0663
			(7.045)	(5.152)	(1.209)	(1.209)	(.037)	(.032)
		3	29.6109	11.3825	.1250	-.1250	.1883	.1150
			(16.19)	(2.592)	(1.531)	(1.538)	(.063)	(.041)

TABLE 2 LS versus ELS method: Contamination = 5%, n = 5000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	5000	1	4.1767	4.1820	-.0012	.0012	.0148	.0149
			(2.529)	(2.535)	(1.008)	(1.008)	(.009)	(.009)
		2	28.2544	20.5225	.0017	-.0017	.0688	.0588
			(12.73)	(9.193)	(1.210)	(1.005)	(.018)	(.015)
		3	138.278	51.0223	-.0481	-.0053	.1944	.1196
			(44.23)	(16.968)	(1.607)	(1.612)	(.034)	(.021)

TABLE 3 LS versus ELS method: Contamination = 5%, n = 10000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	10000	1	3.8879	3.8868	.0020	-.0020	.0103	.0103
			(2.451)	(2.442)	(1.007)	(1.007)	(.007)	(.007)
		2	49.7868	36.2468	.0001	-.0001	.0667	.0573
			(17.960)	(12.995)	(1.206)	(1.206)	(.012)	(.010)
		3	270.248	97.9009	.0139	-.0139	.1936	.1183
			(64.657)	(23.491)	(1.601)	(1.601)	(.025)	(.015)

Note: Standard deviations for chi-squares, parameters, and root mean square of approximation are in parentheses in the tables. Results based on 100 replications (r), except when k=3 due to non-convergence (n = 1,000, k = 3, r = 79; n = 5,000, k =3, r = 97; n = 10,000, k = 3, r = 99).

TABLE 4 LS versus ELS method: Contamination = 10%, n = 1000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	1000	1	4.2305	4.2393	- .0087	.0087	.0315	.0316
			(2.716)	(2.731)	(1.011)	(1.011)	(.024)	(.024)
		2	18.1399	11.9387	.0086	-.0086	.1350	.1089
			(10.274)	(6.632)	(1.398)	(1.398)	(.045)	(.037)
		3	39.3374	23.6486	.5394	.0272	.2772	.2415
			(20.380)	(10.458)	(1.783)	(2.012)	(.090)	(.072)

TABLE 5 LS versus ELS method: Contamination = 10%, n = 5000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	5000	1	4.1767	4.1820	-.0012	.0012	.0148	.0149
			(2.529)	(2.535)	(1.008)	(1.008)	(.009)	(.009)
		2	73.1212	47.4799	.0084	-.0084	.1291	.1049
			(21.596)	(14.038)	(1.005)	(1.005)	(.020)	(.016)
		3	243.548	84.0961	-.4071	.4071	.3231	.1918
			(28.048)	(10.651)	(2.142)	(2.142)	(.016)	(.011)

TABLE 6 LS versus ELS method: contamination = 10%, n = 10000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	10000	1	3.8879	3.8868	.0020	-.0020	.0103	.0103
			(2.451)	(2.442)	(1.007)	(1.007)	(.007)	(.007)
		2	141.967	92.1144	.0032	-.0032	.1286	.1046
			(33.885)	(21.816)	(1.406)	(1.406)	(.016)	(.013)
		3	439.546	152.562	- 1.890	1.890	.3163	.1883
			(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Note: Standard deviations for chi-squares, parameters, and root mean square of approximation are in parentheses in the tables. Results based on 100 replications (r), except when k=3 due to non-convergence (n = 1,000, k = 3, r = 17; n = 5,000, k =3, r = 5; n = 10,000, k = 3, r = 1).

Generalized Least Squares Estimation

The normal generalized least squares (GLS) and elliptical generalized least squares (EGLS) estimation methods are compared in Tables 7 to 12. As the percent non-normal data and kurtosis increased, the chi-square values increased, but the elliptical generalized least squares estimation method computed lower chi-square values. The structure coefficients and root mean square error of approximation values (RMSEA) remained similar across research conditions and were more robust under conditions of extreme percent contamination (10%) and kurtosis (k=3) than the previous least squares estimation methods. The elliptical generalized least squares estimation methods also performed better under these extreme conditions and returned the required 100 replications, except for percent=10%, n=1000, k=3.

TABLE 7 GLS versus EGLS method: Contamination = 5%, n = 1000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	1000	1	4.2199	4.2282	-.0068	.0068	.0108	.0109
			(2.687)	(2.700)	(1.005)	(1.005)	(.013)	(.013)
		2	8.9916	6.5332	-.0094	.0090	.0291	.0199
			(6.098)	(4.379)	(1.107)	(1.103)	(.021)	(.019)
		3	28.9084	10.2459	-.0112	.0092	.0748	.0359
			(15.008)	(5.197)	(1.290)	(1.264)	(.025)	(.017)

TABLE 8 GLS versus EGLS method: Contamination = 5%, n = 5000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	5000	1	4.1577	4.1629	-.0001	.0001	.0046	.0046
			(2.520)	(2.526)	(1.005)	(1.005)	(.005)	(.005)
		2	25.4624	18.2719	.0007	-.0006	.0256	.0134
			(10.827)	(7.674)	(1.110)	(1.106)	(.007)	(.019)
		3	116.339	39.6596	.0006	-.0003	.0740	.0417
			(34.803)	(11.372)	(1.301)	(1.275)	(.011)	(.006)

TABLE 9 GLS versus EGLS method: Contamination = 5%, n = 10000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	10000	1	3.8791	3.8781	.0022	-.0022	.0027	.0027
			(2.443)	(2.434)	(1.006)	(1.006)	(.003)	(.003)
		2	44.6443	32.1243	-.0006	.0007	.0313	.0260
			(15.073)	(10.707)	(1.107)	(1.103)	(.005)	(.005)
		3	222.340	76.1177	-.0050	.0068	.0734	.0422
			(47.854)	(15.914)	(1.293)	(1.005)	(.008)	(.004)

Note: Standard deviations for chi-squares, parameters, and root mean square of approximation are in parentheses in the tables based on 100 replications.

TABLE 10 GLS versus EGLS method: Contamination = 10%, n = 1000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	1000	1	4.2199	4.2282	-.0068	.0068	.0108	.0109
			(2.687)	(2.700)	(1.005)	(1.005)	(.013)	(.013)
		2	16.3307	10.4576	-.0134	.0126	.0518	.0362
			(8.802)	(5.409)	(1.207)	(1.197)	(.020)	(.018)
		3	52.5078	17.0954	.0203	-.0028	.1078	.0554
			(19.686)	(6.044)	(1.548)	(1.500)	(.023)	(.014)

TABLE 11 GLS versus EGLS method: Contamination = 10%, n = 5000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	5000	1	4.1577	4.1629	-.0001	.0001	.0046	.0046
			(2.520)	(2.526)	(1.005)	(1.005)	(.005)	(.005)
		2	62.6112	39.7008	.0008	-.0007	.0536	.0418
			(17.123)	(10.729)	(1.206)	(1.196)	(.007)	(.006)
		3	239.777	75.8848	.0013	-.0102	.1082	.0597
			(42.825)	(13.401)	(1.561)	(.993)	(.009)	(.005)

TABLE 12 GLS versus EGLS method: Contamination = 10%, n = 10000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	10000	1	3.8791	3.8781	.0022	-.0022	.0027	.0027
			(2.443)	(2.434)	(1.006)	(1.006)	(.003)	(.003)
		2	120.921	76.6337	.0020	-.0019	.0537	.0423
			(26.454)	(16.420)	(1.208)	(1.199)	(.006)	(.004)
		3	476.482	150.603	.0020	-.0016	.1084	.0604
			(67.304)	(20.726)	(1.565)	(1.511)	(.007)	(.004)

Note: Standard deviations for chi-squares, parameters, and root mean square of approximation are in parentheses in the tables based on 100 replications (r), except for n = 1,000, k = 3, r = 98.

Maximum Likelihood Estimation

The maximum likelihood (ML) and elliptical re-weighted least squares (ERLS) estimation methods are compared in Tables 13 to 18. As the percent non-normal data and kurtosis increased, the chi-square values increased, but the elliptical re-weighted least squares estimation method computed lower chi-square values. The structure coefficients and root mean square error of approximation (RMSEA) values remained similar across research conditions and were similar to results obtained using the least squares estimation methods. The elliptical re-weighted least squares estimation method however performed better under extreme conditions and returned the required 100 replications, except for percent = 10%, n=1000, k=3.

TABLE 13 ML versus ERLS method: Contamination = 5%, n = 1000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	1000	1	4.2436	4.2473	-.0068	.0068	.0111	.0111
			(2.733)	(2.756)	(1.009)	(1.009)	(.013)	(.013)
		2	9.6928	7.4258	-.0105	.0102	.0309	.0228
			(6.969)	(5.478)	(1.118)	(1.114)	(.023)	(.021)
		3	34.4752	14.4736	-.0160	.0104	.0820	.0464
			(20.240)	(9.083)	(1.339)	(1.314)	(.030)	(.022)

TABLE 14 ML versus ERLS method: Contamination = 5%, n = 5000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	5000	1	4.1790	4.1919	-.0001	.0001	.0047	.0047
			(2.532)	(2.542)	(1.006)	(1.006)	(.005)	(.005)
		2	27.5948	20.926	.0007	-.0005	.0330	.0278
			(12.300)	(9.495)	(1.115)	(1.111)	(.009)	(.008)
		3	140.185	56.576	.0015	-.0010	.0813	.0504
			(47.217)	(20.010)	(1.337)	(1.311)	(.014)	(.009)

TABLE 15 ML versus ERLS method: Contamination = 5%, n = 10000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
5%	10000	1	3.8867	3.8880	.0022	-.0022	.0027	.0027
			(2.449)	(2.442)	(1.006)	(1.006)	(.003)	(.003)
		2	48.4701	36.8498	-.0005	.0008	.0327	.0281
			(17.268)	(13.391)	(1.112)	(1.108)	(.006)	(.005)
		3	268.350	108.588	-.0047	.0050	.0807	.0111
			(64.916)	(27.782)	(1.328)	(1.301)	(.009)	(.013)

Note: Standard deviations for chi-squares, parameters, and root mean square of approximation are in parentheses in the tables based on 100 replications.

TABLE 16 ML versus ERLS method: Contamination = 10%, n = 1000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	1000	1	4.2436	4.2473	-.0068	.0068	.0111	.0111
			(2.733)	(2.756)	(1.009)	(1.009)	(.013)	(.013)
		2	18.5216	13.0090	-.0159	.0152	.0559	.0428
			(10.869)	(7.759)	(1.230)	(1.221)	(.023)	(.021)
		3	65.9481	26.8282	.0514	.0257	.1212	.0726
			(28.302)	(12.471)	(1.671)	(1.636)	(.028)	(.021)

TABLE 17 ML versus ERLS method: Contamination = 10%, n =5000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	5000	1	4.1790	4.1919	-.0001	.0001	.0047	.0047
			(2.532)	(2.542)	(1.006)	(1.006)	(.005)	(.005)
		2	71.6210	49.9599	.0006	-.0007	.0575	.0473
			(21.113)	(15.285)	(1.229)	(1.213)	(.009)	(.007)
		3	309.804	121.616	.0027	-.0110	.1230	.0762
			(65.043)	(27.830)	(1.675)	(1.003)	(.012)	(.008)

TABLE 18 ML versus ERLS method: Contamination = 10%, n = 10000

Contamination	n	k	χ^2_{LS}	χ^2_{ELS}	γ_{LS}	γ_{ELS}	$RMSEA_{LS}$	$RMSEA_{ELS}$
10%	10000	1	3.8867	3.8880	.0022	-.0022	.0027	.0027
			(2.449)	(2.442)	(1.006)	(1.006)	(.003)	(.003)
		2	138.753	96.809	.0019	-.0020	.0576	.0478
			(32.933)	(23.670)	(1.009)	(1.215)	(.007)	(.006)
		3	618.058	242.540	.0029	-.0025	.1235	.0769
			(102.49)	(43.120)	(1.678)	(1.628)	(.010)	(.007)

Note: Standard deviations for chi-squares, parameters, and root mean square of approximation are in parentheses in the tables based on 100 replications (r), except for n=1000, k=3, r = 96.

Conclusions and Recommendations

The elliptical estimation methods performed better overall than the normal estimation methods in the presence of increasing contamination and kurtosis, e.g., the normal least squares (LS) estimation method failed to reach a solution (lacked convergence) under increased kurtosis. The elliptical generalized least squares (EGLS) estimation method overall performed better than the other estimation methods in computing chi-square, structure coefficient, and root mean square error of approximation values under increasing contamination and kurtosis. Previous findings by Bentler (1983a), Harlow and Newcomb (1984), Muthen and Kaplan (1985), and Tanaka (1984) which indicated that ML chi-square estimates were too large, causing the rejection of a true structural equation model too often, was supported in the study. The tendency for increased levels of kurtosis to affect elliptical estimated chi-square statistics, as reported by Harlow (1985), was also substantiated in the present study. In contrast, the findings by Weng and Cheng (1997) that chi-square values computed by LS, GLS, and ML estimators differ, but the effects diminish as sample size increased was not supported, especially under increased kurtosis in this study.

The effects of various estimation methods on the parameter estimate in the structural equation model was found to be minimal. This was supported by Harlow(1985), who concluded that ML and ERLS parameter estimates were comparable in a Monte Carlo simulation study; Muthen and Kaplan(1985), who found no difference between parameter estimates using ML and GLS estimation methods; Henly(1993), who pointed out a striking similarity between ML and GLS estimates; and Wang, Fan, & Willson(1996) who also found the results from ML and GLS estimation methods to be practically identical as sample size increased.

The root mean square error of approximation (RMSEA) was robust across the research conditions and estimation methods. The root mean square error of approximation values were especially robust in the presence of kurtosis using the elliptical estimation methods. The root mean square error of approximation is therefore the preferred inferential approach to assessing model fit because of known distributional properties and determination of confidence intervals for hypothesis testing.

In practice, researchers are often confronted with non-normal data, i.e., skewness and kurtosis. Recommendations based in part on the findings in this study and related research indicate several suggestions. First, determine the sample size and power needed to conduct a test of the structural model using programs by

MacCallum, Browne, and Sugawara (1996) and/or Schumacker & Lomax (1996). Second, based on a comparison of non-normal data transformation methods, use a probit regression transformation to produce an approximate normal distribution of data to handle skewness. Third, use the elliptical generalized least squares estimation method with non-normal kurtotic data. Fourth, report the root mean square error of approximation (RMSEA) and associated confidence interval to test hypotheses concerning model fit. And finally, when reporting chi-square statistics, conduct the Bollen-Stine bootstrap technique to yield a test of the sufficiency of the obtained model chi-square value and/or report the Satorra-Bentler re-scaled chi-square statistic (Chou, Bentler, Satorra, 1991).

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