CHAPTER III

THEORY

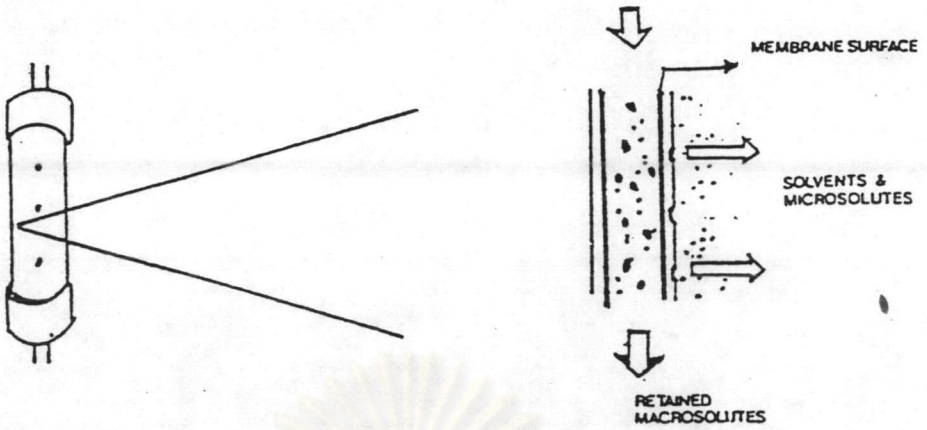
The purpose of this chapter is to lay theoretical ground work to be used as basis for interpretation of the experimental results. In section 3.1 a gereral description of ultrafiltration system based on gel polarization model is given and important parameters of the system are identified. In section 3.2 a general description of mathematical analysis of continuous fermentation process coupling with UF is given.

3.1 Ultrafiltration

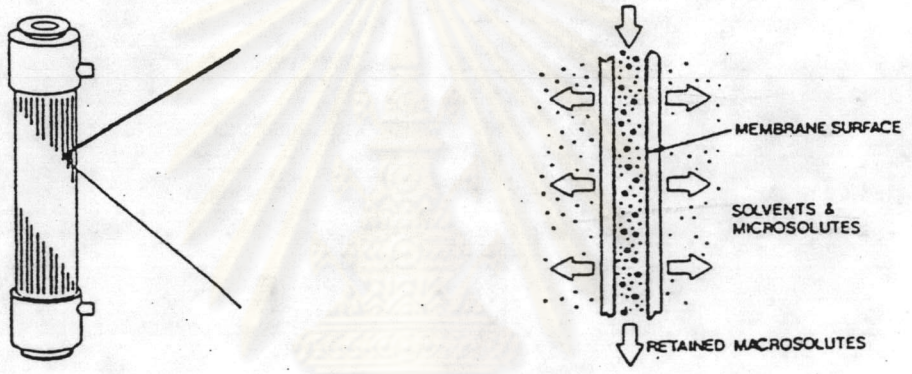
Ultrafiltration (UF) is a pressure-driven membrane separation technique for dissolved and suspended materials based on molecular size. Substances smaller than the pore size of the filter are driven through with the solvent while larger substances are retained. Ultrafiltration is used for separating particles with molecular weights from 500-300,000 (or 10-100 $^{\circ}$A). Pressure exerted on a solution is 1-10 bar as a driven force that causes a flow of solutes and water toward the ultrafilter. Examples of different membrane configurations are showed in Figure 3.1.

Mass Transfer and Gel Polarization

In ultrafiltration, the concentration of retained macrosolutes will build up at the membrane surface due to the removal of solvent. This causes a concentration gradient with the maximum macrosolutes level at the membrane (Fig. 3.2). This phenomenon is known as concentration polarization (Michaels, 1968). As a result of the increased concentration at the membrane surfare, there is a tendency for solute to diffuse away from this point. Under steady state conditions, the



Tubular Module



Hollow Module

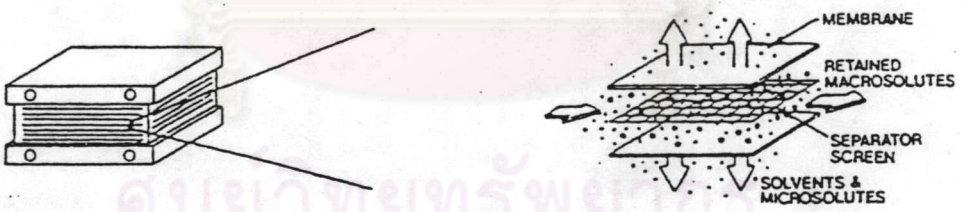
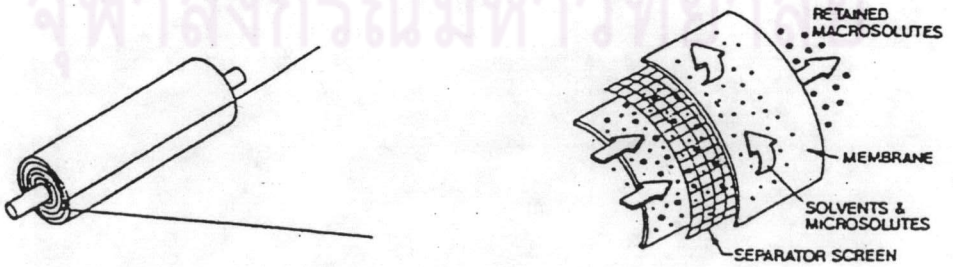


Plate & Frame

Module



Spiral cartridge

Module

Figure 3.1 Examples of different membrane configurations

convective mass transfer toward the membrane balanced by the diffusive movement in the opposite direction, as the following equation.

$$J\,C - D_v \frac{dc}{dx} = 0 \qquad \text{——— (1)}$$

J = permeate flux

C = solute concentration at point x

D_v = solute diffusivity

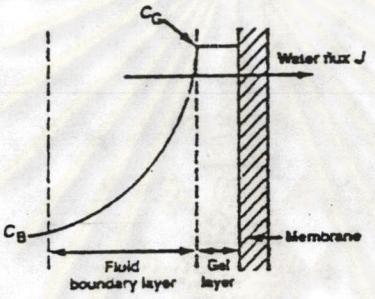


Figure 3.2 Concentration gradient during gel polarization

Equation (1) can be integrated across the solute boundary layer to give

$$J = (D_v/\delta)\ \ln\ (C_w/C_B) \qquad \text{——— (2)}$$

C_w = solute concentration at the membrane surfare

C_B = bulk solute concentration

δ = boundary layer thickness

$$J = K \ln (C_w/C_B) \qquad \text{——— (3)}$$

K = mass transfer coefficient = D_v/δ

Under actual operating conditions, the value of Cw can be increased until the point that the retained solute forms a gel layer (gel polarization) (Michaels, 1968); (Blatt et al, 1970). This gel concentration, C_G. is the maximum value of Cw and may be substituted into equation (3):

$$J = K \ln (C_G/C_B) \qquad \text{——— (4)}$$

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C_G is dependent upon operating pressure, temperature, solubility and pH, Ingham et al (1980) suggest that C_G is actually the concentration at which osmotic back-pressure is high enough to prevent flux.

K is generally not a function of the solute concentration but depend on the driving pressure and any fluid flow across the membrane. Blatt et. al. (1970) showed that K is a function of the fluid velocity (V) across the membrane;

$$S_h = (Kd)/D_v = A\ Re^B\ S_c^{1/3} \qquad \text{——— (5)}$$

d = the fluid channel height on top of the membrane

R_e = the Reynolds number ($\rho dv/\mu$)

S_c = the Schmidt number ($\mu/D_v\rho$)

S_h = the Sherwood number

A, B = constants

B = 0.5 in laminar flow, 1.0 in turbulent flow.

Cross Flow Filtration (see figure 3.3)

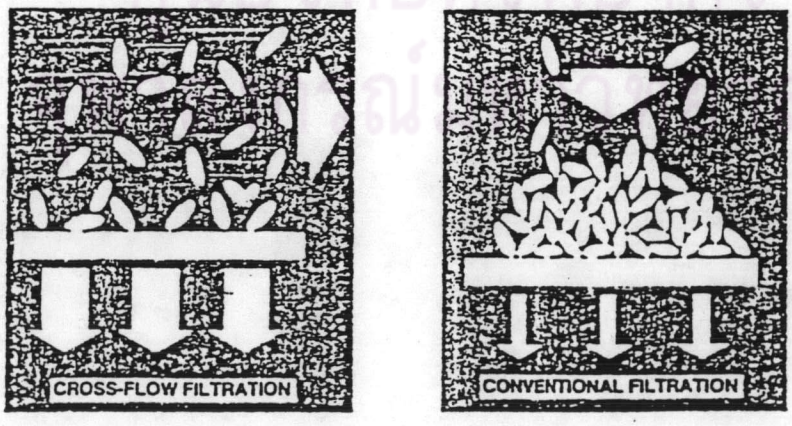

Figure 3.3 Comparision of cross-flow and conventional filtration

The effects of gel polarization can be reduced by crossflow filtration. In cross flow filtration, the feed stream flows tangentialy across the membrane surface. The retained fluid is recirculated over the membrane surface. This operation makes the movement of a solute away from the membrane and reduces the thickness of the gel layer so the problem of clotting in filtration process can be reduced.

Effect of Operating Parameters

The feed stream flows across the ulrafiter and creates a pressure differential from the inlet (Pi) to the outlet (Po)

$$\Delta P = P_1 - P_o \qquad \text{——— (6)}$$

ΔP = Pressure drop

This pressure drop can be related to the flow rate (Q) or velocity (V) across the membrane

in laminar flow : $\Delta P = (C_1 \mu LV)/d^2 = (C_2 \mu LQ)/d^4$

(Poiseuille equation) ——— (7)

μ = viscosity

L = filter length

d = fluid channel height above the membrane

C_1, C_2 = constants dependent on chanel geometry

in turbulent flow : $P = (C_3 f LV^2)/d = (C_4 f L/Q^2)/d^5$

(Fanning equation) ——— (8)

f = a factor based on the Reynold's number

C_3, C_4 = constants dependent on channel geometry

The driving force through the membrane is also dertermined by pressure. This transmembrane pressure is the different between the pressure on the feed side and on the filtrate side of the ultrafilter The differential will be highest at the inlet and reduce to a minimum at the outlet. Figure 3.4 shows cross-flow filtration pressure relationships.

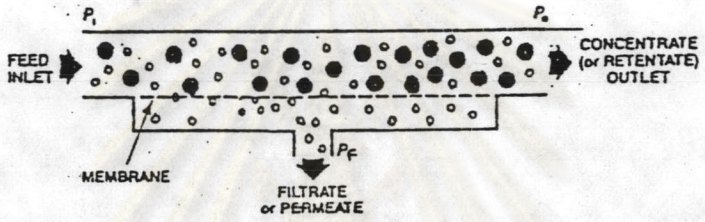

Figure 3.4 Cross-flow filtration pressure relationships

An average driving force (ΔPTM) is;

$$\Delta PTM = \frac{P_i + P_o}{2} - Pf \qquad \text{——— (9)}$$

Generally, the filtrate pressure is negligible and Pf is taken as zero, so

$$\Delta PTM = P_i - (\Delta P/2) \qquad \text{——— (10)}$$

The flux rate will be a function of PTM as difined by:

$$J = \Delta PTM/(R_M + R_G) \qquad \text{——— (11)}$$

R_w = hydraulic resistance created by the membrane

R_G = hydraulic resistance created by the gel layer

From the above equations, Permeate flux depends on four parameters.

1. Pressure (P_1) (from equations 9, 10, 11)

The permeate flux will increase with pressure until the point when gel layer forms and increases overall resistance. Then further increases in pressure will increase the thickness and the resistance of gel layer so the permeate flux will reach a maximum and become relatively constant with pressure.

2. Recirculation Velocity (V) (from equations 5, 8, 10)

The mass transfer coefficient (K) will increase with recirculation velocity (equation 5), moreover an increase in recirculation velocity increases the shear force at membrane surface such that the thickness of gel layer and gel resistance decrease. But the average driving force (PTM) will decrease with the increasing of recirculation velocity (equation 8, 10)

3. Temperature (T)

The mass transfer coefficient increases when temperature increases, so the permeate flux increases with the temperature.

4. Concencentration of Solute (C_B) (from equation 4)

The permeate flux decreases with the solute concentration.

In this study the temperature and the solute concentration are fixed at the points that optimize the productivity of solvent in the fermentation process, therefore only the effects of pressure and recirculated velocity which are suitable for separating the microorganism from the fermentation broth will be studied.

Membrane Rejection The ability of an ultrafiltration membrane to retain a given species is defined by the rejection coefficient,

$$\sigma' = 1 - (C_p/C_B) \qquad \text{——— (12)}$$

C_p = concentration of the species in the permeate side of the membrane at a given instant of time.

If the membrane completely retains the species, the concentration in the permeate would be zero ($C_p = 0$) and its rejection coefficient would be one ($\sigma' = 1$)

3.2 Mathematical Analysis of Continuous Fermentation Process Coupled with Ultrafiltration.

The rate equations for cell mass (x) substrate (s), and metabolic product (P) are

$$\frac{dx}{dt} = (\mu - BD)x \qquad \text{——— (13)}$$

= specific growth ratio

B = bleed ratio

D = dilution rate = F/V

$$\frac{dP}{dt} = \nu x - DP \qquad \text{——— (14)}$$

= specific solvent production rate

$$\frac{ds}{dt} = D(S_o - S) - \frac{1}{Y_{x/s}} x \qquad \text{——— (15)}$$

$$\frac{ds}{dt} = D(S_o - S) - \frac{1}{Y_{p/s}} P \qquad \text{——— (16)}$$

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In the above equations, specific growth rates (μ) and specific production rates (ν) have been expressed by a simple Monod-type equation as a function of limiting substrate concentration.

$$\mu = \frac{\mu_m S}{K_s + S} \qquad \text{——— (17)}$$

However, this equation is valid only in a low-inhibition environment. In practice, the product has a substantially inhibitory effect of cell concentration and solvent (butanol) concentration, so it can be expressed as

$$\mu = f(S, P, X) \qquad \text{——— (18)}$$

$$\nu = g(S, P, X) \qquad \text{——— (19)}$$

From equation (13) we can see that in a total cell recycle system (B = 0) the cell mass builds up. To control cell concentration and control steady state condition in a high-density cell culture, a controlled bleed is necessary,

at steady state

$$\mu = BD \qquad \text{——— (20)}$$

$$\nu = \frac{DP}{X} \qquad \text{——— (21)}$$

From equation (14), the relation between the volumetric productivity (r_p) and the product concentration P is

$$r_p = \frac{dp}{dt} + DP \qquad \text{——— (22)}$$

For the substrate consumption rate, the relation is

$$r_s = D(S_o - S) - \frac{ds}{dt} \qquad \text{——— (23)}$$

As these parameters were rapidly varying with time, the integral productivity and substrate consumption rate for a period $t_1 - t_2$ are

$$\overline{r}_p = \int_{t1}^{t2} \frac{r_p\, dt}{t_2 - t_1} = \int_{t1}^{t2} \frac{DPdt + P_2 - P_1}{t_2 - t_1} \qquad \text{——— (24)}$$

$$\overline{r}_s = \int_{t1}^{t2} \frac{r_s\, dt}{t_2 - t_1} = \int_{t1}^{t2} \frac{D(S_o - S) + S_2 - S_1}{t_2 - t_1} \qquad \text{——— (25)}$$

Yield of production of solvent ($Y_{p/s}$)

$$Y_{p/s} = \frac{r_p}{r_s} \qquad \text{——— (26)}$$

This experimental study was operated with total cell recycling (B = O). Equations (13) and (14) may be used for calculation of specific growth rate (μ) and specific production rate (ν). Equations (22), (23) and (26) may be used for calculation of productivity (r_p), consumption rate (r_s) and production yield ($Y_{p/s}$) respectively.