

การประยุกต์ใช้อัลตราฟิลเตรชั่น เพื่อเพิ่มผลผลิตในกระบวนการหมัก  
อะซิโตน-บิวทานอล แบบต่อเนื่อง



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ศูนย์วิทยทรัพยากร

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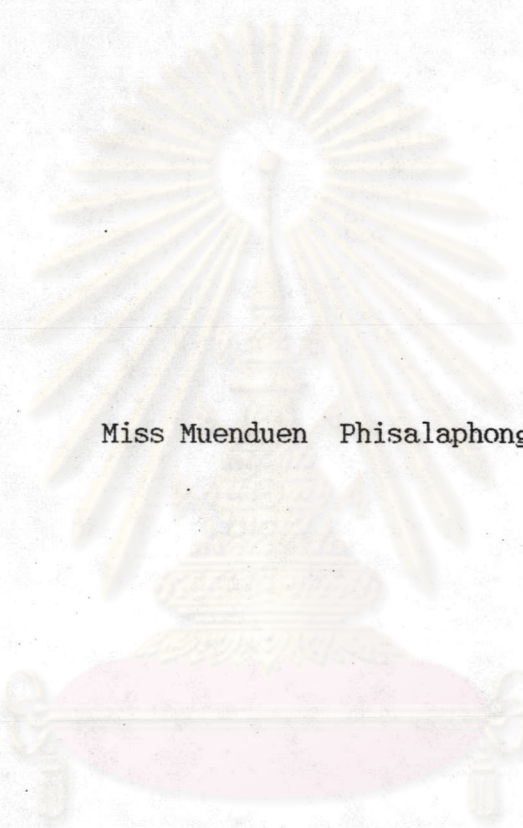
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APPLICATION OF ULTRAFILTRATION FOR IMPROVED PRODUCTIVITY  
IN CONTINUOUS ACETONE-BUTANOL FERMENTATION



Miss Muenduen Phisalaphong

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วิทยานิพนธ์นี้ศึกษาการนำกระบวนการกรองแบบอุลตราฟิลเตรชันมาประยุกต์ ใช้เพื่อเพิ่มผลผลิตในกระบวนการหมักอะซิโตน-บิวทานอล แบบต่อเนื่อง โดยนำแผ่นเยื่อกรองอุลตราฟิลเตรชันที่ทำด้วยเซรามิกส์แบบท่อหลายท่อ (multitubular ultrafilter) ซึ่งมีพื้นที่ผิวการกรองรวม 0.203 ตารางเมตร มาใช้เป็นตัวแยกเซลล์จุลินทรีย์ Clostridium acetobutylicum ATCC 824 จากน้ำหมักแล้วนำเซลล์ย้อนกลับมาใช้ในกระบวนการหมักต่อไป จากการศึกษาพบว่า ความดันและความเร็วในการไหลของน้ำหมักผ่านแผ่นกรองที่เหมาะสมคือ 0.17 กิโลกรัม/เซนติเมตร<sup>2</sup> และ 0.4 เมตร<sup>3</sup>/ชั่วโมง ตามลำดับ จากการทดลองโดยการทดสอบที่ความเข้มข้นของน้ำตาลกลูโคสในสารอาหารตั้งต้นตั้งแต่ 40 กรัม/ลิตร ถึง 60 กรัม/ลิตร และที่อัตราการป้อนสารอาหารต่อปริมาตรรวมในการหมัก (dilution rate) ตั้งแต่ 0.11 ต่อชั่วโมง ถึง 0.55 ต่อชั่วโมง พบว่าได้ผลผลิตสูงสุดเมื่อใช้สารอาหารตั้งต้นที่มีความเข้มข้นของน้ำตาล 42.4 กรัม/ลิตร ที่อัตราการป้อนสารอาหารต่อปริมาตรรวมในการหมัก 0.55 ต่อชั่วโมง โดยให้ผลผลิตของสารละลายผลิตภัณฑ์ 6.06 กรัม/ลิตร-ชั่วโมง สารละลายผลิตภัณฑ์ทั้งหมดมีความเข้มข้น 11.03 กรัม/ลิตร ประกอบด้วย 6.26 กรัม/ลิตร บิวทานอล 4.40 กรัม/ลิตร อะซิโตน และ 0.37 กรัม/ลิตร เอทานอล ที่ความเข้มข้นของเซลล์จุลินทรีย์ 80 กรัม/ลิตร โดยมีอัตราการใช้น้ำตาลกลูโคส 19.30 กรัม/ลิตร-ชั่วโมง อัตราการเปลี่ยนน้ำตาลเป็นสารละลายผลิตภัณฑ์ 0.31

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MUENDUEN PHISALAPHONG : APPLICATION OF ULTRAFILTRATION FOR  
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Ultrafiltration was applied for improved productivity in continuous acetone-butanol fermentation. A multitubular ceramic ultrafilter with 0.203 m<sup>2</sup> surface area was used to separate and recycle cells in a continuous fermentation of Clostridium acetobutylicum ATCC 824. The optimum applied pressure and recirculation flow rate were 0.17 kg<sub>f</sub>cm<sup>-2</sup> and 0.4 m<sup>3</sup> hr<sup>-1</sup> respectively. From the experiments of total cell recycle system with the glucose of concentration varying from 40 to 60 gl<sup>-1</sup> and the dilution rate varying from 0.11 to 0.55 hr<sup>-1</sup>, the maximal solvent productivity was achieved at 42.4 gl<sup>-1</sup> of glucose concentration at a dilution rate of 0.55 hr<sup>-1</sup>. Under total cell recycle, a maximal solvent productivity was attained at about 6.06 gl<sup>-1</sup>hr<sup>-1</sup>. The product solution had a concentration of 11.03 g.l<sup>-1</sup> and consisted the combination of 6.26 gl<sup>-1</sup> butanol, 4.40 gl<sup>-1</sup> acetone and 0.37 gl<sup>-1</sup> ethanol. A dry weight concentration of 80 gl<sup>-1</sup> was obtained with 19.30 gl<sup>-1</sup>hr<sup>-1</sup> glucose consumption and 0.31 production yield.

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CONTENTS

	Page
ABSTRACT (THAI).....	I
ABSTRACT (ENGLISH).....	II
ACKNOWLEDGEMENT.....	III
CONTENTS.....	IV
LIST OF TABLES.....	VI
LIST OF FIGURES.....	IX
NOMENCLATURE.....	XIV
CHAPTER	
I INTRODUCTION.....	1
II LITERATURE REVIEW.....	4
2.1 Acetone-Butanol Production.....	4
2.2 Developement of Acetone-Butanol Fermentation Process.....	16
III THEORY.....	19
3.1 Ultrafiltration.....	19
3.2 Mathematical Analysis of Continuous Fermentation Process Coupling with Ultrafiltration.....	26
IV EXPERIMENTAL.....	29
4.1 Strain.....	29
4.2 Medium.....	29
4.3 Experimetal Equipments.....	29
4.4 Experimetal Procedure.....	32
V RESULTS AND DISCUSSION.....	44
5.1 The Study of Ultrafiltration Membrane Characteristic.....	44
5.2 Application of Ultrafiltration for Cell Recycle in Continuous ABE Fermentation.....	52

Chapter	Page
5.3 Experiment of Cell Recycle System with Constant Dilution Rate.....	78
5.4 Optimization of the Cell Recycle System.....	94
5.5 Comparision of Cell Recycle System with Other Processes.....	98
VI CONCLUSION AND RECOMMENDATION.....	104
REFERENCES.....	107
APPENDIX.....	112
A Experiment Datas.....	112
B The Importation of Butanol and Propylene from 1980-1986...	124
AUTOBIOGRAHY .....	127


  
 ศูนย์วิทยทรัพยากร  
 จุฬาลงกรณ์มหาวิทยาลัย





LIST OF TABLES

Table	Page
1.1 Characteristics of Chemically Pure Fuels.....	3
2.1 Microorganisms which can Produce Butanol by Fermentation....	5
2.2 Raw Materials which can be Used as Carbon Source in Acetone-Butanol Fermentation.....	6
2.3 Sugars which can be Used as Carbon Source in Acetone-Batanol Fermentation.....	7
2.4 Mass Balance of Fermentation.....	12
2.5 Solvent Ratio.....	12
4.1 The Operation of Ultrafiltration System for Studying Effects of Pressure, Recirculation Flow Rate and Biomass Concentration on Permeate Flux.....	34
4.2 The Operation of Cell Recycle System for Studying the Effects of Glucose Concentration and Dilution Rate on Solvent Productivity.....	40
5.1 Permeate Flux against $\ln C_B$ .....	51
5.2 Results of ABE Fermentation from Cell Recycle System with Feeding Glucose Concentration 42.4 g/l (Run K).....	61
5.3 Results of ABE Fermentation from Cell Recycle System with Feeding Glucose Concentration 52.0 g/l (Run L).....	69
5.4 Results of ABE Fermentation from Cell Recycle System with Feeding Glucose Concentration 64.8 g/l (Run M).....	77
5.5 Comparison of ABE Fermentation from Cell Recycle Systems with Feeding Glucose Concentrations 42.4 and 42.3 g/l in Run K and Run N respectively.....	85
5.6 Results of ABE Fermentation from Cell Recycle System with Feeding Glucose Concentrations 43.6 g/l (Run O).....	93

## VII

Table	Page
5.7 Comparison of Solvent productivity ( $\text{gl}^{-1}\text{hr}^{-1}$ ) from Cell Recycle Process with Feeding Glucose Concentrations 42.4, 52.0 and 64.8 g/l.....	95
5.8 Comparison of Butanol productivity ( $\text{gl}^{-1}\text{hr}^{-1}$ ) from Cell Recycle Process with Feeding Glucose Concentrations 42.4, 52.0 and 64.8 g/l.....	96
5.9 Comparison of Glucose Consumption ( $\text{gl}^{-1}\text{hr}^{-1}$ ) from Cell Recycle Process with Feeding Glucose Concentrations 42.4, 52.0 and 64.8 g/l.....	97
5.10 Comparison of ABE Fermentation from Run K and Run P.....	102
5.11 Comparing the Results Obtained from Run K (at $0.55 \text{ hr}^{-1}$ Dilution Rate) with Datas Obtained from the Literatures....	103
A 1 Results of Studying Effects of Pressure, Recirculation Flow rate and Biomass Concentration on Permeate Flux.....	113
A 2 The Results of ABE Fermentation from Cell Recycle System in Run K (with 42.4 g/l Feeding Glucose Concentration).....	115
A 3 The Results of ABE Fermentation from Cell Recycle System in Run L (with 52.0 g/l Feeding Glucose Concentration).....	116
A 4 The Results of ABE Fermentation from Cell Recycle System in Run M (with 64.8 g/l Feeding Glucose Concentration).....	117
A 5 The Results of ABE Fermentation from Cell Recycle System in Run N (with 42.3 g/l Feeding Glucose Concentration).....	118
A 6 The Results of ABE Fermentation from Cell Recycle System in Run O (with 43.6 g/l Feeding Glucose Concentration).....	119
A 7 The Results of ABE Fermentation from Cell Recycle System in Run P (with 43.8 g/l Feeding Glucose Concentration).....	120

## VIII

Table	Page
A 8 Glucose Consumption, Solvent Productivity, Acid Productivity, Cell Productivity and Production Yield in Run K.....	121
A 9 Glucose Consumption, Solvent Productivity, Acid Productivity, Cell Productivity and Production Yield in Run L.....	121
A 10 Glucose Consumption, Solvent Productivity, Acid Productivity, Cell Productivity and Production Yield in Run M.....	122
A 11 Glucose Consumption, Solvent Productivity, Acid Productivity, Cell Productivity and Production Yield in Run N.....	122
A 12 Glucose Consumption, Solvent Productivity, Acid Productivity, Cell Productivity and Production Yield in Run O.....	123
A 13 Glucose Consumption, Solvent Productivity, Acid Productivity, Cell Productivity and Production Yield in Run P.....	123

จุฬาลงกรณ์มหาวิทยาลัย



LIST OF FIGURES

Figure	Page
2.1 Process of N-Butanol Production by Fermentation.....	9
2.2 Biochemical Pathway for Conversion of Sugar into Organic Solvents by <u>Cl. acetobutylicum</u> .....	10
2.3 The Scheme for the Butyl Alcohol Fermentation.....	11
2.4 Glucose Balance of Acetone-Butanol Fermentation.....	13
2.5 Process of N-Butanol Production by Hydroformylation of Propylene.....	15
3.1 Examples of Different Membrane Configurations.....	20
3.2 Concentration Gradient during Gel Polarization.....	21
3.3 Comparison of Cross-Flow and Conventional Filtration.....	22
3.4 Cross-Flow Filtration Pressure Relationships.....	24
4.1 A Ceramic Filter (Type 1M-1).....	31
4.2 Schematic Diagram of the Ultrafiltration System.....	33
4.3 Cell Recycle System.....	35
4.4 Schematic Diagram of the Cell Recycle System.....	36
4.5 Culture Tube and Inoculum Flask.....	38
4.6 Batch Fermentation.....	41
4.7 Schematic of Batch Operation.....	42
5.1 The Effect of Applied Pressure on Permeate Flux of Water and Fermentation Broth.....	46
5.2 Permeate Flux as a Function of Applied Pressure.....	47
5.3 Permeate Flux as a Function of Recirculation Flow Rate.....	49
5.4 Permeate Flux as a Function of Biomass Concentration ( $C_B$ )..	50
5.5 Permeate Flux as a Function of $\ln C_B$ .....	51

Figure	Page
5.6 Time Variation of Butanol, Acetone, Ethanol, Acetic Acid and Butyric Acid Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 42.4 g/l (Run K).....	54
5.7 Time Variation of Glucose, Biomass Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 42.4 g/l (Run K).....	55
5.8 Time Variation of Total Solvent Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 42.4 g/l (Run K).....	56
5.9 Specific Growth Rate, Specific Production Rate and Specific Acid Production Rate Against Time (Run K).....	57
5.10 Glucose Consumption, Total Solvent Productivity and Production Yield Against Time (Run K).....	58
5.11 Total solvent, Butanol, Acetone and Ethanol Productivity as a Function of the Dilution Rate (Run K).....	59
5.12 Total solvent, Butanol, Acetone and Ethanol Concentration as a Function of the Dilution Rate (Run K).....	60
5.13 Time Variation of Butanol, Acetone, Ethanol, Acetic Acid and Butyric Acid Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 52.0 g/l (Run L).....	64
5.14 Time Variation of Glucose, Biomass Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 52.0 g/l (Run L).....	65

Figure	Page
5.15 Time Variation of Total Solvent Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 52.0 g/l (Run L).....	66
5.16 Specific Growth Rate, Specific Production Rate and Specific Acid Production Rate Against Time (Run L).....	67
5.17 Glucose Consumption, Total Solvent Productivity and Production Yield Against Time (Run L).....	68
5.18 Time Variation of Butanol, Acetone, Ethanol, Acetic Acid and Butyric Acid Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 64.8 g/l (Run M).....	72
5.19 Time Variation of Glucose, Biomass Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 64.8 g/l (Run M).....	73
5.20 Time Variation of Total Solvent Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Feeding Glucose Concentration 64.8 g/l (Run M).....	74
5.21 Specific Growth Rate, Specific Production Rate and Specific Acid Production Rate Against Time (Run M).....	75
5.22 Glucose Consumption, Total Solvent Productivity and Production Yield Against Time (Run M).....	76
5.23 Time Variation of Butanol, Acetone, Ethanol, Acetic Acid and Butyric Acid Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Constant $0.55 \text{ hr}^{-1}$ Dilution Rate with 42.3 g/l Feeding Glucose Concentration (Run N).....	80

Figure	Page
5.24 Time Variation of Glucose, Biomass Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Constant $0.55 \text{ hr}^{-1}$ Dilution Rate with 42.3 g/l Feeding Glucose Concentration (Run N).....	81
5.25 Time Variation of Total Solvent Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Constant $0.55 \text{ hr}^{-1}$ Dilution Rate with 42.3 g/l Feeding Glucose Concentration (Run N).....	82
5.26 Specific Growth Rate, Specific Production Rate and Specific Acid Production Rate Against Time (Run N).....	83
5.27 Glucose Consumption, Total Solvent Productivity and Production Yield Against Time (Run N).....	84
5.28 Time Variation of Butanol, Acetone, Ethanol, Acetic Acid and Butyric Acid Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Constant $0.65 \text{ hr}^{-1}$ Dilution Rate with 43.6 g/l Feeding Glucose Concentration (Run O).....	88
5.29 Time Variation of Glucose, Biomass Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Constant $0.65 \text{ hr}^{-1}$ Dilution Rate with 43.6 g/l Feeding Glucose Concentration (Run O).....	89
5.30 Time Variation of Total Solvent Concentration during a Total Cell Recycle System of <u>Cl. acetobutylicum</u> at a Constant $0.65 \text{ hr}^{-1}$ Dilution Rate with 43.6 g/l Feeding Glucose Concentration (Run O).....	90

Figure	Page
5.31 Specific Growth Rate, Specific Production Rate and Specific Acid Production Rate Against Time (Run O).....	91
5.32 Glucose Consumption, Total Solvent Productivity and Production Yield Against Time (Run O).....	92
5.33 Time Variation of Butanol, Acetone, Ethanol, Acetic Acid, Butyric Acid and Total Solvent Concentration during a Batch Fermentation of <u>Cl. acetobutylicum</u> at Feeding Glucose Concentration 43.8 g/l (Run P).....	99
5.34 Time Variation of Glucose, Biomass Concentration during a Batch Fermentation of <u>Cl. acetobutylicum</u> at Feeding Glucose Concentration 43.8 g/l (Run P).....	100
5.35 Specific Growth Rate, Specific Production Rate and Specific Acid Production Rate Against Time (Run P).....	101
B 1 The Importation of Butanol From 1980-1986.....	125
B 2 The Importation of Propylene From 1980-1986.....	126

ศูนย์วิทยทรัพยากร  
จุฬาลงกรณ์มหาวิทยาลัย



## NOMENCLATURE

Dimensions are given in terms of mass (M), length (L), time (t) and temperature (T)

B	=	Bleed ratio (-)
C	=	Solute concentration at point x ( $\text{kg}/\text{m}^3$ )
$C_B$	=	Bulk solute concentration ( $\text{kg}/\text{m}^3$ )
$C_G$	=	Gel concentration ( $\text{kg}/\text{m}^3$ )
$C_P$	=	Solute concentration in permeate ( $\text{kg}/\text{m}^3$ )
$C_w$	=	Solute concentration at membrane surface ( $\text{kg}/\text{m}^3$ )
D	=	Dilution rate ( $\text{hr}^{-1}$ )
$D_V$	=	Solute diffusivity ( $\text{m}^2/\text{s}$ )
d	=	Fluid channel height (m)
J	=	Permeate flux ( $\text{m}^3/\text{m}^2/\text{s}$ )
K	=	Mass transfer coefficient (m/s)
$K_S$	=	Half-rate saturation constant ( $\text{kg}/\text{m}^3$ )
L	=	Filter length (m)
P	=	Product concentration ( $\text{kg}/\text{m}^3$ )
$P_f$	=	Filtrate pressure ( $\text{kg}_f/\text{cm}^2$ )
$P_i$	=	Inlet pressure ( $\text{kg}_f/\text{cm}^2$ )
$P_o$	=	Outlet pressure ( $\text{kg}_f/\text{cm}^2$ )
$\Delta P$	=	Pressure drop ( $\text{kg}_f/\text{cm}^2$ )
$\Delta P_{TM}$	=	Transmembrane pressure ( $\text{kg}_f/\text{cm}^2$ )
Q	=	Flow rate ( $\text{m}^3/\text{sec}$ )
$R_e$	=	Reynold's number
$R_g$	=	Gel resistance ( $\text{m}^{-1}$ )
$R_m$	=	Membrane resistance ( $\text{m}^{-1}$ )

- $r_p$  = Instantaneous production rate ( $\text{kg}/\text{m}^3/\text{hr}$ )  
 $r_s$  = Instantaneous substrate consumption rate ( $\text{kg}/\text{m}^3/\text{hr}$ )  
 $S$  = Residue substrate concentration ( $\text{kg}/\text{m}^3$ )  
 $S_c$  = Schmidt number  
 $S_h$  = Sherwood number  
 $S_o$  = Substrate concentration in the feed ( $\text{kg}/\text{m}^3$ )  
 $T$  = Temperature ( $^{\circ}\text{C}$ )  
 $t$  = Time (hr)  
 $V$  = Volume ( $\text{m}^3$ )  
 $v$  = Velocity (m/sec)  
 $X$  = Biomass concentration ( $\text{kg}/\text{m}^3$ )  
 $Y_{p/s}$  = Yield of production of P :  $r_p/r_s$  (g/g)  
 $Y_{x/s}$  = Yield of cell production :  $r_x/r_s$  (g/g)  
 $\mu$  = Specific growth rate ( $\text{hr}^{-1}$ )  
 $\mu_{\text{max}}$  = Maximum specific growth rate ( $\text{hr}^{-1}$ )  
 $\nu$  = Specific production rate ( $\text{hr}^{-1}$ )  
 $\nu_{\text{acid}}$  = Specific acid production rate ( $\text{hr}^{-1}$ )  
 $\delta$  = Boundary layer thickness (m)  
 $\mu$  = Viscosity ( $\text{N}_s/\text{m}^2$ )  
 $\sigma$  = Rejection coefficient