CHAPTER II

LITERATURE SURVEY

2.1 Flow Problems

It is well known that a flowing fluid with an initially uniform velocity profile undergoes a hydrodynamic development until a fully developed velocity profile before entering a constricted duct is ultimately achieved. The analysis of loss coefficients and pressure drop due to abrupt contraction and expansion has been the subject of extensive study, especially for laminar flow. However, owing to the nonlinearity of the inertia terms which appear in the equations of motion, it has not been possible to find exact solutions, and various types of approximations have been employed.

In 1878, Weisbach published a book on theoretical mechanics in which the analytical and experimental results were presented on resistance to the motion of water flowing through conduits with sudden enlargement and contraction. The head loss in such a system was given as

$$h_{loss} = \xi V^2/2g_c \quad \text{...............(2.1)}$$

Here, V denotes the outlet velocity; g_c, conversion factor; and ξ, coefficient of resistance which takes value of 0.480 for a sudden contraction and $(1/\sigma - 1)^2$ for sudden enlargement.

In 1950, Kays[8] has obtained semi-empirical results on the expansion and contraction coefficients, K_e and K_c for flow through tubes and ducts with abrupt constrictions, respectively. These coefficients whose magnitude is a measure of pressure drop ΔP due to sudden expansion or contraction can be determined from the Darcy equation

$$\Delta P_{expansion} = K_e \rho V^2/2g_c \quad \text{........(2.2)}$$

$$\Delta P_{contraction} = K_c \rho V^2/2g_c \quad \text{........(2.3)}$$

Where ΔP represents the pressure drop; V, the bulk velocity in the flow passage; and ρ the fluid density. His results are only valid for the flow passage of large length-to-hydraulic diameter ratio L/D_h, since the study was performed on a fully developed flow in either the laminar or turbulent flow range. K_c and K_e are plotted against the area ratio, σ , at different Reynolds numbers. A single curve is used to represent the coefficient of both K_c and K_e for the entire laminar region.

In 1952, Kays and London [9] have dealt with graphical results on the contraction and expansion coefficients, K_c and K_e for flows through a bundle of tubes with headers at either end where the velocity distribution is essentially uniform. K_c and K_e are also plotted against the area ratio, σ at dimensionless variable $4(L/D)/Re = \infty$, 0.2, 0.1 and 0.05 for laminar range and at Re = 3000, 5,000, 10,000 and ∞ for turbulent. Therefore, the results are applicable to the flow through a tube bundle of any L/D ratio. For the more general case of any shape of cross-section, Kays derived the following equations:

$$K_e = 1 - 2\sigma/\alpha_2 + \sigma^2(2/\alpha_3 - 1) \qquad \text{...............(2.4)}$$

$$K_c = \frac{1 - \sigma^2(C_c^2/\beta_0) - 2C_c + 2(C_c^2/\beta_1)}{C_c^2} - 1 + \sigma^2 \qquad \text{....(2.5)}$$

Where C_c is the contraction coefficient at the tube entrance and magnitudes of C_c are graphically presented in reference [16] for circular tubes and for parallel plates. α and β are defined as

$$\frac{1}{\alpha} = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^2 dA \qquad \text{and} \qquad \frac{1}{\beta} = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^3 dA \qquad \text{......(2.6)}$$

Many publications [3,4,6,11,14,15,16,18,23,25] reported that the friction factor, f, was obtained to be 24/Re for laminar flow in parallel duct. The literature survey pertinent to the problems of entrance length is presented in [10,19] and will not reported here. In summary, the hydrodynamic entry length for flows through long tubes and ducts can be related to the flow [5,12,17, 19,20,21] by

$$L/D_h = C.Re \qquad \text{.......................(2.7)}$$

where C varies from 0.0138 to 0.07, such as, in Ref.[20] C = 0.138 for the duct of aspect ratio 5:1, and C = 0.0317 for the duct of aspect ratio 2:1.

Bunditkul and Yang [1,2] obtained K_c, K_e and critical constriction length in long parallel channel and obtained correction factor for loss coefficients and Fanning friction factor, C_c, C_e and C_f respectively for short flow constriction in parallel duct. Pressure distributions were also obtained in both cases. The results are attained by numerically integrating the full Navier-Stokes and Energy equations using finite-difference technique.

A comprehensive survey of the literature pertainning to the friction loss and heat transfer performance in the entrance region, the pressure drop in flows across nozzle, orifices and other flow constrictions, and the blood pressure change in cardiovascular systems including aortic valves and arterial stenoses that were published prior to 1977 is summarized in reference [3]. More recent publications [1,2,5,12,13,14,19,24,25] have emphasized numerical solutions by the finite-difference method.

2.2 Theory and Calculation

In fully developed flow in a duct-either laminar or turbulent, ΔP is proportional to the length L and that the following functional relationship is valid:

$$\Delta P/L = \phi\,(V,\ D,\ \rho\ ,\ \mu\ ,\ e) \qquad \text{..............(2.8)}$$

The quantity e is a statistical measure of surface roughness of the duct and has the dimensions of length. With force F, mass M, length L, and time Θ as fundamental dimensions, and V, D, ρ as the set of maximum number of quantities which in themselves cannot form a dimensionless group, the pi theorem leads to

$$\Delta P/4(L/D)(\rho\ V^2/2) = \psi\,(VD\rho/\mu,\ e/D) \qquad \text{.......(2.9)}$$

Where the dimensionless numerical constant 4 and 2 are added here for convenience.

The above dimensionless group involving ΔP has been defined as a friction factor, f

$$f = \Delta P/4(L/D)(\rho V^2/2) \qquad \text{..............(2.10)}$$

Then, eq.(2.9) becomes

$$f = \psi(Re, e/D) \quad ....(2.11)$$

For a duct of noncircular cross section, D is replaced by a " hydraulic diameter," D_h defined by

$$D_h = 4A/p = 4(\text{flow area})/\text{wetted perimeter} \quad ....(2.12)$$

For laminar flow between parallel flat plates - Poiseuille flow [4,16,23] applies in the following form:

$$\Delta P/\Delta x = 12 \mu V/b^2 \quad ....(2.13)$$

Here b is the distance between parallel flat plates.

Since D_h = 2b and eq.(2.13) can be written in the form

$$f = 24/Re \quad ....(2.14)$$

with D_h replacing D in the definitions of f and Re

To determine loss coefficients, K_c and K_e, Equations were shown relation between loss coefficients and various variables which can be measured by experiment.

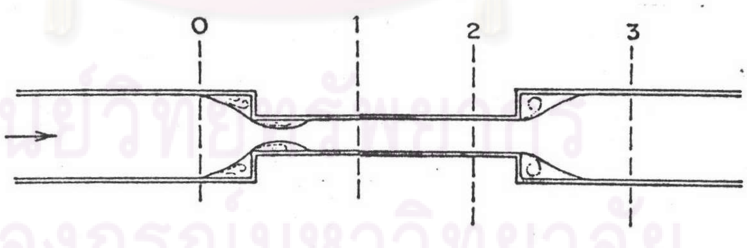

Fig.2.1 Sudden contraction and enlargement

From reference [16], Bernoulli equation defined as

$$dH = d(P/\rho + V^2/2 + Zg_c) = dF \quad ....(2.15)$$

Where dF is the frictional effect. The quantity $(P/\rho + V^2/2 + g_cZ)$ is known as the Bernoulli head, H, and eq.(2.15) for idealized frictionless flow ($dF = 0$) is known as the Bernoulli equation.

The pressure drop associated with sudden contraction and expansion, Fig.2.1, is reported in terms of a decrease in Bernoulli head H, eq.(2.15), and a loss coefficient K referred to the kinetic energy of the flow in the smaller cross section. Since $\Delta Z = 0$ for each of the cases,

$$\frac{P_o - P_1}{\rho_{01}} + \frac{V_o^2 - V_1^2}{2g_c} = K_c \frac{V^2}{2g_c} \qquad \text{............(2.16)}$$

$$\frac{P_2 - P_3}{\rho_{23}} + \frac{V_2^2 - V_3^2}{2g_c} = K_e \frac{V_2^2}{2g_c} \qquad \text{............(2.17)}$$

From continuity since ρAV = constance, and $\rho_0 \approx \rho_1$ and $\rho_2 \approx \rho_3$

Thus $A_1/A_o = V_o/V_1$ and $A_2/A_3 = V_3/V_2$ ....(2.18)

Inserting eq.(2.18) into eq.(2.16) and (2.17), rearranging eq.(2.16) and (2.17) become

$$\Delta P_1 = (1 - \sigma^2) \frac{\rho V^2}{2g_c} + K_c \frac{\rho V^2}{2g_c} \qquad \text{............(2.19)}$$

$$\Delta P_2 = -(1 - \sigma^2) \frac{\rho V^2}{2g_c} + K_e \frac{\rho V^2}{2g_c} \qquad \text{............(2.20)}$$

Where $\sigma = A_1/A_o = A_2/A_3$ = ratio of constriction area to frontal area

$V = V_1 = V_2$ = mean velocity inside constricted duct

ΔP_1 = pressure drop due to abrupt contraction

ΔP_2 = pressure drop due to abrupt expansion

g_c = conversion factor

$\rho = \rho_{01} = \rho_{23}$ = density of fluid

Loss coefficients, K_c, K_e and Fanning friction factor, f are determined by eq.(2.19), (2.20) and eq.(2.10) respectively, in this investigation.