CHAPTER IV

NUMERICAL SOLUTIONS

This chapter is concerned with the numerical results obtained from the solution scheme described in Chapter III. A computer program has been developed to investigate the interaction problem between an elastic circular plate and two types of poroelastic medium, namely a homogeneous poroelastic half-space and a multi-layered poroelastic half-space. Convergence and stability of numerical solutions are investigated. The accuracy of the present solution is verified by comparing with the existing solutions given in the literature. Numerical results are presented in this chapter to demonstrate the applicability of the present solution scheme and to portray the influence of governing parameters on the interaction problem. The discussion on these results is also given in this chapter.

4.1 Numerical Solution Scheme

The solution scheme described in Chapter III is implemented into a computer program. The tasks performed by the computer program can be summarized as:

1. The vertical displacement is assumed in the terms of generalized coordinates, equation (3.104). The plate is then discretized into Ne ring elements.

2. The influence functions are determined to establish the flexibility equation, equation (3.115) for an impermeable plate and equation (3.120) for a permeable plate. These equations are then solved for contact stresses and pore pressure jumps.

3. The strain and kinetic energies of the plate are derived. The Lagrangian funtional and the Lagrange 's equations of motion are established resulting in a system of linear simultaneous equations.

4. The simultaneous linear equations are solved for the generalized coordinates α_n.

5. The plate displacements, contact stresses and pore pressure jumps are obtained by back substituting the generalized coordinates α_n into equations (3.104), (3.1113) and (3.114), respectively.

In this thesis, two types of problem are considered, i.e., the dynamic interaction of a plate-homogeneous poroelastic soil and a plate-multi-layered poroelastic soil. The main difference between the two problems is the evaluation of the influence functions, G^{ij} in equations (3.115) and (3.120). For a homogeneous poroelastic half-space, solutions of the influence functions, in Hankel transform domain, are given explicitly in Section 3.2.1 together with the arbitrary functions in Appendix B. To calculate the influence functions of multi-layered poroelastic half-space, it is necessary to develop a computer program to compute the stiffness matrices corresponding to each layer and the underlying half-space for specified values of Hankel transform parameter ξ and the frequency of excitation ω. These matrices are assembled into the global stiffness matrix, equation (3.103). The required influence functions of a multi-layered poroelastic half-space are obtained by solving the global stiffness equation for each specified value of ξ and ω.

The major computation effort performed by the computer program is the evaluation of those influence functions. It is found that the influence functions G^{ij} for both homogeneous and multi-layered poroelastic half-spacess appear in terms of semi-infinite integrals with complex-valued integrand. The integral with respect to ξ is evaluated by using a globally numerical quadrature scheme. The scheme subdivides the interval of integral and employs a 21-point Gauss-Kronrod rule to estimate the integral over each subinterval. The error for each subinterval is estimated by comparison of the results obtained from a 21-point Gauss-Kronrod rule with those from a 10-point Gauss-Kronrod rule. The subinterval with the largest estimated error is then bisected and this procedure is applied to both halves. This bisection procedure is continued until the error criterion is reached.

4.2 Convergence and Numerical Stability of Present Solution Scheme

The convergence and stability of the numerical solution scheme described in the previous section are investigated with respect to the following parameters:

1. The upper limit of integration, ξ_L, used in the numerical integration of the semi-infinite integrals of influene functions.

2. The number of terms N used in the displacement representation given by equation (3.104)

3. The number of ring elements Ne used to discretize the circular area S into equally annular elements.

Table 1 presents the convergence of non-dimensional influence functions of a homogeneous poroelastic half-space with respect to ξ_L. The uniform vertical loading of intensity f_o is applied over a circular area of radius a at a depth a below a free surface of a half-space of material HB. The properties of material HB are defined in Table 3. The non-dimensional frequency, δ=1, where $\delta = \omega a / \sqrt{\rho\mu}$ is used in the comparison. The configuration of the loading is shown in Figure 4. It is found from the results shown in Table 1 that the influence functions are converged for $\xi_L \geq 100$.

Table 2 shows the influence of N and Ne on the non-dimensional central displacement $\bar{w}_p(0)$ $[= w_p(0)\mu/(\delta f_o)]$ of an elastic plate resting on the surface of a homogeneous poroelastic half-space of material HB subjected to a uniformly distributed load f_o for different values of N and Ne. The elastic plate with non-dimensional relative flexibility parameter γ=100, where $\gamma = \mu a^3 / D$, and Poisson 's ratio, v_p=0.25 is considered. It appears from Table 2 that the accurate numerical results are obtained when $N \geq 6$. As for number of ring elements used to discretize S, the convergence is achieved for Ne=20. All subsequence numerical solutions are then determined by employing Ne=20 and N=8.

4.3 Comparison with Existing Solutions

The accuracy of the present solution scheme is verified by comparing the solutions obtained from the present scheme with the existing solutions. Figures 6 to 9 present a comparison between the influence functions of a homogeneous poroelastic half-space given by Zeng and Rajapakse (1999) with those obtained from the exact stiffness matrix scheme outlined in Chapter III. The axisymmetric vertical loading applied at the level $z'=a$ in a homogeneous poroelastic half-space as shown in Figure 4. The present analysis are obtained from a multi-layered poroelastic half-space consisting of 10 layers with a thickness of $0.2a$ and an underlying half-space of identical properties. The present solutions are in excellent agreement with those given by Zeng and Rjapakse (1999).

Figures 10 and 11 present a comparison of vertical compliances of impermeable and fully permeable rigid plates with different depth of embedment ratio (h/a = 0, 1, 2, 5). The non-dimensional vertical compliance of a rigid plate is defined by $\bar{C}_v = (\Delta_z / P_o)/C_v^o$ in which Δ_z is the vertical displacement of a rigid plate and P_o is the magnitude of vertical load. In addition, C_v^o is the vertical compliance of a rigid circular plate resting on an ideal elastic half-space under static loading. Note that $C_v^o = (1-\nu)/4\mu a$, where μ and ν denote shear modulus and Poisson 's ratio of the half-space (Selvadurai, 1979). Solutions corresponding to the present analysis are obtained from a multi-layered poroelastic half-space consisting of 10 layers with a thickness of $0.2a$ and an underlying half-space. Each layer and the underlying half-space have the same properties as in Zeng and Rajapakse (1999). It can be observed that the present solutions agree very closely with the results given by Zeng and Rajapakse (1999). The numerical stability and the accuracy of the numerical evaluation of the influence functions for a multi-layered poroelastic half-space are confirmed through these comparisons.

Gucunski and Peek (1993) presented solutions for an elastic circular plate resting on the surface of a homogeneous elastic half-space subjected to a uniformly distributed vertical load f_o. Figures 12 and 13 show a comparison of the non-dimensional displacement profiles $\bar{w}_p(r)$ $[= w_p(r)\mu / \delta f_o]$ obtained from the present scheme with those reported by Gucunski and Peek (1993). It can be seen from these figures that the two sets of solutions agree very closely at all points in the plate.

4.4 Numerical Results and Discussion

The numerical solutions presented in sections 4.2 and 4.3 confirm the convergence, numerical stability and accuracy of the presented formulation. Numerical results for two different types of poroelastic medium , i.e., (i) homogeneous poroelastic half-space and (ii) multi-layered poroelastic half-space are presented in this section to demonstrate the influence of various parameters on the dynamic response of the plate.

4.4.1 Response of Circular Plate in Homogeneous Poroelastic Half-Space

Numerical results for vertical vibrations of an elastic circular plate embedded in a homogeneous poroelastic half-space are presented in this section. Four poroelastic materials, identified as materials HA, HB HC and HD and a dry elastic material (HE), are considered in the numerical study presented in this subsection. The properties of these materials are given in Table 3. The massless plate considered in the interaction problem is subjected to a uniformly distributed load of magnitude f_o. The configuration of the problem considered in this subsection is shown in Figure 5. The non-dimensional frequency, δ, and the non-dimensional relative flexibility parameter, γ, defined in the previous section, are employed in this subsection. In addition, the Poisson 's ratio of the plate, ν_p, is kept constant with a value 0.25 for all numerical results.

The influence of relative flexibility on the plate response is investigated. Figure 14 presents non-dimensional displacement profiles, $\bar{w}_p(r)$, of an impermeable circular plate resting on a homogeneous poroelastic half-space for different γ (γ = 0, 1, 10, 100, 1000 and 2000) and δ=1. The case of an impermeable circular plate buried in a homogeneous poroelastic half-space (h/a=1) is presented in Figure 15. The influence of the relative flexibility on the displacement profiles is similar for both surface and buried plates. Both real and imaginary parts of $\bar{w}_p$ for flexible (γ =1, 10, 100, 1000 and 2000) plates increase with increasing γ and maximum at the center of the plate before monotonically decreasing to the plate edge. It is also found that the effect of relative flexibility is negligible for $\gamma \geq 1000$ in all cases.

Figures 16, 17 and 18 show the non-dimensional vertical displacement at the center of an elastic circular plate in a homogeneous poroelastic half-space for different depths of embedment (h/a = 0, 1, 2, 5). The solutions are presented for both impermeable and fully permeable plates. The properties of the poroelastic material are those of material HB defined in Table 3. Numerical results presented in Figures 16 to 18 indicate that the central displacement of a circular plate depends very significantly on the frequency of the excitation and the depth of embedment for both permeable and impermeable plates. The variation of the displacements of a surface plate (h/a = 0) with non-dimensional frequency is smooth for both real and imaginary parts. However, for an embedded plate (h/a = 1, 2, 5), both real and imaginary parts show oscillations with δ due to the effect of the standing waves generated between the free surface and the embedded plate as also noted by Zeng and Rajapakse (1999). Comparison of the central displacement for a circular plate in these figures and a rigid plate in Figures 10 and 11 indicate that the central displacement of an elastic plate are always greater than those of a rigid plate. This is due to the fact that the rigid plate is stiffer than the elastic one. However, the differences between the central displacement of elastic and rigid plates decrease as the depth of embedment increases. In addition, the difference in the central displacement of permeable and impermeable plates increases for δ>2. It can be observed that the influence of hydraulic boundary condition is more significant for an elastic plate.

Figures 19 and 20 present a comparison of the central displacements of an impermeable circular plate in an ideal elastic material and three different poroelastic materials. The solutions are presented for both surface (h/a=0) and embedded (h/a=1) plates. For δ<1, the influence of different material properties on the central displacement of the embedded plate is small when compared to the surface plate. It becomes significant for a higher frequency. For surface plate, both real and imaginary parts of $\overline{w}_p(0)$ depend on the material type over the entire frequency range considered. For both surface and embedded plates, the magnitude of the imaginary part of $\overline{w}_p(0)$ decreases with increasing b at δ>1. The results shown in Figure 20 indicate that the ideal elastic solutions cannot be used to approximate the response of the plate interacting with poroelastic media except for the case of embedded plate at very low frequencies.

4.4.2 Response of Circular Plate in Multi-Layered Poroelastic Half-Space

The dynamic response of an elastic plate buried in a multi-layered poroelastic half-space is considered in this section. The configuration of the problem is shown in Figure 21. A non-dimensional frequency defined as $\delta = \omega a\sqrt{\rho^{(1)} / \mu^{(1)}}$ is used hereafter where $\rho^{(1)}$ and $\mu^{(1)}$ are mass density and shear modulus of the top layer of a multi-layered half space, respectively. Two systems of multi-layered poroelastic medium identified as two- and three-layered systems are considered in this subsection. For a two-layered system, a multi-layered poroelastic half-space consists of one layer on an underlying half-space as shown in Figure 21a. There are five cases considered for the two-layered system as shown in Figure 22. The thickness of the first layer is set to $h_1 = a$ for all cases and the material properties for each layered system in Figure 22 are given in Table 3. For a three-layered system, a multi-layered poroelastic medium consists of two poroelastic layers on an underlying half-space as shown in Figure 21b. Two poroelastic layered system identified as the layered systems 3A and 3B and a dry elastic layered medium (3C) are considered in this system. The material properties of the three-layered system are given in Table 4. In addition, b = 1.5 x 10^6, 7.5 x 10^6 and 4.5 x 10^6 Ns/m^4 for the first and second layers and the underlying half space of the system 3B, respectively, and b = 0 for the dry layer and all layers of the system 3A. Note that only the parameters μ, λ and ρ are required in the representation of dry elastic layered medium, system 3C.

The influence of material property on the plate response is further investigated. Figures 24 to 27 present the non-dimensional displacement at the center of an elastic plate (γ=100) interacting with a two-layered poroelastic half-space. The results are shown for both fully permeable and impermeable plates. The results in these figures confirm the fact that the plate response is significantly influenced by various material properties. The influence of material properties is comparatively larger in the case of an impermeable plate when compared to the case of a permeable plate for δ>2. For a permeable embedded plate (h/a=1), the solutions are nearly identical for the cases 2B, 2C and 2D. However, there is more discrepancy among these solutions for the case of an impermeable plate. It is noted that the solutions of surface plate of cases 2A and 2E and a buried plate of case 2E are identical because the water is not existed at the level of the plate.

A more complicated problem of the dynamic plate-multi-layered half-space interaction, as shown in Figure 21b, is considered next. Figures 28 and 29 show the non-dimensional central displacement of a rigid ($\gamma=0$) and elastic ($\gamma=100$) plates embedded at depth $h=a$ in a three-layered poroelastic half-space. The amplitude of the solutions shown in Figures 28 and 29 are also presented in Figure 30. The results shown in these figures once again indicate that the poroelastic material properties and the frequency of excitation have significant influence on the displacements of both rigid and elastic plates. It is evident that differences in displacements of both real and imaginary parts are observed for a plate in different types of poroelastic materials. The difference in the vertical displacements of a plate corresponding to layered systems 3A and 3B is mainly due to the presence of internal friction in the latter system, i.e., the parameter b.

The influence of the layer thickness on the non-dimensional central displacement of fully permeable and impermeable plates is presented in Figures 31 and 32, respectively. The three-layered system of type 3B is considered in these two figures for different values of the thickness of the second layer are considered, i.e., h_2/a=0, 0.2, 0.5, 1, 2, 5, 10 and 20. Note that the thickness of the first layer is kept constant, $h_1/a=1$. It can be observed that the frequencies at the peak responses, for both real and imaginary parts, are smaller as the layer thickness increases. The response is more significantly influenced by the thickness of the second layer in the frequency range 0<δ<2.5 when compare to a higher range.

It is also useful to consider the influence of the depth of saturation since water is normally saturated at a certain depth below its free surface of the natural soil. The impermeable plate embedded in a half-space ($h/a=1.0$) with different depths of saturation, H, as shown in Figure 23 is examined. The non-dimensional central displacements are presented in Figure 33 for $H/a=$ 0, 1, 1.5, 5 and 20. The case of H/a=0 represents a problem in which an impermeable plate embedded in a saturated half-space. At δ<1, the results from saturated, partially saturated and dry half-space are nearly identical. At higher frequency, the solution of the plate in a saturated half-space is substantially difference from that of a partially saturated medium. Such behavior can be explained that at low frequency, pore pressure in a saturated and partial saturated half-spaces have sufficient time to drain and thus avoids carrying stresses that may be imposed by the deformation of the solid skeleton.

Figure 34 shows the non-dimensional central displacement of an impermeable circular plate in a multi-layered poroelastic half-space case 2A. The solutions are presented for different depths of embedment, i.e., h/a= 0, 1, 2, 3, 5 and 20. From these results, it is found that the dependence of the plate response on the depth of embedment is qualitatively similar for plates in a homogeneous and a multi-layered poroelastic half-spaces. However, oscillation in the response of a buried plate in this figure is comparatively larger when compared to that in a homogeneous poroelastic half-space.

Figure 35 presents the profiles of contact stresses, $T_z \pi a^2 / P$, and pore pressure jumps, $T_p \pi a^2 / P$, under an impermeable plate resting on a three-layered poroelastic half-space for δ = 0.5 and 2.0. The results in this figure demonstrate the load transfer mechanism between a vertically loaded plate and a multi-layered poroelastic half-space. The real part of T_z is slightly dependent on frequency and nearly independent on material type. The imaginary parts of T_z and T_p show considerable dependence on frequency and material type with increasing frequency. Both real and imaginary parts of T_z are singular whereas those of T_p approach zero near the edge of the plate. The real part of T_p is comparatively small but the imaginary part is larger at δ = 2.0. Thus, the pore pressure generated under the disk is negligible at a low frequency and that the load is mainly transferred through the solid skeleton. At a higher frequency, both stress and pore pressure are generated to carry the applied load. This behavior is also noted in the case of a rigid plate in a homogeneous poroelastic half-space (Zeng and Rajapakse, 1999).