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VERTICAL VIBRATIONS OF CIRCULAR PLATE IN MULTI-LAYERED POROELASTIC MEDIA

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วิทยานิพนธ์ฉบับนี้เกี่ยวข้องกับ การศึกษาพฤติกรรมการสั่นในแนวตั้งของแผ่นพื้นยึดหยุ่นวงกลมกับตัวกลางชนิดโพโรอีลาสติก ตัวกลางที่พิจารณามีพฤติกรรมตามทฤษฎีพลวัตของ Biot สำหรับตัวกลางชนิดโพโรอีลาสติกและพิจารณาทั้งกรณีตัวกลางชนิดโพโรอีลาสติกเนื้อเดียวและตัวกลางชนิดโพโรอีลาสติกหลายชั้น แผ่นพื้นวงกลมมีพฤติกรรมตามทฤษฎีแผ่นพื้นบางและรับแรงกระทำแบบสมมาตรกับแนวแกน ผิวสัมผัสระหว่างแผ่นพื้นกับตัวกลางที่รองรับถูกสมมติว่าเรียบและพิจารณาทั้งในกรณีที่ของเหลวสามารถซึมผ่านได้และไม่สามารถซึมผ่านผิวสัมผัสได้ การทรุดตัวในแนวตั้งของแผ่นพื้นถูกแสดงในรูปอนุกรมกำลังของพิกัดในแนวรัศมีโดยมีตัวพิกัดอิสระเป็นตัวแปรไม่รู้ค่า หน่วยแรงสัมผัสและแรงดันน้ำใต้แผ่นพื้นจะถูกจัดให้อยู่ในรูปของตัวพิกัดอิสระในสมการแสดงการทรุดตัวของแผ่นพื้น พิกัดอิสระจะสามารถหาค่าได้โดยสร้างสมการการเคลื่อนที่ของแผ่นพื้นซึ่งประยุกต์จากสมการการเคลื่อนที่ของ Lagrange พฤติกรรมของแผ่นพื้นสามารถทราบได้โดยการแทนค่าตัวพิกัดอิสระกลับเข้าไปในสมการแสดงการทรุดตัวของแผ่นพื้น โปรแกรมที่ใช้ในการศึกษาตามขั้นตอนดังกล่าวได้ถูกพัฒนาขึ้น จากผลที่เสนอในวิทยานิพนธ์ฉบับนี้พบว่า ระดับความลึกของแผ่นพื้น ความถี่ของแรงกระทำและความยึดหยุ่นของแผ่นพื้นมีผลต่อพฤติกรรมการสั่นของแผ่นพื้นอย่างมาก นอกจากนี้ คุณสมบัติของวัสดุชนิดโพโรอีลาสติกและความทึบน้ำของแผ่นพื้นจะมีผลต่อการสั่นของแผ่นพื้นเมื่อความถี่ของแรงกระทำสูง ถ้าความถี่ของแรงกระทำต่ำ แรงกระทำส่วนใหญ่จะถูกรับโดยส่วนเนื้อดิน แต่ที่ความถี่สูง แรงกระทำจะถูกรับโดยทั้งดินและน้ำ

ภาควิชา \_\_\_\_\_ วิศวกรรมโยธา \_\_\_\_\_ ลายมือชื่อนิสิต ยโสธร ทรัพย์เสถียร  
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KEY WORD: DYNAMIC SOIL-STRUCTURE INTERACTION / POROELASTICITY / VERTICAL VIBRATIONS / CIRCULAR PLATE / MULTI-LAYERED HALF-SPACE

YASOTHORN SAPSATHIARN : VERTICAL VIBRATIONS OF CIRCULAR PLATE IN MULTI-LAYERED POROELASTIC MEDIA. THESIS ADVISOR : TEERAPONG SENJUNTICHAJ, Ph.D., 80 pp. ISBN 974-03-1398-1.

This thesis is concerned with vertical vibrations of an elastic circular plate resting on or buried in a poroelastic medium. The poroelastic medium is governed by Biot 's poroelastodynamic theory and considered as a homogeneous half-space and a multi-layered half-space. The plate is subjected to axisymmetric time-harmonic vertical loading and its response is governed by the classical thin-plate theory. The contact surface between the plate and the half-space is assumed to be smooth and either fully permeable or impermeable. The vertical displacement of the plate is represented by an admissible function containing a set of generalized coordinates. Contact stresses and pore pressure jumps are established in terms of generalized coordinates through the solution of flexibility equations based on the influence functions corresponding to vertical and pore pressure loads. The generalized coordinates are obtained by establishing the equations of motion of the plate through the application of Lagrange 's equations of motion. Finally, the response of the plate can be obtained by back substituting the generalized coordinates into the assumed displacement function. A computer program based on the above method is developed. The accuracy and numerical stability of the present solution scheme are verified by comparing with the existing studies. Numerical results indicate that the plate response is significantly influenced by the depth of embedment, the frequency of excitation and the flexibility of the plate. In addition, the poroelastic material properties and the hydraulic boundary condition of the plate influence the plate response at high frequencies. The applied load is mainly carried through the solid skeleton at low frequencies and through both solid and fluid phases at higher frequencies.

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## LIST OF SYMBOLS

$a$	radius of circular plate;
$b$	parameter accounting for the internal friction of the medium;
$E_p$	Young 's modulus of plate;
$e$	dilatation of the solid matrix;
$\mathbf{F}^{(n)}$	generalized force vector for the $n$ th layer;
$f(r)$	externally applied loading;
$G^{zj}$	influence functions for vertical displacement;
$G^{pj}$	influence functions for relative fluid displacement in the $z$ -direction;
$h_p$	thickness of plate;
$J_n$	Bessel function of the first kind of order $n$ ;
$\mathbf{K}^{(n)}$	stiffness matrix of the $n$ th layer;
$L_p$	Lagrangian function;
$M$	Biot 's parameters accounting for compressibility of the medium;
$m$	density-like parameter;
$N$	number of terms of generalized coordinates;
$N_e$	number of ring elements;
$p$	excess pore pressure;

$q_i$	fluid discharge in the $i$ -direction;
$r$	radial coordinate;
$\mathbf{T}^{(n)}$	external force vector at the $n$ th interface;
$T_p$	kinetic energy of plate;
$\bar{T}_p$	pore pressure jump at the contact surface;
$\bar{T}_z$	contact stress at the contact surface;
$t$	time variable;
$\mathbf{U}^{(n)}$	generalized displacement vector for the $n$ th layer;
$U_p$	strain energy of plate;
$u_i$	displacement of the solid matrix in the $i$ direction;
$w_i$	fluid displacement relative to the solid matrix in the $i$ direction;
$w_p$	vertical displacement of elastic circular plate;
$z$	vertical coordinate;
$\alpha$	Biot 's parameters accounting for compressibility of the medium;
$\alpha_n$	generalized coordinates;
$\delta$	non-dimensional frequency;
$\delta_{ij}$	Kronecker delta;
$\varepsilon_{ij}$	strain component of the solid matrix;



$\gamma$	non-dimensional relative flexibility parameter;
$\zeta$	variation of fluid volume;
$\lambda$	constant of the bulk material;
$\mu$	shear modulus of the bulk material;
$\nu_p$	Poisson 's ratio of plate;
$\xi$	Hankel transform parameter;
$\rho$	mass density of the bulk material;
$\rho_f$	mass density of the pore fluid;
$\rho_p$	mass density of plate;
$\sigma_{ij}$	total stress component of the bulk material;
$\omega$	circular frequency