INTRODUCTION

Dense subsemigroups of semigroups of certain types were studied by Hall [1] and by Higgins [2]. Included in [2], Higgins also gave a necessary and sufficient condition on a set X such that S has a proper dense subsemigroup where S is any one of the following transformation semigroups :

(i) the partial transformation semigroup on X,

(ii) the full transformation semigroup on X and

(iii) the symmetric inverse semigroup on X.

A continuation of Higgin's work concerning transformation semigroups was given by Hirunmustsuwan [3]. She characterized the following transformation semigroups on a set X having proper dense subsemigroups in term of the cardinality of X :

(i)' the semigroup of all 1-1 transformations of X and

(ii)'' the semigroup of all onto transformations of X.

Certain matrix semigroups having proper dense subsemigroups were also characterized in [3]. Tranformation semigroups are considered important in the field of semigroups. The first purpose of this research is to characterize each of the following well-known linear transformation semigroups on a vector space V over a field F which has a proper dense subsemigroup in terms of the dimension of V and the multiplicative structure of F.

(a) the multiplicative semigroup of all linear transformations of V,

(b) the multiplicative group of all 1-1 onto linear transformations of V,

(c) the multiplicative semigroup of all 1-1 linear transformations of V and

(d) the multiplicative semigroup of all onto linear transformations of V.

Absolutely closed semigroups have been studied widely (e.g.,[1], [4], [5], [6], [7], [8], [9] and [10]). It is very well-known that every inverse semigroup is absolutely closed. In particular, every symmetric group on a set is absolutely closed. It was proved in [1], [5], [6] and [7] that every full transformation semigroup on a set is absolutely closed. Included in [1], Hall also proved that every partial transformation semigroup on a set is absolutely closed. Limbupsiriporn generalized these results in [10] by proving that for any set X, every ideal of the full transformation semigroup on X and every ideal of the partial transformation semigroup on X is absolutely closed. Moreover,it was also proved in [10] that the transformation semigroups in (i) and (ii) are both closed in B_X, the semigroup of binary relations on X. The linear transformation semigroups (a), (b), (c) and (d) are subsemigroups of T_V, the full transformation semigroup on V, P_V, the partial transformation semigroup on V and B_V. We know that the linear transformation semigroup (b) is absolutely closed since it is a group. It is natural to ask whether each of the linear transformation semigroups (a), (c) and (d) is absolutely closed. We do not give the answer for this in the research. However, we show that each of the linear transformation semigroups (a), (c) and (d) is closed in T_V, P_V and B_V which are standard extensions of them.

The preliminaries and notations of this work are given in Chapter I. In Chapter II we characterize each of the linear transformation semigroups (a), (b), (c) and (d) which has a proper dense subsemigroup. We prove in Chapter III that the linear transformation semigroups (a), (c) and (d) are closed in all of T_V, P_V and B_V.