INTRODUCTION

Limit theorems for sums of independent random variables have been studied by many mathematicians. The most general limit theorems are treated in [8]. The major problems may be described as follows.

Let (X_{nj}), $j = 1,2,3,..., j_n$, $n = 1,2,3,...$, be a double sequence of random variables such that

(a) for each n, X_{nj} , $j = 1,2,3,...,j_n$ are independent, and

(b) (X_{nj}), $j = 1,2,3,..., j_n$, $n = 1,2,3,...$, are infinitesimal.

Let

$$S_n = X_{n1} + X_{n2} +...+ X_{nj_n} - A_n ,$$

where (A_n) is a sequence of real numbers.

Problem1. What are the possible limit distribution functions of the distribution functions of S_n ?

Problem2. Find a necessary and sufficient condition for convergence of the distribution functions of S_n.

Answers to both problems are now well-known. The class of possible limit distribution functions asked in problem 1 coincides with the class of infinitely divisible distribution functions, which includes normal distribution functions as special cases. Hence the answer to problem 2 include limit theorem for convergence of distribution functions of S_n to normal distribution function as a special case as well.

Recently Bethmann [1] generalizes a limit theorem for the case of normal limit distribution function to the cases in which the number of summands is random. He was able to obtian a random sums version of Feller-Lindeberg

theorem. It is natural to try to extend the study along the line of [1] to cover the cases where the limit distribution function may be other infinitely divisible distribution functions. In this study we try to do that. However, we restrict ourselves to the cases where finiteness of variances of X_{nj}'s are assumed.

In Chapter I we summarize known results and notations used in our work. Chapter II deals with the concepts of random sums and random infinitesimal. A necessary and sufficient condition for convergence of distribution functions of random sums is given in terms of accompanying random distribution functions. Chapter III contains our main result. In chapter IV, we specialize our main result to the case where limit distribution functions is the standard Normal distribution functions and relate our result to that of [1].