

สถานะวอร์เท็กซ์ในระบบควมแน่นแบบโบส-ไอน์สไตน์ของแก๊สที่ถูกกัก



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VORTEX STATES IN BOSE-EINSTEIN CONDENSATES OF A TRAPPED GAS

Mr. Kobchai Tayanasant

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Accepted by the Faculty of Science, Chulalongkorn University in Partial
Fulfillment of the Requirements for the Doctor's Degree

Wanchai Phothiphichitr
..... Dean of Faculty of Science
(Associate Professor Wanchai Phothiphichitr, Ph.D.)

THESIS COMMITTEE

David Ruffolo
..... Chairman
(Associate Professor David Ruffolo, Ph.D.)

Virulh Sa-yakanit
..... Thesis Advisor
(Professor Virulh Sa-yakanit, F.D.)

Wichit Sritrakool
..... Member
(Associate Professor Wichit Sritrakool, Ph.D.)

Rujikorn Dhanawittayapol
..... Member
(Rujikorn Dhanawittayapol, Ph.D.)

Suthat Yoksan
..... Member
(Professor Suthat Yoksan, Ph.D.)

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อินทิเกรตตามวิถีของอนุภาคหลายตัว เราสามารถคำนวณหาพลังงานที่สถานะพื้นได้ผลตรงกับการ
คำนวณที่ได้จากวิธีอื่น นอกจากนั้นเราได้ตรวจสอบสถานะถูกกระตุ้นของระบบนี้โดยการเปลี่ยนความ
เข้มของศักย์เก็บกัก พบว่าความหนาแน่นของกลุ่มแก๊ส ที่ควบแน่นมีการสั่นตามเวลา ความถี่ของการ
สั่นนี้มีค่าตรงกับสถานะถูกกระตุ้นที่ต่ำที่สุดที่สามารถวัดได้จากการทดลอง ทำที่สุดเราศึกษากลุ่มแก๊ส
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We study the static and dynamic properties of the trapped Bose gas at the ground state by using the many-body path integral techniques. We obtain the ground state energy of the system and it is in agreement with the results obtained by other methods. The excitation of the condensate by changing the trap strength is investigated. We calculate the density of the system and find that it oscillates with time. The frequency of oscillation we derived coincides with the lowest-lying excitation found in the experiment. Finally, we study the excitation of the condensate with vortex and find the fluctuation in density together with the precession of the vortex around the center of the condensate.

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Student's signature.....*Kobchai Tayanasanti*
Advisor's signature.....*Virulh Sa-yakanit*

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