INTRODUCTION

Let us begin this thesis by recalling some definitions.

Definition 1: An algebra A over a field K is a ring A(with no identity element) which is at the same time a vector space over K. Moreover the scalar multiplication in the vector space and the ring multiplication are required to satisfy the axiom

$$\alpha(ab) = (\alpha a)b = a(\alpha b) \quad , \quad \forall \alpha \in K, \forall a,b \in A.$$

Definition 2: An algebra A over a field K is nilpotent if there exists a positive integer m such that $A^m = \{0\}$.

Definition 3: Let A be an algebra with multiplication o and B be an algebra with multiplication * . Then the algebras A and B are isomorphic iff there exists a linear, 1-1, function f of A onto B such that $f(x \circ y) = f(x) * f(y)$.

Definition 4: The center C of an algebra A is the set

$$C = \{x \in A \mid xy = yx = 0 , \quad \forall y \in A\} .$$

By the left-center C_L of A and the right-center C_R of A we mean that

$$C_L = \{x \in A \mid xy = 0 , \forall y \in A\}$$

and

$$C_R = \{x \in A \mid yx = 0 , \forall y \in A\}.$$

<u>Proposition 5</u>: Let A and B be finite dimensional algebras over a field K with multiplication o and * respectively. Suppose that these two algebras are isomorphic, and let $f : A \to B$ be an isomorphism, then f takes the center (left center, right center) $C(C_L, C_R)$ of A isomorphically onto the center (left center, right center) $C'(C'_L, C'_R)$ of B.

The proof of this proposition can be found in [1] page 57.

<u>Definition 6</u>: A field k is algebraically closed if every polynomial in $k[X]$ of degree ≥ 1 has a root in k.

This thesis is a continuation of Prapa's Thesis ([1]).

In Chapter I, we prove that the theorem in [1] is true for arbitrary fields. This chapter also gives the classification of 3-dimensional nilpotent algebras A over an algebraically closed field K of characteristic $\neq 2$ with dimension $A^2 = 1$ and $A^3 = \{0\}$.

In Chapter II, we present some partial results in classifying the nilpotent algebras of dimension 4 over an arbitrary field.