CHAPTER 5

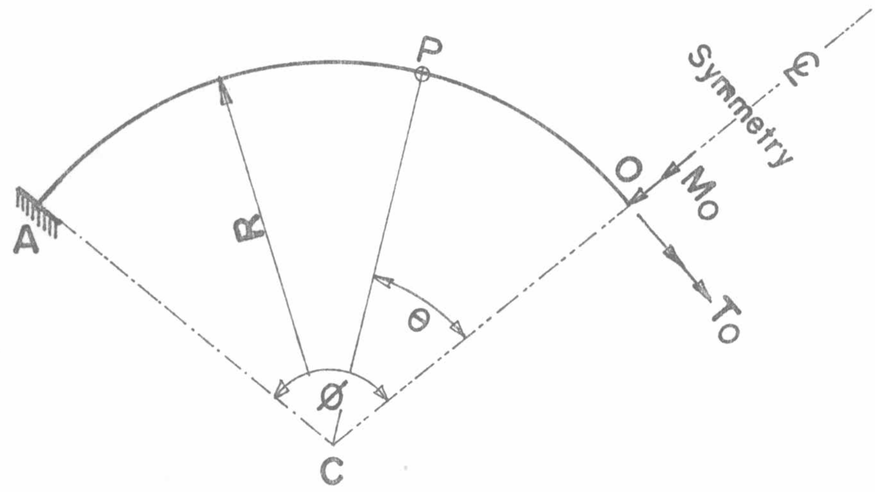
DEFLECTIONS

5.1 Foreword

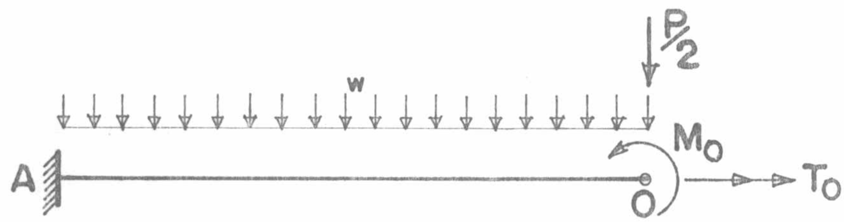
The necessity for assessing deflections of S-beams and Z-beams may not arise. At any rate expressions for centre-span deflections have been incorporated into this investigative endeavour. The development of these formulae, although burdensome, renders materialisation of a comparision between theoretical and experimental values. The derivation reclining on the strain energy principle necessitates placement of a fictitious concentrated load P at centre-span in conjuction with the authentic uniformly distributed load w on the beam. In this manner symmetry remains valid and the system thus identifies with second-degree indeterminacy. The application of Castigliano's second theorem leads to establishment of expressions for M_o and T_o at centre-span. Institution of expressions for M and T for any spot between centre-span and support ensues. A re-application of the theorem succeeded by allocation of zero to the load P finalises the ordeal.

5.2 The S-beam

Referring to Figures 5.1 the bending moment M and the torsional moment T at any point between the centre-span and the support are written in terms of R, Ø, w, P, and redundants M_o and T_o as follows:



(a) Plan



(b) Elevation

FIGURE 5.1 Half-beam under Action of Uniform Load, Fictitious Load, and Redundants

$$M = M_o \cos\theta + T_o \sin\theta - \frac{1}{2} PR \sin\theta - wR^2(1-\cos\theta) \qquad (5.1)$$

$$T = M_o \sin\theta - T_o \cos\theta - \frac{1}{2} PR(1-\cos\theta) - wR^2(\theta-\sin\theta) \qquad (5.2)$$

Employment of the strain energy approach yields the following expressions for the redundants:

$$M_o = B_1PR + B_2wR^2 \qquad (5.3)$$

$$T_o = B_3PR + B_4wR^2 \quad (5.4)$$

wherein

$$B_1 = \frac{f_1f_2 - f_3f_4}{4f_5} \quad (5.5)$$

$$B_2 = \frac{2f_1f_6 - f_3f_7}{f_5} \quad (5.6)$$

$$B_3 = \frac{2f_4f_8 - f_2f_3}{8f_5} \quad (5.7)$$

$$B_4 = \frac{f_7f_8 - f_3f_6}{f_5} \quad (5.8)$$

$$f_1 = (1+m)\,\phi - \frac{1}{2}(1-m)\sin 2\phi \quad (5.9)$$

$$f_2 = (1+3m) - (1-m)\cos 2\phi - 4m\cos\phi \quad (5.10)$$

$$f_3 = (1-m)(1-\cos 2\phi) \quad (5.11)$$

$$f_4 = (1+m)\,\phi - \frac{1}{2}(1-m)\sin 2\phi - 2m\sin\phi \quad (5.12)$$

$$f_5 = (1+m)^2\phi^2 - \frac{1}{2}(1-m)^2(1-\cos 2\phi) \quad (5.13)$$

$$f_6 = (1+m)(\sin\phi - \frac{1}{2}\phi) - \frac{1}{4}(1-m)\sin 2\phi - m\phi\cos\phi \quad (5.14)$$

$$f_7 = \frac{1}{4}(3+5m) + \frac{1}{4}(1-m)\cos 2\phi - (1+m)\cos\phi - m\phi\sin\phi \quad (5.15)$$

$$f_8 = 2(1+m)\,\phi + (1-m)\sin 2\phi \quad (5.16)$$

The bending and torsional moments M and T now become explicit. Presentation of the explicit expressions for M and T is omitted owing to presence of a very large number of terms.

The centre-span deflection is given by

$$\Delta_o = \frac{2R}{EI}\int_o^A M \frac{\partial M}{\partial P}\, d\theta + \frac{2mR}{EI}\int_o^A T \frac{\partial T}{\partial P}\, d\theta$$

This mathematical operation leads finally to:

$$\Delta_o = (D_1 + D_2 - D_3 - D_4 + D_5 + D_6)\cdot \frac{wR^4}{EI} \tag{5.17}$$

in which

$$D_1 = \frac{1}{2}(1+m)\phi\ \{2B_1(B_2+1) + B_4(2B_3-1)\} \tag{5.18}$$

$$D_2 = \{\frac{1}{2}\phi + 2B_1 \cos\phi + (2B_3-1)\sin\phi\}\, m\phi \tag{5.19}$$

$$D_3 = \{2(1+m)B_1 - mB_4\}\sin\phi \tag{5.20}$$

$$D_4 = \{m(B_2+1) + (1+m)(2B_3-1)\}(1-\cos\phi) \tag{5.21}$$

$$D_5 = \frac{1}{4}(1-m)\{2B_1(B_2+1) - B_4(2B_3-1)\}\sin 2\phi \tag{5.22}$$

$$D_6 = \frac{1}{4}(1-m)(1-\cos 2\phi)\{2B_1B_4 + (B_2+1)(2B_3-1)\} \tag{5.23}$$

5.3 The Z-beam

The imposition of a fictitious concentrated load P at centre-span of the uniformly loaded Z-beam refers to Figure 5.2. The bending and torsional moments at any location between centre-span and support can be written in terms of the redundants as follows:

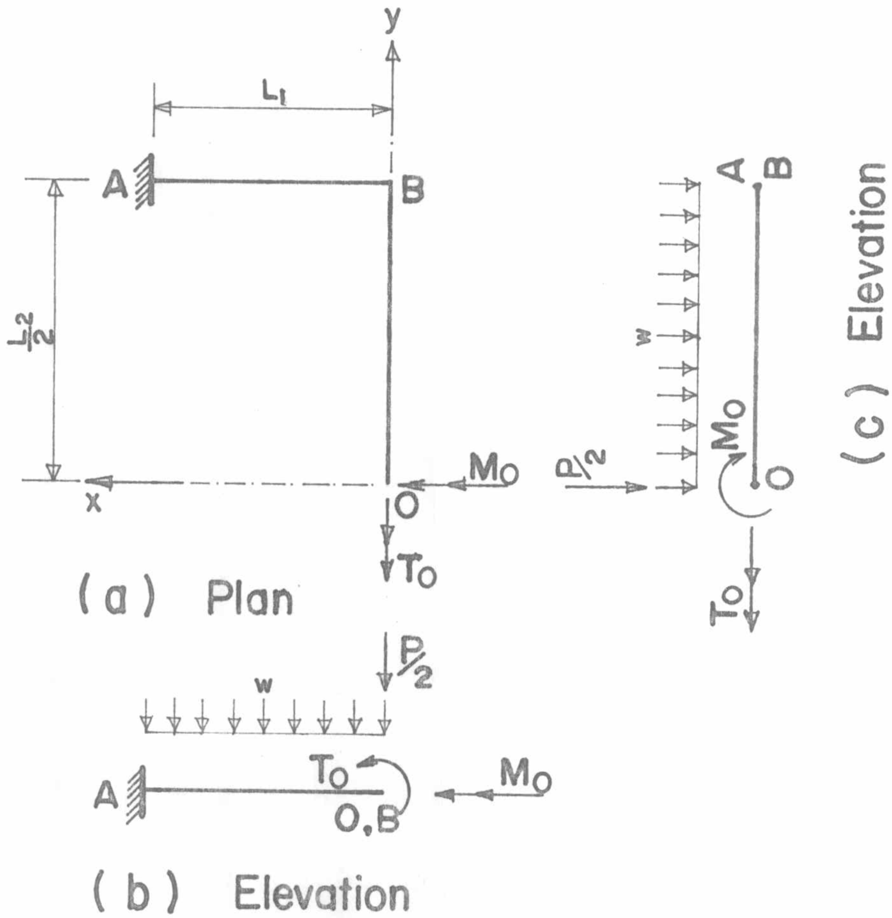


FIGURE 5.2 Half-beam under Action of Uniform Load, Fictitious load, and Redundants

for the transverse part,

$$M = M_o - \frac{1}{2} Py - \frac{1}{2} wy^2 \qquad (5.24)$$

$$T = T_o \qquad (5.25)$$

and for the longitudinal part,

$$M = T_o - \frac{1}{2} Px - \frac{1}{2} wL_2 x - \frac{1}{2} wx^2 \qquad (5.26)$$

$$T = \frac{1}{4} PL_2 + \frac{1}{8} wL_2^2 - M_o \qquad (5.27)$$

The use of the strain energy principle gives

$$M_o = \frac{1}{8} \frac{(1+4km)}{(1+2km)} PL_2 + \frac{1}{24} \cdot \frac{(1+6km)}{(1+2km)} wL_2^2 \qquad (5.28)$$

$$T_o = \frac{1}{2} \frac{k^2}{(2k+m)} PL_2 + \frac{k^2}{6} \cdot \frac{(2k+3)}{(2k+m)} wL_2^2 \qquad (5.29)$$

and eventually leads to formulation of the centre-span deflection:

$$\Delta_o = \left[\frac{1}{384} + \frac{1}{48} \frac{km}{(1+2km)} + \frac{k^3}{24} \frac{(2k^2 + 2k + 3km + 4m)}{(2k + m)} \right] \frac{wL_2^2}{EI} \qquad (5.30)$$

which pursues a far simpler form than that governing the S-beam.

PART II

EXPERIMENTAL ENQUIRY