CHAPTER 4

APPLICATIONS OF THE GREEN'S FUNCTIONS

4.1 Correlation Functions

The four Green's functions for the normal spinel ferrite obtained in eqs. (3.31a)-(3.31d) are in terms of the correlation functions $\langle A^- A^+\rangle_{\underline{k}}$, $\langle B^- B^+\rangle_{\underline{k}}$, $\langle A^- B^+\rangle_{\underline{k}}$ and $\langle B^- A^+\rangle_{\underline{k}}$. Using the definition of the correlation function (2.60) where t goes to zero, we can obtain the four correlation functions in terms of the absolute temperature and the applied magnetic field as follows

$$\langle A^- A^+\rangle_{\underline{k}} = \frac{\langle A^z\rangle}{2b}\left[2b + (E_- - g_B\mu_B H)\coth\frac{E_-}{2k_B T} - (E_+ - g_B\mu_B H)\coth\frac{E_+}{2k_B T}\right]$$

$$- \frac{\langle A^z\rangle^2}{2b}\left(\frac{N_A}{N_B}\right)^{1/2} J(0)\left(\coth\frac{E_-}{2k_B T} - \coth\frac{E_+}{2k_B T}\right)$$

$$+ \frac{\langle A^z\rangle\langle B^z\rangle}{2b}\left(2\sigma''_{\underline{k}} - \alpha'_B \frac{1}{N_B}\sum_{\underline{k}'} J(\underline{k}\ )\ \langle B^- A^+\rangle_{\underline{k}'}\right.$$

$$\left. + 2\alpha'_B \frac{1}{N_B}\sum_{\underline{k}'} \sigma''_{\underline{k}'}\langle B^- B^+\rangle_{\underline{k}'}\right)$$

$$\times\left(\coth\frac{E_-}{2k_B T} - \coth\frac{E_+}{2k_B T}\right), \qquad (4.1a)$$

$$\langle B^-B^+\rangle_{\underline{k}'} = \frac{\langle B^z\rangle}{2b}\left[2b + (E_- - g_A\mu_B H)\coth\frac{E_-}{2k_BT} - (E_+ - g_A\mu_B H)\coth\frac{E_+}{2k_BT}\right]$$

$$- \frac{\langle B^z\rangle^2}{2b}\left(\frac{N_B}{N_A}\right)^{1/2} J(0)\left(\coth\frac{E_-}{2k_BT} - \coth\frac{E_+}{2k_BT}\right)$$

$$+ \frac{\langle A^z\rangle\langle B^z\rangle}{2b}\left(2\sigma'_{\underline{k}} - \alpha'_A\frac{1}{N_A}\sum_{\underline{k}'} J(\underline{k})\langle A^-B^+\rangle_{\underline{k}'}\right.$$

$$\left. + 2\alpha'_A\frac{1}{N_A}\sum_{\underline{k}'}\sigma'_{\underline{k}'}\langle A^-A^+\rangle_{\underline{k}'}\right)$$

$$\times\left(\coth\frac{E_-}{2k_BT} - \coth\frac{E_+}{2k_BT}\right), \qquad (4.1b)$$

$$\langle A^-B^+\rangle_{\underline{k}'} = -\frac{\langle B^z\rangle^2}{2b}\,\alpha'_B\frac{1}{\sqrt{N_AN_B}}\sum_{\underline{k}'} J(\underline{k}-\underline{k}')\,\langle B^-A^+\rangle_{\underline{k}'}$$

$$\times\left(\coth\frac{E_-}{2k_BT} - \coth\frac{E_+}{2k_BT}\right)$$

$$- \frac{\langle A^z\rangle\langle B^z\rangle}{2b} J(k)\left(\coth\frac{E_-}{2k_BT} - \coth\frac{E_+}{2k_BT}\right), \qquad (4.1c)$$

$$\langle B^-A^+\rangle_{\underline{k}'} = -\frac{\langle A^z\rangle^2}{2b}\,\alpha'_A\frac{1}{\sqrt{N_AN_B}}\sum_{\underline{k}'} J(\underline{k}-\underline{k}')\,\langle A^-B^+\rangle_{\underline{k}'}$$

$$\times\left(\coth\frac{E_-}{2k_BT} - \coth\frac{E_+}{2k_BT}\right)$$

$$- \frac{\langle A^z\rangle\langle B^z\rangle}{2b} J(\underline{k})\left(\coth\frac{E_-}{2k_BT} - \coth\frac{E_+}{2k_BT}\right), \qquad (4.1d)$$

where $\sigma'_{\underline{k}} = J'(\underline{k}) - J'(0)$, $\quad \sigma'_{\underline{k}'} = J'(\underline{k}-\underline{k}') - J'(\underline{k}')$,

$$\sigma''_{\underline{k}} = J''(\underline{k}) - J''(0), \quad \sigma''_{\underline{k}'} = J''(\underline{k}-\underline{k}') - J''(\underline{k}'), \qquad (4.2)$$

and b, E_-, E_+ are defined in eqs. (3.34a)-(3.34c).

These correlation functions have to be self-consistently calculated if the magnetization and the resonance susceptibility as will be studied in the next sections are to be obtained. For high temperatures where coth $E_{\pm}/2k_BT = 2k_BT/E_{\pm}$, the above correlation functions become, assuming zero applied magnetic field $H = 0$,

$$\langle A^-A^+\rangle_{\underline{k}'} = \langle A^z\rangle - \frac{\langle A^z\rangle^2}{a^2-b^2}\left(\frac{N_A}{N_B}\right)^{1/2} 2k_BT\,J(0) + \frac{\langle A^z\rangle\langle B^z\rangle}{a^2-b^2}\,2k_BT$$

$$\times\left(2\sigma''_{\underline{k}} - \alpha'_B\frac{1}{N_B}\sum_{\underline{k}'} J(\underline{k}')\langle B^-A^+\rangle_{\underline{k}'}\right.$$

$$\left. + 2\alpha'_B\frac{1}{N_B}\sum_{\underline{k}'}\sigma''_{\underline{k}'}\langle B^-B^+\rangle_{\underline{k}'}\right), \qquad (4.3a)$$

$$\langle B^-B^+\rangle_{\underline{k}'} = \langle B^z\rangle - \frac{\langle B^z\rangle^2}{a^2-b^2}\left(\frac{N_B}{N_A}\right)^{1/2} 2k_BT\,J(0) + \frac{\langle A^z\rangle\langle B^z\rangle}{a^2-b^2}\,2k_BT$$

$$\times\left(2\sigma'_{\underline{k}} - \alpha'_A\frac{1}{N_A}\sum_{\underline{k}'} J(\underline{k}')\langle A^-B^+\rangle_{\underline{k}'}\right.$$

$$\left. + 2\alpha'_A\frac{1}{N_A}\sum_{\underline{k}'}\sigma'_{\underline{k}'}\langle A^-A^+\rangle_{\underline{k}'}\right), \qquad (4.3b)$$

$$\langle A^-B^+\rangle_{\underset{\sim}{k}'} = -\frac{\langle B^z\rangle^2}{a^2-b^2}\, 2k_BT\, \alpha_B' \frac{1}{\sqrt{N_AN_B}} \sum_{\underset{\sim}{k}'} J(\underset{\sim}{k}-\underset{\sim}{k}')\, \langle B^-A^+\rangle_{\underset{\sim}{k}'}$$

$$-\frac{\langle A^z\rangle\langle B^z\rangle}{a^2-b^2}\, 2k_BT\, J(\underset{\sim}{k}), \tag{4.3c}$$

$$\langle B^-A^+\rangle_{\underset{\sim}{k}'} = -\frac{\langle A^z\rangle^2}{a^2-b^2}\, 2k_BT\, \alpha_A' \frac{1}{\sqrt{N_AN_B}} \sum_{\underset{\sim}{k}'} J(\underset{\sim}{k}-\underset{\sim}{k}')\, \langle A^-B^+\rangle_{\underset{\sim}{k}'}$$

$$-\frac{\langle A^z\rangle\langle B^z\rangle}{a^2-b^2}\, 2k_BT\, J(\underset{\sim}{k}), \tag{4.3d}$$

where $\sigma'_{\underset{\sim}{k}}$, $\sigma''_{\underset{\sim}{k}}$, $\sigma'_{\underset{\sim}{k}'}$ and $\sigma''_{\underset{\sim}{k}'}$ are given by eq. (4.2) and a, b are given by eqs. (3.34b), (3.34c), respectively.

The above is a set of nonlinear simultaneous equations which in general, can be solved only by numerical means.

4.2 Magnetization

The magnetization of the normal spinel ferrite can be written in the usual way as

$$M = N_A g_A \mu_B \langle A^z\rangle + N_B g_B \mu_B \langle B^z\rangle, \tag{4.4}$$

where N_A, N_B are the number of ions in the A, B sublattices, respectively, g_A, g_B are the g-factors for spin of ions in the A, B sublattices, respectively, μ_B is the Bohr magneton and $\langle A^z\rangle$, $\langle B^z\rangle$ are the average value of the z-component of spins A, B, respectively.

The calculation of $\langle A^z \rangle$ and $\langle B^z \rangle$ can be done by writing the correlation functions in the form

$$\psi_A(a) = \langle e^{aA^z} A_1^- A_1^+ \rangle = A(A+1)\Omega_A(a) - D_a \Omega_A(a) - D_a^2 \Omega_A(a), \quad (4.5a)$$

$$\psi_B(b) = \langle e^{bB^z} B_1^- B_1^+ \rangle = B(B+1)\Omega_B(b) - D_b \Omega_B(b) - D_b^2 \Omega_B(b), \quad (4.5b)$$

where $$D_a = \frac{d}{da}, \quad D_b = \frac{d}{db},$$

$$\Omega_A(a) = \langle e^{aA^z} \rangle, \quad \Omega_B(b) = \langle e^{bB^z} \rangle. \quad (4.6)$$

The two quantities $\Theta_A(a)$ and $\Theta_B(b)$ can also be expanded in terms of those quantities defined in eq. (4.6) such as

$$\Theta_A(a) = A(A+1)(e^{-a}-1)\Omega_A(a) + (e^{-a}+1)D_a\Omega_A(a) - (e^{-a}-1)D_a^2\Omega_A(a), \quad (4.7a)$$

$$\Theta_B(b) = B(B+1)(e^{-b}-1)\Omega_B(b) + (e^{-b}+1)D_b\Omega_B(b) - (e^{-b}-1)D_b^2\Omega_B(b). \quad (4.7b)$$

We now introduce the functions $\psi_A(\underset{\sim}{k},a)$ and $\psi_B(\underset{\sim}{k},b)$ which are related to $\psi_A(a)$ and $\psi_B(b)$ by the relations

$$\psi_A(a) = \frac{1}{N_A} \sum_{\underset{\sim}{k}} \psi_A(\underset{\sim}{k},a) = \frac{1}{N_A} \sum_{\underset{\sim}{k}} \langle e^{aA^z} A_k^- A_k^+ \rangle, \quad (4.8a)$$

$$\psi_B(b) = \frac{1}{N_B} \sum_{\underline{k}} \psi_B(\underline{k},b) = \frac{1}{N_B} \sum_{\underline{k}} \langle e^{bB^z} B_k^- B_k^+ \rangle . \qquad (4.8b)$$

In terms of the Green's functions, $\psi_A(\underline{k},a)$ and $\psi_B(\underline{k},b)$ can be written as

$$\psi_A(\underline{k},a) = \langle e^{aA^z} A_k^- A_k^+ \rangle = \lim_{\epsilon \to 0} i \int \frac{(-2i)\,\mathrm{Im}\langle\langle A_k^+ ; e^{aA^z} A_k^- \rangle\rangle_{E=\omega+i\epsilon}}{e^{h\omega/k_B T} - 1} \times e^{i\omega t}\, d\omega , \qquad (4.9a)$$

$$\psi_B(\underline{k},b) = \langle e^{bB^z} B_k^- B_k^+ \rangle = \lim_{\epsilon \to 0} i \int \frac{(-2i)\,\mathrm{Im}\langle\langle B_k^+ ; e^{bB^z} B_k^- \rangle\rangle_{E=\omega+i\epsilon}}{e^{h\omega/k_B T} - 1} \times e^{i\omega t}\, d\omega . \qquad (4.9b)$$

Upon substituting eqs. (3.31a) and (3.31d) into eqs. (4.9a) and (4.9b), respectively, we get the following relationships

$$\langle e^{aA^z} A_k^- A_k^+ \rangle = \Theta_A(a)\, \frac{1}{2} \left[\frac{A+\sqrt{A^2+4n}}{2\sqrt{A^2+4n}} \coth \frac{E_+}{2k_B T} - \frac{A-\sqrt{A^2+4n}}{2\sqrt{A^2+4n}} \coth \frac{E_-}{2k_B T} - 1 \right], \qquad (4.10a)$$

$$\langle e^{bB^z} B_k^- B_k^+ \rangle = \Theta_B(b)\, \frac{1}{2} \left[\frac{A+\sqrt{A^2+4n}}{2\sqrt{A^2+4n}} \coth \frac{E_-}{2k_B T} - \frac{A-\sqrt{A^2+4n}}{2\sqrt{A^2+4n}} \coth \frac{E_+}{2k_B T} - 1 \right], \qquad (4.10b)$$

where E , A and n are given by eqs. (3.34a)-(3.34c), (3.33) and (3.32), respectively.

Substituting eqs. (4.10a) and (4.10b) into eqs. (4.8a) and (4.8b) gives

$$\psi_A(a) = \Theta_A(a)\,\frac{1}{N_A}\sum_{\underline{k}}\left[\frac{1}{2}\left(\frac{A+\sqrt{A^2+4n}}{2\sqrt{A^2+4n}}\coth\frac{E_+}{2k_BT} - \frac{A-\sqrt{A^2+4n}}{2\sqrt{A^2+4n}}\coth\frac{E_-}{2k_BT} - 1\right)\right], \quad (4.11a)$$

$$\psi_B(b) = \Theta_B(b)\,\frac{1}{N_B}\sum_{\underline{k}}\left[\frac{1}{2}\left(\frac{A+\sqrt{A^2+4n}}{2\sqrt{A^2+4n}}\coth\frac{E_-}{2k_BT} - \frac{A-\sqrt{A^2+4n}}{2\sqrt{A^2+4n}}\coth\frac{E_+}{2k_BT} - 1\right)\right]. \quad (4.11b)$$

Introducing the two functions

$$\Phi_A = \frac{1}{N_A}\sum_{\underline{k}}\left[\frac{1}{2}\left(\frac{A+\sqrt{A^2+4n}}{2\sqrt{A^2+4n}}\coth\frac{E_+}{2k_BT} - \frac{A-\sqrt{A^2+4n}}{2\sqrt{A^2+4n}}\coth\frac{E_-}{2k_BT} - 1\right)\right], \quad (4.12a)$$

$$\Phi_B = \frac{1}{N_B}\sum_{\underline{k}}\left[\frac{1}{2}\left(\frac{A+\sqrt{A^2+4n}}{2\sqrt{A^2+4n}}\coth\frac{E_-}{2k_BT} - \frac{A-\sqrt{A^2+4n}}{2\sqrt{A^2+4n}}\coth\frac{E_+}{2k_BT} - 1\right)\right], \quad (4.12b)$$

eqs. (4.11a) and (4.11b) can be written as

$$\psi_A(a) = \Phi_A \Theta_A(a), \tag{4.13a}$$

$$\psi_B(b) = \Phi_B \Theta_B(b), \tag{4.13b}$$

where $\Theta_A(a)$ and $\Theta_B(b)$ are given by eqs. (4.7a) and (4.7b), respectively.

Comparing eqs. (4.13a) and (4.13b) with eqs. (4.5a) and (4.5b), yields the following two differential equations for $\Omega_A(a)$ and $\Omega_B(b)$:

$$D_a^2\Omega_A(a) + \frac{(1+\Phi_A)e^a + \Phi_A}{(1+\Phi_A)e^a - \Phi_A} D_a\Omega_A(a) - A(A+1)\Omega_A(a) = 0, \tag{4.14a}$$

$$D_b^2\Omega_B(b) + \frac{(1+\Phi_B)e^b + \Phi_B}{(1+\Phi_B)e^b - \Phi_B} D_b\Omega_B(b) - B(B+1)\Omega_B(b) = 0. \tag{4.14b}$$

Callen has shown[32] that the solutions to the above equations are of the forms

$$\Omega_A(a) = \frac{\Phi_A^{2A+1}\, e^{-Aa} - (1+\Phi_A)^{2A+1}\, e^{(A+1)a}}{\left[\Phi_A^{2A+1} - (1+\Phi_A)^{2A+1}\right]\left[(1+\Phi_A)e^a - \Phi_A\right]}, \tag{4.15a}$$

$$\Omega_B(b) = \frac{\Phi_B^{2B+1}\, e^{-Bb} - (1+\Phi_B)^{2B+1}\, e^{(B+1)b}}{\left[\Phi_B^{2B+1} - (1+\Phi_B)^{2B+1}\right]\left[(1+\Phi_B)e^b - \Phi_B\right]}, \tag{4.15b}$$

from which $\langle A^z \rangle$, $\langle B^z \rangle$ or $\Theta_A(a)$, $\Theta_B(b)$ can be found by differentiation. Using eqs. (4.15a) and (4.15b), we get

$$\langle A^z \rangle = D_a \Omega_A(0) = \frac{(A-\Phi_A)(1+\Phi_A)^{2A+1} + (A+1+\Phi_A)\Phi_A^{2A+1}}{(1+\Phi_A)^{2A+1} - \Phi_A^{2A+1}} , \quad (4.16a)$$

$$\langle B^z \rangle = D_b \Omega_B(0) = \frac{(B-\Phi_B)(1+\Phi_B)^{2B+1} + (B+1+\Phi_B)\Phi_B^{2B+1}}{(1+\Phi_B)^{2B+1} - \Phi_B^{2B+1}} , \quad (4.16b)$$

where Φ_A and Φ_B are given by eqs. (4.12a) and (4.12b), respectively.

At low temperatures where Φ_A and Φ_B are small, eqs. (4.16a) and (4.16b) can be expanded in powers of Φ_A and Φ_B to give

$$\langle A^z \rangle = (A+\Phi_A) + (2A+1)\Phi_A^{2A+1} - (2A+1)^2\Phi_A^{2A+2} + \ldots , \quad (4.17a)$$

$$\langle B^z \rangle = (B+\Phi_B) + (2B+1)\Phi_B^{2B+1} - (2B+1)^2\Phi_B^{2B+2} + \ldots . \quad (4.17b)$$

This gives the magnetization as

$$M_{low} = N_A g_A \mu_B \; (A+\Phi_A) + (2A+1)\Phi_A^{2A+1} - (2A+1)^2\Phi_A^{2A+2} + \ldots$$

$$+ N_B g_B \mu_B \; (B+\Phi_B) + (2B+1)\Phi_B^{2B+1} - (2B+1)^2\Phi_B^{2B+2} + \ldots . \quad (4.18)$$

While at high temperatures, $\langle A^z \rangle$ and $\langle B^z \rangle$ can be expanded in terms of Φ_A^{-1} and Φ_B^{-1} to give

$$\langle A^z \rangle = \frac{1}{3} A(A+1) \Phi_A^{-1}, \qquad (4.19a)$$

$$\langle B^z \rangle = \frac{1}{3} B(B+1) \Phi_B^{-1}, \qquad (4.19b)$$

Therefore, the magnetization at high temperatures has the form:

$$M_{high} = \frac{1}{3} N_A g_A \mu_B A(A+1) \Phi_A^{-1} + \frac{1}{3} N_B g_B \mu_B B(B+1) \Phi_B^{-1}. \qquad (4.20)$$

Dependence of the magnetization on the spin quantum numbers A and B, temperature, coupling constants and applied magnetic field H can be obtained by substituting eqs. (4.12a) and (4.12b) into eqs. (4.18) or (4.20). The explicit dependence can only be obtained after the four correlation functions $\langle A^- A^+ \rangle_{\underline{k}'}$, $\langle B^- B^+ \rangle_{\underline{k}'}$, $\langle B^- A^+ \rangle_{\underline{k}'}$ and $\langle A^- B^+ \rangle_{\underline{k}'}$ have been evaluated since they appear in the Φ_A and Φ_B.

4.3 Resonance Susceptibility

Consider the case where the system is perturbed by a circularly polarized field $\underline{H}_1$ at right angles to the uniform steady field $\underline{H}$ chosen in the negative z-direction. The components of $\underline{H}_1$ are

$$H_{1x} = H_1 \cos\omega t, \quad H_{1y} = -H_1 \sin\omega t, \quad H_{1z} = 0. \tag{4.21}$$

The perturbing Hamiltonian $\mathcal{H}_1$ is therefore

$$\mathcal{H}_1 = -g_A\mu_B \sum_i \underline{H}_1\cdot\underline{A}_i - g_B\mu_B \sum_j \underline{H}_1\cdot\underline{B}_j \tag{4.22}$$

$$= -g_A\mu_B H_1 \sum_i \left(A_{ix}\cos\omega t - A_{iy}\sin\omega t\right)$$

$$- g_B\mu_B H_1 \sum_j \left(B_{jx}\cos\omega t - B_{jy}\sin\omega t\right). \tag{4.23}$$

Using $\cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$ and $\sin\omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$, eq. (4.23) becomes

$$\mathcal{H}_1 = V e^{-i\omega t} + V^* e^{i\omega t}, \tag{4.24}$$

where

$$V = -\frac{1}{2} g_A\mu_B H_1 \sum_i A_i^- - \frac{1}{2} g_B\mu_B H_1 \sum_j B_j^-, \tag{4.25a}$$

$$V^* = -\frac{1}{2} g_A\mu_B H_1 \sum_i A_i^+ - \frac{1}{2} g_B\mu_B H_1 \sum_j B_j^+. \tag{4.25b}$$

If we consider the perturbation to be switched on adiabatically, eq. (4.21) may be written as

$$\mathcal{H}_1 = \sum_\omega e^{\epsilon t} e^{-i\omega t} V_\omega, \quad (\epsilon > 0, \epsilon \to 0) \tag{4.26}$$

Due to such a perturbation, the linear steady-state response of a system observable, $\delta\langle A\rangle$ is therefore[42],

$$\delta\langle A\rangle = 2\pi \sum_{\omega} e^{\epsilon t}\, e^{-i\omega t} \langle\langle A;V_{\omega}\rangle\rangle_{E=\omega+i\epsilon}. \quad (\epsilon > 0, \epsilon\to 0) \quad (4.27)$$

Introducing the magnetization of a ferrimagnetic material as defined by

$$M_{\pm} = M_x \pm iM_y = N_A g_A \mu_B \sum_i A_i^{\pm} + N_B g_B \mu_B \sum_j B_j^{\pm}, \qquad (4.28)$$

then the deviation of M due to the perturbation should be, according to eq. (4.27),

$$\delta\langle M_{\pm}\rangle = 2\pi \sum_{\omega} e^{\epsilon t}\, e^{-i\omega t} \langle\langle M_{\pm};V_{\omega}\rangle\rangle_{E=\omega+i\epsilon}, \quad (\epsilon > 0, \epsilon\to 0) \quad (4.29)$$

Using eq. (4.24) we have

$$\delta\langle M_{\pm}\rangle = 2\pi e^{\epsilon t}\left[e^{-i\omega t} \langle\langle M_{\pm};V\rangle\rangle_{E=\omega+i\epsilon} + e^{+i\omega t} \langle\langle M_{\pm};V^{*}\rangle\rangle_{E=-\omega+i\epsilon}\right],$$

$$(\epsilon > 0, \epsilon\to 0) \qquad (4.30)$$

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[42] S.V. Tyablikov, op. cit., pp. 238-242.

$$\delta\langle M_+\rangle = 2\pi\, e^{\epsilon t}\left[e^{-i\omega t}\langle\langle M_+;V\rangle\rangle_{E=\omega+i\epsilon} + e^{+i\omega t}\langle\langle M_+;V^*\rangle\rangle_{E=-\omega+i\epsilon}\right].$$

$$(\epsilon > 0, \epsilon \to 0) \qquad (4.31)$$

The linear complex magnetic susceptibility per spin, $\chi_\pm(\omega)$, is defined by

$$\delta\langle M_\pm\rangle = N\chi_\pm(\omega)H_1\, e^{\pm i\omega t}, \qquad (4.32)$$

where $N = N_A+N_B$ and

$$\chi_\pm(\omega) = \chi'(\omega) \pm i\,\chi''(\omega), \qquad (4.33)$$

$\chi'(\omega)$ and $\chi''(\omega)$ being called the magnetic dispersion and absorption factors, respectively.

Comparing eq. (4.31) with eq. (4.32) gives

$$NH_1\chi_+(\omega) = 2\pi\langle\langle M_+;V\rangle\rangle_{E=\omega+i\epsilon}, \quad (\epsilon > 0, \epsilon\to 0) \qquad (4.34)$$

where the factor $e^{\epsilon t}$ has been taken as unity.

Substituting eqs. (4.25a) and (4.28) into eq. (4.34), we get

$$\chi_+(\omega) = -\pi\Bigg[(g_A\mu_B)^2\frac{N_A}{N}\sum_i\sum_{i'}\langle\langle A_i^+;A_{i'}^-\rangle\rangle_E + (g_B\mu_B)^2\frac{N_B}{N}\sum_j\sum_{j'}\langle\langle B_j^+;B_{j'}^-\rangle\rangle_E$$

$$+ (g_A\mu_B)(g_B\mu_B)\frac{N_A}{N}\sum_i\sum_j\langle\langle A_i^+;B_j^-\rangle\rangle_E$$

$$+ (g_A\mu_B)(g_B\mu_B)\frac{N_B}{N}\sum_i\sum_j\langle\langle B_j^+;A_i^-\rangle\rangle_E\Bigg]_{E=\omega+i\epsilon}. \quad (\epsilon > 0, \epsilon\to 0) \qquad (4.35)$$

The Green's functions appearing in this equation are the Green's functions of eqs. (3.30a)-(3.30d) with $a = b = 0$.

Using the Fourier transforms defined in eqs. (3.23)'s and taking $\underset{\sim}{k} = 0$, eq. (4.35) may be written as

$$\chi_+(\omega) = -\pi\Big[(g_A\mu_B)^2\frac{N_A^2}{N}\langle\langle A_k^+;A_k^-\rangle\rangle_E + (g_B\mu_B)^2\frac{N_B^2}{N}\langle\langle B_k^+;B_k^-\rangle\rangle_E$$
$$+ (g_A\mu_B)(g_B\mu_B)\frac{N_A^{3/2}N_B^{1/2}}{N}\langle\langle A_k^+;B_k^-\rangle\rangle_E$$
$$+ (g_A\mu_B)(g_B\mu_B)\frac{N_A^{1/2}N_B^{3/2}}{N}\langle\langle B_k^+;A_k^-\rangle\rangle_E\Big]_{k=0,\,E=\omega+i\epsilon} .$$
$$(\epsilon > 0, \epsilon \to 0) \qquad (4.36)$$

Evaluating eqs. (3.30a)-(3.30d) at $\underset{\sim}{k} = 0$ and substituting the resulting expressions into eq. (4.36) give

$$\chi_+(\omega) = -\Big[N(E)\ /\ (E-\omega_1)(E-\omega_2)\Big]_{E=\omega+i\epsilon}, \quad (\epsilon > 0, \epsilon \to 0) \qquad (4.37)$$

where the numerator expression $N(E)$ is given by

$$N(E) = (E-g_B\mu_B H)(g_A\mu_B)^2\frac{N_A^2}{N}\langle A^z\rangle + (E-g_A\mu_B H)(g_B\mu_B)^2\frac{N_B^2}{N}\langle B^z\rangle$$
$$- J(0)\Big[(g_A\mu_B)^2\frac{N_A^{5/2}N_B^{-1/2}}{N}\langle A^z\rangle^2 + (g_B\mu_B)^2\frac{N_A^{-1/2}N_B^{5/2}}{N}\langle B^z\rangle^2\Big]$$

(continued)

$$- \alpha_B \sum_{\underset{\sim}{k}'} J(\underset{\sim}{k}')\langle B^- A^+\rangle_{\underset{\sim}{k}'} \Big[(g_A\mu_B)^2 \frac{N_A^2 N_B^{-1}}{N} \langle A^z\rangle$$

$$+ (g_A\mu_B)(g_B\mu_B) \frac{N_A^{3/2} N_B^{-1/2}}{N} \langle B^z\rangle \Big]$$

$$- \alpha_A \sum_{\underset{\sim}{k}'} J(\underset{\sim}{k}')\langle A^- B^+\rangle_{\underset{\sim}{k}'} \Big[(g_B\mu_B)^2 \frac{N_A^{-1} N_B^2}{N} \langle B^z\rangle$$

$$+ (g_A\mu_B)(g_B\mu_B) \frac{N_A^{-1/2} N_B^{3/2}}{N} \langle A^z\rangle \Big]$$

$$- (g_A\mu_B)(g_B\mu_B) J(0) \left(\frac{N_A^2}{N} + \frac{N_B^2}{N} \right) \langle A^z\rangle\langle B^z\rangle \qquad (4.38)$$

and where ω_1, ω_2 are, respectively, equal to E_+, E_- defined by eqs. (3.34a)-(3.34c) evaluated at $\underset{\sim}{k} = 0$, i.e.,

$$\omega_1 = c + d, \qquad (4.39a)$$

$$\omega_2 = c - d, \qquad (4.39b)$$

where

$$c = \frac{1}{2} (g_A + g_B)\mu_B H + \frac{1}{2} \Bigg[\left(\frac{N_B}{N_A}\right)^{1/2} J(0)\langle B^z\rangle + \left(\frac{N_A}{N_B}\right)^{1/2} J(0)\langle A^z\rangle$$

$$+ \alpha_A' \frac{1}{N_A} \langle A^z\rangle \sum_{\underset{\sim}{k}'} J(\underset{\sim}{k}')\langle A^- B^+\rangle_{\underset{\sim}{k}'}$$

$$+ \alpha_B' \frac{1}{N_B} \langle B^z\rangle \sum_{\underset{\sim}{k}'} J(\underset{\sim}{k}')\langle B^- A^+\rangle_{\underset{\sim}{k}'} \Bigg], \qquad (4.40a)$$

$$d = \frac{1}{2}\left(\left[\left(\frac{N_B}{N_A}\right)^{1/2} J(0)\langle B^z\rangle - \left(\frac{N_A}{N_B}\right)^{1/2} J(0)\langle A^z\rangle\right.\right.$$

$$+ \alpha_A'\frac{1}{N_A}\langle A^z\rangle \sum_{\underline{k}'} J(\underline{k}')\langle A^-B^+\rangle_{\underline{k}'}$$

$$\left.\left. - \alpha_B'\frac{1}{N_B}\langle B^z\rangle \sum_{\underline{k}'} J(\underline{k}')\langle B^-A^+\rangle_{\underline{k}'}\right]^2 + 4n(\underline{k}=0)\right)^{1/2}, \quad (4.40b)$$

where

$$n(\underline{k}=0) = \left[J(0)\langle B^z\rangle + \alpha_A'\langle A^z\rangle \frac{1}{\sqrt{N_A N_B}} \sum_{\underline{k}} J(\underline{k}')\langle A^-B^+\rangle_{\underline{k}'}\right]$$

$$\times \left[J(0)\langle A^z\rangle + \alpha_B'\langle B^z\rangle \frac{1}{\sqrt{N_A N_B}} \sum_{\underline{k}'} J(\underline{k}')\langle B^-A^+\rangle_{\underline{k}'}\right]. \quad (4.40c)$$

Equation (4.37) can be written as

$$\chi_+(\omega) = -\left[N(E) / (\omega_1-\omega_2)\right]\left[(E-\omega_1)^{-1} - (E-\omega_2)^{-1}\right]_{E=\omega+i\epsilon}.$$

$$(\epsilon > 0, \epsilon \to 0) \qquad (4.41)$$

Instead of using the identity

$$\lim_{\epsilon\to 0, (\epsilon > 0)} (x \pm i\epsilon)^{-1} = P(1/x) \mp i\pi\delta(x), \qquad (4.42)$$

where P denotes the principal part and $\delta(x)$ is the Dirac delta function, to get the following expressions for the dispersion and absorption factors of the magnetic susceptibility, namely,

$$\chi'(\omega) = -\frac{N(\omega)}{\omega_1-\omega_2}\left[\frac{1}{\omega-\omega_1}-\frac{1}{\omega-\omega_2}\right], \tag{4.43a}$$

$$\chi''(\omega) = \pi\left[\frac{N(\omega_1)}{\omega_1-\omega_2}\,\delta(\omega-\omega_1)-\frac{N(\omega_2)}{\omega_1-\omega_2}\,\delta(\omega-\omega_2)\right], \tag{4.43b}$$

we shall assume that the evaluation of ω_1 and ω_2 gives both a real and imaginary part, i.e.,

$$\omega_1 = \mathrm{Re}\,\omega_1 + i\,\mathrm{Im}\,\omega_1, \tag{4.44a}$$

$$\omega_2 = \mathrm{Re}\,\omega_2 + i\,\mathrm{Im}\,\omega_2. \tag{4.44b}$$

Calling the real part E_o and the imaginary part Γ, we find that

$$\frac{1}{E-\omega_1} = \frac{1}{E-E_{o1}-i\Gamma_1} = \frac{(E-E_{o1})}{(E-E_{o1})^2+\Gamma_1^2} + i\,\frac{\Gamma_1}{(E-E_{o1})^2+\Gamma_1^2}, \tag{4.45a}$$

$$\frac{1}{E-\omega_2} = \frac{1}{E-E_{o2}-i\Gamma_2} = \frac{(E-E_{o2})}{(E-E_{o2})^2+\Gamma_2^2} + i\,\frac{\Gamma_2}{(E-E_{o2})^2+\Gamma_2^2}. \tag{4.45b}$$

Substituting eqs. (4.45a) and (4.45b) into eq. (4.41) and equating real and imaginary parts, we find that the dispersion and absorption factors of the complex magnetic susceptibility are given by

$$\chi'(\omega) = -\frac{N(E)}{\omega_1-\omega_2}\left[\frac{E-E_{o1}}{(E-E_{o1})^2+\Gamma_1^2}-\frac{E-E_{o2}}{(E-E_{o2})^2+\Gamma_2^2}\right]_{E=\omega+i\epsilon}, \tag{4.46a}$$

$$\chi''(\omega) = \frac{N(E)}{\omega_1-\omega_2}\left[\frac{\Gamma_1}{(E-E_{o1})^2+\Gamma_1^2} - \frac{\Gamma_2}{(E-E_{o2})^2+\Gamma_2^2}\right]_{E=\omega+i\epsilon} \qquad (4.46b)$$

instead of by eqs. (4.43a) and (4.43b).

Equations (4.46a) and (4.46b) are preferable to eqs. (4.43a) and (4.43b) since the observed line shapes have more of a Lorentzian line shape than a Dirac delta function shape. In the absence of an applied magnetic field, since one of the two resonant energies E_{o1} or E_{o2} would in general be negative, only one absorption peak would be observed. The width of the peak would be a measurement of Γ while the position of the peak would be a measurement of E_o. The values of the coupling constants or the importance of the correlation functions in determining the magnetic properties of the spinel ferrites could be determined by comparing the measured width of the resonance peak with the imaginary part of eqs. (4.39a) or (4.39b) and by comparing the position of the peak with the real part of the equations.