CHAPTER III

SOLUTION PROCEDURE

In order to investigate the effect of uniform axial compression on minimum weight of an internally stiffened cylindrical shell, one wants to know that when the uniform axial load is changed (either increase or decrease) how this new load will effect on the corresponding minimum weight of the stiffened cylindrical shell.

In this work, the author increases the uniform axial compression and finds the corresponding minimum weight of the stiffened cylindrical shell by using the method proposed by V. Ungbhakorn[1] several times. Then graph of load against minimum weight is plotted. From the plotted graph, one can study the effect of uniform axial compression on the minimum weight of internally stiffened cylindrical shells.

Since this work is considered to be continued from V. Ungbhakorn's research and in order to save computer time, the author selects the result of his research in Appendix C, case 1 for TSRR (tee stringer and rectangular ring), as a starting point in this research. From case 1 in Appendix C of V. Ungbhakorn's research, an internally stiffened circular cylindrical shell is under a uniform axial compression of 800 lbs./in. The minimum

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[1] Ungbhakorn, V, *op. cit.*

weight of this shell was found to be 703.4 lbs. The shell under consideration has the following properties:

$$R = 95.5 \text{ in.}, \; L = 291 \text{ in.}, \; \bar{N} = 800 \text{ lbs/in.},$$

$$E = E_x = E_y = 10.5 \times 10^6 \text{ psi.},$$

$$\rho_{sk} = \rho_x = \rho_y = 0.101 \text{ lbs/ in}^3.,$$

$$\nu = 0.33, \; \sigma_0 = 50{,}000. \text{ psi.}$$

Stiffeners are tee stringer and rectangular ring (TSRR) and they are attached inside the shell. They have the following properties:

$$C_x = 1.097, \quad c_{fx} = 1., \quad k_s = 0.35,$$

$$C_y = 0., \quad c_{fy} = 0., \quad k_r = 0.$$

MG (minimum gauge) = 0.02 in.

Then the author increases the uniform axial load up to 900 lbs/in. or 12.5 % increases from the original load , and he tries to find the corresponding minimum weight of the stiffened cylindrical shell by using V. Ungbhakorn's method. There are two stages in his method: Phase 1 and Phase 2.

In Phase 1, non-linear programming search technique of Nelder and Mead combines with golden section method[2] (search by golden section) are used to prepare charts and tables(see guide line for data generation in Appendix D). In order to move a simplex toward the minimum $\bar{W}$ in $\bar{\lambda}_{xx} - \bar{\lambda}_{yy}$ space for each Z, and a pair of $(\bar{\alpha}_x - \bar{\alpha}_y)$, $\bar{K}_{xx_{cr}}$ at every vertex of the simplex is evaluated. To achieve this, the well-known and the most efficient unidimensional

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[2] Himmelblau, David M., Applied Nonlinear Programming(New York, McGraw-Hill Book Co., Inc., 1972), pp. 42-44.

search technique, the golden section, is employed. To find $\bar{K}_{xx_{cr}}$ for each point or vertex in the $\bar{\lambda}_{xx}-\bar{\lambda}_{yy}$ space when m is an integer, the golden section has to be applied twice. The process is as follows:

During the optimum seeking procedure, at a point in the $\bar{\lambda}_{xx}-\bar{\lambda}_{yy}$ space, all quantities, except m and $\bar{\beta}$ in equation (16) are known. First, m is treated as a continuous variable and the golden section is applied in equation (17) to find $\bar{\beta}$ for $\bar{K}_{xx_{cr}}$. From equation (17) one can compute m. In general, this m will not be an integer. Then m is considered to be two consecutive integers, except 0., bounding the non-integer m found previously. From these two m's, by using equation (16), one can find $\bar{\beta}$'s and then $\bar{K}_{xx_{cr}}$'s. The value of $\bar{\beta}$ and the corresponding m, giving the smaller $\bar{K}_{xx_{cr}}$, will be taken as the solution for $\bar{K}_{xx_{cr}}$ at this point. There is no convergence problem in finding the minimum $\bar{W}$ with this method. Instructions and computer listings used in Phase 1 are given in Appendix E.

In Phase 2, the design charts and tables are used to attain the minimum $\bar{W}$ satisfying all constraints. Steps in designing for minimum weight of the stiffened cylindrical shell with TSRR are as follows:

1. For each Z, locate the minimum weight parameter $\bar{W}$ in the $\bar{\alpha}_x-\bar{\alpha}_y$ space (charts or tables) and the corresponding $\bar{\lambda}$'s. Since the expression for the stress in the rings is based on thin ring theory, $\frac{R}{d_{wy}}$ must be greater than 20. This implies that $\bar{\alpha} < \frac{R}{20h}$. One then follows step 2 through 7 such that no constraints are

violated. If any constraint is violated one must increase the weight and repeat the procedure. Note that in many cases minimum $\overline{W}$ is a line rather than a point.

2. Calculate the stresses in the skin, rings, and stringers by using equation (8).

If all stresses are less than or equal to the yield stress or certain limiting stress level the next step is continued. Otherwise, one must move to the next higher $\overline{W}$ and repeat step 2. Note that since the skin is in a biaxial state of stress, one should use an appropriate yield criterion.

3. The stringer and ring heights are computed from the definitions of $\bar{\alpha}_x$ and $\bar{\alpha}_y$.

$$d_{wx} = \frac{(1+C_{fx} \cdot k_s)h}{(1+4C_{fx} \cdot k_s)^{\frac{1}{2}}} \bar{\alpha}_x \; ; \qquad d_{wy} = \frac{(1+C_{fy} \cdot k_r)h}{(1+4C_{fy} \cdot k_r)^{\frac{1}{2}}} \bar{\alpha}_y$$

4. The ratios of the stiffener thickness to the stiffener spacing are determined from the definitions of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$.

$$\frac{t_{wx}}{l_x} = \frac{E\bar{\lambda}_{xx} h}{E_x(1-\nu^2)(1+C_{fx}k_s)d_{wx}} \; ; \quad \frac{t_{wy}}{l_y} = \frac{E\bar{\lambda}_{yy} h}{E_y(1-\nu^2)(1+C_{fy}k_r)d_{wy}} \, .$$

5. From the constraint of skin wrinkling

$$|\sigma_{xxsk_{cr}}| > |\sigma_{xxsk}|$$

one has,

$$l_x < h\sqrt{\frac{\pi^2 E}{3(1-\nu^2)|\sigma_{xxsk}|}}$$

6. From the selected l_x, calculate the stringer web thickness, t_{fx}, from step 4. Then the stringer flange thickness and width are determined from

$$t_{fx} = C_{fx} \cdot t_{wx} \quad ; \quad b_{fx} = k_s \cdot d_{wx} \, .$$

7. From the constraints of stringer flange buckling

$$\left|\sigma_{xxsf_{cr}}\right| > \left|\sigma_{xx_{st}}\right|$$

one has,

$$l_y < \frac{d_{fx}}{\sqrt{\frac{12(1-\nu^2)}{\pi^2 E_x}\left(\frac{d_{fx}}{t_{fx}}\right)^2\left|\sigma_{xxst}\right| - 0.425}}$$

where $d_{fx} = \frac{1}{2}(b_{fx} - t_{fx})$.

If quantity under the radical sign is negative, then any l_y will satisfy this constraint. The selected l_y must be checked to ensure that panel instability must not occur.

For small k_s (i.e. d_{fx} is small), the stringers are equivalent to the bulb-head stringers; therefore there will be no stringer flange buckling. Thus, one will not have the above expression for l_y, but l_y is determined on the basis of panel instability alone, with number of rings being greater than three.

8. From the selected l_y, calculate t_{wy} from step 4. Next the ring flange thickness and width are determined from

$$t_{fy} = c_{fy} \cdot t_{wy}, \quad b_{fy} = k_r \cdot d_{wy}.$$

The simultaneous occurrence of general instability, panel instability, and local instabilities can be avoided by proper choice of l_x and l_y.

9. Check the local stringer web buckling

$$\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{3(1-\nu^2)}\left(\frac{t_{wx}}{d_{wx}}\right)^2 \qquad \text{for } \frac{l_y}{d_{wx}} \geqslant 1.$$

$$\sigma_{xxsw_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)}\left(\frac{t_{wx}}{d_{wx}}\right)^2\left(\frac{d_{wx}}{l_y} + \frac{l_y}{d_{wx}}\right)^2 \qquad \text{for } \frac{l_y}{d_{wx}} < 1.$$

If $\left|\sigma_{xxsw_{cr}}\right| > \left|\sigma_{xxst}\right|$, one goes to the next step. Otherwise, the weight must be increased and step 2 through 8 are repeated.

10. Calculate the weight of the stiffened shell by

$$W = 2\pi RLh\rho_{sk}\overline{W}$$

11. Repeat the above steps for a number of Z values (h) and plot W against h (see guide line for data generation in Appendix D). At least three values of h are needed. From the plot one can located the absolute minimum weight with the corresponding value of h and hence Z.

12. With the value of Z for minimum weight in step 11, one then generates the required data (design charts and tables) and repeats step 1 through 10 to finalize the dimensions. This last step is performed only when the exact minimum weight configuration is desired.

In reality, it will be very difficult to prepare charts and tables, if computer time(for scientific research) is not available such as in our country. The cost for hiring computer time in our country is very expensive. Thus, in order to save computer time (or money) in doing this research, the author tries to search for the minimum $\overline{W}$ in $\bar{\lambda}_{xx}-\bar{\lambda}_{yy}$ space for each Z, and a pair of $(\bar{\alpha}_x, \bar{\alpha}_y)$, by combining Phase 1 and Phase 2 together without preparing charts. First, he selects the arbitrary pairs of $(\bar{\alpha}_x, \bar{\alpha}_y)$ for each Z at random. The value of $\bar{\alpha}_x$ and $\bar{\alpha}_y$ must be distributed to cover all possible ranges. Those pairs of $(\bar{\alpha}_x, \bar{\alpha}_y)$ are used in Phase 1. Then the results from Phase 1 are used to search for minimum $\overline{W}$ in Phase 2. Each pair of $(\bar{\alpha}_x, \bar{\alpha}_y)$ which satisfies all constraints will give the roughly design result for minimum $\bar{W}$. Next, the author selects the new value for each pair of $(\bar{\alpha}_x, \bar{\alpha}_y)$ from the above suitable design results, then he uses these pairs of $(\bar{\alpha}_x, \bar{\alpha}_y)$ in Phase 1 again. Results from Phase 1 are used in Phase 2 and the minimum $\bar{W}$ received by these results will be less (or more) than the first time. The author repeats his research in this way until the minimum $\bar{W}$ is obtained.

The author then increases axial load up to 1000 and 1100 lbs/in. respectively and at the same time he tries to search for the corresponding minimum weight of each load by using the same method mentioned above.

Finally, graphs between uniform axial compression , $\bar{N}$, and the corresponding minimum weight, W , are plotted. These graphs will give the relation or the effect of uniform axial compression on minimum weight of internally stiffened cylindrical shells.